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The Solow–Pasinetti debate on productivity measurement: Review and reformulation

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A B S T R A C T

It is beyond doubt that Solow's proposal for "an elementary way of segregating variations in output per head due to technical change from those due to the availability of capital per head" (Solow, 1957, p. 312) leading to the 'residual', and hence, TFP growth, has been a crucial development of Neoclassical economics. This notwithstanding, the critique of (and alternative to) Solow's proposal advanced by Pasinetti (1959) has not been equally acknowledged. The debate re-emerged when a posthumous note by Richard Stone (1998[1960]) triggered a further exchange between the authors. This paper aims at retracing the key conceptual aspects of the discussion, pointing to some limitations of Pasinetti's original implementation of his measure of productivity changes, and providing an Input–Output generalisation based on Pasinetti's notion of hyper-integrated labour. Seen in this light, Pasinetti's computable measure of technical change provides a theoretically sound alternative to perform productivity analyses from a Classical perspective.

> "In this day of rationally designed econometric studies and super-input-output tables, it takes something more than the usual "willing suspension of disbelief" to talk seriously of the aggregate production function" (Solow, 1957, p. 312)

> "In a production system, saving labour is the ultimate meaning of technical progress" (Pasinetti, 1981, p. 207)

1. Introduction

The literature concerning the measurement of productivity and technical progress is vast and long-standing, encompassing many, and very different, theoretical and empirical approaches. A comprehensive review of such literature is out of the scope of this paper (see, e.g. Hulten, 2010). However, it is worth devoting some time to one specific debate, which is at the origin of a *crucial bifurcation* of approaches to the subject.

The Solow–Pasinetti debate on productivity measurement consists of an exchange between Robert Solow and Luigi Pasinetti, following Solow's seminal article 'Technical Change and the Aggregate Production Function' (Solow, 1957; Pasinetti, 1959). The debate re-emerged when a posthumous note by Richard Stone (1998) triggered a further exchange between the authors (Pasinetti, 1998; Solow, 1998).

The crucial bifurcation this debate led to may be better understood by enquiring about the notion of productivity itself. It is of course true that "to measure is not to understand" (Salter, 1966, p. 1), and this is particularly so as regards productivity analysis:

> "One of the reasons why interpretative analysis of productivity has been slow to develop has been the interminable controversy over what is productivity and what do we really wish to measure. The word now carries a multitude of meanings; to some it measures the personal efficiency of labour; to others, it is the output derived from a composite bundle of resources; to the more philosophic, it is almost synonymous with welfare; and in one extreme case it has been identified with time. I personally believe that much of this discussion has proved fruitless and *only served to confuse the issues of measurement with the issues of interpretation*. Unless there is a revolution in statistical techniques and information, *only one type of productivity concept is measurable*. This is the concept of output per unit of input".
> (Salter, 1966, p. 2, italics added)

Far from being a trivial statement, the position taken by Salter (1966) was not the usual one at the time (and clearly even less

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nowadays). In a multisectoral economy, a scalar measure of physical input per unit of output is not straightforward to obtain, given the multitude of output-input productivity ratios present in the economy. It is necessary to solve the aggregation of commodities, *or* the reduction of some of them in terms of others.

This is particularly difficult for capital goods, which by being reproducible, are themselves subject to productivity improvements. In fact, in the summary record of the debate at the 1958 Conference on the Theory of Capital, Kaldor noticed two radically different positions on this issue:

> "*One extreme case* was to assume that there was no technical progress in the production of capital goods but that these always required the same amount of real resources. This was obviously quite unrealistic. *At the other extreme*, one could say that a unit of capital was whatever unit was capable of producing a given output in a given year — ignoring both longer and shorter output streams. Here any distinction between the quantity of capital and its productivity was washed away by the definition itself. Any idea that capital might have varying productivities was lost; its output was always constant".
> (Hague, 1961, p. 304, italics added)

The first 'extreme case' corresponds to the traditional Total Factor Productivity (TFP) treatment of the 'quantity of capital', in which a TFP growth measure is assumed to capture disembodied efficiency changes, independently from capital deepening, which is assumed to require the same amount of *real* resources (e.g. 'waiting') per unit of saving. This is the approach pioneered by Solow (1957).

The second 'extreme case' consists in measuring capital goods in 'units of capacity', i.e. as a set of composite commodities of heterogeneous physical content, specific for each final product of the economy. But then, if capacity is defined in terms of the final output *actually* produced, at every moment the number of units of commodity-specific capacity would coincide with the number of units of each final product.

Hence, by adopting such measuring rod, the 'quantity of capital' in real terms would not be needed anymore and, at the same time, each of these composite commodities would change their physical composition from one period to the next, though retaining their function as commodity-specific 'productive capacities'. This route was precisely the one taken by Pasinetti (1959), all throughout his approach to structural economic dynamics.[1] *This* is the approach advocated in this paper.

Our main aim, besides going through the key points of the Solow–Pasinetti debate, is to take advantage of Pasinetti's theoretical developments beyond his original article — tracing connections with and shortcomings of his 1959 contribution — in order to extend and generalise the index of technical change introduced in his 1959 paper. We do so not only by recasting Pasinetti's indicators in terms of Input–Output magnitudes — as attempted by Stone (1998) — but also, and crucially, by adopting the vertically hyper-integrated sector (or growing subsystem) as a disaggregated (but systemic) unit of analysis (Pasinetti, 1988). Our methodological contribution is illustrated empirically, hoping that it may lead to further empirical studies adopting this productivity measurement framework.

After this brief introduction, the rest of the paper is organised as follows. Section 2 introduces the debate between Solow and Pasinetti on productivity measurement, whereas Section 3 explains and specifies Pasinetti's (1959) original index of technical change. Sections 4 and 5 highlight some shortcomings of the original index and hint at features of Pasinetti's later works useful to reformulate it. Section 6 presents an Input–Output reformulation of Pasinetti (1959), whereas Section 7 illustrates empirically the novel indicators derived for the case of Italy. Concluding remarks in Section 8 close the paper.

[1] See Pasinetti (1963, 1973, 1981, 1986, 1988, 1989, 1993) for details.

2. The debate between Solow and Pasinetti on productivity measurement

At the end of the 1950s, Robert Solow published his famous paper 'Technical Change and the Aggregate Production Function' (Solow, 1957), in which he described "an elementary way of segregating variations in output per head due to technical change from those due to the availability of capital per head" (Solow, 1957, p. 312). The paper was an attempt to make an explicit distinction between shifts in the aggregate production function and movements along it, also providing an empirical application for the US economy between 1909 and 1949.

