Giornale Italiano di Educazione alla Salute, Sport e Didattica Inclusiva / Italian Journal of Health Education, Sports and Inclusive Didactics - Anno 6 n. 1 - ISSN 2532-3296 ISBN 978-88-6022-436-1 - gennaio - marzo 2022 - CC BY-NC-ND 3.0 IT-
https://doi.org/10.32043/gsd.v6i1.604

GETTING A GRIP ON GEOMETRY: DEVELOPING A TANGIBLE MANIPULATIVE FOR INCLUSIVE QUADRILATERAL LEARNING

AVERE UNA PRESA SULLA GEOMETRIA: SVILUPPO DI UN MANIPOLATORE TATTILE PER L'APPRENDIMENTO INCLUSIVO DEI QUADRILATERI

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Abstract:

A joint project of design-based educational researchers, mechatronic engineers, and digital accessibility experts has created a new genre of pedagogical technologies — hybrid material–digital multimodal artifacts for collaborative learning of sensorily diverse students in inclusive classrooms. Here we present the Quad, a manipulable quadrilateral hand-held object that is linked in real time to its digital screen-based simulation, whose own transformation, in turn, activates content-oriented voice description and output sonification. Pilot studies with blind and visually-impaired student-participants suggest the Quad's potential in grounding geometric reasoning, insight, and generalization in exploratory haptic–proprioceptive investigation. In its conception and development, the Quad exemplifies the ethical, philosophical, and theoretical perspectives of its collaborating designers respecting all children's universal right to access and participate in cultural practices, including techno–scientific activities. As researchers, we harness technological innovations to realize moral obligations and, through that, to promote the study of human perception, action, and cognition.

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We look forward to mutual growth of our research program along with parallel efforts advancing an Italian project to promote teachers' pre-service training and professional development surrounding implications of the embodiment turn in the cognitive sciences for school-based instructional practice.

Il presente lavoro descrive un progetto congiunto di ricercatori educativi basati sul design, ingegneri meccatronici ed esperti di accessibilità digitale che ha creato un nuovo genere di tecnologie pedagogiche - artefatti multimodali ibridi materiali-digitali per l'apprendimento collaborativo ed inclusivo di studenti con disabilità sensoriali. Viene presentato il *Quad*, un oggetto quadrilatero manipolabile che è collegato in tempo reale alla sua simulazione digitale basata sullo schermo, la cui trasformazione, a sua volta, attiva la descrizione vocale orientata al contenuto e la sonificazione dell'output. Studi pilota con studenti partecipanti non vedenti e ipovedenti suggeriscono il potenziale del Quad nel fondare il ragionamento geometrico, la comprensione e la generalizzazione nell'indagine esplorativa aptico-propriocettiva. Nella sua concezione e sviluppo il Quad esemplifica le prospettive etiche, filosofiche e teoriche dei suoi progettisti, rispettando il diritto universale di tutti i bambini ad accedere e partecipare alle pratiche culturali, comprese le attività tecno-scientifiche. Come ricercatori, si cerca di sfruttare le innovazioni tecnologiche per realizzare obblighi morali e, attraverso ciò, per promuovere lo studio della percezione, dell'azione e della cognizione umana. Ci si augura una crescita del nostro programma di ricerca insieme agli sforzi paralleli promossi da quella italiana che promuove la formazione pre-servizio degli insegnanti e lo sviluppo professionale basato sulle teorie embodied nelle scienze cognitive per la pratica educativa basata sulla scuola.

1. Introduction

The design and evaluation of educational products for students with diverse abilities requires interdisciplinary collaborations that draw on both engineering expertise in human–computer interaction and scholarship in cognitive–developmental psychology, each with its unique foci on atypical populations. This set of knowledge and skills is rarely found in a single institution. The authors of this paper are researchers from three universities distributed across the United States of America as well as an author based in Italy. We depend on each other to develop quality products for multiple stakeholders of educational enterprises, including foremost students with special needs but also their teachers, caretakers, parents/guardians, and siblings. This paper reports on the USA team's first joint design-based research effort, a project in which we are creating multisensory interactive resources particularly for visually impaired and blind students studying geometry in inclusive classrooms.

The collaboration builds on three independent longstanding research programs that arrived at a confluence around this particular project. One of these research programs, *embodied design*, developed at the Embodied Design Research Laboratory (EDRL, https://edrl.berkeley.edu), University of California Berkeley, has looked to create interactive media that offer students opportunities to learn mathematical concepts by drawing on their natural sensorimotor capacities in negotiation with disciplinary models (Abrahamson, 2009, 2012, 2014; Abrahamson, Nathan, et al., 2021, Abrahamson, Tancredi et al., under review). When embodied-design efforts began attending to the special needs of atypical students with diverse abilities, *special-education embodied design* (*SpEED*) was formulated as a framework informing the creation of interactive educational resources bearing unique interaction solutions (Tancredi et al., 2021, 2022).

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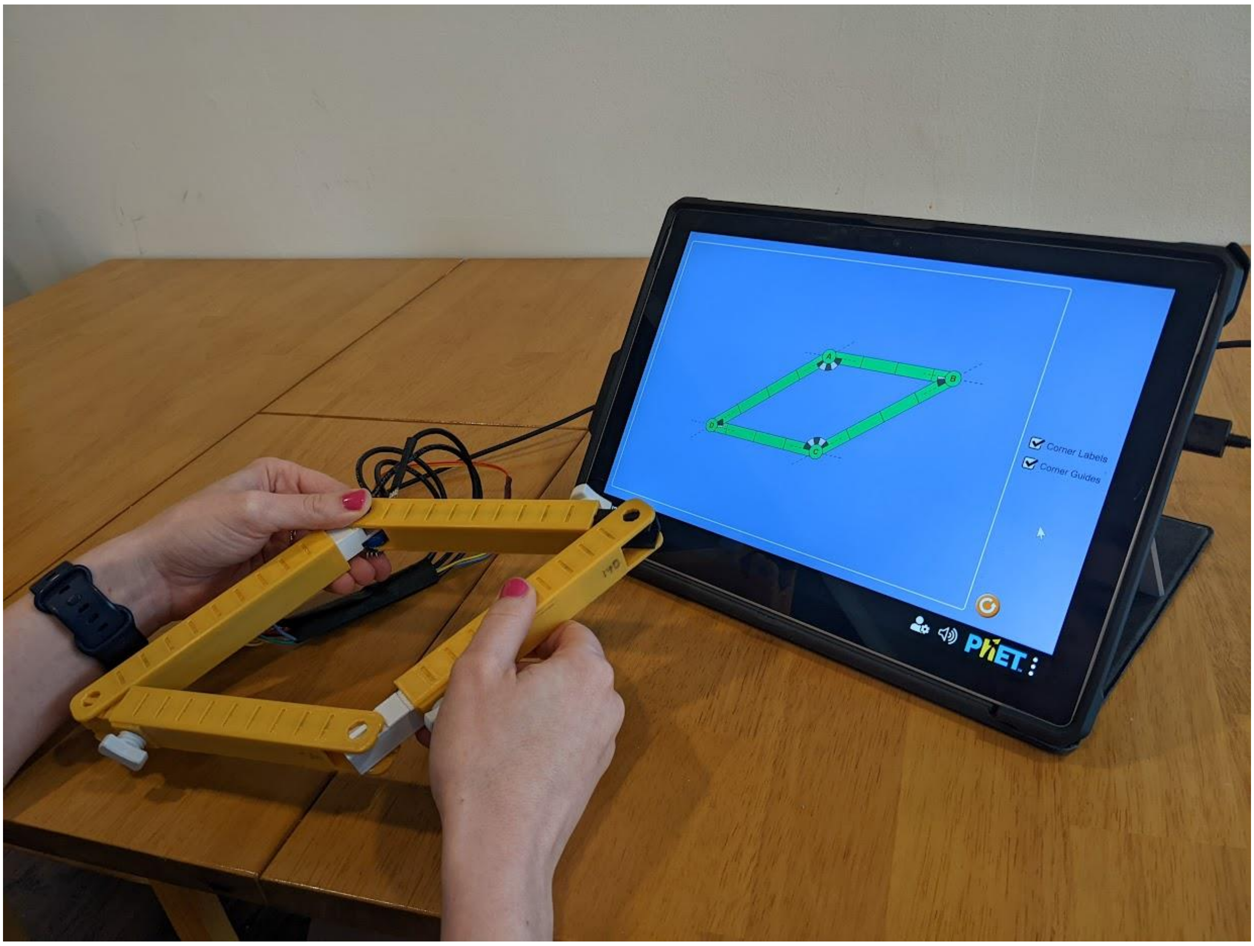

