The risk in the within horizon: a test applied to Dollar Cost Averaging.

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Abstract

Dollar Cost Averaging refers to an investment methodology in which a set dollar amount is invested in a risky asset at equal time intervals over a holding period. Our paper compares the advantages and risk of this strategy from the point of view of a saver. Many theories focused on the inefficiency of this strategy compared to other non discretionary strategies in terms of performance but, in the real world, DCA is often used for its straightforwardness. Besides we offer a comparison between DCA and Lump Sum focusing on the risk the investor bears during the entire investment horizon and not only at the end of the period. This risk in the within horizon is measured in particular with First Passage Time Probability and Expected Minimum Portfolio Value applied to portfolios simulated with Monte Carlo and different types of Bootstrap (block, stationary and residual sampling).

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Introduction

Dollar Cost Averaging refers to an investment methodology in which a set dollar amount is invested in a risky asset at equal time intervals over a holding period.

Since the '50 this simple dynamic investment strategy became famous among practitioners either for the simple implementation either for the easy comprehension on the client's side; in particular the automatic feature avoids emotional pitfalls associated with investing and thus makes DCA very appealing.

In the United States this simple strategy could be assimilated at the 401 (K) which allows a worker to save a portion of the wage for retirement and have the savings invested while deferring income taxes on the saved money and earnings until withdrawal; normally, the employees can choose among plans with different opportunity/risk profiles that emphasize stocks, bonds, money market investments, or some mix of the above. Moreover also in the rest of the world the accumulation plans assumed a particular relevance. The provident Baby Boomers for example used the accumulation plan to assure a certain level of consumption during the retirement. In fact many pension systems like US, Canada and Italy adopted a pay as you go pension scheme; it means that the active workers contribute with their income to the "pool" the government uses to pay out the retirement income to retirees. It is evident that in many countries the size of the retired population is growing faster than the next generation of active workers, implying that the Government will pay just a smaller amount as retirement. For this reason financial institutions and families are adopting customized accumulation plan (PAC) similar to dollar cost strategies.

The advantage of DCA is the easy replication by any saver and the opportunity to close the position before the deadline if the level of risk overcomes some thresholds although the basic rule of this strategy requires continuing the plan in any market's scenario. For this reason, this paper wants to test empirically the advantage, in terms of efficiency, of the DCA strategy looking at the level of risk in the within horizon; we say that the DCA strategy is more efficient than the Lump Sum (LS, defined as one time investment at the beginning of the strategy) if the possible lower returns seem to be more than compensated by a reduction in terms of risk and if the investor always bears less risk for the entire investment horizon. We measure the level of efficiency of both strategies simulating 50,000 scenarios and calculating some risk indicators monthly (First Passage Time Probability and Expected Minimum Portfolio Value). Our results show that the risk of DCA is always lower than LS during the entire investment horizon; that could be the reason why investors are attracted by DCA.

The remainder of the paper is organized as follow. In section 2 we present a literature review on both DCA and some popular risk indicators. In section 3 we present data and methodology. In Section 4 we present the results obtained using historical data and simulation. We conclude in section 5.

1. Literature Review

The literature about DCA is not very wide and can be divided in two main branches: behavioural finance theories, which analyze DCA with respect to the investor utility function and computational works which analyze the strategy with respect to performance and risk.

The earlier works tried to analyze the DCA from the point of view of the behavioural finance theory which states that investors are not rational in the traditional sense of Von Neumann and Morgenstern (1947) and they suffer from loss aversion. Costantinides (1979) showed for the first time, within a theoretical framework, that Dollar Cost Averaging plans are sub optimal in terms of expected utility. Subsequently, several works confirmed that Dollar Cost Averaging almost always produces lower returns than Lump Sum in diversified portfolios, and almost never reduces risk meaningfully. From a behavioural point of view, Statman (1995) confirmed that DCA "*is an embarrassment to the role of standard finance as a positive theory of financial behaviour*" and described four elements to justify the investor's preference: prospect theory, aversion to regret, cognitive errors and self control. Later, Dybvig (1988) in his "Inefficient dynamic portfolio strategies or how to throw away a million dollars in stock market", following the idea of Cox and Leland (1982), computed empirically the cost of inefficiency of the most important dynamic investment strategies and revealed that they are substantially non efficient.

On the other hand, several studies adopted simulation techniques to test empirically the level of efficiency of the DCA. Abysekera and Rosenbloom (2000) use Monte Carlo simulation to test if the DCA strategy leads to superior returns finding that there is not a clear advantage of DCA over LS even if they considered different combinations of expected returns on stock, risk-free rates and levels of volatility. Their study shows that for assets with low volatility the LS is a superior investment strategy but for high volatility assets the results are less clear; moreover, LS exposes the investor to greater risk in terms of VaR. Unfortunately in the last decade there are a variety of empirical works that confirm the inefficiency of VaR as a good risk measure, in particular because VaR is sub-additive in the tail region, the most relevant region for risk management. Finally, the mean return from the DCA strategy will be approximately the average of the current risk-free return and the expected return of stocks and if one incorporates transaction costs into the model, the LS strategy outperforms the DCA strategy in the majority of the cases.