Based on Hicks' classification, Solow considered "[s]hifts in the production function [. . .] as neutral if they leave marginal rates of substitution untouched [. . .], simply increas[ing] or decreas[ing] the output attainable from given inputs" (Solow, 1957, p. 312). His conclusions were that, in the period considered, shifts in the production function had been almost neutral, with an acceleration of technical change after 1929. More precisely, over the whole period output per man hours doubled, with 12.5% of this increase due to a higher capital-labour ratio and 87.5% due to 'technical change' — the now well known 'Solow residual'.

Pasinetti's comment on Solow's argument came two years later in the *Review of Economics and Statistics* (Pasinetti, 1959, 'On Concepts and Measures of Changes in Productivity'), criticising Solow's attempt to evaluate technical change "and to introduce capital into the picture by making use of theoretical notions like the production function", since "these attempts [. . .] have neglected an important characteristic of capital — that it is reproducible and that its process of production is also subject to technical change" (Pasinetti, 1959, p. 270).

But Pasinetti did not limit himself to criticise Solow's theoretical approach, clearly in sharp contrast with his own. He also put forward a methodological proposal for dealing with the issue of technical change in a more complete and consistent way, and then implemented it for the case of the US economy between 1929 and 1950 — i.e. the period in which Solow recognised an acceleration of technical progress accompanied by an increase in capital intensity.

More specifically, Pasinetti considered not only the process of producing final (consumption) commodities, but also that of producing the corresponding productive capacity. In this way, he derived a simple index of the direction of technical change by computing the variation in the ratio of output per man hours in the consumption goods sector to the hypothetical output per man hours that would be necessary to reproduce the corresponding productive capacity. When such a ratio is constant through time technical progress is neutral in the sense of Harrod.[2] On the contrary, changes in such a ratio reflect capital or labour saving technical progress, according to their sign.

We can notice that Pasinetti adopted Harrod's — and not Hicks' as Solow did — criterion for classifying technical progress. As he pointed out, "[t]he richest [. . .] [criterion] for 'neutrality' — in terms of information content — seems therefore to be the first one [due to Harrod] [. . .] which conveys information on the effects of technical progress on capital-intensity, i.e. on the proportion between the labour which must be locked-up in the means of production and the labour which is currently required" (Pasinetti, 1981, p. 214).

Moreover, Solow identified capital intensity with the capital-labour ratio, which Pasinetti calls 'degree of mechanisation', while for Pasinetti capital intensity is given by the capital-net output ratio. In fact, when this distinction is recognised:

> "Solow's conclusions are therefore ambiguous and contradictory. What one can simply say is that the technological change that took place in the U.S. economy, from 1909 to 1949, was accompanied by an increase

[2] See the discussion in Harrod (1973, pp. 52–57).

> in the degree of mechanisation (and not in capital intensity), and by a decrease (not an increase!) in capital intensity".
> (Pasinetti, 1981, p. 184n)

Solow's reply and Pasinetti's rejoinder appeared on the very same issue of the *Review of Economics and Statistics*.

Solow argued that Pasinetti's method cannot be accurate in general, the only exception being the irrelevant case "when Q is produced by K and L in fixed proportions, and no one ever wastes any K or L" (Pasinetti, 1959, p. 283, Solow's Comment). Moreover, he pointed out that Pasinetti's "doubling the number of the commodities in the model increases its realism by 100 per cent [. . .] [I]f there are really 1000 commodities worth distinguishing we only decrease the unrealism by about one-tenth of one per cent" (Pasinetti, 1959, p. 283, Solow's Comment).

However, in explicitly considering productive capacity, Pasinetti is not going from a one- to a two-sector model, but rather accounting for the fact that technical change takes place also in the production of capital goods, and that this has implications that must be acknowledged:

> "My position on this issue is not one of more or less aggregation [. . .] What I do say is that, at whatever level of aggregation our analysis may be carried on, [. . .] an evaluation of changes in productivity cannot leave without an explicit consideration the technical change which may occur in the production process of capital".
> (Pasinetti, 1959, pp. 285-6)

Solow also rejects Pasinetti's statement that technical change in the capital goods industry is always capital-saving for the consumption goods industry. In fact, his idea on this issue is the traditional, Neoclassical one, i.e. technical progress in the production of productive capacity does not save labour:

> "[w]hat it saves is an abstract 'waiting'. It now takes less saving to add a robot to the stock of capital than it did before".
> (Pasinetti, 1959, p. 284, Solow's Comment)

It is our contention that one of the main drawbacks of many — both Neoclassical and other — analyses of technical change is that they fail in recognising changes in productivity as a physical, technological phenomenon, which is "always ultimately labour saving" (Pasinetti, 1981, p. 212n). In these analyses, technical change is often identified with 'real cost reductions' (e.g. Harberger, 1998, p. 2) emerging from a theory of value added, rather than, as in Pasinetti's framework, from a theory of the (physical) net output in which quantities are *reduced* (in the sense of Leontief, 1967) to their (concurrent and co-existing) labour content.

Interestingly enough, Solow's comment to the quotation above is that it "is a true statement and an interesting statement. But it mixes up, *as such statements must*, technological and non-technological facts" (Pasinetti, 1959, p. 284, italics added). Solow is referring to the fact that changes in the composition of demand influence Pasinetti's aggregate measure of technical change, and also that changes in the rate of profit result in *non*-neutral changes.

What Solow points out is of course true: any aggregate measure is influenced by changes in the sectoral composition of the economic system — from here the necessity of moving to sectoral measures. Moreover, changes in the rate of profit of course cause changes in capital intensity. However, Pasinetti's idea of measuring and classifying technical progress is strongly based, as we will see below, on the evolution of physical quantities. This point is raised in a very effective way in Pasinetti's reply to Solow's Comment:

> "[A]part from short-run fluctuations, by far the largest part of changes in productivity over time have been shown to be due to technical change and only to a minor extent to changes in income distribution. I have simply suggested, therefore, an approach that focuses the investigation on the first cause, as opposed to the neo-classical analysis which focuses it on the second".
> (Pasinetti, 1959, p. 285)

As will be seen in more detail, Pasinetti's (1959) paper, and the following debate with Solow, already contains, though sometimes in a still naive and embryonic way, many ideas and insights that will be further developed and incorporated in his approach to the analysis of structural economic dynamics and technical progress (Pasinetti, 1973, 1981, 1988).

The 1959 paper came back to the fore again forty years later, in 1998, when a note written by Richard Stone in 1960 was posthumously published in *Structural Change and Economic Dynamics*. Stone's note starts from Pasinetti's (1959) original measure of productivity, setting it up in Input–Output terms. The last paragraph of such a note is particularly relevant for our purposes:

> "In this analysis consumption and assets used up are reduced to their labour content and in this way made comparable. Technical progress is said to be capital saving, neutral or labour saving according as $\beta_1/\beta_0 \gtreqless 1$. The point of this note is that such statements can be based on data which are actually being provided by input–output analysts without any reference to the form of production functions except at the specific points of time under comparison".
> (Stone, 1998, p. 231)

The publication of Stone's (1998) note caused a further exchange between Pasinetti and Solow, showing that the original sources of disagreement had not disappeared in the course of those 40 years. But there clearly emerges that Pasinetti's (1959) paper has been written in the very same period in which he was working on his PhD Thesis, the first elaboration of his vertically hyper-integrated framework. Far away from being two independent works, they are two faces of the same coin, reflecting the intellectual turmoil that would have led to the formulation of the idea of vertical hyper-integration itself. Richard Stone, in 1960, had perfectly foreseen the natural development of Pasinetti's (1959) analytical apparatus: setting it up into Input–Output terms.

