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# Analysis of standardized tests and pre-service teacher education: reflections on developed teachers' specialized knowledge

Federica Ferretti<sup>1</sup>, Francesca Martignone<sup>2</sup>, George Santi<sup>3</sup>

<sup>1</sup>University of Ferrara, Italy; [federica.ferretti@unife.it](mailto:federica.ferretti@unife.it)

<sup>2</sup>University of Eastern Piedmont, Italy; [francesca.martignone@uniupo.it](mailto:francesca.martignone@uniupo.it)

<sup>3</sup>University of Macerata, Italy; [grpsanti@gmail.com](mailto:grpsanti@gmail.com)

*In this paper we describe a pre-service teacher professional development program based on Meta-Didactical Transposition model. We focus on the encounter between the praxeologies of researchers/teacher educators and pre-service primary teachers. We show how specific tasks for teachers and information provided by a standardized assessment database (GESTINV) can be used to establish a link between Italian National Evaluation System and pre-service teacher professional development. We present some aspects of mathematics specialized knowledge that pre-service teachers declare to be improved during the educational program. Pre-service teachers' reflections on their specialized knowledge are framed into MTSK model.*

*Keywords: Teacher education, pre-service primary teachers, teacher knowledge, MTSK model, standardized tests.*

## Introduction

The use of standardized tests in mathematics for both international surveys (such as PISA and TIMSS) and national evaluations of school systems is widespread, and the results of these tests are also often disseminated in the press and commented on by policymakers and researchers (Doig, 2006). In order to not confine these data to the ranking of students, schools and nations it is essential to develop the dialogue between standardized assessment and educational research (De Lange, 2007). The Standardized Assessment data can also become effective and usable tools for improving teaching and learning processes. Our study fits in this stream of thought; in particular, we will show educational potentials of Italian Standardized Assessment by using theoretical tools to interpret the quantitative data they provide and the macro phenomena that emerge from the complexity of educational systems (Bolondi et al., 2019; Ferretti, & Bolondi, 2019). Quantitative data should be interpreted and intertwined with a qualitative analysis of tasks. This is possible by means of an encounter between Standardized Assessment, mathematics education research and teacher education. In this paper we present an example of pre-service teachers' professional development that profits from data collected by the Italian National Evaluation Institute for the School System (INVALSI). We will show how to create a link between the Italian National Evaluation System and pre-service teacher professional development programs in order to improve mathematics teaching school practices. In our study we argue how the analysis of standardized assessment results should impact the improvement of the teaching and learning of mathematics. We have been working for many years in this perspective and we have carried out educational projects using data from INVALSI tests as an object of reflection for teachers (Martignone, 2017; Ferretti et al., 2018; Ferretti, et al., 2020, Santi et al., in press). It must be stressed, however, that this use of INVALSI tests is favored by the fact that

in Italy the framework of INVALSI tests is closely linked to the National Guidelines and it takes into account the research in mathematics education. In this paper we describe some teacher education activities carried out during a pre-service primary teacher professional development course. During a university course teacher educators/researchers shared with pre-service teachers some theoretical tools in order to conduct an a priori qualitative analysis of tasks from INVALSI tests. As final reflections, after the introduction of the Mathematics Teacher Specialized Knowledge model - MTSK Model (Carrillo-Yañez et al., 2018), the educators/researchers asked pre-service teachers about which aspects of mathematics specialized knowledge the course activities brought out and developed

