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The interplay between theory and practice in the development of a model for inclusive mathematics education

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We present the first results of the OPEN-MATH project that aims at the accomplishment of inclusive mathematics learning environments via the construction of the Open Activity Theory Lesson Plan (OATLP). We built a theoretical framework for inclusion that stems from the networking of differentiation for all, featured as open learning and the theory of objectification. We describe the interplay between theory and practice. Its outcome is a conceptual framework for inclusive mathematics and the development of the OATLP model across 3 implementations involving an Italian middle school class.

Keywords: Inclusion, Differentiation, Open learning, Theory of objectification.

Introduction

The debate about inclusion in mathematics education has broadened in recent years, polarizing on two complementary aspects: those related to the questioning of the term itself, and its meaning in relation to mathematics education, and those related to classroom practices (Roos, 2019). In the project OPEN-MATH, funded by the Free University of Bozen, we tried to build a link between these two polarizations, defining inclusion from the perspective of mathematics education and trying to arrive at a set of design principles that can enhance the participation and learning of each one in the classroom. For that purpose, we decided to combine two approaches, that of the theory of objectification (Radford, 2021), which serves as background theory to define learning in mathematics, and the didactical differentiation (Tomlinson, 2014), which instead provides the inclusive perspective regarding classroom management. We need to recall that in Italy, students with and without SEN attend the same classrooms since 1977. Therefore, the perspective taken in the project starts from the assumption that inclusion does not only mean that all students can attend the same class, but that the educational environment must allow meaningful learning and participation for each student, according to her own characteristics (Aiscow, 2016). In the next pages we present the theoretical background of the project and the developing of the Open Activity Theory Lesson Plan (OATLP) that allowed us to define the first set of design principles for inclusive mathematics education in the classroom combining the theory of objectification and didactical differentiation. What we want to show, is how these two theoretical approaches can shape classroom activity to foster the inclusion of each student according to his or her characteristics.

Theoretical Framework

Inclusive Education

Inclusive education has been conceptualized in several ways. In literature, we find a certain consensus on a general distinction between narrow and broad definitions (Nilholm & Göransson, 2017). Narrow definitions focus on students with disabilities, their presence in mainstream schools and classes and the needed support. Broad definitions are about school systems and school communities and their commitment and capacity of welcoming all students with all their individual differences, granting participation and effective learning processes. Tomlinson (2014) encourages a view that assumes difference as the norm in learning and that places differentiation in the normality of instructional design for all. A broad idea of inclusion poses a great challenge to the way learning processes can be supported in schools both considering all students' differences and granting their participation to a common learning project. Differentiation has been discussed by several authors as a tool that can contribute to tackle the challenge. Within the broad understanding of inclusion that we advocate in our work, we will follow Tomlinson's approach to differentiation. Open education (Demo, 2016) is a learning strategy that accomplishes differentiation for all by promoting students' opportunity to organize the learning process for their own, working on different tasks at the same time in the same space: this is consistent with the broad idea of inclusion presented in Tomlinson's Differentiation framework. Students are expected to be active in their learning processes, aware about the way they learn and to take decisions according to the activities they are exposed to. We can describe three possible areas of decision-making for pupils in terms of learning: 1. organization (spaces, times, learning partners), 2. methodology (how to solve a task), 3. aims and objectives (content and goals). In the project, we used open learning in relation to organization and methodology using learning stations.