It is also quite clear that writing this paper has been a very important step in the genesis of Pasinetti's approach to economic analysis:

> "My 1959 paper [. . .] originated as a paper for a seminar at the *Harvard Economic Research Project*, directed by Wassily Leontief, who was one of my supervisors while I was at Harvard University".
> (Pasinetti, 1998, p. 233)

Leontief's legacy, as well as Sraffa's, is absolutely evident when carefully analysing Pasinetti's work.

3. Pasinetti's original measure

Pasinetti's (1959) analysis — as hinted above — starts from Solow's (1957) paper, precisely from an analysis of technical progress "along traditional lines". Solow is criticised for not having considered the reproducible character of capital, and therefore the fact that technical progress can take place in its production too.

Thus, Pasinetti provides an extension of the analysis also including the production of productive capacity. He abandons a 'real' measure of capital, defining it in terms of capacity, i.e. with reference to the final (consumption) commodity for whose production process it is employed. First of all, "[t]he unit of measurement with which capital is [usually] measured is itself not independent of the rate of profit" (Pasinetti, 1959, p. 271). But more importantly, this redefinition allows to focus attention, when dealing with the problem of measuring productivity changes, on the evolution of three ratios: Q/L, C/N and C/Q, where C is the capacity necessary for reproducing Q, Q is the quantity of the consumption commodity which is actually produced, L is the (direct) labour employed in its production process and N is the quantity of labour which would be necessary in order to reproduce the whole existing productive capacity.

Pasinetti (1959, p. 273) proposes to evaluate the direction of technical change by analysing the (relative) movements through time of Q/L and C/N, i.e. by computing the ratio:

$$\beta = \frac{Q/L}{C/N} \tag{1}$$

If a unit of capacity is defined as the composite commodity required exactly to reproduce one unit of the consumption commodity *at the time observations are made*, then there will be as many units of capacity as there are units of the consumption good in the net output. Therefore $Q = C$, the last ratio, Q/C, is constant through time and equal to unity, and β becomes:

$$\beta = \left.\frac{Q/L}{C/N}\right|_{Q=C} = \frac{N}{L} \tag{2}$$

According as to whether $d\ln\beta \gtreqless 0$, technical change is labour-saving, neutral or capital-saving, respectively. Pasinetti (1959) bases his notion of neutrality on Harrod's conception (Pasinetti, 1959, p. 274). To see this, he derives an equation for the value at current (production) prices of output of consumption goods and an equation for the value at current prices of productive capacity:

$$p_q Q = (\tau + r)p_c C + w a_q Q \tag{3}$$

$$p_c C = w a_k C \tag{4}$$

where $a_q = L/Q$, $a_k = N/C$ and τ (assuming linear depreciation) stands for the reciprocal of the length of life of the capital good.[3]

According to Harrod, in fact, the direction of technical progress can be classified on the basis of the movements of the capital/net output ratio for a constant profit rate:

$$\kappa = \frac{p_c C}{p_q Q} = \frac{a_k C}{(\tau + r)a_k C + a_q Q} = \frac{N}{(\tau + r)N + L} \tag{5}$$

Notice that when r is constant through time:

$$d\ln\kappa = \frac{\kappa}{\beta} d\ln\beta \tag{6}$$

i.e., the direction of changes in the capital/net output ratio κ always corresponds to the direction of movement of the index of technical change β.

4. Shortcomings of the original measure

The original index β proposed and measured by Pasinetti suffers from some shortcomings due to both the simplifying assumptions he adopted and the kind of data used for its computation.

First of all, it must be stressed that Pasinetti chose *not* to use Input–Output data, but rather aggregate figures from National Accounts. This is quite understandable given the aim of the paper, which was basically intended to be a critique of Solow's (Neoclassical) approach to the study of technical progress, and not an empirical analysis of the phenomenon. Since the aim was theoretical, rather than specifically empirical, it was much more convenient to use as manageable data as possible and to accordingly choose consistent simplifying assumptions.

Pasinetti assumes that all industries in the economic system produce either a capital or a consumption commodity. Moreover, he also implicitly assumes that capital goods are produced by means of labour alone — in fact, these are the very same assumptions that Pasinetti adopted in his doctoral thesis and in his 1981 book. The analogy becomes clearer when we compare Eqs. (3) and (4) with the price equations of a growing subsystem with the technology of Pasinetti (1981): they share exactly the same characteristics.[4]

Clearly, while these assumptions can be accepted within a work aiming at reaching theoretical conclusions, when adopted in an empirical analysis they lead to results which are crude approximations of the magnitudes that are to be computed.

Moreover, and closely connected to the kind of data used for computations, the analysis carried out in Pasinetti's (1959) paper is an aggregate one. No sectoral measures are proposed or computed. This choice is not of course due to Pasinetti's denial of the importance of multisectoral analyses, but to the fact that performing an analysis of that kind was beyond the aim and scope of the paper — as stressed by Pasinetti himself in the quotation provided in Section 2: "My position on this issue is not one of more or less aggregation" (Pasinetti, 1959, p. 285). Nonetheless, it is clear — especially when one considers Pasinetti's more recent scientific production — that going from an aggregate to a multi-sectoral analysis is the natural development of the approach suggested in the paper we are discussing. And it is also clear — especially when one reads Stone's (1998) paper — that translating Pasinetti's (1959) framework into Input–Output terms is the way of improving it and giving it new life.

Finally, Pasinetti's original index of technical change mainly depends on nominal, rather than physical, magnitudes — quite obviously, given the restrictions imposed by the kind of data used for the estimation of β. The amount of labour that would be necessary for the reproduction of the existing capital stock (N) is estimated as the ratio of the capital stock at current prices to the average wage in the capital-producing sector. The latter is obtained as the ratio of the wage bill in the capital-producing industries to the corresponding labour force.