## **Theoretical framework**

Mathematical knowledge lives in the institutional dimension where mathematical objects emerge from socio-cultural activities shared by individuals belonging to one or more institutions. The development of mathematics, teaching and learning of mathematics and mathematics teachers' education are characterized by the dialectics between the personal and institutional relation to knowledge (Chevallard, 1985). The Anthropological Theory of Didactics (Chevallard, 1985) conceives human activity as a praxeology, which is made up of a set of tasks that drive the practice (praxis), the techniques that allow individuals to solve the problems, and the knowledge and discourses (logos) that ground the techniques. To describe and interpret the practices of mathematics educators/researchers and those of teachers who are engaged in teachers' education activities, in our study we use theoretical lenses from Meta-didactical Transposition Model (Aldon et al., 2013; Martignone, 2015). The teacher professional development program described in this paper can be seen as an instantiation of the Meta-Didactical Transposition model. In regard to the institutional dimension, the teacher education process involves the Italian Ministry of Education via the INVALSI institute, Italian Schools and Universities. Researchers/teacher educators and teachers share didactical praxeologies and reflect on them. Researchers bring to the fore tasks and techniques related to the epistemology of mathematics and mathematics education studies. As we will show in the next paragraph, the shared praxeology is linked to the analysis of mathematical and pedagogical issues in INVALSI tasks about rational numbers, but not just this. Theoretical lenses for reflecting on mathematics teacher specialised knowledge are shared and discussed. In specific, the MTSK model (Carrillo-Yañez et al., 2018) was introduced. MTSK model distinguishes between mathematical knowledge (MK) and pedagogical content knowledge (PCK), both of which are considered as sub-domains of the teacher's specialized knowledge. As concerns the MK, are part of the Knowledge of Topic (KoT) the knowledge of definitions, properties, procedures, representations, etc. The knowledge about how to connect activities in different domains of mathematics is part of the Knowledge of the Structure of Mathematics (KSM). As knowledge of Practices in Mathematics (KPM), we can identify, for example, the knowledge of how to prove, justify, define, make inferences and inductions, give examples and counterexamples. Also Pedagogical Content Knowledge is divided into three sub-domains. The Knowledge of Mathematics Teaching (KMT): knowledge of theories of mathematics teaching or knowledge of teaching resources, materials and technologies, but also knowledge of strategies for introducing and representing contents and concepts, etc. The Knowledge of Features of Learning Mathematics (KFLM): knowledge of theories of mathematics learning or knowledge of the way in which pupils interact with mathematics. The Knowledge of Mathematics

Learning Standards (KMLS): knowledge of expected learning outcomes and teaching goals in different school segments. The MTSK model in addition to detailing these subdomains of Mathematical Knowledge and PCK explicitly highlights the centrality of teachers' beliefs about mathematics and mathematics teaching-learning. For this reason, in our opinion, MTSK model is suitable to be used by teachers to explore, reflect, discuss on what they think/believe about their specialized knowledge developed during the education program. Therefore, in our study we show that MTSK model can be used by pre-service teachers to reflect on their knowledge during a teacher education program. The interpretive lenses provided by MTSK model have become part of the logos of shared meta-didactical praxeologies. Moreover, the MTSK model becomes a theoretical object shared between researchers and teachers to reflect on tasks and practices. Following MDT model, in our teacher educational program the researcher/teacher educator play the role of broker (Rasmussen et al., 2009) who belongs both to the community of mathematics experts and to the community of the teacher education program. The broker facilitates the sharing of knowledge and practices from one community to the other by drawing on boundary objects (Bowker and Star, 1999). Boundary objects are meaningful tools, ideal or material, in both the communities and they put them in touch, although with different nuances and uses that characterize their respective praxeologies. Boundary objects can be material artefacts, digital technologies, mathematical procedures etc. In our teaching education programs the database of INVALSI tests, GESTINV, acts as a boundary object between communities of researchers and future teachers. GESTINV ([www.gestinv.it](http://www.gestinv.it)) is a structured database containing all the data of the INVALSI standardized assessments from the 2008 surveys of all surveyed domains. In detail, it contains 2121 items from the INVALSI mathematics tests. There are more than 25,000 registered users in GESTINV and an average of 200 accesses per day. These data, together with its structured information in line with the theoretical framework of INVALSI tests, promote Gestinv as a tool to be implemented in the design of teacher education models (Ferretti et al., 2020). There are many ways in which the database can be used; different forms of research can be carried out within it (National Guidelines, scholastic year, school grade, content, percentage of correct/wrong/invalid answer). In our research, GESTINV plays an important role in providing teachers and researchers with an interactive tool to access a wide range of information and feedback concerning the processes of learning and teaching mathematics. In particular, the results of the INVALSI tests highlight and quantify relevant macro-phenomena, which can be interpreted according to the methods and results of research in mathematics education. By means of specific tasks for teachers the information provided by GESTINV can be used to establish the link between the data of the standardized assessments and pre-service teacher professional development (PTPD), assuming the role of boundary object. In this paper we show some teachers' a-posteriori reflections on specialized knowledge emerged and developed in the analysis of INVALSI tasks selected by means of GESTINV. The pre-service teachers use theoretical lenses from MTSK model to state which aspects of specialized knowledge are improved at the end of the PTPD program.

## Methodology

In this paper we show some activities carried out during a pre-service primary teacher professional development university course (conducted in presence in 2019). This course lasted 40 hours and each activity consists of the following phases:

*Introduction.* The teacher educators address the mathematical contents selected for the educational activity from an epistemological and didactic point of view and they also present GESTINV. The teacher educators share with future teachers' possible interpretative tools and research results that can be useful in the analysis of INVALSI tasks and data. As we already stressed INVALSI tests' theoretical framework is linked to the National Guidelines and also for this reason the macro-phenomena emerging from the INVALSI tests are meaningful for the analysis carried out with future teachers.