The Theory of Objectification

According to the Theory of Objectification, thinking is a *praxis cogitans* (Radford, 2021). Conceptual objects, thinking, learning, and meaning in mathematics are intertwined in reflexive mediated activity that unfolds as joint labour. Learning is a specific *praxis cogitans* termed *process of objectification* that allows the student to notice, find and encounter the cultural object. The artifacts that mediate reflexive activity and accomplish the objectification processes are called *semiotic means of objectification*: objects, tools, linguistic devices, and signs that individuals intentionally use to carry out their actions and attain the goal of their activities. Radford, resorting to a dialectic materialistic stance, conceives embodiment as a sensuous cognition, that is, a multimodal sentient form (perceptual, sensible, and imaginative) of responding to the world sprouting from cultural and historical activity. In this respect, objectification unfolds as a materialization of mathematical knowledge in the student's sensuous cognition and can happen in various ways, according to the individual characteristics of the student. The dialectic interplay between a cultural-historical environment, the individual and reflexive activity gives rise to a double-sided construct: objectification-subjectification. Subjectification, the counterpart of objectification, is related to the production of subjectivities as they engage in the reflexive mediated activity. The Theory of Objectification outlines a dialectical co-production between individuals and their cultural and historical reality. The individual, according to Radford, is continuously inscribing herself in the social world, producing her subjectivity according to the possibilities given by her environment.

A conceptual framework for inclusive mathematics learning

According to the combining-coordinating strategy (Bikner-Ahsbahs & Prediger, 2014), we networked the Theory of Objectification and Differentiation, featured as open learning, to draw a conceptual framework for inclusive learning in mathematics with the following elements (Demo et al., 2021):

Definition of inclusion. Inclusion is conceived as the dialectical and critical positioning (subjectification) of all students in the cultural-historical practice of mathematics, who act, feel, and think according to their individual distinctive traits to pursue their project of life. The process of subjectification described is equated with meaningful participation and learning, which can be defined only with respect to a cultural practice.

Mathematical activity. Mathematical reflexive mediated activity, in its multimodal acceptation, is the meeting point of the social and individual dimension of mathematical learning. The notion of sensuous cognition allows us to keep together social interaction and individual self-determination. Semiotics means of objectification allow multimodal activity both as open learning, making available to different students different learning path, and joint labor with respect to common learning goals.

Teaching-learning model. Starting from the Theory of Objectification we have developed an inclusive lesson plan that intertwines social interaction and individual self-determination. We have added to the original Activity Theory design (Radford, 2021), which alternates phases of small group work and of whole classroom discussion, elements of individual differentiated work, stations work, made according to Open Learning. Multimodality characterizes every aspect of the cycle, giving the possibilities to different students to experience different ways to reach the same learning goal. The outcome is what we have called Open Activity Theory Lesson Plan (OATLP) and it is presented in Figure 1.

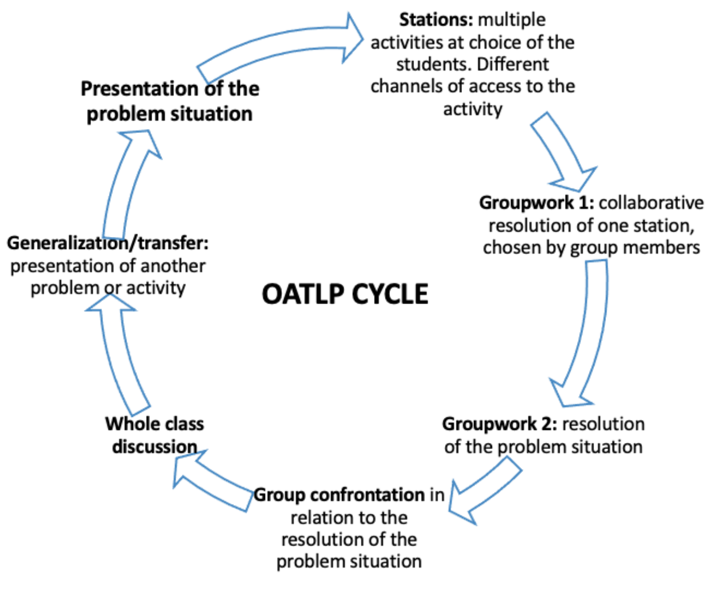

Figure 1: The OATLP Cycle and its different phases.