By calling K the value at current prices of the existing stock of capital; W and W_M the total wage bill and the wage bill in the capital goods sector, respectively; with w and w_M the corresponding average wage rates; and M the employment in the capital goods sector, one can write:

$$w_M = \frac{W_M}{M}, \qquad N = \frac{K}{w_M} = K\frac{M}{W_M}$$

and therefore

$$\beta = \frac{N}{L} = \frac{K}{L}\frac{M}{W_M} = \frac{K}{W}\left(\frac{M}{L}\frac{W}{W_M}\right)$$

Therefore, when computed in this way, β is given by the product of two components: the capital/wages ratio and a scale factor. Looking at this scale factor more in detail, we see that in its turn is the product of two components: the ratio of employment in investment goods industry to employment in the consumption goods industry, and the ratio of overall wage bill to the wage bill of the capital goods industry. In this way, not

[3] These equations can be derived either within a traditional Neoclassical framework, using an aggregate production function whose factors are paid their marginal products, or, "perhaps much better, in other theoretical frameworks, such as the Leontief models or the dynamic growth models which pay more attention to fixed coefficients and to idle capacity" (Pasinetti, 1959, p. 275).

[4] In fact, by combining the equation sets (II.5.4) and (II.6.3) in Pasinetti (1981, pp. 39–41), the price equations for a growing subsystem i are:

$$p_i X_i = (1/T_i + \pi_i)p_{k_i} X_{k_i} + w a_{ni} X_i \tag{7}$$

$$p_{k_i} X_{k_i} = w a_{nk_i} X_{k_i} \tag{8}$$

with $\tau_i = 1/T_i$ (the reciprocal of the length of life of the subsystem-specific capital good i). The parallel with Eqs. (3) and (4) becomes apparent.

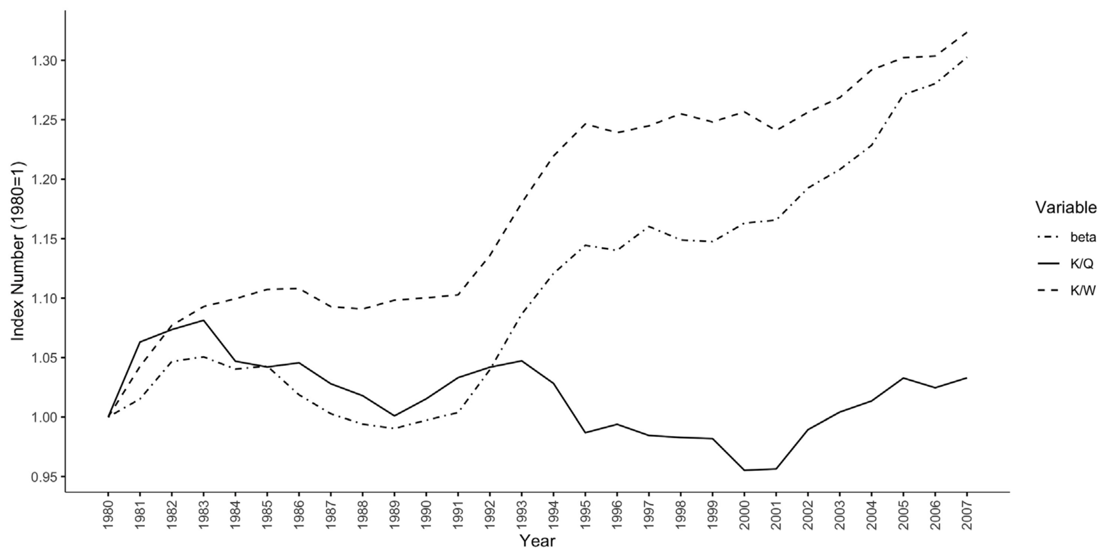

Fig. 1. Dynamics of fixed capital to wages (K/W), fixed capital to net output (K/Q) and β (beta) for Italy (1980–2007).

only β strongly depends on nominal magnitudes, but it is also going to show a co-movement with the capital/wages ratio.

In fact, by reproducing Pasinetti's (1959) original empirical exercise for the Italian economy between 1980 and 2007, this co-movement clearly appears, as displayed in Fig. 1.

The solid line representing the fixed-capital-net-output ratio (K/Q) experiences only mild changes (less than $\pm$ 5 percentage points from 1985 onwards), while the fixed-capital-wages ratio (the dashed line K/W) increases until mid 1980s, remains nearly constant until the beginning of 1990s, and sharply increases afterwards. As to β (the dash-dotted line), though being close to K/Q during the first ten years, this is no longer so afterwards and, in fact, β clearly co-moves with K/W during the whole period.

5. Towards a reformulation

The above-mentioned shortcomings are basically connected with Pasinetti's (1959) empirical implementation of his theoretical ideas. The necessity of using manageable data in order to get ready estimates also compelled the choice of the simplifying assumptions and forced the computation of rough approximations of the theoretical magnitudes.

It is our contention, however, that the ideas at the basis of Pasinetti's (1959) theoretical proposal are correct and worth being developed with the aid of the theoretical developments that followed on the one hand, and Input–Output data and techniques, on the other.

In particular, we want to stress four theoretical features of Pasinetti's (1959) proposal which deserve particular attention — and which have then been further developed by Pasinetti himself.

First of all, an analysis of technical change cannot deny the fact that progress does not only take place in the production of consumption commodities, but also affects the process of (re)production of capital goods. Stressing the importance of this phenomenon — in sharp contrast with the Neoclassical approach followed by Solow (1957) — was the principal aim of Pasinetti's (1959) paper.

Secondly, and closely connected to the previous point, Pasinetti (1959) uses a definition of net output different from the traditional one, including only consumption commodities.[5] New investments, together with replacements, are part of the means of production and not of the net output. This idea has been very important in the elaboration of the concept of *growing* subsystems, and allowed Pasinetti to go into dynamic analysis and studying the problem of capital accumulation.

Another feature which has been introduced in the 1959 article, though present throughout Pasinetti's works, is that of measuring the stock of capital in terms of units of productive capacity, rather than in ordinary physical units. In this way, it is possible to study the problem of capital accumulation separately from that of the composition of the stock of capital itself, and in close connection with the evolution of final demand for consumption commodities.

Finally, and most importantly, productivity accounting must be based on the evolution of *physical*, and not nominal, magnitudes. In fact, the concept of productivity is a purely physical-technical one, and the *measurement* of its evolution through time has nothing to do with changes in income distribution and market prices.

In order to develop these ideas and implement them empirically, Section 6 reformulates the analytical apparatus taking into account the intrinsically multisectoral nature of productivity measures.

First, we are going to change the kind of data used for computing productivity measures. In particular, we will use Input–Output data. More specifically, in order to incorporate pure joint production from the very beginning, we will use the set of Supply-Use Tables (SUT) instead of single-product Input–Output Tables.

Second, we will change the unit of analysis: our sectoral measures will not be computed at the single-industry level, but will refer to growing subsystems, i.e. to vertically hyper-integrated sectors (see Pasinetti, 1988). This step is the analytical counterpart of incorporating the above-mentioned redefinition of the concept of net output: in fact, this very redefinition is at the basis of the hyper-integrated repartition of economic activities leading to the — analytical — construction of growing subsystems.

