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ESSAYS ON THE ECONOMICS OF EDUCATION: FROM THEORY TO AGENT-BASED MODELLING

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Abstract

This thesis is a collection of three works on the economics of education. Starting from exploring the formalisation of the human capital theory, it proposes a new approach for studying the educational choices at the tertiary level, by means of an Agent-Based Model designed for the Italian context and rigorously calibrated by performing an experiment on its input parameters.

In detail, the first chapter provides a historical review on the role of education, from the point of view of some major economists, from Adam Smith to Amartya Sen, touching Mill, Marshall and the founding fathers of the economics of education: Schultz, Mincer and Becker. The aim is to highlight that a multidimensional approach to education was present even in the first contributions on the topic; yet, precious concepts on education have been left behind in the process of formalisation of a unitary theory such as the human capital theory and, as a consequence, they do not find a corresponding application in empirical research.

A new methodological approach in the field of education economics is provided in the second chapter. The work explores the determinants behind university enrolment decisions, by modelling the choice of young Italians to attend university or to drop out and enter the labour market, by making use of an Agent-Based Model (ABM). The model implies that the individual preference to enrol at university depends upon (i) economic motivations, represented by the expectations on future income, which are formed on the basis of information gathered through interaction within individuals' social network; (ii) influence from peers; (iii) perceived effort of obtaining a university degree. Simulated over 100 periods of time and 100 Monte Carlo experiments, the model is able to provide a realistic representation of the rate of enrolment to tertiary education in Italy.

Finally, in the third chapter, we carry out a calibration experiment on the ABM developed in the second chapter. In the benchmark model, we assume that individual income is distributed as a two-parameter lognormal distribution. In order to provide a more accurate representation of Italian personal income, using the same data from the Bank of Italy employed in chapter 2, we fit to the data three Beta-type distributional forms: the generalised beta distribution of the second kind (GB2), the Singh-Maddala distribution and the Dagum distribution. The GB2 distribution always outperforms the other models; therefore, we take the average of the GB2 parameters estimates across the years considered (2002-2014) and use it as input to re-simulate the model. Results are only slightly different than the reference model, confirming the decreasing trend characterising the rate of transfer from secondary to tertiary education in Italy.

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Contents

List of Figures

List of Tables

Chapter 1

A historical review on the role of education: from human capital to human capabilities

1.1 Introduction

When reading about human capital theory, literature is extremely abundant. It is widely recognised that the theory of human capital, in the strict sense, spread out around 1960, resulting in that branch of economics known as the economics of education. In the same way, it is undeniable among economists and historians of economic thought that the theory of human capital is deeply rooted in at least two centuries before, starting in 1776 with Adam Smith's Wealth of Nations (Smith, 1880). The debate on the concept of human capital covering the time span that from Smith reaches the well known Jacob Mincer's doctoral thesis (Mincer, 1958) has been particularly intense over the Sixties when indeed human capital became a concept integrated within the economic science. The outcome of this debate was an extremely prolific research activity characterised by different perspectives, sometimes even in conflict with each other. In this pluralism, some concepts evolved and matured, eventually defining the human capital theory as we know it today; some others have been left behind.

According to its classical formalisation, we can define human capital as a set of skills and characteristics that increase a worker's productivity and that have sources in e.g. innate ability, schooling and training. This definition only captures some of the concepts related to education and training, object of the historical debate; some other concepts, in particular, a multidimensional approach that we can find already in Smith's thought, have lost consideration and retrieved only with the formalisation of another theory: the human capability developed by Amartya Sen. Even though these aspects have been rediscovered, the empirical analyses populating research in the field of education still rely on the key concepts of the theory of human capital: education can be assimilated to an investment, whose rate of return can be compared with other types of investment. The rate of returns to education is usually computed on the basis of the returns estimated through the Mincerian wage equation, which is based on the assumption that schooling decisions will maximise the net present discounted value of the individual. A few examples of empirical analysis of this kind include Angrist and Keueger (1991) and Card and Krueger (1992), which study data related to the U.S., De la Croix and Vandenberghe (2004) who focus on Belgium, Harmon, Oosterbeek, and Walker (2000) studying the U.K., Cainarca and Sgobbi (2012) and Fiaschi and Gabbriellini (2013) focusing on Italy. Adopting different econometric techniques, these works have all foundation in the Mincerian model and thus in the human capital theory in the strict sense.