Figure 1 The Quad: an inter-linked cross-modal technological system for exploring geometric shapes, created through collaboration between three research groups. The basic design idea for a haptic transformable quadrilateral prototype was created by EDRL. On the left, the tangible manipulable quadrilateral (TMQ) created by the CHROME lab. On the right, the TMQ's real-time "digital twin" linked into a tablet environment, created by PhET simulations.

SPEED perspectives resonated well with the tenets and objectives of the PhET Interactive Simulations, a major campaign at the University of Colorado, Boulder (https://phet.colorado.edu/), leading to a collaborative design between EDRL and PhET to build a Mathematics Imagery Trainer for Proportion (Howison et al., 2011), a digital interface that is accessible to students with visual impairments via fingers-to-screen interaction and auditory feedback (PhET Interactive Simulations, 2021). PhET predates SpEED as a world-renown provider of interactive digital content for mathematics and science studies, and its accessibility engineering framework stems from a coherent credo respecting universal access to equitable pedagogical resources. The third party involved in the current project, in addition to EDRL and PhET, is CHROME—Collaborative Haptics, Robotics, and Mechatronics—a laboratory based at Saint Louis University in Missouri (https://sites.google.com/slu.edu/gorlewicz-lab/). CHROME Lab specializes in developing haptic interfaces for interacting in digital environments, which could expand the sensory offerings of PhET-based SpEED solutions into touch-based physical exploration with auditory feedback. Here we report on the EDRL/PhET/CHROME development of the tangible components of the *Quad* (from "quadrilateral"; see Figure 1), which, in its completion, will be a trans-media system of "smart" interactive objects for inclusive geometry learning across sensory modalities and digital media. As we will explain, the Quad links a sensor-infused manipulable material device with its real-time screen-based transformable simulation, enabling simultaneous multi-sensory output, including auditory information in the form of voice description and sonification feedback.

Following further elaboration on the SpEED framework (Section 2), we will describe the Quad engineering work and empirical studies to date (Section 3), and then step back to situate the project within PhET's broader considerations for accessible educational solutions (Section 4). We will end with views from the Italian perspective (Section 5) and then offer some closing words (Section 6)

2. SpEED: A View from EDRL

2.1 What is SpEED?

Special Education Embodied Design (SpEED) (Tancredi et al., 2021, 2022) is a design-based research framework that takes up embodied cognition theory to imagine more accessible educational experiences for marginalized learners (Abrahamson et al., 2019). Embodied cognition theorizes cognition as not merely the machinations of a computer-like brain, but rather as including and emerging from the body and bodily activity (Newen et al., 2018). SpEED brings this perspective together with disability studies' commitments to celebrate human variation and challenge ideas of what is normal (Ferguson & Nusbaum, 2012) to analyze and reimagine the kinds of learning opportunities we offer students with sensory, motor, and cognitive differences. Design-based research (Cobb et al., 2003) generates both contexts to empirically evaluate embodied cognition theory in the context of diverse populations and useful educational tools and practices (Odom et al., 2005). Popular educational accessibility frameworks like Universal Design for Learning call for giving students multiple ways to engage, act, express themselves, and represent ideas (CAST, 2018). SpEED can serve this vision by expanding the tools and perspectives on offer to celebrate and leverage learners' diverse ways of being in the world.

2.2 SpEED Research to Date

Embodied design creates the conditions for the development of new perceptuomotor schemes, drawing these into relation with disciplinary ways of knowing (Abrahamson, 2014) and has been fruitfully applied to multiple domains of mathematical teaching and learning from ratio and proportion to parabolas to trigonometry (Alberto et al., 2021). SpEED emerged through dialogue among embodied design projects focusing on designing learning opportunities informed by differences in learners' embodied experiences:

- Balance Board Math, which uses the modality of balance to support sensory seeking learners who crave lots of movement in learning about functions and graphing (Tancredi et al., in press);
- SignEd|Math, which integrates the semiotic mode of sign language into the design of tablet activities for learning about ratio and proportion for Deaf signers (Krause & Abrahamson, 2020);
- The Magical Musical Mat, which enables non-speaking children on the autism spectrum and their verbal peers to have a symmetrical interaction where they use touch to improvise musical sounds together (Chen et al., 2020);
- The audio-haptic Mathematics Imagery Trainer for Proportion, which takes up shared modalities of engagement and communicational resources for blind and sighted students to learn together about ratio and proportion (Abrahamson et al., 2019; Tancredi et al., 2022).

Recently, SpEED has been applied with an eye towards ever-more populations, including students with motor disorders, blind and visually impaired students, students with intellectual disability, and students with multiple disabilities, and to a broader set of problems in education, from expressive language to data visualization to arithmetic (Krause et al., in progress). The Quad, focusing on the content domain of geometry learning, represents an expansion of the technological and ideological scope of SpEED, working towards new levels of reach and applicability.

2.3 SpEED Parameters

SpEED distinguishes 3 key design parameters that shape learners' bodily interactions with instructional designs: media, modality, and semiotic mode (Tancredi et al., 2022). *Media* (e.g., Kress, 2001) specify the materials being used, such as pen and paper. *Modality* (e.g., Edwards & Robutti, 2014) refers to the specific sensory and motor systems used to engage with these media, such as the visual sensory modality, or the manual motor modality. *Semiotic mode* (e.g., Kress, 2001) denotes the meaning system(s) at play, be they language, symbol, or forms of movement.