To clarify, a vertically hyper-integrated sector or growing subsystem is a unit of sectoral analysis obtained by logically repartitioning gross outputs, circulating and fixed capital inputs and employment into as many parts as there are commodities composing the net output (i.e. commodities satisfying only *final* uses of an economic system).[6]

[5] In fact, note that when he computes the 'capital–output' ratio in Pasinetti (1959, p. 277, Table 2) the values 3.125595 (for 1929) and 2.6574374 (for 1950) are computed with Q (net output) taken to be real consumption goods production, and *not* consumption *plus* gross investment demand.

[6] In a closed economy, the net output of a vertically hyper-integrated sector will correspond to a final consumption commodity. In an open economy, it will also include exports of that product, as these will enter the production process of *another* country, being a *final* use for the producing economy.

Essentially, to each final commodity in the system there will correspond a hyper-integrated labour input coefficient, a hyper-integrated vector of productive capacity and a hyper-integrated vector of gross outputs, which are comprehensively required to (re)produce only that final commodity. Hyper-integrated sectors represent an extension introduced by Pasinetti (1988) to Sraffa's notion of 'subsystem' (Sraffa, 1960, Appendix A, p. 89) for economies which are not necessarily in a self-replacing state. Pasinetti's *growing* subsystems crucially include *new (capacity-generating) investments* as part of the productive requirements of each sector, rather than being included in the net output of the whole economy. This treatment of expansion requirements allows to render each part of the system truly autonomous (for details, see Garbellini, 2010, pp. 51–53).

6. An Input–Output reformulation[7]

In his original measure discussed in Section 3, Pasinetti (1959) was concerned with the evolution of the ratios Q/L and C/N, where Q is the quantity of final consumption commodity actually produced, C is the productive capacity necessary for reproducing Q, L is the (direct) labour employed in its production and N "can be interpreted as the quantity of labour which would be necessary for reproducing the existent capacity, with the technique available at the time observations are made" (Pasinetti, 1959, p. 273). While Q/L is the labour productivity in the production of net output, C/N measures labour productivity in the reproduction of capacity.[8] Hence:

> "A change through time of Q/L can be assumed by itself to be an indication of change in productivity only if C/N changes in the same proportion. If C/N does not change in the same proportion at least two parts of the change have to be distinguished — a neutral effect equal to the proportional change of that ratio which has changed the least, and a labour saving effect — or alternatively a capital saving effect — given by the excess of the proportional change of Q/L over C/N — or alternatively of C/N over Q/L"
> (Pasinetti, 1959, p. 273)

Thus, as has been done in Eq. (1) of Section 3, by defining:

$$\beta = \frac{Q/L}{C/N} \tag{9}$$

we may assess the direction of technical change, according to the movement of β.

Nevertheless, as has been emphasised in Section 3, a key point of this formulation lies in measuring capital goods in units of capacity C currently required to reproduce a given (hyper-integrated) net output Q. In fact, by adopting this unit of measurement for capital goods, $C = Q$ in every period and $\beta = N/L$.

Clearly, β was originally conceived in the context of an economy producing a single final commodity, without taking into account the complexities of inter-industry relations. However, as soon as general interdependence is accounted for, such an aggregate index could never be purely 'technical', as it would depend on compositional changes in the vector of final uses. At best, the original measure could be conceptually thought of as a subsystem-specific index. Moreover, this indicator should mirror the evolution of the capital/net output ratio, reflecting the overall capital intensity of the system, in the sense of Harrod (1948).

[7] This section partially draws upon Section 3.2 of Garbellini and Wirkierman (2014).

[8] It should be clear that C/N measures a counter-factual, as N corresponds to a measure of *current* and *co-existing* labour, no reference at all being made to series of *dated* labour quantities.

To translate the logic of Pasinetti's (1959) index β and the productivity index Q/L into a multisectoral formulation with the growing subsystem or vertically hyper-integrated sector (Pasinetti, 1988) as a disaggregated unit of analysis, we need to establish a correspondence between Input–Output magnitudes and Q, L, C, and N.[9]

To begin with, define the labour content of the net output of a growing subsystem i as[10]:

$$L_\eta^{(i)} = \boldsymbol{\eta}^T \hat{\boldsymbol{c}} \boldsymbol{e}_i = \eta_i c_i \tag{10}$$

$$\boldsymbol{c} = \boldsymbol{q} - (\boldsymbol{U}_q - \boldsymbol{F}_{k_q})\boldsymbol{e} \tag{11}$$

$$\boldsymbol{\eta}^T = \boldsymbol{l}^T (\boldsymbol{V}_q - \boldsymbol{U}_q - \boldsymbol{F}_{k_q})^{-1} \tag{12}$$

where $\boldsymbol{c} = [c_i]$ is the vector of hyper-integrated net output (consisting of final uses that do not re-enter the circular flow), while $\boldsymbol{\eta}^T = [\eta_i]$ is the vector of vertically hyper-integrated labour coefficients, each of them (i.e. η_i) reflecting the direct, indirect and *hyper*-indirect labour requirements to reproduce a unit of commodity i for final uses.[11]

Note that vector $\boldsymbol{c}$ is the physical residual of subtracting a comprehensive measure of means of production, including the current *gross* flow of circulating ($\boldsymbol{U}_q$) as well as fixed ($\boldsymbol{F}_{k_q}$) capital inputs, from the vector of gross outputs by commodity (as represented by vector $\boldsymbol{q}$).

Hence, from (10) we may compute the hyper-integrated labour productivity of growing subsystem i as:

$$\alpha_\eta^{(i)} = \frac{c_i}{L_\eta^{(i)}} = \frac{1}{\eta_i} \tag{13}$$

its evolution being approximated by the proportional rate of change $d \ln \alpha_\eta^{(i)} \approx \Delta\%\alpha_\eta^{(i)}$, while a system-wide weighted average of productivity changes may be thus obtained by computing:

$$\rho^* = \frac{\sum_i \Delta\%\alpha_\eta^{(i)} L_\eta^{(i)}}{\sum_i L_\eta^{(i)}} \tag{14}$$

which represents Pasinetti's (1981, pp. 101–2) 'standard rate of growth of productivity', generalised to an Input–Output framework.[12]

As regards the *stocks* of productive capacity of the economic system, these may be defined at the level of each hyper-integrated sector i by means of its column vector of vertically hyper-integrated productive capacity $\boldsymbol{m}_i^*$ (Pasinetti, 1988, pp. 127–8). In our empirical framework, each of these columns — stacked in a matrix denoted by $\boldsymbol{M} = [\boldsymbol{m}_i]$ — may be computed as:

$$\boldsymbol{m}_i = \boldsymbol{S}(\boldsymbol{V}_q - \boldsymbol{U}_q - \boldsymbol{F}_{k_q})^{-1} \boldsymbol{e}_i \tag{15}$$

where $\boldsymbol{S}$ is a commodity × industry matrix containing fixed and circulating capital stocks (both domestically produced *and* imported) in volume terms required to support the production of the gross output vector $\boldsymbol{q}$.[13]

[9] For a detailed methodological exposition of the mathematical expressions that follow, please see Garbellini and Wirkierman (2014).