Other studies tried to demonstrate empirically the inefficiency of the DCA strategy. Harrigton (2001), for example, considers the impact of transaction costs and taxes on returns. Knight and Mandell (1993) compared the DCA to alternative strategies, optimal rebalancing and buy and hold, using historical stock market returns and Monte Carlo simulations and demonstrated that "nobody gains from Dollar Cost Averaging". In 2003 Leggio and Lien studied the DCA compared to other alternative strategies looking either to the performance and the risk using the Sortino, Sharpe and Upside Potential ratios; unfortunately they are indicators of risk at maturity while investors are often interested also in the risk associated with the portfolio strategy in the within horizon. Balvers and Mitchell (1997) evaluated the return's autocorrelation in the inter-temporal portfolio plans because DCA exploits mean reversion properties.

Other works denied the superiority of alternative investment techniques. Milevsky and Posner (1999) use the tools of stochastic calculus and Brownian bridges to show that DCA is superior to Lump Sum, especially for volatile securities, only when the investment ends up with a zero return or with a loss. Vora and McGinnis (2000) focused on the retiree requirement hypothesizing a dollar cost disinvestment; the concept below is easy, the pensioner needs more liquidity during his oldness, for this reason they applied the reverse of DCA: periodically the investor moves a fixed dollar amount from a stock or bond portfolio into cash for consumption. Marshall (2000) also recognized the advantage of DCA and compared it with another periodic investment plan, the Value Averaging, obtaining that the latter provides superior expected returns. The Value Averaging strategy states that investors contribute to their portfolios in such a way that the portfolio balance increases by a set amount, regardless of market fluctuations.

Generally many authors focused on the risk valuation at maturity but as analyzed by Kritzman and Don Rich (2002) "the investors are affected by exposure to loss throughout the investment period, not just at its conclusion"; their work evaluated the "first passage probability" and the "continuous value at risk" for currency hedging and leveraged hedge funds, revealing that within horizon risk is far greater (by a magnitude) than end of horizon risk. Dubil (2004) found that not only DCA reduces costs but also offers significant risk reduction than non-averaged alternatives. The risk reduction is greater for longer holding horizons and riskier underlying assets. Dubil (2005) reached similar results with Monte Carlo simulation, noting that "the return on Lump Sum versus Dollar Cost Averaging depends crucially on the sequence of stock returns". The main question is whether the market shows an up or down trend.

The most used risk adjusted performance measures in risk management are indicators that value the risk at maturity; this means they give us the probability that our portfolio falls above a fixed threshold only on the last observation of the entire horizon. As illustrated in Leggio and Lien (2003) the risk level is very important for the investor that wants to value his portfolio opportunities. For this reason, to compare the Dollar Cost Averaging with Value Averaging and Lump Sum, the authors exploited different risk indicators among which there are the Sharpe Ratio (defined as the excess return per unit of standard deviation), the Sortino ratio (that replaces the standard deviation with the downside risk measure) and the traditional Value at Risk (VaR). Value at Risk measures the maximum loss of the portfolio given a precise probability over a fixed period of time, but it is well known that VaR does not work for extreme market fluctuations because it is usually based on a "normal" asset returns that exhibit skinner tails, while the losses under extreme price fluctuations are larger and less frequent that under "normal" conditions. To solve the problem Artzner et al. (1997) proposed the use of the Expected Shortfall defined as the conditional expectation of loss given that the loss occurs. The main advantage of this indicator is that it could be applied at any distribution either historical or simulated.

2. Risk Indicators

Our paper wants to focus on the evolution of the risk during the implementation of the investment strategy. A wrong idea which is quite spread among investors is that if you increase the investment horizon the probability of loss reduces; on the contrary some works showed that increasing the investment horizon does not reduce risk but actually increases the magnitude of potential losses. To understand this concept Samuelson (1963) reminds a situation he had with a colleague who refused to take a bet on a single flip of a coin but agreed to a series of 100 flips, unfortunately just flipping a coin 100 times does not change the probability of head and tail, on the contrary rises the amount of loss he could incur. Samuelson proved that the conventional time diversification model focuses only on the shortfall probability at the end of the investment horizon and completely ignores the exponential increase of the monetary amount of loss. Later, Bodie (1995) compared this situation with an investor who wants to protect his exposure buying a put option; clearly the put premium increases as the time horizon increases, consequently the risk rises with the maturity. Besides it is also evident that when we consider a risky asset that follows a Brownian motion the standard deviation increases with the square root of time. Moreover the investor looks at the risk periodically. Usually, an institutional investor has periodical thresholds he cannot overstep; likewise a retail investor observes carefully the probability of loss regarding family savings.

Below we illustrate the most important risk indicators evaluated in our paper for both DCA and LS.