The aim of this work is to provide a review with a historical perspective that could highlight the presence of a multidimensional approach to the role of education in the thought of a few major economists, along the centuries; nevertheless, it has been almost forgotten and it was missing even in the contributions of those economists defined as the "founding fathers" of the human capital theory, upon which the empirical literature still relies on, with a void on methodologies that could possibly apply the broader concept of human capabilities.

This contribution is mainly inspired by the work of Spalletti (2009), whose book not only analyses the concept of education in the field of the history of economic thought, but also shows how the evolution of the notion of education is characterised by discontinuity and contradictions, all by keeping an objective point of view that allows for a critical consideration of the economic changes occurred in the past. Although the work of Spalletti (2009) is in Italian and, therefore, it naturally has a narrow audience and limited possibility of diffusion with respect to similar works, Spalletti (2009) is able to realise an extensive review and a complete examination of the topic.[1] In his book, Spalletti (2009) only sketches the contribution of Amartya Sen, without a deep analysis of his thought. Adding on that, the present work will enlarge and discuss the Sen's approach, showing how, when compared to Smith, it has much more than only a common ground with the Scottish author.

This work has roots also in the work of M. J. Bowman (1966) which has been able to highlight the evolution of the choice of investing in education from a classical and Marxian passive perspective, related to income distribution, to an active one, where finally education can be considered as a real investment. Indeed it is not a coincidence that M. J. Bowman (1966) writes about a "revolution" in the human investment.

By making a selection of the most interesting historical contributions related to education, the paper also completes and expands the research of R. S. Bowman (1990) by examining more recent contributions.

This work does not strive for being a complete and extended review of the history of economic thought on the role of education; it does not want to cover all existing contributions nor the whole possible time span. Instead, it intends to underline that precious concepts on education have been left behind in the process of formalisation of a unitary theory, and, as a consequence, they do not find a corresponding application in empirical research. Starting from Smith, we will find a close connection to

[1] See for example Blaug (1975) and Teixeira (2000). In some way, also these two articles are subjected to some limitations: the former to English classical political economy only and the latter only analyses the time span 1960-1997. Besides, even though Teixeira (2000) presents a full and elegant study on the emergence and consolidation of the economics of education, including the analysis of five major economics journals, his work keeps a general profile, lacking an evaluation of the single thought of the main economists.

his ideas in the work of Sen, where a multidimensional description of the concept of human capital is present again; all this by analysing the thought of some main author, whose ideas were determinant in the evolution of human capital theory, and by deeply investigating when and how it was possible to consider education in terms of human capital.

The paper is organised in six sections that chronologically analyse the selected authors, by reporting extracts from their work. Final remarks are included in Sections 1.7, which concludes and leaves an input for future research.

1.2 A "decent and orderly" population

We start our *excursus* by studying the thought of Adam Smith. In order to understand Smith's view on education, it is essential to analyse his thought on the division of labour and on the related concept of productive and unproductive labour.

> The greatest improvement in the productive powers of labour, and the greater part of the skill, dexterity, and judgement with which it is anywhere directed, or applied, seem to have been the effects of the division of labour. (Smith, 1880, p. 5)

This quotation represents the very start of Book 1 of *The Wealth of Nations* and in some way summarises Smith's description of the reality he observes in those years characterised by the Industrial Revolution and which is fully narrated in the following Chapters. Smith notices that there is an increase in skills and dexterity which is given by the reduction of tasks required by a worker's activity. He seems to imply that the division of labour has fostered productive qualities in the worker. A more clearly "productivistic" interpretation can be extrapolated from the tenth Chapter of the first book:

> When any expensive machine is erected, the extraordinary work to be performed by it before it is worn out, it must be expected, will replace the capital laid out upon it, with at least the ordinary profits. A man educated at the expense of much labour and time to any of those employments which require extraordinary dexterity and skill, may be compared to one of those expensive machines. The work which he learns to perform, it must be expected, over and above the usual wages of common labour, will replace to him the whole expense of his education, with at least the ordinary profits of an equally valuable capital. It must do this, too, in a reasonable time, regard being had to the very uncertain duration of human life, in the same manner as to the more certain duration of the machine. The difference between the wages of skilled labour and those of common labour is founded upon this principle. (Smith, 1880, p. 106)