Stations (Demo, 2016; Tomlinson, 2014) are one of the possible strategies related to the implementation of open education and represent a way to put differentiation into practice by developing classroom environments in which learning processes are multimodal, decentralized and plural. Different learning activities related to a main didactical objective are structured in different stations. Students can move from station to station and choose which ones to complete and with whom. Decision making is enhanced with respect to organization of times (1) and to methodology (2): in fact, students can decide how long to work on each station, and to avoid one or more activities. The activities are connected to the same learning goal but exploit different means to reach them. A "passport" is used for the student to take note of the stations completed, the difficulties encountered, and what enabled them to learn in the most effective or enjoyable way, but it also allows the teacher to keep track of and understand individual differences in learning mathematics.

The evolution of the OATLP model during the research project.

In this section we show how the structure of OATLP cycle has gone through subsequent changes when applied to different topics, with the same classroom during the school year 2020/2021. The changes have been made accordingly to the analysis of the processes of objectification-subjectification, of the levels of participations and of the level of self-determination experienced by the students. The process of reworking and reflection on the OATLP cycle was carried out in accordance with the principles of Educational Design Research (McKenney & Reeves, 2019) in order to understand and develop how the educational intervention could be adapted to a real classroom context maintaining a connection with the theoretical principles defined above. The model has been implemented in a lower secondary school class of 17 students, among which 4 students have special educational needs, during the mathematics lessons. Throughout the experimentation, the OATLP model has been modified both in its structure and in its time scheduling. The set of every cycle's analysed data consists in videorecording of groupwork, collected students' materials and interviews with six chosen student and with the teacher.

In this contribution we analyse the main transformations of the OATLP model across 3 of the 5 cycles we implemented in school, each cycle is related to a different topic, and we addressed each topic only once. The addressed topics are, from cycle number 1 to cycle number 5: Ratios and Proportions, Circle and Circumference, Pythagorean theorems, Area estimation, Quadrilaterals. The first three cycles introduce a new topic, the 4th and the 5th work more on problem solving in relation to already faced topic.

Circle and circumference

Design of the activities

<u>Objectives in the national indications for mathematics</u>: The student is required to know definitions and properties of the circle and to calculate the area of the circle and the length of the circumference, knowing the radius.

<u>Stations</u> (120 min): Focus on the definition of circle and circumference and the elements that characterise them. The activities are designed differentiating according to different approaches to knowledge (Sousa & Tomlinson, 2011). E.g., in station 1 students have to draw a circle using a rope and a pen, in station 4 they make a drawing representing circle and circumference and in station 5 they are asked to define the figures verbally.

Groupwork (80 min): the students are exposed to a problem that contains 3 questions. The aim is to distinguish the definitions of circle and circumference and to calculate the area of the circle. In the last question they are asked to calculate an area corresponding to ¾ of the area of the circle.

Changes made compared to the previous intervention

Four different roles are assigned to students for the groupwork (mediator, designer, verbalist and controller); a procedure is established to handle the request for help during station activity (three coloured cards representing independent work, need for a classmate, need for the teacher); Stations are made shorter, and their objective becomes more precise and defined. Introduction of a framework defining different individual attitudes to learning that have informed the design of the stations.

Retrospective analysis

Strengths: stations are more effective and differentiated; the cards used to ask for help are appreciated both by the teacher and the students. In group work, according to the teacher, students begin to self-organize and divide tasks among them independently.

Critical issues: tight timeframes scheduled for activities hinder the completion of the OATLP cycle and do not acknowledge the students' need to express their different attitudes to learning; the connection between stations and group work needs to be refined; in groupwork, more attention needs to be given on division of roles and collaboration aimed at learning and not just at completing the task.

Pythagorean Theorem

Design of the activities

Objectives in the national indications for mathematics: The student is required to know the Pythagorean Theorem and its applications in mathematics and concrete situations.