2.1 Conditional Expected Shortfall

The probability of shortfall is the probability that the terminal value of the investment falls below the pre-fixed threshold. Expected shortfall or the Tail Conditional Expectation was first proposed by Artzner et. al. (1997) as alternative to Value at Risk that is more sensitive to the shape of the loss distribution in the tails. The standard definition of VaR states that:

$$VaR_\alpha = V_0 - B \tag{1}$$

$$\Pr\{V_T > B\} = \alpha \tag{2}$$

where:
B = threshold level
V_0 = initial value of the portfolio
V_T = final value of the portfolio

The formula was corrected by Acerbi and Tasche (2001) in order to obtain a risk indicator which is sub-additive and so coherent. This indicator is an unconditional measure of risk; it is the expected value of the shortfall, whether there is one or not. All outcomes that exceed the threshold are treated equally (as zero shortfalls), no matter what their magnitude is. Practically the expected shortfall at $\alpha\%$ level is the average return of the portfolio in the worst $\alpha\%$ of the cases.

To compute a conditional measure, we divide each unconditional probability by the probability of a shortfall:

$$CES = E\left[B - V_T \mid V_T \leq (B)\right] \tag{3}$$

2.2 First Passage Time Probability

Mark Kritzman (1994) observed that "*although the investor is less likely to loose money over a long horizon than over a short horizon, the magnitude of the potential loss increases with the duration of the investment horizon*"; this means that the probability of observing a loss during the entire investment horizon (within horizon probability) is always grater than the probability of observing a loss only at the end of the investment horizon and that the former increases as the investment horizon lengthens while the latter decreases.

For this reason we introduce a new risk measure called First Passage Time Probability which represents the probability to have a first occurrence of a loss at any time within the entire investment horizon. This formula was originated by Ginzburg et al. (1982) and Burgman et al. (1983) in an environmental setting in order to measure the risk of extinction for some animal species.

In the financial context the First Passage Time Probability $q_{(FPTP)}$ is defined as[1]:

$$q_{(FPTP)} = \Phi\left[\frac{\log(B/V_0) - \mu T}{\sigma\sqrt{T}}\right] + \Phi\left[\frac{\log(B/V_0) + \mu T}{\sigma\sqrt{T}}\right] * (B/V_0)^{2\mu/\sigma^2} \tag{4}$$

where:
$\Phi[\cdot]$ = cumulative normal distribution function
B/V_0 = cumulative percentage loss (in periodic units)
T = maturity
μ = annualized expected return (in continuous units)
σ = annualized standard deviation of continuous returns
$q_{(FPTP)} = 1 - p = \Pr\{V_t \leq B$ at least once for $t = 0, 1, ...T\}$

Clearly, the previous formula presupposes that returns are normally distributed.The first part of formula 4 gives the probability of loss at the end of the period, while the second part is the probability of observing a loss at any time before maturity; at the same time this second part can be seen as the probability that the investment falls down the threshold almost once during the entire investment period but ends up with a gain at the end of the period. By definition the probability of loss thought out an investment horizon must exceed the probability of loss at the end of the horizon.

[1] The mathematical derivation of the formula is described in Karlin and Taylor (1975)

If we hypothesize to observe our investment monthly, this approach gives us the probability that the portfolio falls below a given threshold B/V_0, each month. This value can be found counting, month by month, all the paths that fell down this barrier at least once and dividing this value by the total number of paths. Kritzman and Rich (2002) computed for the first time the within horizon probability of loss using the FPTP applied to currency hedging and leveraged hedge funds. To understand this concept a useful figure is shown by Kritzman and Don Rich (2002)[2].

Unlike Trainor (2005)[3], this study is able to empirically derive this result for a set of simulated portfolio returns with Montecarlo and Bootstrap methods. Our final result is inline with Trainor (2005), but we changed some hypothesis: first of all he shows only a single indicator which refers to the end of the investing horizon, while we show the evolution of this risk indicator during the entire period. Second, he applies a formula based on the idea that returns are normally distributed (which is in contrast with real data), while we present results based on the real (empirical) probability distribution.

2.3 Expected Minimum Portfolio Value

"*The expected minimum portfolio value measures the largest loss that is expected at some stage over that period*"[4]. It was first employed by McCarthy (1996) in an ecological/ environmental setting.

$$P_{EMPV} = \Pr\{V_t \leq B \text{ at least once for } t = 0, 1, ...T\} \tag{5}$$

Equation 5 states quite simply that the expected minimum portfolio value (fraction) is equal to the area to the left of the cumulative distribution curve $q(f)$ as a function of f on [0,1]. This value is the smallest value of the portfolio that is expected at any time within the time interval 0 to T.

3. Data and Methodology

3.1 Data

Our sample is based on data from DataStream International. We use common equity indexes like DJ Euro Stoxx, DJ Euro Stoxx Small and DJ Euro Stoxx Large because their liquidity is similar, on average, to the risky assets in which PAC or 401(K) invest. We collected the price index monthly from December 1987 to January 2009, for a total of 266 observations (the price is adjusted for dividend yields). The Dow Jones Euro Stoxx Index is a subset of the Dow

[2] Kritzman, M., & Rich, D. (2002). The mismeasurement of risk. Financial Analysts Journal, 58, pag. 92.