[10] All throughout the rest of the paper, vectors are indicated by lower case boldface characters (e.g. $\boldsymbol{z}$), and have to be intended as column vectors unless explicitly transposed (e.g. $\boldsymbol{z}^T$); matrices are indicated by upper case boldface characters (e.g. $\boldsymbol{X}$), except for lower case characters with a hat (e.g. $\hat{\boldsymbol{z}}$), indicating diagonal matrices with the vector elements on the main diagonal. Moreover, $\boldsymbol{e} = [1 \dots 1]^T$ is the sum vector and $\boldsymbol{e}_i = [0 \dots 1 \dots 0]^T$, with 1 in the ith position, is a column selector vector.

[11] In expressions (10)–(12), $\boldsymbol{U}_q$ is the commodity × activity Use matrix for domestic output at basic prices in volume terms, $\boldsymbol{V}_q$ is the commodity × activity Make matrix in volume terms, $\boldsymbol{F}_{k_q}$ is the matrix of *gross* fixed capital formation by product of origin and industry of destination, $\boldsymbol{l}^T$ is the vector of employment units by industry of origin, and $\boldsymbol{q}$ is the vector of gross output by commodity in volume terms.

[12] See also Garbellini and Wirkierman (2014, section 3.1) for a detailed presentation.

[13] See Garbellini and Wirkierman (2014, section 2.3) for a more detailed mapping into empirical objects of the System of National Accounts.

Therefore, when considered at the level of the single hyper-integrated sector, the scalar magnitude L of expression (1) may be associated with $L_{\eta}^{(i)}$ in (10), while Q and C correspond to c_i. Finally, we may define:

$$N_{\eta}^{(i)} = \boldsymbol{\eta}^T \boldsymbol{m}_i c_i \tag{16}$$

i.e. the quantity of *co-existing* vertically hyper-integrated labour that would be necessary for the reproduction of the existing productive capacity with the technique actually in use.

In this way, the disaggregated index for the direction of technical change in each vertically hyper-integrated sector i may be written as:

$$\beta^{(i)} = \frac{c_i / L_{\eta}^{(i)}}{c_i / N_{\eta}^{(i)}} = \frac{N_{\eta}^{(i)}}{L_{\eta}^{(i)}} = \frac{\boldsymbol{\eta}^T \boldsymbol{m}_i c_i}{\eta_i c_i} = \frac{\boldsymbol{\eta}^T \boldsymbol{m}_i}{\eta_i} \tag{17}$$

while the economy-wide index of capital intensity is given by:

$$\beta^* = \frac{Q/L}{C/N} = \frac{N}{L} = \frac{\sum_i N_{\eta}^{(i)}}{\sum_i L_{\eta}^{(i)}} = \frac{\boldsymbol{\eta}^T \boldsymbol{M} \boldsymbol{c}}{\boldsymbol{\eta}^T \boldsymbol{c}} \tag{18}$$

Note that the series of subsystem-specific indexes $\beta^{(i)}$ as well as the aggregate index β^* are 'pure numbers'.[14] Moreover, it is worth stressing that while β^* depends on the composition of final consumption $\boldsymbol{c}$ (its movement through time thus depending on compositional changes in net output), sectoral indexes $\beta^{(i)}$ are intrinsically 'technical', since they are independent of the structure of final uses.[15] The intrinsically technical character of subsystem magnitudes with an overall average that depends on the composition of final demand is also present in $\Delta\%\alpha_{\eta}^{(i)}$ and ρ^* (expressions (13) and (14) above, respectively).

Hence, by considering the tuple $(\Delta\%\alpha_{\eta}^{(i)}, \beta^{(i)})$ for each hyper-integrated sector i, as well as the system-wide measures (ρ^*, β^*) we may assess the evolution of productivity as well as the direction of technical change in the spirit of Pasinetti's (1959) original proposal.

7. An illustrative empirical exploration: Italy (1999–2007)

In this section we present a brief illustration of the measures previously introduced for the case of the Italian economy throughout 1999–2007. Yearly series of square 30 × 30 (commodity × industry) Supply-Use Tables at the 2-digit NACE Rev. 1 level, as well as gross fixed capital stock and flow matrices and labour input data have been obtained from the Italian National Institute of Statistics (ISTAT).[16]

7.1. Aggregate trends

Tables 2 and 3 display levels and rates of change, respectively, of selected aggregate variables for the period 1999–2007. Their description is given in Table 1.

For the aim of analysing co-movement trends among variables, the full period 1999–2007 is divided into three sub-periods: 1999–2000, 2000–2003 and 2003–2007. The first two years are presumably the end of a trend that comes from previous years, the 2000–2003 sub-period is characterised by negative productivity growth (as measured by ρ^*, and computed according to (14)), while the contrary occurs in the final 2003–2007 sub-period.

The transition between 1999 and 2000 is characterised by the highest values for ρ^* (2.46%) and ρ_{tfp} (1.76%) of the whole period, i.e. high productivity growth and increasing surplus from the value added side. This has been accompanied by a mild decrease in the wage share Ω_W *and* real wage rate $w/\bar{c}_p^*$, together with the highest increase in employment (1.80%) throughout the 1999–2007 period.

Moreover, note that the ratio of the money wage rate to the per-capita average consumption basket in nominal terms (w/c_p^*) experiences a decrease (from 1.88 to 1.83). Thus, given that the money wage rate and employment are increasing, this must be due to the rising consumption per-capita.

The sub-period 2000–2003 is characterised by negative productivity growth as well as a decrease in the real wage rate though accompanied by a mild increase in the wage *share*. The ratio of the money wage to average per-capita consumption remains constant with a rising money wage rate and employment, indicating an increase in nominal per-capita (average) consumption.

Between 2000 and 2003, capital intensity of the system shows the highest increase of the whole period, either measured at current statistical prices (S^*/C) or by using vertically hyper-integrated labour coefficients as aggregators (β^*), so the direction of technical change has clearly been non-neutral.

The negative trend of productivity growth is reverted in the 2003–2007 sub-period, though experiencing continuous decline. The rhythm of employment creation has also been reduced though the real wage rate has experienced the highest increase of the whole 1999–2007 period during 2003–2006. Real wage increase with mild wage share increase have been accompanied by a rising trend in the ratio of money wage rate to per-capita average consumption. Technical change has been capital intensity-increasing (both S^*/C and β^* have risen), though to a lesser extent than during 2000–2003.