This statement provides a sort of definition of human capital. Three main points can be underlined. First, Smith affirms that the dexterity and skills of an educated worker can be compared to a specialised machine, hence to fixed capital. Second, he states that the future earnings of an educated man are expected to at least compensate for the cost of education and training; therefore, he is suggesting that the cost of education constitutes an investment and gives the idea that one could choose to undertake education in view of a future return. Third, Smiths clearly makes a difference between skilled and unskilled labour and their related wages.

Ultimately, Smiths seems to affirm that human abilities are a sort of capital in which one can invest in through education. He is even more precise in the following passage:

> Fourthly, of the acquired and useful abilities of all the inhabitants or members of the society. The acquisition of such talents, by the maintenance of the acquirer during his education, study, or apprenticeship, always costs a real expense, which is a capital fixed and realised, as it were, in his person. Those talents, as they make a part of his fortune, so do they likewise of that of the society to which he belongs. The improved dexterity of a workman may be considered in the same light as a machine or instrument of trade which facilitates and abridges labour, and which, though it costs a certain expense, repays that expense with a profit. (Smith, 1880, p. 279)

Here, he leaves little doubt to the fact that fixed capital, which in his interpretation includes individuals' acquired abilities and talents, has as objective to increase labour productivity. In these passages, the interpretation of Smith's thought let transpire a view of education as an actual investment, through which one can accumulate a kind of capital, that Smith does not yet define as human capital, but that we can recognise as such.

Then, why does the concept of human capital date back to the second half of the twentieth century? Why is not Smith acknowledged to have built the foundation of human capital theory?[2] As a matter of fact, a common point of view is that neither Smith nor other classical authors went beyond a general and qualitative approach to the subject.[3] Smith did not attempt to empirically measure the returns to those

[2] Admittedly, a literature sustaining the presence of the concept of human capital in *The Wealth of Nations* actually exists. Among the most significant contributions we find Miller (1966),Kiker (1966), R. S. Bowman (1990) and Schultz (1992).

[3] This does not mean that they did not take into account at all the possibility to measure human capital. Malthus (1964) clearly states that the intangible aspect characterising human resources determines an issue in the measure of value.

> All the knowledge acquired by a superior education and superior talents, on account of a similar susceptibility, would have a greater claim to be included in the estimate [of value].
> In short, if we include under the denomination of wealth all the qualities of the mind and body which are susceptible to be hired, we shall find that by the restriction of the term wealth, to that which has exchangeable value, we have advanced but little towards removing the confusion and uncertainty attendant upon the former definition;
> The person who buys instruction, buys an amount of wealth, which it must be presumed is equal in value to what he has paid for it, while the self-taught person, who is in possession of much superior knowledge, has acquired no wealth. (Malthus, 1964, pp. 24-25)

skills and dexterity that he writes about.[4] According to R. S. Bowman (1990), eventually, Smith did not treat any aspect related to education as an investment[5] and, coherently with his distinction between productive and unproductive labour, that will be discussed later in the chapter, he only attributes a marginal role to human capital formation. Our work embraces the vision of Spalletti (2009), supported also in Schultz (1992) and based on the idea that the Enlightenment had an influence on Adam Smith. As Schultz (1992) affirms, Smith's confidence in human resources collides with a view of the human being seen as a (physical) capital. The reason why the Scottish author did not contribute to complete a precise theory of human capital can be ascribed to this idea. Together with this explanation, it is necessary to remind also the aforementioned distinction between productive and unproductive labour. In the third Chapter of Book Two, Smith clearly states:

> There is one sort of labour which adds to the value of the subject upon which it is bestowed: there is another which has no such effect. The former, as it produces a value, may be called productive; the latter, unproductive labour. Thus the labour of a manufacturer adds, generally, to the value of the materials which he works upon, that of his own maintenance, and of his master's profit. The labour of a menial servant, on the contrary, adds to the value of nothing. (Smith, 1880, p. 332)

If before there was no room for doubts on the productivity perspective upon which that sort of definition of human capital lies on, in the light of the latter quotation we must take a backward step in the interpretation of Smith's words and, certainly, we cannot make such a definite and strong statement as we previously did. Basically, Smith identifies all the jobs in the field of services as unproductive, although their importance, utility or prestige, in such a way that the investment motive implication is now lost.