To examine the relations among these parameters, let us consider the example of learning to assemble an *IKEA*™ table. The default media available for such a task typically consists of a printed instructional booklet, as well as some household tools such as a screwdriver. The booklet medium calls upon specific modalities, such as the visual sensory modality. The booklet uses several semiotic modes accessible through the visual modality: diagrams, symbols, and icons, as well as some written language. If we were to change the media used for this task, we may very well find ourselves dealing with a different set of modalities and semiotic modes. For example, perhaps instead of the instruction manual, one might instead call a friend who has built such a table before. The new medium at hand would be the body of your friend. This might elicit additional modalities such as the auditory modality, listening to the friend's speech, or the tactile/haptic modality if the friend were to guide one's hand. This alternate configuration of media and modalities transforms the semiotic modes at play, now spanning spoken language, gesture, and bodily actions. These different configurations offer different kinds of learning experiences; for example, in the former, consulting the manual and completing steps is somewhat constrained to happen sequentially, whereas in the latter, it is quite possible for one's friend to give instructions and feedback mid-action.

The Quad is a new form of instructional media, designed with attention to offering dynamic interactions through modalities and semiotic modes available to blind and visually impaired learners in inclusion contexts (Table 1). The details of this new media will be elaborated in Section 3.

Media	**Modalities**	**Semiotic modes**
Linked tangible and digital quadrilaterals	Tactile, haptic, kinesthetic, visual, auditory, manual-motor	Gesture; spatial manipulations; dynamic visuals; geometry terms such as "quadrilateral" and "parallel"

Table 1 SpEED Parameters of the Quad

2.4 SpEED Principles

The Quad applies the basic principles of SpEED (Tancredi et al., 2022) as follows:

*Principle 1: Learning happens through the body's sensorimotor engagement with the world***.** The quad invites users to manipulate mathematical objects as physical objects. Rather than a 2D picture on the page of a textbook, quadrilaterals become something with which one can interact. Shapes are not merely seen or differentiated; they become transformable one into the other through active manipulation, revealing the relations among them.

Principle 2: Learning begins from learners' existing embodied resources: their prior sensorimotor experiences, frequent practices, and abilities. The Quad builds from blind and visually impaired learners' extensive experiences with touch- and auditory-based ways of exploring and knowing, including experiences with tactile graphics, braille, and audio description. Beyond popular learning tools for these populations, it also brings in these learners' capacity to physically manipulate objects and differentiate sounds to expand the multimodal layers of information available to them.

Principle 3: Instruction must flexibly adapt to learners' sensorimotor diversities. A goal of the Quad is to generate a flexible tool that is ready to be used in inclusive education settings with learners of a maximal range of profiles together. Thus, building from the embodied resources of blind and visually impaired learners, the Quad offers a broad range of ways to interact, be it through physically manipulating the tangible, moving points or sides on a touchscreen, or using a cursor, as well as a range of feedback forms, including feeling the physical tool, seeing the shape, color, and sound feedback associated with different shape properties, and descriptions that can be turned on and off.

3. Developing the Quad: A View from CHROME

3.1 Who is CHROME?

The Collaborative Haptics Robotics and Mechatronics (CHROME) Lab employs engineering theory and praxis to increase accessibility across multiple foci. CHROME Lab is particularly interested in haptics — the science of touch — and in designing, building, and evaluating haptic interfaces to support embodied learning experiences. With the rapid shift of educational content to the digital space, embodied learning experiences are decreasing, being narrowed to visual-based interactions displayed on screen. CHROME Lab has developed various technologies and techniques to support multimodal, and touch-based access to educational content — particularly visual-based content such as graphics (Gorlewicz et al., 2019), with a specific focus on bringing touch back via vibrations (Gorlewicz et al., 2020; Tennison et al., 2016, 2019, 2020, 2021), wearable interfaces (MacGavin et al., 2021), and tangible manipulatives (Lambert et al., 2022). The Quad was the first tangible manipulative prototype developed by CHROME Lab that was specifically rooted in the three basic principles of SpEED (Section 2).

Several other research initiatives have investigated meaningful ways to reintroduce touch into education (Ding & Gallacher, 2018; Martinez et al., 2016). Briefly, devices such as the Haptic Paddle and Hapkit demonstrated the efficacy of 3D printed devices to elucidate complex STEM topics that include dynamics (Martinez et al., 2019). Other systems have highlighted the importance of haptics in kinesthetic learning systems across STEM disciplines (Buzzi et al., 2015; Grow et al., 2007). Sallnäs et al. (2007) showed the effectiveness of adding haptics via commercially available haptic devices (e.g., Phantom Omni) to interactive mathematics simulations. Similarly, a paired learning tool called

'Clicks' was developed as a modular geometry device that connected with a tablet application to demonstrate various shapes in two and three dimensions (Adusei & Lee, 2017).

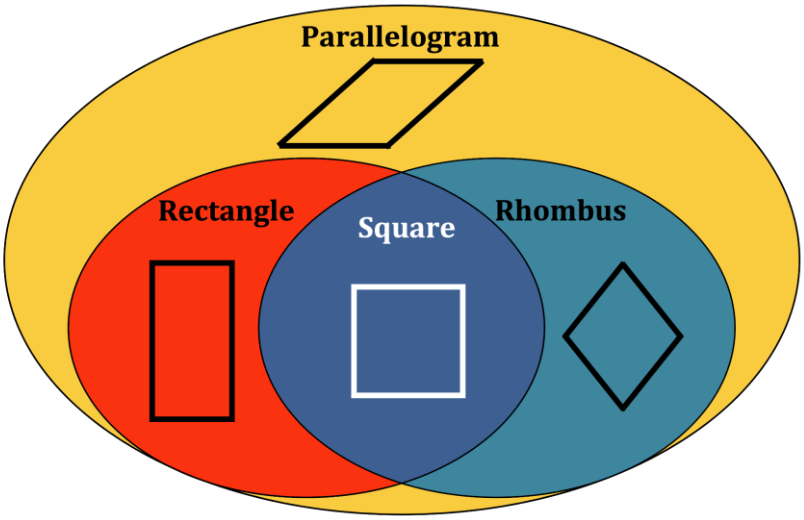

Figure 2 A four-sided shape can take many forms; the Quad design highlights these configurations and relationships

The Quad expands upon these prior research initiatives, with a focus on the development of an inclusive learning tool for geometry that can flexibly adapt for use in different learning contexts spanning physical, tangible, digital, and virtual environments (see Section 4) to support learners with a wide range of needs. A quadrilateral geometry (see Figure 2) was chosen as the initial design, because it serves as a simple basis for complex and foundational geometric properties (Birgin & Ozkan, 2022): 1. 90° (right) angles; 2. angle congruence across diagonals; 3. parallel lines and parallelism through transformation; and 4. explicit shape definitions (such as why all squares are rectangles but all rectangles are not necessarily squares).