[3] Trainor, W. J., (2005). Within horizon exposure to loss for dollar cost averaging and lump sum investing. Financial Services Review, 14, 319-330

[4] Thompson, C. J., McCarthy, M. A., (2008). Alternative measures to value at risk. The Journal of Risk Finance, 9, 81-88.

Jones Stoxx 600 Index. With a variable number of components, the index represents large, mid and small cap companies of 12 Eurozone countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, the Netherlands, Portugal and Spain. The Dow Jones Euro Stoxx Large/Small Index represents large/small capitalisation companies with a variable number of components across the Eurozone[5].

Previous works try to compare the efficiency of DCA with respect to LS, or other investment strategies, assuming the investor has the entire amount available at the initial time. Of course this assumption is necessary for the comparison, but one of the most advantages of DCA, as we stated previously, is just that the investor does not need the entire amount at the beginning, but only a small sum at each investing period. Previous authors suppose to develop the DCA strategy in such a way that the initial amount is invested in a risk free asset and they assume to move a fraction of the riskless investment into the risky asset time by time during the entire averaging period. Similarly, other works hypothesize to borrow the amount needed month by month at the risk free rate. These two alternatives are quite the same because the comparison between DCA and LS is strongly affected by the level of the risk free rate. In our work, like Lei and Li (2007), we consider that the cash position does not increase at the risk free rate, this choice is based on the assumption that usually the retail investor does not own all the amount of money at the beginning of the investment period but he does not need to borrow it since he can take a fraction of his monthly wage. In this framework, the results of the DCA are not pumped up by the riskless rate; we know that the comparison with the LS is not very correct from a mathematical point of view, but at the same time we are considering the "worst case scenario" for DCA: if this excercise shows some advantages for the DCA, these advantages would be even greater considering also the contribute of the risk free rate (in fact we show that with a risk free rate a bit greater than zero the results are completely in behalf of DCA).

To evaluate the return and risk of the portfolios based on the LS or the DCA strategy we simulated the evolution of the risky asset using different simulation techniques.

3.2 Methodology

3.2.1 Data Resampling

The comparison between investment strategies requires considering the evolution of different portfolios invested in the same risky asset. As emphasized by Abysekera and Rosenbloom (2000), the profitability of this strategy compared to the lump sum depends heavily on the path followed by the risky asset. In fact, it is intuitive that a decreasing path favours an automatic investment plan; at the same time an increasing path privileges the lump sum investment as the investor buys all his stocks at the minimum price. But the investor does not know the future market pathway, moreover when he supposes to invest over a

[5] http://www.stoxx.com/indices/index_information.html?symbol=SXXE

relative long period (in our case 20 years) the terminal value of the risky asset is decisive. For this reason we simulated different possible scenarios of the risky asset using different models based on historical data. We chose two main types of models: Monte Carlo and Bootstrap. The first model is quite spread because its implementation is very easy, it is based on two main parameters, the drift and the variance rates and it assumes a Gaussian distribution of the asset returns: in this paper we used historical moments as inputs. The inefficiency related to this simulation technique is that the distribution of the resulting data (which is normal) is different from the original sample (fatter tails and skewness). For this reason we ran other simulations using bootstrap methods. Efron and Tibshirani (1986) for the first time illustrated this "*computer-based-method, which substitutes considerable amounts of computation in place of theoretical analysis*"[6]. The simple concept below bootstrap is that we sample randomly single - or block of - elements (with replacement or not) from historical data that are assumed to be i.i.d. (independent and identically distributed) in order to obtain a new possible evolution of the risky asset. Replicating these steps thousand times we obtain a new distribution that owns the same characteristics of the original distribution.

Econometric theory developed a great variety of bootstrap methods; taking into account the characteristics of our original data set we decided to apply three different methods: block bootstrap, stationary bootstrap and residual sampling.

3.2.2 Block Bootstrap and Stationary Bootstrap

The block bootstrap for time series consists of a sequence of randomly chosen blocks of data sampled from the original time series. Each block has the same length and each simulation counts the same number of blocks. This is done to capture the dependence structure of subsequent observations; we use a 6 months length block because over longer periods the autocorrelation of returns is not statistically significant. The block bootstrap has turned out to be a powerful method for dependent data. It does not achieve the accuracy of the bootstrap for i.i.d. data but it outperforms the subsampling. It works reasonably well under very weak conditions on the dependency structure and so it is eligible for a very broad range of applications. For the block bootstrap no specific assumption is made on the structure of the data generating process.

The Stationary Bootstrap was first proposed by Politis and Romano (1994); it is quite similar to Block Bootstrap, but it is based on resampling (with replacement) blocks of random length from the original distribution; the length of each block has a geometric distribution. The Stationary Block Bootstrap is applied to stationary weakly dependent time series.