It is interesting to ask to what extent productivity increases (as measured by ρ^*) have accrued to real wage growth (as measured by $\Delta\%(w/\bar{c}_p^*)$). For the whole 1999–2007 period, ρ^* has exceeded $\Delta\%(w/\bar{c}_p^*)$ by a yearly average of 0.25 percentage points, though it is interesting to notice that when productivity is falling (2000–2003), the real wage decreases to a lesser extent (their yearly average difference is −0.53 percentage points). Hence, productivity movements amplify those of the real wage rate in both directions, though the overall trend suggests that only 60% of productivity growth accrues to wages, on average.

Finally, it emerges from Table 3 that when Pasinetti's (1959) original measure β is replaced by β^* — as defined in (18) — its co-movement and order of magnitude clearly resembles the ratio S^*/C of total capacity (domestically produced plus imported) to net output (in hyper-integrated terms).[17] In fact, by looking at the evolution depicted in Fig. 2, β^* closely resembles aggregate capital intensity (measured by S^*/C), being detached from the capacity-to-wages ratio (measured by S^*/W), and supporting Pasinetti's (1959) intuition on the adequacy of β^* as an index of the direction of technical change.

7.2. Sectoral trends

While the tuple (ρ^*, β^*) depends on the composition of hyper-integrated net output, sectoral measures $(\Delta\%\alpha_{\eta}^{(i)}, \beta^{(i)})$ are strictly 'technological', in the sense that do not depend on neither the composition of final uses nor relative prices (and thus, income distribution).

[14] In fact, it is straightforward to show that the absolute level of both measures can be computed starting from nominal magnitudes, since the effect of prices cancels out. For a given price vector of basic statistical prices ($\boldsymbol{p}_s$), we have:

$$\frac{\boldsymbol{\eta}^T \widehat{\boldsymbol{p}}_s^{-1} \widehat{\boldsymbol{p}}_s \boldsymbol{M} \widehat{\boldsymbol{p}}_s^{-1} \widehat{\boldsymbol{p}}_s \boldsymbol{c}}{\boldsymbol{\eta}^T \widehat{\boldsymbol{p}}_s^{-1} \widehat{\boldsymbol{p}}_s \boldsymbol{c}} = \frac{\boldsymbol{\eta}^T \boldsymbol{M} \boldsymbol{c}}{\boldsymbol{\eta}^T \boldsymbol{c}} = \beta^*$$

$$\frac{\boldsymbol{\eta}^T \widehat{\boldsymbol{p}}_s^{-1} \widehat{\boldsymbol{p}}_s \boldsymbol{m}_i p_i^{-1}}{\eta_i p_i^{-1}} = \frac{\boldsymbol{\eta}^T \boldsymbol{m}_i}{\eta_i} = \beta^{(i)}$$

[15] In this sense, while $\beta^{(i)}$ adequately reflects the direction of technical change in each growing subsystem, the interpretation of β^* as indicating the 'type' of technical change at an aggregate level is not warranted. See Pasinetti (1981, p. 214).

[16] As regards particular characteristics of the dataset, as well as data preparation and estimation procedures, please refer to Wirkierman (2012, Appendix C).

[17] Their correlation coefficient between 2000 and 2007 is 0.978.

Table 1
Dictionary of empirical variables.
Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT.

Variable	Description	Units
L	Units of employment	(10^3 ULE)
W/L	Money wage rate	(10^3 MU/ULE)
Ω_W	Wage share (adjusted by employment/employee ratio)	(%)
w/c_p^*	Wage rate to per-capita average consumption basket in nominal terms	(Ratio)
S^*/W	Capacity to wages ratio	(Ratio)
S^*/C	Capacity to domestic final consumption ratio	(Ratio)
β^*	Aggregate capital intensity	(Ratio)
$\Delta\%L$	Growth of industry employment	(Rate of growth in per cent)
$\Delta\%(w/\overline{c}_p^*)$	Growth of ratio of wage rate to per-capita average consumption basket of the base year at constant prices (i.e. real wage rate growth)	(Rate of growth in per cent)
ρ_{tfp}	Total Factor Productivity (TFP) growth rate	(Rate of growth in per cent)
ρ^*	Standard rate of productivity growth	(Rate of growth in per cent)
$\Delta\%\beta^*$	Aggregate direction of technical change	(Rate of growth in per cent)

Notes: MU: monetary units; ULE: units of full-time labour equivalent, measured in thousand of man-years.

Table 2
Selected Aggregate-Level Variables, Italy (1999–2007).
Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT.

Variable	1999	2000	2001	2002	2003	2004	2005	2006	2007	Mean
L	22 994.70	23 412.30	23 828.60	24 132.20	24 282.90	24 373.00	24 411.60	24 788.70	25 026.40	24 138.93
W/L	20.26	20.86	21.59	22.15	22.86	23.64	24.45	25.22	25.82	22.98
Ω_W	46.34	45.90	45.82	45.86	46.11	46.02	46.47	47.20	46.73	46.27
w/c_p^*	1.88	1.83	1.82	1.82	1.82	1.85	1.86	1.87	1.86	1.85
S^*/W		14.37	14.33	14.54	14.63	14.90	15.16	15.18	15.46	14.82
S^*/C		6.31	6.33	6.51	6.61	6.70	6.76	6.74	6.78	6.59
β^*		6.37	6.42	6.58	6.67	6.67	6.80	6.78	6.87	6.64

Table 3
Selected Aggregate Variables, Rates of Change (in %), Italy (1999–2007).
Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT.

Variable	99-00	00–01	01–02	02–03	03–04	04–05	05–06	06–07	Mean 00–03	Mean 03–07	Mean 99-07
$\Delta\%L$	1.80	1.76	1.27	0.62	0.37	0.16	1.53	0.95	1.22	0.75	1.06
$\Delta\%(w/\overline{c}_p^*)$	−0.24	0.17	−0.31	−0.40	1.26	1.31	1.06	0.10	−0.18	0.93	0.37
ρ_{tfp}	1.76	0.11	−0.83	−1.42	0.73	−0.23	0.46	0.33	−0.71	0.32	0.11
ρ^*	2.46	−0.24	−1.27	−0.62	1.69	1.18	0.92	0.82	−0.71	1.15	0.62
$\Delta\%\beta^*$		0.92	2.41	1.89	−0.16	2.22	−0.52	1.42	1.74	0.74	1.17

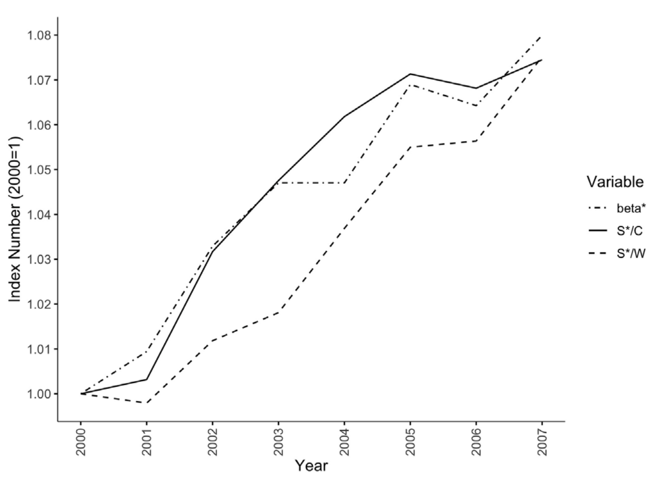

Fig. 2. Dynamics of capacity to wages (S^*/W), capacity to net output (S^*/C) and β^* ($beta^*$) for Italy (2000–2007).