What is then Smith's idea of education in relation to this distinction? Considering the manufacturing work, the one that Smith finds productive, Smith observes that child labour is quite common in England: parents find convenient to set their children to work very soon, even at a younger age than 6 or 7, thus not providing them with any form of education. Therefore, from the point of view of labour productivity, the Scottish economist explicitly asserts that in this case education is unnecessary:

Malthus acknowledges that "the qualities of the mind" should be included in the computation of value, yet this would not be suitable for the definition of value, meant as the exchange of goods and services.

[4]It must be acknowledged that most likely it was not possible to perform an empirical analysis as we intend it today and surely there was no wide availability of data and techniques, as proper data collection only started in the XX century.

[5]However, Smith analysed the substitution of unskilled for skilled labour and observed daily and weekly wages in Europe. He noticed that wages of specialised workers, such as mechanics and artisans, were only slightly larger than the wages of common workers, like for example those working in the clothing industry. In addition, the lifespan of a worker was not long and, therefore, that small difference in wages would not had been enough to compensate the worker for the cost of his education and training.

> Long apprenticeships are altogether unnecessary. The arts, which are much superior to common trades, such as those of making clocks and watches, contain no such mystery as to require a long course of instruction. The first invention of such beautiful machines, indeed, and even that of some of the instruments employed in making them, must, no doubt, have been the work of deep thought and long time, and may justly be considered as among the happiest efforts of human ingenuity. But when both have been fairly invented and are well understood, to explain to any young man, in the completest manner, how to apply the instruments and how to construct the machines, cannot well require more than the lessons of a few weeks: perhaps those of a few days might be sufficient. (Smith, 1880, p. 129)

Certainly, Smith's conception about apprenticeship must be contextualised in the historic era he lives in and definitely could not be truthful nowadays when instead software and technologies require high levels of knowledge and skills that can be acquired only through traineeships. In addition, still concerning productive labour, Smith once again observes the effects of the division of labour:

> In the progress of the division of labour, the employment of the far greater part of those who live by labour, that is, of the great body of the people, comes to be confined to a few very simple operations, frequently to one or two. (...) The man whose whole life is spent in performing a few simple operations, of which the effects are perhaps always the same, or very nearly the same, has no occasion to exert his understanding or to exercise his invention in finding out expedients for removing difficulties which never occur. He naturally loses, therefore, the habit of such exertion, and generally becomes as stupid and ignorant as it is possible for a human creature to become. The torpor of his mind renders him not only incapable of relishing or bearing a part in any rational conversation, but of conceiving any generous, noble, or tender sentiment, and consequently of forming any just judgement concerning many even of the ordinary duties of private life. (Smith, 1880, p. 365)

He identifies in the division of labour distortionary effects in the intellectual capabilities of workers, such that he justifies institutional intervention in the field of education.[6] In this case, education is essential to contrast the negative consequences of routine work and for purposes of social control:

> A man without the proper use of the intellectual faculties of a man, is, if possible, more contemptible than even a coward, and seems to be

[6] He takes as a reference the educational system in force in Scotland, where the "Scottish Educational Act" dating back to 1696 established a strong connection between education and parish, as each parish had to build a school. In this system, teacher's salary was only for a small fixed part a public cost, while the rest was made of a private contribution directly asked to pupils and their families, so to push teachers to a better performance and not become lazy on the job. Smith supported this school system against the English one, based on the *endowment approach*: schools were financed by rich owners so that a fund always guaranteed teachers' wage, without taking into account the quality of their job.