3.2 Quad Design Principles

The design rationale of The Quad was rooted in the SpEED framework detailed in Section 2.4, with a specific focus on supporting embodied interactions in learning the foundational geometric principles stated above. CHROME Lab takes an iterative, human-centered approach to design with the goal of rapid generation of concepts targeted at determining preferred design parameters. The research team looks forward to leveraging the design acumen developed through this project toward taking on more advanced concepts in geometry curriculum. In particular, characterizing students' emergent multimodal sense-making strategies with two-dimensional shapes will inform how the team expands its technological offerings toward three-dimensional geometries of solids, which draws on perceptuomotor cognitive foundations in two-dimensional content.

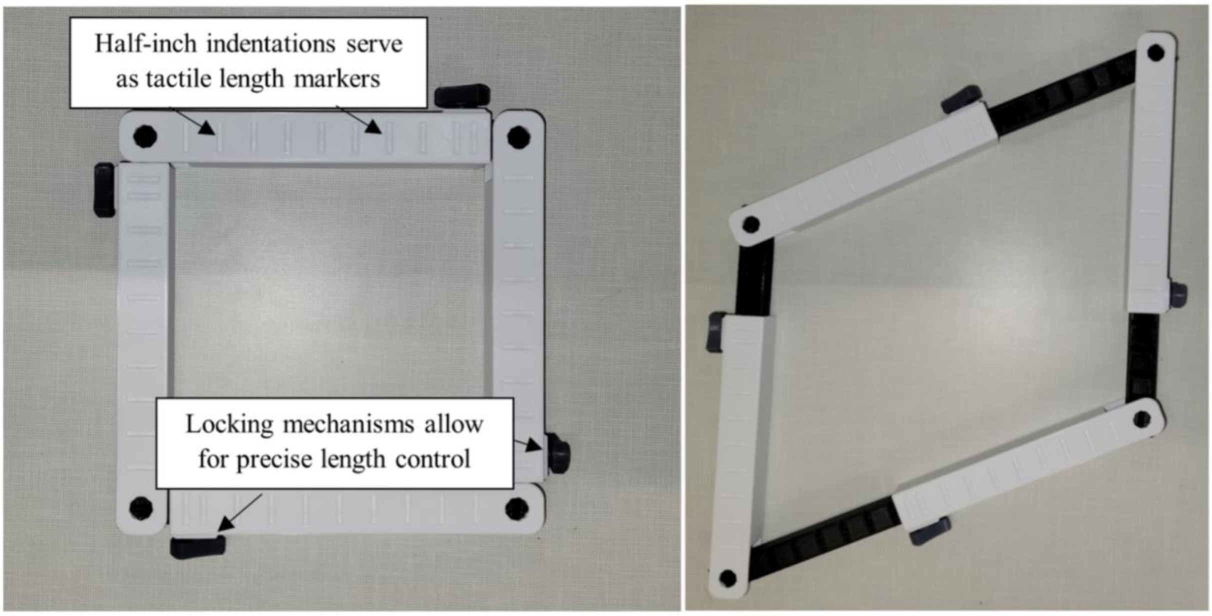

Figure 3 Features of the tangible manipulative quadrilateral

Here, we present the tangible manipulative component of the multimodal Quad system, which we will refer to as TMQ (tangible manipulative quadrilateral). The TMQ (see Figure 3) is a 3D-printed reconfigurable device that allows a learner to create and explore four-sided shapes. The TMQ emphasizes the connections between different quadrilaterals (as highlighted in Figure 2) through movement of its extensible sides and flexible vertices. For example, a learner could create a parallelogram by tilting the side lengths of the TMQ, or they could transform to a trapezoid simply by extending the bottom length. The current model of the TMQ (see Figure 3) is 15cm square at its base configuration with flat lengths. The TMQ expands to a 23cm square, and each vertex angle measure can be adjusted from 35° to 135°, allowing a learner to quickly create and transform between numerous quadrilaterals. From testing, we received and implemented further design ideas, such as locking mechanisms along the lengths to give a learner more precision in exploration, and tactile indentations along the lengths to serve as tangible measurements.

The TMQ is low in complexity (just three unique parts) and rich in usability and functionality for learners. The TMQ can be used in two formats: as a stand-alone manipulative or as a smart tangible (see Section 3.4) paired with an interactive PhET simulation on quadrilaterals (see Section 4). The former supports easy, quick, tangible explorations of quadrilaterals, while the latter supports a unique bi-directional exploration between a simulation and a tangible instance of the quadrilateral, thus creating a conceptually coherent system of interactive resources with visual, auditory, and haptic features.

The ideation process from concept to tangible was rapid, and was co-designed with students with blindness or visual impairments (BVI) to ensure an inclusive design from the onset. By 3D-printing the TMQ, we are able to iterate quickly on design feedback, as well as more easily share the device with collaborators.

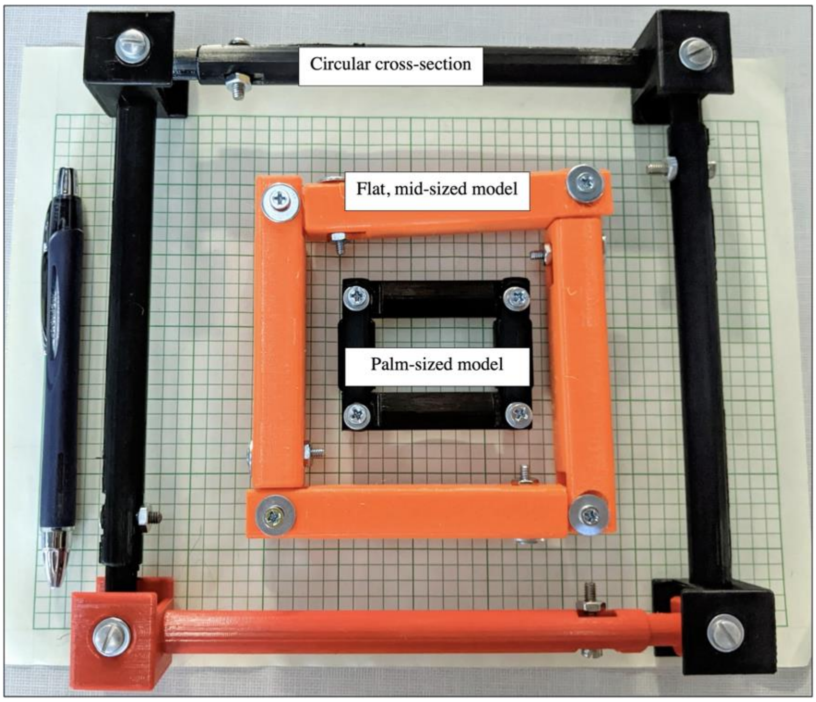

Figure 4 Early models of the Quad varied in size and cross-sectional shape

Initial prototypes (see Figure 4) focused on testing features and preferences for students. At first, we created a palm-sized model, about 5cm square. This TMQ enabled students to explore the concept of a reconfigurable learning tool in a small form factor. The small TMQ, had round lengths and was compared against a larger, 12cm square TMQ with flat lengths. The larger size and flat lengths enabled learners to get both hands on the device and set it flat on a tabletop. Finally, we tested those with a larger still, 30cm TMQ, again with round lengths so that a user could grip their fingers around the device. Feedback showed that the flat length, middle sized model was preferred: usage was two-handed, and students would place both hands flat on the device to understand the configuration. To aid in exploration and creation of exact shapes, we added a locking mechanism along the lengths as well as indentations to serve as tactile length markers. The current model was shown, earlier, in Figure 2.