[6] B. Efron B., & Tibshirani R.(1986). Bootstrap Methods for Standard Errors, Confidence Intervals, and Other Measures of Statistical Accuracy. Statistical Science, Vol. 1, No. 1, pp. 54-75

3.2.3 Residual Resampling

Residual sampling is another technique that is able to generate a sample with the same degree of autocorrelation of the original distribution data. This method resamples the residuals from an AR(1) model on the original return series. First of all we estimated the model $r_t = \beta_0 + \beta_1 r_{t-1} + e_t$ where β_0 represents the mean return after considering the first-order return autocorrelation, β_1 represents the first-order return autocorrelation coefficient and $\{e_t\}$ the sequence of residuals. Then we simulated the sample return series over time using $r_{t-1} = r_0 = \widehat{\beta}_0$ and a sequence of residuals $\{e_t^*\}$ bootstrapped with replacement from the original series of residuals. We discarded the first 100 observations from the simulated return series in order to stabilize the simulated sample. This approach preserves the empirical density function of the original time series.

4. Results

We tested the differences between LS and DCA implementing all the models presented in the previous section (plus two other Bootstrap methods) and using all the data from the three Euro Stoxx indices, nevertheless we decided to present only the data of the Euro Stoxx Index and the simulations coming from Monte Carlo, Residual Sampling and Stationary Bootstrap because the results obtained from all the methods are pretty much the same. It is important to notice that the data simulated with different Bootstrap methods are similar with each other, but they are quite different from the data coming from Monte Carlo; this result is due to the fact that the Bootstrap is able to generate samples that are closer to the original distribution, on the other hand Monte Carlo is based on a theoretical distribution (the normal probability distribution) which is different from the real one.

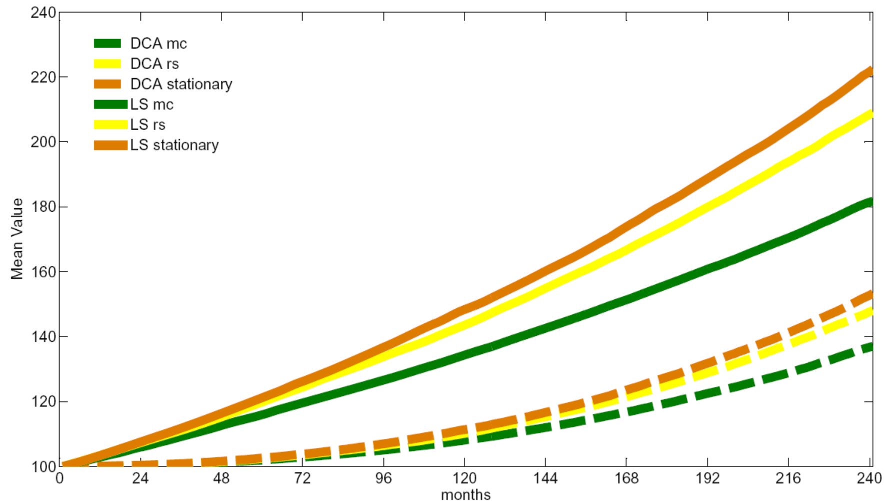
In addition we tested our models for different investment horizons (5, 10 and 20 years), but we present here only the 20 years strategy because also these results are very similar.

In order to compare the two strategies in terms of efficiency we observed monthly the evolution of the respective portfolios, evaluating step by step some return and risk indicators. Unlike previous authors we supposed that the averaging period overlaps the investment horizon because in the real word this is often the case of a retail investor that earmarks a part of his salary for the investment. Other works took into account shorter averaging periods that lasted from 1 to 5 years even if the investment horizon was longer (even 20 years).

Table 1 shows some return and risk indicators referred to the end of the investing horizon. The results obtained with different simulation methods agree even if the outcomes of Monte Carlo are quite far from the ones obtained with Bootstrap. At the end of the period we observe a small reduction of the average return paired with a consistent reduction of its standard deviation. Also the other risk indicators confirm that the DCA strategy has a lower shortfall probability and a lower conditional expected shortfall which means that the probability to suffer a loss is less that in LS case and also the average amount associated with a loss is smaller. We also evaluated the average duration of loss for the two strategies that represents the length of time in which the portfolio remains below its initial value. In this case we observe that with the implementation of the DCA this time reduces.

Figure 1 and Figure 2 show the evolution, month by month, of the mean value and its standard deviation of the two portfolios based on the LS strategy (continuous lines) and the DCA strategy (dotted lines).

Exhibit 1: Mean Value



Like Trainor (2005) and Dubil (2004), but unlike Lei and Li (2007), we observe a little reduction in the return for the automatic investment plan, but this reduction is more than compensated with a sensible decrease in the dispersion of the terminal value (risk) of the portfolio. The return of the DCA portfolio is, on average, the 70% of the return of the LS portfolio, but its standard deviation is less than half (40% on average) and the result holds for the entire investment horizon.

The evolution of the shortfall probability of the two strategies shows different results if compared to previous works. May be this result is due to the fact that we chose a longer averaging period. As we stated previously we evaluated this risk indicator choosing a threshold equal to the initial sum available for the investment.