> mutilated and deformed in a still more essential part of the character of human nature. Though the state was to derive no advantage from the instruction of the inferior ranks of people, it would still deserve its attention that they should not be altogether uninstructed. The state, however, derives no inconsiderable advantage from their instruction. The more they are instructed the less liable they are to the delusions of enthusiasm and superstition, which, among ignorant nations, frequently occasion the most dreadful disorders. An instructed and intelligent people, besides, are always more decent and orderly than an ignorant and stupid one. They feel themselves, each individually, more respectable and more likely to obtain the respect of their lawful superiors, and they are therefore more disposed to respect those superiors. They are more disposed to examine, and more capable of seeing through, the interested complaints of faction and sedition, and they are, upon that account, less apt to be misled into any wanton or unnecessary opposition to the measures of government. (Smith, 1880, p. 371)

From this last passage from Book 5 of *The Wealth of Nations*, the suggestion that education can somehow affect labour productivity or growth is completely lost. It emerges instead a sort of “theory of social control”, which can be encountered also in other classical economists: educating the masses would determine a moderate behaviour resulting in a “decent” population, as the author writes. In some way, we find here an anticipation of the concept of (positive) externalities,[7] in terms of crime prevention and promotion of ethical behaviours.

Nevertheless, the interpretation cannot stop to such a utilitarian vision. In the last lines of the cited passage, individuals are also seen in an active perspective, as agents with the power of understanding and criticising, and education is the key to unblock those abilities that would allow people to express their personality and interests. What comes to light from that quotation is what Spalletti (2009) defines as *sympathy*, an unavoidable interest of the human being toward other individuals. The interpretation of Spalletti (2009) goes far beyond the restricting and utilitarian vision of R. S. Bowman (1990) and highlights a moral structure in which education is a fundamental mean through which individuals develop their own characteristics as part of the society. Therefore, not only education contributes to contrast “stupidity” originated from the mechanical repetition of work tasks, but it is also an instrument for realising a positive order which is essential for exercising individual rights. Smith also believed that failures of politics could be ascribed to politicians’ low cultural level and inability to shape awareness in the whole society. The working class, lacking any form of economic and political education was unable to influence the political conduct to contrast the growth of power of the few capitalists, which were well aware of economic mechanisms.

The type of education Smith refers to is the so-called “literary education”, made of three essential skills: reading, writing and accounting. As mentioned above, he does not place much importance in apprenticeship, and this type of education has

[7]The theory of economic externalities was formalised for the first time by the British economist Pigou (1932) even though Sidgwick (1885) first articulated the idea of spillover costs and benefits.

therefore only a social purpose. Since, according to Smith, a few days of practice are enough to learn a mechanic job, the labour productivity standpoint seems now missing. If we decide to agree with R. S. Bowman (1990), education would be unnecessary in order to perform a job and certainly not connected at all with the possibility to gain future returns. However, once again this work supports the idea of Spalletti (2009), according to whom the attention of Smith toward the employment aspect is still visible and affects his judgement on education as well as his construction of an ideal society.[8]

1.3 The role of education in the classical economics

Among other classical economists, very little can be said about David Ricardo, whose theories do not let emerge an overall view on the topic of education. In his *On the principles of political economy and taxation*, Ricardo only employs the concept of education in relation to a Malthusian view on the growth of population. In particular, he states that in the developing countries, education is necessary together with a good government to ensure happiness among the population, in the sense that where natural resources are abundant, education would help guarantee an increase in wealth greater than the increase of population.[9] This utilitarianism with social purposes is present also in Malthus, who, beyond his view on demographic control, provided another remarkable intuition on education. In fact, we find in Malthus the same social motivations identified in Smith: the poorest social classes would have been more pacific and orderly if they have received an education.[10] Once again, we find a trace of the idea of externality that is undoubtedly deeply rooted in the

[8]In addition, even if we put aside the typical view on productivity, one could argue that education which is not acquired with a "productivistic" scope, thus education intended as the Smithian literary education, cannot but improve the job skills and the overall quality of the work, by developing those that nowadays we call *soft skills*.