3.3 User Testing: Exploration Methods

Having established a consistent TMQ design, we then investigated the usage of the device: how would a student use the TMQ? We tested the base TMQ seeking to understand the exploration strategies a learner with BVI might employ when using the TMQ.

In our first study, Exploration Methods, we had 7 participants with a range of vision loss from low vision to total blindness. These students were 18–22 years old and had completed a high school geometry course. The guiding purpose of this study was to understand usage of the TMQ: participants were given the TMQ and encouraged to explore its functionality and create shapes.

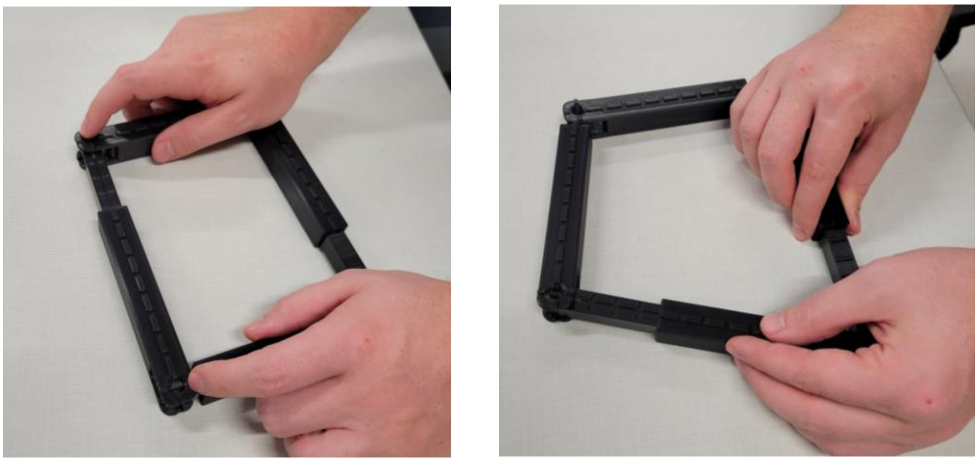

Figure 5 Learners dominantly used both hands in exploration to quickly garner the configuration of the TMQ, both through mirrored-hand action (left) and single-hand stabilization (right)

We observed four main characteristics of exploration: 1. one- versus two-handed exploration; 2. usage of the TMQ on or off the tabletop; 3. rotation of the device; and 4. tilting strategies for angle modification — single-hand stabilization or mirrored-hand action (Figure 5). All participants were able to manipulate the TMQ into three or more distinct shapes, easily creating parallelograms and right-angle quadrilaterals yet struggling with trapezoids. Overall, most students mirrored their hand motions while using both hands in exploration. Students primarily kept the device flat on the table to aid in maintaining a global perception of its configuration.

In the last part of the study, we asked participants to transform the TMQ between shapes in three different scenarios, with an emphasis on using smooth motion when possible: (1) transforming from any rectangle to any parallelogram; (2) mirroring that parallelogram while maintaining side lengths; and (3) transforming from the smallest possible square to the largest possible. The first task was completed smoothly by all learners. Some required clarification on how to achieve a mirrored shape (e.g. “reverse the shape, make it point the other way”), and most completed the mirroring in multiple separate steps rather than smooth motions. Similarly, most participants completed the expansion in multiple steps, expanding the left and right sides together, then the top and bottom.

3.4 Smart Quad Design and Testing

In order to connect to multiple modalities, a “smart” version of the TMQ (Smart TMQ) was created to enable future communications with software applications on a touchscreen tablet. This allows a learner to quickly transition between shapes as 2D renderings displayed on the touchscreen and 3D representations from the Smart TMQ.

Giornale Italiano di Educazione alla Salute, Sport e Didattica Inclusiva / Italian Journal of Health Education, Sports and Inclusive Didactics - Anno 6 n. 1 - ISSN 2532-3296 ISBN 978-88-6022-436-1 - gennaio - marzo 2022 - CC BY-NC-ND 3.0 IT-
https://doi.org/10.32043/gsd.v6i1.604

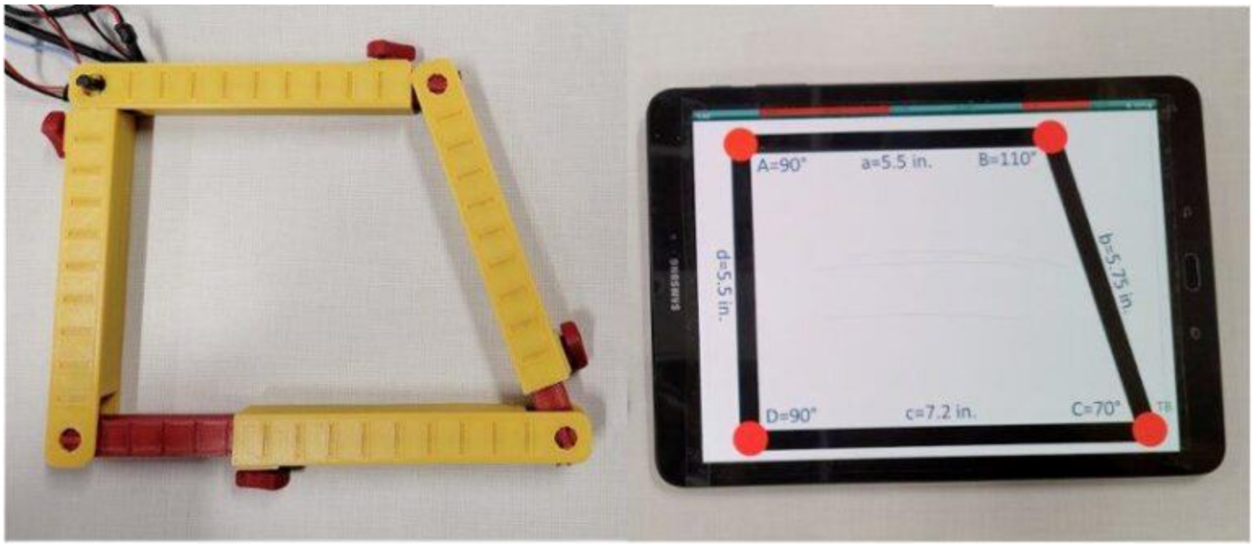

Figure 6 . The Smart TMQ (on the left) "talking to" an Android tablet. This design phase bridged over from the basic TMQ toward a digitally enhanced version that could communicate with the PhET simulation environment

The Smart TMQ is identical in design as the base Quad, with added functionality (see Figure 6). The Smart TMQ is embedded with length and angle sensors: four Force-Sensitive Linear Potentiometers (2730, Pololu Robotics and Electronics) and one rotary potentiometer (EVU-F2AF30B14, Sparkfun), giving a user access to exact length and angle information. An Arduino Uno microcontroller was used for acquisition and calibration of sensors and connection to digital environments. Validation of the accuracy of the sensors was assessed by comparing the physical value of length and angle to the sensor-acquired value. Each length was expanded and contracted 10 times and measured every 1cm. For the angle sensors, the same process was repeated at 45° increments, and the sensors demonstrated consistent accuracy within 2% of the physical values.