*Analysis of an example.* The teacher educators analyze an example of INVALSI tasks selected by using GESTINV.

*Group activity.* The teacher educators ask teachers to analyze INVALSI tasks on specific topics. Teachers work in subgroups of a maximum of 4/5 people. They choose mathematical contents with reference to the Italian National Guidelines of primary school. They should identify one or more learning difficulties. The small group activity takes place in line with the characteristics of a Community of Inquiry in the sense of Jaworski (2006). The group activity aims at the construction of a multimedia product, an artefact, the design of an activity for the students, etc. that highlights the reflections, beliefs and convictions of the future teachers involved.

*General discussion.* The subgroups present their productions to the whole group. Each presentation is discussed in order to highlight beliefs and convictions, address doubts, difficulties and unclear contents. Then teacher educators presented and discuss with teachers the MTSK model.

At the end of the course, we administered an open-ended questionnaire to the pre-service teachers in which they were asked if and how, in their opinion, the activities carried out during the course had increased their knowledge by referring to each sub-domain of the MTSK. Specifically, the teachers answered this question: "Using the MTSK model, explain which topics covered during the course, which tasks and activities contributed, in your opinion, to the development of your specialist knowledge". The teachers knew that answers would not have been assessed. Data relating to 52 questionnaires administered to the pre-service teachers have been collected and analyzed. The gathered data were analyzed by an inductive content analysis (Patton, 2002); by using the MTSK theoretical lens, a top-down were performed.

### **INVALSI tasks analysed by pre-service teachers**

In this section, we describe an example which is part of a broad prospective primary teacher academic course, in which different contents were involved and macro-phenomena and teaching practices were analyzed and framed with different constructs of mathematics education. The tasks selected, according to cross-research allowed by GESTINV according to categories described above, belong to INVALSI standardized tests administered to all Italian grade 5 students. Figure 1 shows the texts of the INVALSI tasks and the national percentages of correct, wrong and invalid answers.

<p>D25. Observe the following representations of numbers.</p> <p style="text-align: center;">50%     <math>\frac{1}{2}</math>     0,2     <math>\frac{5}{10}</math></p> <p>Circle all those that represent the same number.</p>	<p>D18. Which of the following equalities is true?</p> <p><input type="checkbox"/> A. <math>\frac{1}{2} = 0,2</math></p> <p><input type="checkbox"/> B. <math>\frac{1}{2} = 0,5</math></p> <p><input type="checkbox"/> C. <math>\frac{1}{2} = 1,2</math></p> <p><input type="checkbox"/> D. <math>\frac{1}{2} = 1,5</math></p>												
<p><b>National answers rate – Item D25, Grade 5 INVALSI test 2016</b></p> <table border="1"> <thead> <tr> <th>Correct Answers</th> <th>Wrong Answers</th> <th>Invalid Answers</th> </tr> </thead> <tbody> <tr> <td>37.4%</td> <td>56.1%</td> <td>6.5%</td> </tr> </tbody> </table>	Correct Answers	Wrong Answers	Invalid Answers	37.4%	56.1%	6.5%	<p><b>National answers rate – Item D18, Grade 5 INVALSI test 2011</b></p> <table border="1"> <thead> <tr> <th>Correct Answers</th> <th>Wrong Answers</th> <th>Invalid Answers</th> </tr> </thead> <tbody> <tr> <td>47%</td> <td>51.9%</td> <td>1.2%</td> </tr> </tbody> </table>	Correct Answers	Wrong Answers	Invalid Answers	47%	51.9%	1.2%
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**Figure 1: INVALSI items and national rates ([www.gestinv.it](http://www.gestinv.it))**