Stations (150 min): Focus on the statement of the Pythagorean theorem, from a 'geometric' point of view (equivalence between areas). Some examples are given in Figure 2

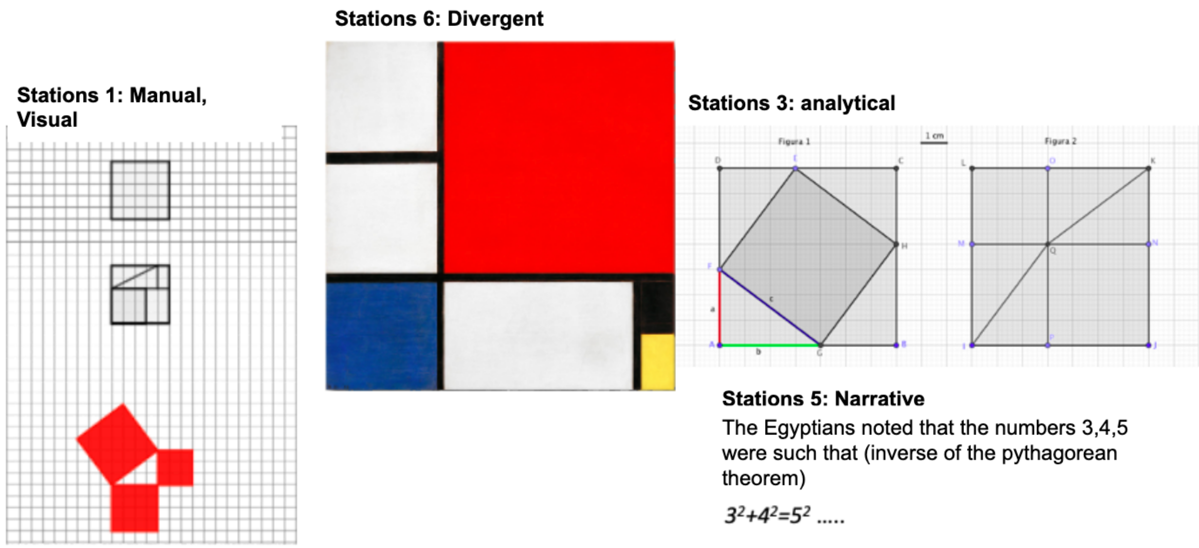

Figure 2: Parts of the stations proposed in relation to the Pythagorean Theorem.

Group (220 min): A double task is proposed: the application of the theorem to a given problem and an activity of problem posing related to the theorem. The groups then exchange the invented problems, solve them, and give feedback to the group that invented it. The feedback must be about the mathematical correctness, clarity, beauty, and interest of the problem. Invention of another problem that can be solved with the Pythagorean theorem is requested at the end of the confrontation.

Changes made compared to the previous intervention

We expanded the timeframe of the OATLP cycle, 2 weeks instead of one; greater attention is given to aspects related to the confrontation among students and a moment of confrontation between groups is introduced; the task for groupwork presents open questions that can adapt to different solving strategies and cognitive approaches; introduction of problem posing activities in the task for the groupwork.

Retrospective analysis

Strengths: Stations are appreciated by students for the variety of attitudes they encompass, and both students and the teacher notice the potentialities of problem posing in relation to the understanding of the Pythagorean Theorem.

Critical issues: insufficient connection between stations and group work, in this OATLP cycle the geometric aspects (stations) and the algebraic aspects (groupwork) of the Pythagorean theorem that needs to be made explicit.

Problems solving on area estimation

Design of the activities

Objectives in the national indications for mathematics: The student is required to determine the area of simple figures by breaking them down into elementary figures, e.g., triangles, or by using the most common formulas and to estimate the area of a figure, including curved lines, by default and by excess.

Stations (150 min): The stations try to emphasize different moments of problem solving: reading and understanding the text, choosing a strategy, implementing it, checking the results, etc. For instance, one station requests to invent a problem starting from the image of a polygon, another to rewrite the text of a given problem before solving it, to make a drawing of the situation, or simply trying to solve a problem and justifying the chosen procedure.