With the smart TMQ, we then investigated what affordances our device might offer over or in conjunction with multimodal, touchscreen-based renderings of quadrilaterals. In our second study, Comparison of Mediums, 18 participants (16 sighted–hearing, blindfolded, and 2 individuals who are BVI) completed two tasks using the Smart TMQ and a touchscreen tablet and additionally, embossed graphics for individuals who are BVI.

The first task was shape identification: on each medium, how accurately could participants identify randomized quadrilaterals: various trapezoids, right-angle quadrilaterals, and parallelograms. Participants were given either the Smart TMQ preset into a specific shape configuration, the touchscreen tablet displaying a shape (as shown in Figure 2, right), or embossed printouts (only participants who are BVI) of a shape and asked to identify the shape. The second task was shape recreation: participants were presented with a shape on the touchscreen (or an embossed graphic for participants who are BVI) and asked to replicate the geometry on the Smart TMQ as accurately as possible. Length and angle values were available as audio readouts on the touchscreen and recorded on the Smart TMQ via the Arduino Uno upon completion.

In the identification task, learners were about 1.4 times more successful in identifying the correct shape when using the Smart TMQ as opposed to the touchscreen or the embossed graphic. Participants here dominantly used both hands to quickly garner a global understanding of the TMQ shape, and mainly used one finger on the touchscreen or embossed graphic to either trace the shape or find length and angle information.

Shape (total observations)	Overall	Angles	Side Lengths
All Shapes (54)	94%	94%	93%
Parallelogram (54)	90%	89%	91%
Trapezoid (54)	94%	93%	94%
Right-Angle Quadrilateral (54)	96%	97%	96%

table 2 Recreation success rates using the Smart TMQ.

In the shape-recreation task, all participants were able to manipulate the Smart TMQ into a desired shape with an overall angle and length accuracy rate of 94%. Error rates by shape are further broken down in Table 2. Here, participants also began to lean into the tangible device more fully, using the space created between the adjustable lengths to determine how much they had adjusted the device. With a focus on fine-tuning individual angles and sides, we observed participants moving back and forth frequently and quickly between the touchscreen and the Smart TMQ to gather the correct measurements.

These studies supported our design approach: users were more successful in correctly identifying quadrilaterals on the Smart TMQ than on either of the two-dimensional mediums (touchscreen and embossed graphics). We observe several affordances the TMQ offers: First, the extendable side lengths were critical for participants to transform between shapes and investigate scaling. Second, two-handed exploration and manipulation enables users to establish 1) relationships between changes in angles and changes in lengths, 2) the ability to provide kinesthetic references (e.g., usage of fingers for taking measurements), and 3) the ability to quickly create and transform between shapes which highlights their geometric relationships.

When prompted for thoughts on the device and its potential uses, many recounted their geometry learning experiences, stating that such a device "would have been really helpful in class" and "better than shapes on a page." Overall, users enjoyed exploring shapes using the TMQ and its ability to transform between shapes rapidly.

3.5 Discussion & Ongoing Research

With these findings in mind, we are working on several advancements to the Quad system. First, we can enhance the TMQ itself by leveraging other dynamic non-visual feedback (e.g. vibrotactile feedback when a right angle is achieved or audio callouts of length / angle values when desired) to assist in shape creation accuracy. Second, we are crafting a peer-to-peer experience by creating an actuated Quad such that the configurations that one student creates on a Smart TMQ are replicated and explorable on another device, the Actuated TMQ. Next, we are expanding on the idea of a math learning tool: creating a modular "Quad" that is no longer constrained to 4 sides, allowing a learner to explore more numerous and complex shapes (line segments, triangles, convex / concave shapes). Finally, we are continually making advancements to the Quad–PhET Simulation pairing (Section 4), to create a robust 2D–3D learning environment and plan to test these with younger students.

4. Designing for Inclusion: A View from PhET

PhET Interactive Simulations is a research and development group in the Department of Physics at the University of Colorado Boulder (PhET Interactive Simulations, 2022). The PhET group creates and conducts research on the design, development, and use of interactive science and mathematics simulations, published as Open Educational Resources (OER) (Perkins, 2020). Started in 2002, the PhET project has published more than 150 interactive simulations translated into 93 languages, and collectively run over 1 billion times by students, teachers, and parents around the world.

The PhET team has developed a consistent pedagogical design approach for creating these simulations (e.g., Moore et al. 2014): each simulation is highly interactive and provides real-time feedback, allows actions that are difficult or impossible in real life, are intuitive to use, and are designed to implicitly scaffold learners into cycles of productive exploration and discovery (Podolefsky et al., 2013). In the process of exploring the simulation, learners engage in sensemaking (Odden & Russ, 2018), often through mechanistic reasoning (Russ, 2006) — identifying features, actions, relationships between features, and articulating causal relationships between the representations in the simulation.