[9]"In those countries where there is abundance of fertile land, but where, from the ignorance, indolence, and barbarism of the inhabitants, they are exposed to all the evils of want and famine, and where it has been said that population presses against the means of subsistence, a very different remedy should be applied from that which is necessary in long settled countries, where, from the diminishing rate of the supply of raw produce, all the evils of a crowded population are experienced. In the one case, the evil proceeds from bad government, from the insecurity of property, and from a want of education in all ranks of the people. To be made happier they require only to be better governed and instructed, as the augmentation of capital, beyond the augmentation of people, would be the inevitable result. No increase in the population can be too great, as the powers of production are still greater. In the other case, the population increases faster than the funds required for its support. Every exertion of industry, unless accompanied by a diminished rate of increase in the population, will add to the evil, for production cannot keep pace with it." (Ricardo, 1821, pp. 93-94)

[10]According to Malthus (1914), if the lower classes of people, received an education, they

> would become more peaceable and orderly, would be less inclined to tumultuous proceedings in seasons of scarcity, and would at all times be less influenced by inflammatory and seditious publications, from knowing how little the price of labour and the means of supporting a family depend upon a revolution. (Malthus, 1914, p. 260)

centuries before its formalisation.

On the same line followed by Smith and Malthus, Mill gives his contribution on the topic of education by starting from the assumption that uneducated people could cause potential social instability, and, therefore, that the final purpose of education is to reach the mass. In particular, he alights on the inability of the poorest classes to understand the necessity of education:

> The uncultivated cannot be competent judges of cultivation. Those who most need to be made wiser and better, usually desire it least, and if they desired it, would be incapable of finding the way to it by their own lights. It will continually happen, on the voluntary system, that, the end not being desired, the means will not be provided at all, or that, the persons requiring improvement having an imperfect or altogether erroneous conception of what they want, the supply called forth by the demand of the market will be anything but what is really required. (Mill, 1909, p. 953)

Moreover, Mill also recognises a market failure resulting from a natural monopoly in education in favour of skilled workers. Like Smith, he analyses wage differentials and adds that when compared to wages of common labour, wages of skilled workers amply exceed what is sufficient to compensate for the cost of training and education. This is due to the lack of competition among groups of workers caused by the inability of the majority of the masses to pay for expensive courses and support themselves, which makes impossible to gain skills and learn specialised labour.

> But, independently of these or any other artificial monopolies, there is a natural monopoly in favor of skilled laborers against the unskilled, which makes the difference of reward exceed, sometimes in a manifold proportion, what is sufficient merely to equalize their advantages. [...] But the fact that a course of instruction is required, of even a low degree of costliness, or that the laborer must be maintained for a considerable time from other sources, suffices everywhere to exclude the great body of the laboring people from the possibility of any such competition. Until lately, all employments which required even the humble education of reading and writing could be recruited only from a select class, the majority having had no opportunity of acquiring those attainments. (Mill, 1909, pp. 391 - 392).

The segmentation of labour market identified by Mill prevents competition to work and justifies the role of public intervention to guarantee a basic level of knowledge to the whole society, including children coming from the poorest families, by free-access education provided by the State. However, this might be insufficient given the fact that, when free to choose, the mass is not able to see education as essential and consequently to judge itself on it, and therefore it becomes reasonable to rely on the government to somehow "force" the population to become instructed, by making education compulsory.

> With regard to elementary education, the exception to ordinary rules may, I conceive, justifiably be carried still further. There are certain primary elements and means of knowledge, which it is in the highest degree desirable that all human beings born into the community should acquire during childhood. If their parents, or those on whom they depend, have the power of obtaining for them this instruction, and fail to do it, they commit a double breach of duty, towards the children themselves, and towards the members of the community generally, who are all liable to suffer seriously from the consequences of ignorance and want of education in their fellow-citizens. It is therefore an allowable exercise of the powers of government to impose on parents the legal obligation of giving elementary instruction to children. This, however, cannot fairly be done, without taking measures to insure that such instruction shall be always accessible to them, either gratuitously or at a trifling expense. (Mill, 1909, p. 954).[11]

Mill goes beyond the starting ideas of Smith also in the observation of the consequences of industrialisation. He also notices the reduction of workers' tasks in an ever smaller number of simple operations and the substitution of small firms with large ones, mainly employing unskilled labour. Yet, he acknowledges an increase in the demand for skilled labour given by the recently born joint-stock companies requiring educated labour to employ as the managerial class. In Mill we find a confirmation of the investment motive of education that emerged from the analysis of Smith's thought. He writes:

> But the technical or industrial education of the community; the labour employed in learning and in teaching the arts of production, in acquiring and communicating skill in those arts; this labour is really, and in general

[11]Nonetheless, Mill (1909) does not underestimate the risk that the government could influence and educate masses according to a precise scheme with the only purpose to control opinions and ideas:

> One thing must be strenuously insisted on; that the government must claim no monopoly for its education, either in the lower or in the higher branches; must exert neither authority nor influence to induce the people to resort to its teachers in preference to others, and must confer no peculiar advantages on those who have been instructed by them. Though the government teachers will probably be superior to the average of private instructors, they will not embody all the knowledge and sagacity to be found in all instructors taken together, and it is desirable to leave open as many roads as possible to the desired end. It is not endurable that a government should, either de jure or de facto, have a complete control over the education of the people. To possess such a control, and actually exert it, is to be despotic. A government which can mould the opinions and sentiments of the people from their youth upwards, can do with them whatever it pleases. Though a government, therefore, may, and in many cases ought to, establish schools and colleges, it must neither compel nor bribe any person to come to them; nor ought the power of individuals to set up rival establishments, to depend in any degree upon its authorization. It would be justified in requiring from all the people that they shall possess instruction in certain things, but not in prescribing to them how or from whom they shall obtain it. (Mill, 1909, p. 956)

> solely, undergone for the sake of the greater or more valuable produce thereby attained, and in order that a remuneration, equivalent or more than equivalent, may be reaped by the learner, besides an adequate remuneration for the labour of the teacher, when a teacher has been employed. (Mill, 1909, p. 40)

In addition, if Smith left some doubts about his interpretation of education in relation to productivity, Mill leaves no space for questions when he speaks about productivity.

> The intelligence of the workman is a most important element in the productiveness of labour. So low, in some of the most civilized countries, is the present [1848] standard of intelligence, that there is hardly any source from which a more indefinite amount of improvement may be looked for in productive power, than by endowing with brains those who now have only hands. (Mill, 1909, p. 187)

He still adopts the distinction between productive and unproductive labour that we find in Smith and explicitly classifies as productive the activity of learning, as long as it is addressed to the acquisition of manufacturing skills.

> labour expended in the acquisition of manufacturing skill, I class as productive, not in virtue of the skill itself, but of the manufactured products created by the skill, and to the creation of which the labour of learning the trade is essentially conducive. The labour of officers of government in affording the protection which, afforded in some manner or other, is indispensable to the prosperity of industry, must be classed as productive even of material wealth, because without it, material wealth, in anything like its present abundance, could not exist. Such labour may be said to be productive indirectly or mediately, in opposition to the labour of the ploughman and the cotton-spinner, which are productive immediately. They are all alike in this, that they leave the community richer in material products than they found it; they increase, or tend to increase, material wealth. (Mill, 1909, pp. 48-49)

Even a stronger interpretation can be found in his *Essays on some unsettled questions of political economy*, where, in complete opposition to Smith, he acknowledges that the labour of those employed in education should be considered as productive and thus included in the estimation of national wealth.[12]

Ultimately, Mill can be considered the classical economists that most of all contributed in a remarkable way to go beyond his predecessors and, in particular, that added a notable contribution to Smith's ideas on the economics of education. An overall and cohesive view on education is still lacking, but the interest and significance that he shows on the topic in all his works cannot be neglected. He expands

[12] "Every classification according to which a basket of cherries, gathered and eaten the next minute, are called wealth, while that title is denied to the acquired skill of those who are acknowledged to be productive labourers, is a purely arbitrary division, and does not conduce to the ends for which classification and nomenclature are designed." (Mill, 2000, p. 58)

Smith's thoughts from the point of view of productivity, education as an investment and social control purposes, and, in addition, he deals for the first time with the imperfections of the market of education (although perhaps it is not suitable to speak about a proper market yet). But he also goes beyond that and recognises that "active" capacities come from education, not only contributing to a social utilitarianism perspective through don'ts but also releasing the ability of dos.

Although with a more modern view, Mill like Smith before him leaves in some way incomplete the discourse on education, once again renouncing to an attempt to measure human capital.