Group (90 min): A problem on the estimation of the area is proposed. Students have to find two different methods, justifying their choices and to explain differences between methods (Figure 3).

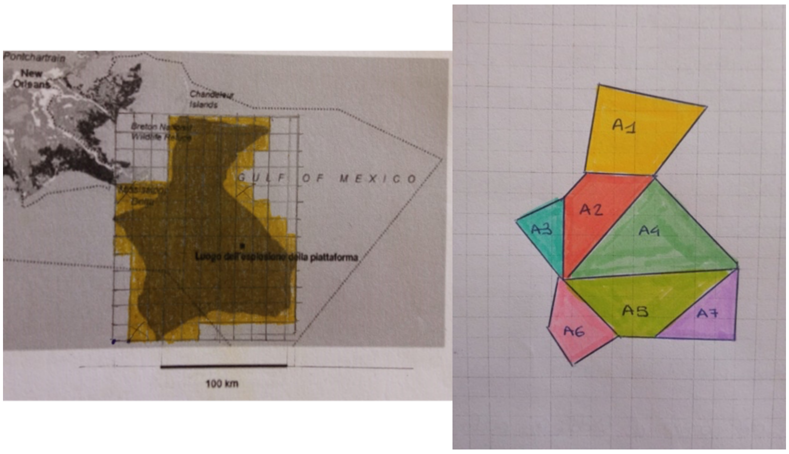

Figure 3: Two different methods implemented by students to calculate the area of the figure.

Changes made compared to the previous intervention

OATLP is designed specifically to work on the ability of problem solving and not in relation to a specific mathematical object like the circle or the Pythagorean Theorem. In this respect, stations and groupwork have a stronger relationship, which can be found in the metacognitive reflection on the different strategies of problem solving.

Retrospective analysis

<u>Strengths</u>: During groupwork students recall the activities done during the stations and build on them to face the new task.

<u>Critical issues</u>: we decide to introduce in the next OATLP cycle a moment of collective resolution (in group) of one of the stations, chosen by the members of the group. This allows students to compare different solving methods and overcome difficulties encountered. Furthermore, we acknowledge the need for a stronger connection between the stations and the groupwork.

Discussion and conclusions

The OATLP cycle is the result of its subsequent implementations with the lower secondary school class involved in the OPEN-MATH project. It is intended as a model that promotes inclusive mathematics education in school. The research project developed in a constant dialogue between the theoretical design principles and classroom activities in a real learning context. In their confrontation, both the theoretical principles and the structure of the model modified: the Theory of Objectification allowed us to focus on multimodality of mathematics education and the relation between subject and mathematical culture. From the theory of objectification come the focus on problem solving and the design of moment of interaction where students are asked to justify their choices and strategies, but also the attention in the choice and role of artifacts in station and groupwork. Didactical differentiation, through Open learning methodology, allowed us to focus on the individual in the process of encountering mathematical culture. In particular, Open Learning allows us to consider the specific learning approaches and bring to the fore strategies to manage the differentiated classroom, for instance through stations, the use of the passport, or the introduction of specific roles during groupwork. These aspects go beyond the specificity of OATLP model and allow to define a set of general design principles that will be a reference for further research on the efficacy of OATLP:

- Alternate modes of work (group, individual, whole class).
- Provide a plurality of semiotic means of objectification and of activities starting from the needs of students in the classroom.
- Work on explication and justification of solving processes and argumentative practices.
- Explain and discuss with students how to help and how to request for help during an activity.
- Encourage effective ways of cooperation and collaboration, for which explicit work with the class is necessary.

The design principles listed and the OATLP cycle are the result of the first phase of the project OPEN-MATH and are the starting point of an interdisciplinary dialogue among the two involved theories: the reciprocal relation of the design principles and the link between each one of them with inclusion in mathematics, and with the specific constructs defining it operationally, will be object of further studies.

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