As we can see in the Figure 1, in both tasks less than half of the students nationwide gave the correct answer. INVALSI provides percentages for each option chosen by the students and these data are available in the GESTINV database. These tasks were identified by teachers through the GESTINV database, using the "Guided Search" function by searching for tasks from the grade 5 INVALSI tests concerning rational numbers and with correct response rates below 50%. Among the items returned by the database, the items D25 and D18 were selected because they clearly show some possible errors related to the representations of rational numbers. By means of the qualitative analysis of the tasks and the quantitative results, researchers/TE and pre-service teachers can share ideas and reflections. Starting from the analysis of these data researchers/teacher educators and pre-service teachers discuss about students' difficulties in conceptualizing and making sense of decimal and fractional notations (some of the main findings of research in the field were deepened, e.g. Iuculano & Butterworth, 2011, Ni & Zhou, 2005). As a matter of fact, they notice that quantitative results show that many Italian students gave the wrong answers, in particular in D18 we can notice that 33.5% of the students chose option C. The teacher educators/researchers address the mathematical content selected for the task from an epistemological and educational point of view. Furthermore, they discuss with pre-service teachers the main research findings of the research that can help frame the macro-phenomena that emerged (Ball, 1993; Empson & Levi, 2011). As highlighted in literature, often, many difficulties are related to the management of the different representations of rational numbers, both in terms of the comparison and ordering of fractions and their comparison with decimals. They agreed that, in order to answer both tasks correctly, it is necessary to properly perform treatments between different semiotic representations (Duval, 2006).

### **Pre-service teachers' reflections**

In this paragraph we show some excerpts from pre-service teachers' answers to the questionnaire administered at the end of the teacher educational program. Pre-service teachers state that the analysis of the INVALSI tasks was useful in strengthening their knowledge about the process of teaching and learning rational numbers. They stress the importance of discussions about students' learning processes, possible students' difficulties and effective/ineffective mathematics teaching practices. By using the interpretative lenses given by MTSK model, they declare an increase in their Mathematics Teacher Specialized Knowledge, in particular in their Pedagogical Content Knowledge (PCK) declined into three subdomains: Knowledge of Mathematics Teaching (KMT), Knowledge of Features of Learning Mathematics (KFLM) and Knowledge of Mathematics Learning Standards (KMLS).

For reasons of brevity, we will only provide a few examples of statements for each PCK sub-domain.

As far as KMT is concerned, some pre-service teachers declared:

- PT\_17: As far as fractions are concerned, [...] I have become aware that they need to be taught from the beginning to represent the same concept in different semiotic representations.
- PT\_18: Knowing the different representations of a mathematical concept means recognizing its connections and underlying structure. In this case, we can see how each number is represented differently by a percentage, a fraction or a decimal number. Through the analysis of this item we can see the strong connection between percentages, fractions and decimal numbers, which every teacher needs to know very well in order to foster the most effective teaching-learning process possible from a mathematical point of view.
- PT\_21: I have therefore understood that the teacher's task will be to offer children different representations of the same mathematical object and will have to make them aware of this knowledge. [...]

Below we provide excerpts referring to KFLM:

- PT\_17: Thanks to this activity, I became aware that for students one representation is not the same as another.
- PT\_39: I have seen that students have difficulties in both processing and converting, leading to misconceptions about fractions.

Finally, here are some pre-service excerpts referring to their KMLS:

- PT\_21: As stated in the National Guidelines, children at the end of primary school will need to be able to handle all the different representations and so it is up to the teacher to structure pathways and activities to help students achieve this understanding.
- PT\_23: With this topic, I got to know the standards of knowledge and skills possessed by the pupils, as well as what learning objectives are required at the end of primary and secondary school. This activity has also underlined that it is necessary to respect the design of the objectives as much as possible, defined in such a way that does to create inconsistencies in later grades.

As we can see in these excerpts, pre-service teachers reflect on their PCK, in terms of KMT, KFLM and KMLS, and recognize aspects of their knowledge that have emerged and developed.

## **Conclusions and further directions**

In this paper we presented an example of a teachers' professional development program for primary pre-service teachers in which meta-didactic praxeologies, linked to the analysis of selected items from the national standardized assessment INVALSI test, were shared. We have shown how the use of GESTINV database, as a boundary object in professional development paths, allows an improvement of pre-service mathematics teacher specialized knowledge. As a concluding activity of the program

the pre-service teachers were asked, using the MTSK model, to identify what specialized knowledge they think/believe they had developed during the professional development program. The MTSK model was used by pre-service teachers to reflect on teaching and learning processes. MTSK model become part of logos component of meta-didactical praxeology and it could also be considered a theoretical boundary object between the two communities of researchers/educators and pre-service teachers. This aspect as well as the analysis of the pre-service teachers' declarations concerning the MK domain will be objects of our next studies. As literature highlights (i.e. Doig, 2006), standardised assessments, designed with the aim of assessing mathematical learning at the system level, are increasingly having implications from an educational, didactic, cultural-historical and political perspective at both global and local levels. Using data from standardized assessments in pre-service teachers program fits into this line of research and it increases the potential of the educational implications of linking standardized assessments to mathematics education.

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