Table 4 reports the average movement of key sectoral variables, allowing to conclude that when focusing on the sectoral direction of technical change, i.e. the dynamics of $\beta^{(i)}$ in (17), we have that if $\Delta\%\beta^{(i)} > 0$, then $\Delta\%(c_i/L_\eta^{(i)}) > \Delta\%(c_i/N_\eta^{(i)})$, implying that total labour productivity increases faster than the reduction in labour content required to reproduce subsystem's i productive capacity. In terms of Pasinetti (1981, p. 209), this pattern corresponds to 'capital-intensity increasing' technical progress. In the case under study, it results from our computations that all growing subsystems but Education (MM) and Business Services (KK) follow this upward trend.

8. Concluding remarks

The Solow–Pasinetti debate has been a milestone in the literature on how to measure productivity changes and singling out the direction of technical change. Whereas the conceptual disagreement between the two authors is still characterising alternative approaches to these issues, the strand of literature following Pasinetti (1959) — and then Stone (1998) — was still lacking a rigorous translation of Pasinetti's original idea into multi-sectoral, Input–Output terms.

The aim of the present paper, besides going through the key points of the Solow–Pasinetti debate, has therefore been that of taking advantage of Pasinetti's further theoretical developments in order to extend and generalise the measure presented in his 1959 paper. By doing so, it was possible to show that the original, aggregate measure was biased and in particular that it co-moves with the capital/wages — rather than with the capital/net output — ratio. We thus used the concept of growing subsystem to establish a correspondence between the index

Table 4
Average sectoral dynamics of net output, labour, productivity and direction of technical change.
Source: Own computation based on Supply-Use Tables (SUT) and National Accounts Data, ISTAT.

(Country: Italy; Period: 1999–2007; Average values)

Hyper-integrated sector	Levels		Yearly average rate of change (in %)				
	$L_{\eta}^{(i)}/L$ (1)	$\beta^{(i)}$ (2)	$\Delta\%c_i$ (3)	$\Delta\%L_{\eta}^{(i)}$ (4)	$\Delta\%\alpha_{\eta}^{(i)}$ (5)	$\Delta\%\beta^{(i)}$ (6)	(5)–(6)
Dynamic subsystems: $\Delta\%\alpha_{\eta}^{(i)} > \rho^*$ and $\Delta\%L_{\eta}^{(i)} > 0$							
DG:Chemicals	1.43	9.38	3.59	0.43	3.40	0.31	3.09
DL:Electr. Machinery	1.54	5.42	2.89	0.71	2.14	0.60	1.54
DA:Food-Tobacco	5.72	6.81	2.16	0.04	2.14	1.65	0.49
DK:Machinery n.e.c.	3.87	6.52	4.18	2.05	2.10	0.51	1.59
DH:Plastics	0.67	7.51	2.73	0.77	1.96	1.81	0.15
JJ:Finance	1.58	6.63	3.65	1.79	1.79	1.16	0.63
II:Transport-Comm.	5.42	6.95	2.91	1.36	1.55	1.40	0.15
DM:Transport Equip.	1.84	7.49	2.09	0.48	1.54	0.81	0.73
DE:Paper-Printing	0.98	7.13	1.81	0.59	1.24	1.42	−0.18
DJ:Metals	1.78	6.56	6.75	5.76	0.91	0.52	0.39
NN:Health	7.91	2.83	2.56	1.77	0.79	3.04	−2.26
Dynamic productivity/labour expelling subsystems: $\Delta\%\alpha_{\eta}^{(i)} > \rho^*$ and $\Delta\%L_{\eta}^{(i)} < 0$							
DD:Wood	0.19	5.91	0.09	−2.80	2.93	1.09	1.84
DC:Leather	1.32	5.69	−1.53	−3.59	2.16	1.91	0.24
DB:Textiles	3.75	5.81	−0.58	−2.36	1.87	1.59	0.28
DN:Manufacture n.e.c.	1.92	5.80	−1.15	−2.53	1.43	0.59	0.85
LL:Public Admin.	9.20	10.57	1.25	−0.17	1.43	2.21	−0.77
DI:Non-met. minerals	0.70	7.25	0.06	−0.96	1.08	1.43	−0.35
AA:Agriculture	1.92	5.45	0.82	−0.05	0.88	2.24	−1.36
Productivity lagging subsystems: $0 < \Delta\%\alpha_{\eta}^{(i)} < \rho^*$							
CB:Mining non-energy	0.03	9.40	2.21	1.97	0.55	0.83	−0.28
MM:Education	6.76	1.86	0.58	0.11	0.47	−0.15	0.62
GG:Trade	16.03	5.48	1.11	0.99	0.13	2.16	−2.03
PP:Household Services	3.28	0.00	2.93	2.92	0.01	0.00	0.01
Productivity decreasing subsystems: $\Delta\%\alpha_{\eta}^{(i)} < 0$							
FF:Construction	0.63	4.35	−0.61	−0.55	−0.06	0.72	−0.78
HH:Hotel-Restaurant	7.87	4.69	1.98	2.38	−0.37	1.09	−1.46
EE:Energy	0.73	15.80	1.04	1.71	−0.46	0.29	−0.75
BB:Fishing	0.19	2.80	−0.71	0.13	−0.80	1.08	−1.88
OO:Personal Services	3.64	5.21	1.50	3.10	−1.46	0.45	−1.91
KK:Business Services	8.81	16.02	1.42	2.93	−1.46	−0.63	−0.82
DF:Coke-Petroleum	0.29	11.55	−1.12	7.10	−5.59	2.27	−7.86

originally presented by Pasinetti (1959) and the multi-sectoral approach developed by Pasinetti (1981, 1988), defining a set of indicators which can be actually computed on the basis of Input–Output data.

By computing — for the case of Italy over the period 1999–2007 — Pasinetti's original measure β, a set of sectoral measures $\beta^{(i)}$, as well as an aggregate index β^*, we showed that the latter, when computed from Input–Output data, provides the correct measure that Pasinetti (1959) had in mind to analyse the phenomenon of technical change, while β actually co-moves with the capital/wages ratio over the whole period. Moreover, the sectoral indicators show that, over the period considered, aggregate measures are insufficient to capture the great variability of sectoral performances in terms of productivity and employment.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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