Figure 3 shows that the shortfall probability seems to be higher for the LS portfolio (even if it is equal at the initial time) and the difference between the two strategies increases with time. This strange result is due to the fact that we hypothesized that, for the DCA, the sum not invested in the risky asset does not earn any interest ("worst case scenario"). If we change this assumption (as previous authors did) and choose a risk free rate greater than zero we obtain

Exhibit 2: Standard Deviation of Mean Value

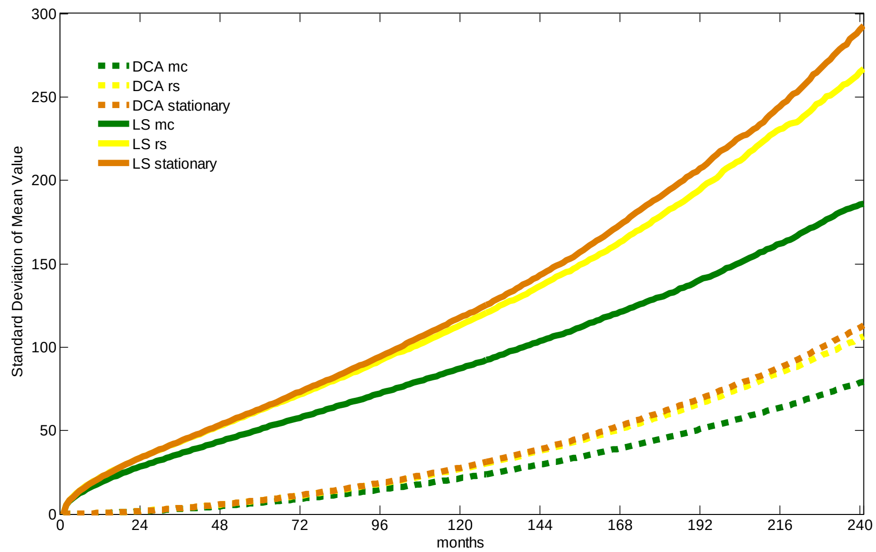


Exhibit 3: Shortfall Probability

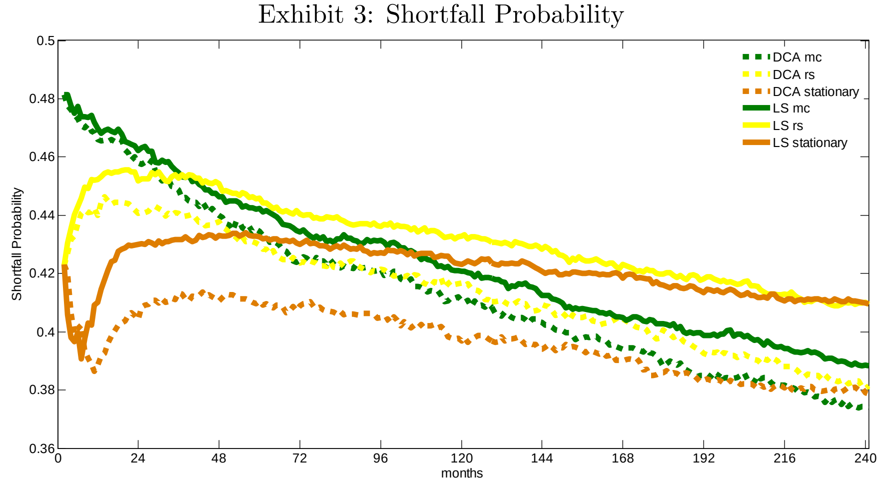
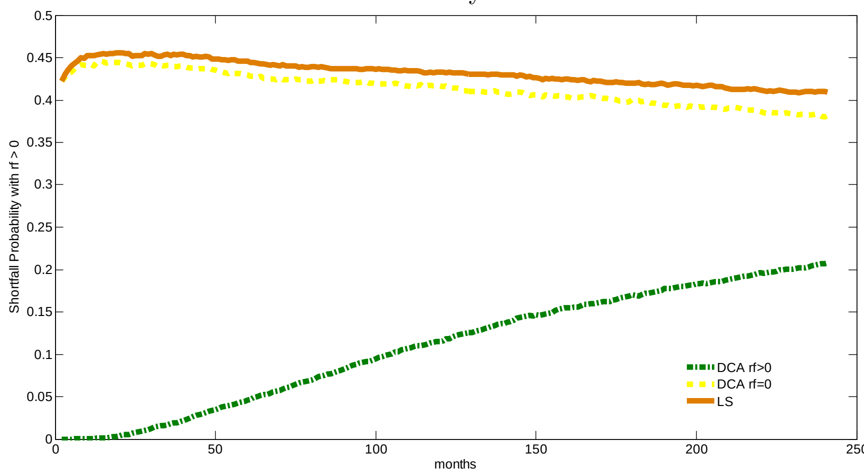


Exhibit 4: Shortfall Probability with risk free rate > 0



a very different result. Figure 4 shows that the shortfall probability for the DCA portfolio turns increasing instead of decreasing and, most important, its value reveals to be very lower with respect to the LS. This means that a small increment in the risk free rate corresponds to a great gain in terms of shortfall probability. The reduction of risk is directly related to the level of the risk free rate.