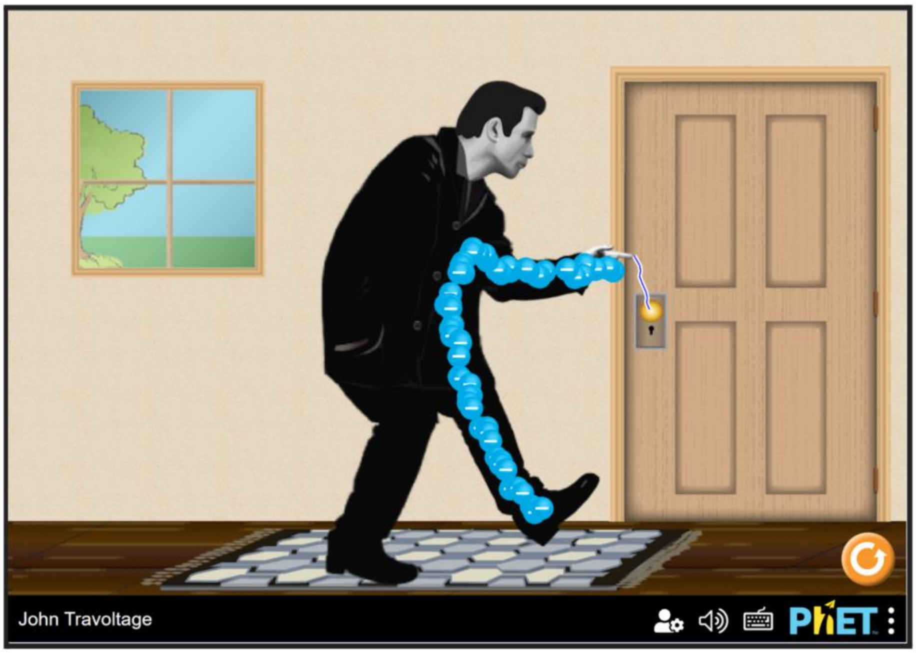

Figure 7 A PhET simulation "John Travoltage": A grayscale character is positioned pointing toward a door while standing on a carpet. A cursor is poised to drag his left foot forward and backward while large blue electrons flow from his foot to his finger, discharging into the doorknob.

As an example of a PhET simulation, we introduce John Travoltage (PhET Interactive Simulations, 2021; Tomlinson, et al. 2019). In this simulation (see Figure 7), a character, John, stands on a rug by a door. Rubbing his foot results in negative charges, represented as small blue circles, transferring onto his body; more foot rubbing results in more charges transferring onto John's body. Moving John's arm towards the doorknob results in a shock. The farther John's hand is away from the doorknob, the more charge must be accumulated on his body before a discharge occurs. With the simulation, learners can be introduced to transfer-of-charge concepts and explore the relationship between the amount of charge on John's body and the distance between his hand and the doorknob that results in a shock.

4.1 Inclusive Design at PhET

In 2014, PhET launched the Inclusive Design Initiative. With the goal of creating the most enjoyable inclusive learning experiences at scale, we began investigating and implementing new design and development strategies to increase access to simulations for people across the full span of sensory, mobility, and cognitive needs. Our approach to creating inclusive simulations centers on inclusive design, which is a disposition (Treviranus, 2018a), approach (Treviranus & Roberts, 2006), and a collection of strategies (IDRC, n.d.) that seeks to create flexible technologies capable of simply adapting or being simply adapted to meet the full spectrum of human needs in interaction and communication. Inclusive design emphasizes the diversity and uniqueness of each individual through inclusive technologies created through inclusive processes, and in so doing, continuously striving for the broadest possible human benefits. Inclusive design rejects notions of an "average" or "typical" user and does not accept designs as adequate without addressing the needs of those "at the margins" (Clarkson et al., 2013; Treviranus, 2018a). Instead, inclusive designers partner with those with a range of needs in co-design, inspiring evolutions of designs that increase the flexibility and usefulness of the resulting technology for everyone (Treviranus, 2018b). Our work draws from the frameworks and strategies of inclusive design, structured as STEM-centered technology co-design, with inclusive learning experiences and tools as the outcome.

To enable PhET simulations to meet the needs of learners with diverse needs, PhET needed to inclusively design features that increase the ways information was conveyed by the simulations (i.e., increase display features) and increase the ways learners could interact with simulations (i.e., increase input features). To do this, PhET has developed open-source technical infrastructure and design frameworks and strategies for a broad range of new display and input features, including:

*Visual Display Features***.** Pan & Zoom allows users to zoom-in to view the visual display up close, and to pan around the screen while zoomed in. This supports learners with visual impairments, and those on smaller devices. Interactive Highlights are bold pink borders around graphical objects that have input focus during navigation with alternative input (e.g., keyboard navigation), enabling the user to know where they are in the navigation order. This feature is useful for anyone using alternative input methods.

Auditory Display Features. Sound & Sonification uses non-speech sound to convey information (Tomlinson et al., 2021). Interactive Description is verbal description built into the simulation delivered by screen reader software, often used by people with significant visual impairments (Smith & Moore, 2020). Voicing is verbal description built into the simulation delivered directly from the web browser without the need for additional assistive technologies (Fiedler, et. al. 2022). Interactive Description and Voicing provide up-to-date descriptions of the current state of the simulation during interaction, and allow access to a brief summary of the simulation, additional details, and exploration and navigation hints. Sound & Sonification provides a more immersive experience useful for any learner. Interactive Description supports non-visual access to the simulations for people with visual impairments, and Voicing supports pre-literate learners and those with print disabilities who would like to hear object names and on-screen text read aloud, and anyone who would like to hear these or additional descriptive information read aloud during simulation use.

Haptic Display Features. Vibratory Haptics utilizes an actuator built into commercial mobile devices to create a vibration that can be used to support exploration of a simulation's static graphics on touchscreen devices for people with visual impairments, and indicators of important state changes during simulation use for all learners. Our work in haptic display (Tennison et al. 2021) has been in collaboration with the CHROME lab, and is available in a prototype iOS app.

Traditional and Alternative Input Types. Traditional input types include use of a mouse cursor and touch for touchpads and touchscreens. Alternative input types include the use of keyboards as well as switch devices and others. Alternative input types enable users of various assistive technologies to navigate and interact with the simulations.

With these features implemented into our example simulation, John Travoltage, learners can explore the simulation visually or non-visually relying on Interactive Descriptions rather than the visual display. Learners can interact using their mouse, touching a touch screen, or with their keyboard, and be supported in their exploration through sound, description, or both. The outcome is an inclusively designed learning tool — flexible, customizable, created using modern web technologies, and published as an openly licensed free resource for learners worldwide. The full list of PhET simulations with inclusive features can be found on the PhET website.

4.2 Quadrilateral: A New Kind of Learning Tool

Our prior work with inclusive simulations centered on enabling a multiplicity of ways to experience and interact with what is displayed visually, inherently privileging the visual modality over other modalities such as auditory or tactile modalities. While there are many benefits to the resulting inclusive simulations, we also wanted to explore more egalitarian approaches to inclusive learning. Specifically, where the different modalities are considered equally, and without one modality taking priority over the others. We think that simulations centered on naturalistic movement or embodiment can potentially be designed as just such an egalitarian inclusive learning tool. Our efforts in the creation of Quadrilateral provide an opportunity to investigate a new kind of inclusive learning tool, centered on pedagogically relevant and naturalistic human movement (Abrahamson et al., 2019; Victor, 2014).

To do this, we capitalize on emerging web technologies enabling new frontiers for blending physical and virtual worlds. With wireless connectivity (e.g., Bluetooth; Mozilla, 2022) or open-source computer vision (OpenCV, 2022) using webcams ubiquitous in commercial devices, we can now connect essentially any object to a simulation. With physical / virtual coupling capabilities, we can create and explore uniquely inclusive learning tools, such as the Quad system.