Although the probability of a terminal shortfall is lower for the DCA portfolio, the difference in terms of absolute values is not very pronounced: the shortfall probability of the terminal value of the DCA is, on average, the 93% of the corresponding probability for the LS. The real reduction in terms of risk is more evident looking at the expected shortfall and the conditional expected shortfall indicators. This result is in line with previous works.

Figure 5 and Figure 6 show that if there is a shortfall the reduction of the DCA portfolio value is always smaller, and, opposed to Trainor (2005), the result holds for the entire investment horizon.

In order to give a real measure of the risk associated with these two investment strategies and to compare their relative efficiency we evaluated an indicator called First Passage Time Probability. As stated previously this measure takes into account the risk that the investor bears during the entire holding period and not only the final risk. The results we obtained are in a way quite similar to those referred to the shortfall probability; if we assume a risk free rate equal to zero we notice that the difference between DCA and LS is not very evident (DCA is a bit worse than LS), but if we increment the risk free rate of a small amount the results are very different.

Figure 7 shows that when the risk free rate increases the total risk of the portfolio falls quickly down. We measured not only the probability that a loss may occur during the investment period, but also how long this loss lasts. To

Exhibit 5: Expected Shortfall

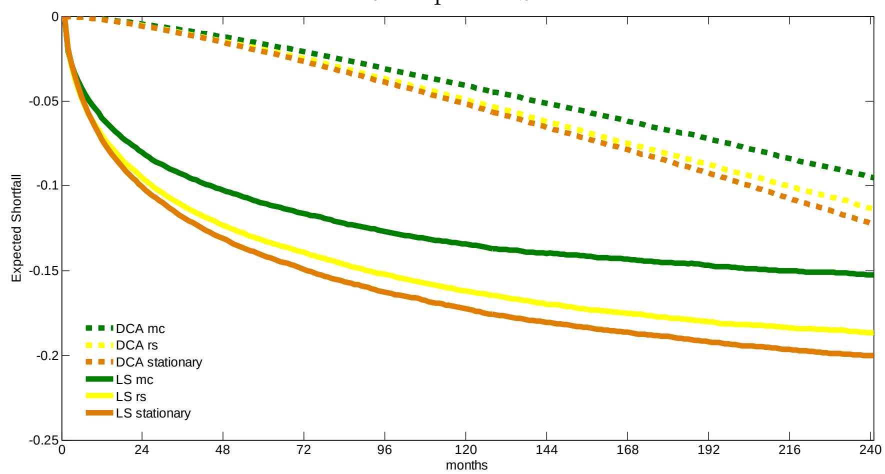


Exhibit 6: Conditional Expected Shortfall

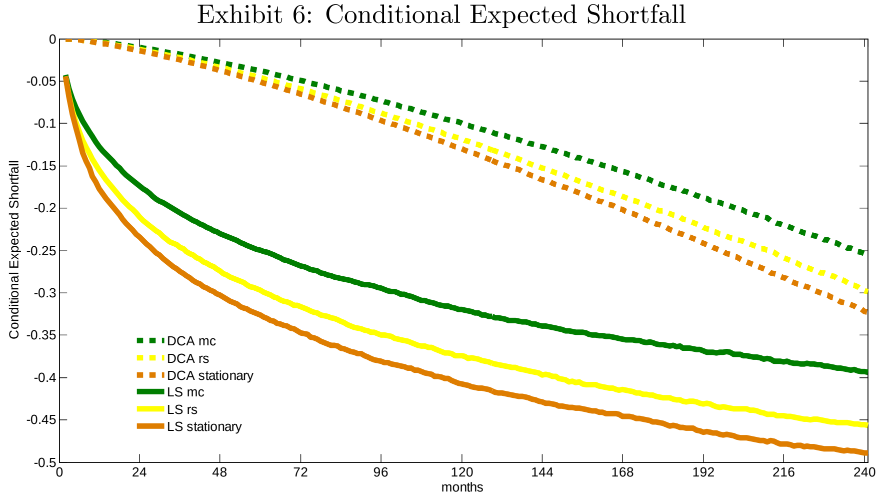
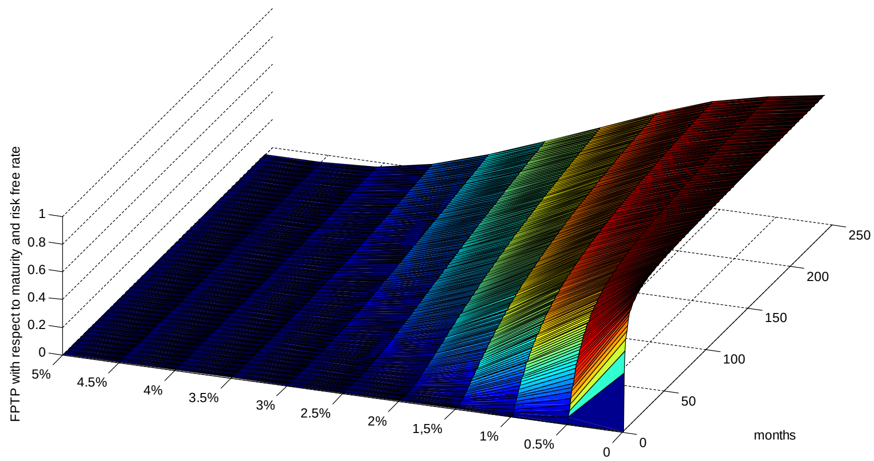


Exhibit 7: First Passage Time Probability with respect to time and risk free rate



this extent we evaluated, for each path, the number of times that the portfolio value falls and remains under the threshold (which is set equal to the initial value); we observed that on average the loss associated with the LS strategy is 8 months longer than the one associated with the DCA.

Another risk indicator directly related to the First Passage Time Probability is the EMPV (see previous section) which is the expected minimum value of the portfolio during the entire holding period. This indicator shows a tangible advantage for the DCA, in fact the minimum value expected in the case of the DCA portfolio is, on average, 15 percentage points greater than the LS. Thus, all the risk indicators in this paper confirm that the investor bears less risk using DCA than LS.

5. Conclusions

This work takes into account an investment strategy called Dollar Cost Averaging and tries to evaluate its efficiency in terms of risk (the risk that the investor bears during the entire investment horizon). Previous works explained the sub optimality of DCA on the basis of its final result in terms of risk and return. This paper analyses this automatic investment plan from the point of view of a retail investor who has two main needs: (i) first of all he does not own a huge initial sum to invest as a whole, but he usually withdraws a part of his salary month by month; (ii) second, he is not only interested in the final result but also in the quantity of risk he has to bear during the entire investment period. In some cases he may also take into account the possibility to stop the investment plan or to liquidate his position if the real risk oversteps a certain level.

Exhibit 8: Comparison of an LS and DCA strategy

	LS MC	DCA MC		LS stationary	DCA stationary		LS rs	DCA rs	
Average terminal value	181.76	137.01	-44.75	222.43	153.28	-69.15	209.02	148.03	-60.98
Standard deviation	185.85	79.10	-106.74	292.60	112.93	-179.67	266.61	106.34	-160.27
Shortfall probability	0.388	0.374	-0.014	0.409	0.378	-0.030	0.409	0.381	-0.027
Expected shortfall	-0.152	-0.095	0.057	-0.200	-0.122	0.077	-0.186	-0.113	0.073
Conditioned expected shortfall	-0.393	-0.255	0.138	-0.489	-0.323	0.165	-0.456	-0.298	0.158
Expected minimum portfolio value	64.19	78.68	14.49	57.49	74.12	16.62	59.30	75.58	16.28
Average duration of loss (months)	36.22	28.90	7.32	36.28	27.37	8.91	37.18	29.00	8.18

Note: The strategies are based on a 100 € initial investment. The risk free rate used to evaluate DCA is null. Shortfall threshold is set at the initial investment. For MC simulation we used historical return and standard deviation.

To do so the paper shows the evolution, during the entire holding period, of a particular indicator called First Passage Time Probability which is able to give an expression of the real risk (end of horizon plus within risk).

The results show that DCA is able to reduce this kind of risk either for high or low volatile assets and the results does not depend on the length of the investment horizon. This result is more interesting if we take into account that according to Dalbar's Quantitative Analysis of Investor Behaviour study 2008 "*investors don't have the patience necessary to ride out the rough period*", in fact the average time an equity mutual fund holder stays invested is only 3.1 years.

The comparison between DCA and LS strategy required the creation of a paper portfolio in which a risky and a riskless asset coexist: month by month a portion of the riskless asset is transferred to the risky asset. In order to obtain a more realistic result we assumed a risk free rate equal to zero (worst case scenario analysis). A very interesting result comes from the fact that if the risk free rate grows (just of a small amount) either the shortfall probability or the First Passage Time Probability of DCA decrease by a huge amount.

Further research could take into account another topic which is worth of interest: the comparison between DCA and another automatic investment strategy called Value Averaging that seems to be even more efficient in terms of risk than DCA. Important results could come evaluating the accumulation plan considering also the investor's work - life cycle.

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I hereby declare that

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has presented the paper

The risk in the within horizon*

at the Conference

"Financial Instability. A New World Framework?"

An interdisciplinary analysis of the new risk scenario"

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Le sottoscritte Giuliana Borello e Francesca Pampurini, autrici del lavoro intitolato "***The risk in the within horizon: a test applied to Dollar Cost Averaging***" pubblicato in "***Challenges for analysis of the economy, the business, and social progress***" da Universitas Szeged Press, Szeged 2009 e presentato alla conferenza internazionale "International Risk Management Conference" IRMC 2009

D I C H I A R A N O

che sebbene l'intero lavoro sia frutto della collaborazione di tutti gli autori, i paragrafi 1, 2 e 3 sono attribuiti a Giuliana Borello, il paragrafo 4 è attribuito a Francesca Pampurini, mentre Introduzione e Conclusioni sono state redatte congiuntamente.

Milano, 5 novembre 2012

(Giuliana Borello)

(Francesca Pampurini)