The Quad transmodal system consists of two components (see Figure 1): a web-based virtual interactive component and a tangible component. The virtual interactive component displays an interactive quadrilateral (closed, four-sided) object. The tangible component can be the TMQ (see Section 3), a physical quadrilateral object with extendable sides. On a tablet device with the virtual quadrilateral, learners can use their fingers to grab and drag the sides and vertices of the object. When doing so, the visual display (e.g., color), haptic vibrations, and Sound & Sonification provide feedback about the state (geometry) of the quadrilateral object, while Voicing describes the current geometry as well as important state changes.

When using the TMQ, learners can use their hands to explore the object’s shape at any time and grab the sides and vertices to expand and contract the side lengths and angles of the object. Sound & Sonification and Voicing, delivered from a nearby device (e.g., mobile phone or laptop) provide information about the current geometry and important state changes during interaction. If the tangible quadrilateral is equipped with an actuator, it can provide haptic vibrations to indicate changes of state as well.

Importantly, the virtual and physical components are coupled such that interaction with the tangible object results in corresponding changes to the virtual object, and can also be experienced independently, where interaction with the virtual object can occur through touch-based, cursor-based (e.g., mouse, trackpad, etc.), and focus-based (e.g., keyboard input, switch devices, etc.) input, and interaction with the tangible object can occur without reliance on the visual display, providing haptic and auditory feedback during interaction.

The resulting learning tool can be used by individual learners, or collaboratively by pairs or small groups of learners. Learners can include those with and without disabilities as well as mixed-ability scenarios, such as those found in inclusive classrooms. Learning activities can center on the virtual component, using the tangible component primarily as an input device, center on the tangible component, or utilize both the virtual and tangible components in different ways. Multiple kinds of tangible objects can be coupled with the virtual component. The TMQ has been explored as a 3D printed tangible that couples with the PhET *Quadrilateral* simulation (Lambert, et. al. 2022) but other tangibles are possible as well; we have explored other tangibles including a high-tech fully actuated quadrilateral object that can be controlled by interacting with the virtual component, and a low-tech set of four children's blocks with each block serving as a corner of a quadrilateral. Each choice of tangible presents its own set of tradeoffs in development time and resources, accessibility, and capabilities and limitations during use. Through the use of open source code, modern web technologies, common commercial devices, and simple household or low-cost craft/maker materials, Quad is an example of a new kind of inclusive learning tool that is ultimately an open physical and virtual ecosystem.

4.3 Future of Quadrilateral

Central to our continued efforts into the design, development, and investigation of Quad, we will conduct co-design activities with learners with diverse needs, seeking input on design features, previously unnoticed opportunities for learning across modalities, and further ways to engage learners with sensory, mobility, and cognitive disabilities in the design of innovative inclusive learning tools. The design, development, and investigations of a coupled virtual / physical multimodal embodied learning tool creates opportunities for identifying specific design features and design process needs — some features and needs will be unsurprising, others new, distinct, and emergent from the complexity of the learning tool and designing inclusively across a virtual / physical learning tool. From the identification of these needs, we can develop design frameworks to support creators of novel multimodal technologies and educational materials, expanding the potential for future widescale design, development, research, and dissemination of similarly inclusive and embodied learning experiences.

Through collaborations with the Embodied Design Research Laboratory and CHROME Lab, we aim to investigate together important questions in the area of how learners with sensory, mobility, and cognitive differences collaborate together across modalities, and how learning tools and associated activities can enable, support, and potentially enhance the experience.

5. Getting Teachers Up to SpEED: A View from Italy

While educational researchers have been developing pedagogical resources that draw on embodiment, educational practitioners are only beginning to consider implications of this paradigm shift for teaching and teacher training (Abrahamson et al., 2022; Gomez Paloma, Damiani, 2015). In Italy, the educational milieu has begun to tentatively explore boundaries, dialogues, and potential collaborations between neuroscience, pedagogy, and teaching. (Cuccaro et al., 2022; D'anna et. al., 2020; Gomez Paloma, 2017, 2020; Minghelli &, D'anna, 2021). These efforts have led to an emerging professional-development framework that looks to transform teachers' epistemological beliefs through supporting their reflective implementation of embodiment resources (Gomez Paloma & Damiani, 2021). Recent turns in national policy that emphasize

inclusive education and children's holistic physical-socio-emotional well-being could make for auspicious opportunities to introduce embodiment theory and practice as offering a coherent framework for implementing the policy.

Against this backdrop, Italian scholarship and practice are well poised to avail from ideas put forth by the US collaboration of EDRL, PhET, and CHROME with respect to educational design for students with diverse sensory and motor capacity. First, an intellectual alliance with international research teams could bolster a fledgling Italian drive to offer pragmatic alternatives to the local pedagogical regimen, which is tethered to conservative educational views. Second, by evidencing the positive impact of educational resources inspired by embodiment, Italian scholars stand to position embodiment as a viable theoretical alternative (Damiani et al., 2020). Third, an engaged conversation on embodiment and education may inform policy more broadly, such as regarding architectural design of educational spaces (Gomez Paloma et al., 2020). In all this, international collaborations could bear reciprocal benefits in tightening the field's understanding of how best to bring embodiment research to practice.

6. Closing Words

The Quad system began as a humble prototype cobbled together from four chopsticks and connected by four rubber bands into a manipulable square. It then became a 3D-printed object, and later received extensible sides, hence the tangible manipulable quadrilateral (TMQ). Next, the Quad went mechatronic through digital inserts that "spoke" from the TMQ to a tablet, displaying the TMQ's shifting structure in real time and then also offering audio read-outs depicting its state and transformation according to our pedagogical objectives. On the Quad horizon are an actuator-equipped TMQ, so that TMQs can remote-form each other and/or their linked digital twins, enabling BVI students to co-explore geometric shapes with each other and with sighted students.

What ethnomethods will collaborating BVIs, each manipulating their own Quad, sort out together to coordinate their mechatronically yoked motor actions? What embedded algorithms and protocols could support these coordinations? What might this all look like when we level up to voluminous geometrical objects, such as polyhedra? Future research will seek to answer these and many more emerging questions.

As the multi-team project collaborative considers next engineering moves, we are also increasingly cognizant that we each follow our own local codex for inclusive design, as the careful reader must have discerned in considering the respective sets of guiding principles discussed by SpEED (Section 2), PhET (Section 4), and CHROME (Section 3). Moving forward, we seek to negotiate, align, and consolidate these principles into a shared framework drawing from a coherent set of philosophical, theoretical, and empirical work.

We further recognize a vital need to disseminate, along with the artifacts and activities of our design, a training program that prepares current and next generations of teachers to implement these educational resources in their inclusive classrooms. We concur with our Italian colleagues that a holistic approach to teachers' pre-service training and in-service professional development best consists of an integrated introduction to embodiment theory, so that they consider the epistemological rationales motivating the pedagogical designs (Abrahamson et al., 2022).

There is much work ahead. The future of inclusive design is in our hands as much as in the hands of students and teachers.

Acknowledgements

This material is based on work supported by the National Science Foundation under DRL-1814220 (PhET) and DRL 1845490 (CHROME).

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