1.4 Education and the neoclassical economists

With Alfred Marshall, the evolution of human capital theory makes another important step forward. Marshall explicitly argued that his predecessors did not take into enough consideration that "human faculties" are as much important as any other capital in terms of production factors.[13] Like the other economists above analysed, Marshall is interested in studying the effects of technical change and industrialisation but unlike Smith, he observes that the introduction of technologies and machines replaced routine tasks with more complex operations, instead of increasing simple and recurring operations and that this new machinery together with innovative production methods and large-scale production required particular competencies and therefore, increased the need for division of labour. This view is definitely more modern and similar to the current utilisation of technologies. In particular, Marshall goes beyond the Millian identification of a recent-born managerial class; he understands the existence of a growing demand for managers with abilities of judgement, intelligence, general knowledge, and responsibility. He distinguishes *general ability* from *specialised ability*, where the former denotes higher skills and according to the British economist, was becoming more and more important with respect to the latter, which identifies the manual ability to work with particular materials and processes. In relation to this classification, and unlike Smith's ideas, Marshall stresses the importance of both general and technical education and underlines that the latter has been underrated in its power to develop artistic and other skills useful in particular occupations.[14] Marshall's contribution is unique and innovative also on productivity. He describes a cumulative process in which education raises labour productivity and the national wealth out of which higher wages are paid. In other words, higher wages led to an increase in investment in human capital, which increased productivity and wages of the next generation of workers; the process would

[13] "The older economists took too little account of the fact that human faculties are as important a means of production as any other kind of capital." (Marshall, 2013, p. 191)

[14] "Technical education has in like manner raised its aims in recent years. It used to mean little more than imparting that manual dexterity and that elementary knowledge of machinery and processes which an intelligent lad quickly picks up for himself when his work has begun; (...) Technical education is however outgrowing its mistakes; and is aiming, firstly, at giving a general command over the use of eyes and fingers (though there are signs that this work is being taken over by general education, to which it properly belongs) ; and secondly at imparting artistic skill and knowledge, and methods of investigation, which are useful in particular occupations, but are seldom properly acquired in the course of practical work." (Marshall, 2013, pp. 173 - 174)

be facilitated by government support.

Marshall attempts to give a definition of human resources, which are included in wealth, nevertheless, he shows some hesitation as in the many versions of his *Principles of economics*, he provides different definitions of "personal capital". Initially, he writes:

> we have already defined Personal wealth to consist firstly of those energies, faculties and habits which directly contribute to making people industrially efficient...if they are to be reckoned as wealth at all, they are also to be reckoned as capital. Thus Personal wealth and Personal capital are convertible; and it seems best to follow here the same course as in the case of wealth, ...to raise no objection to an occasional broad use of the term [capital], in which it is explicitly stated to include Personal capital ...

However, in the fifth and subsequent editions of the *Principles* he deletes that paragraph and with it his determined line of thought. He changes the definition as follows:

> we may define personal wealth so as to include all those energies, faculties, and habits which directly contribute to making people industrially efficient; together with those business connections and associations of any kind, which we have already reckoned as part of wealth in the narrower use of the term. Industrial faculties have a further claim to be regarded as economic in the fact that their value is as a rule capable of some sort of indirect measurement.(Marshall, 2013, p. 48)

According to Blandy (1967), the problem here was definitional and not conceptual. In other words, we cannot deny the presence of a formalised and well-defined concept of human capital, although there is still some hesitation in naming it as we know it today. Within the debate on Marshall's view, Blandy (1967) argues that interpreting the work of Marshall as a denial of human capital would mean ignoring the real content of his theory.

Apart from the connotations of education analysed so far, Marshall as well does not present a theory without contradictions or open questions; his thought shows the intention to make a breakthrough but his reflection does not seem mature enough and still presents many elements which characterised classical economists; he seems to naturally continue and expand their discourse on education. Aligned with the thought of the authors above discussed, he underlines in a much stronger way the importance of education in relation to its indirect effects, such as the development of meritocracy and social skills.

> But a good education confers great indirect benefits grades of even on the ordinary workman. It stimulates his mental activity; it fosters in him a habit of wise inquisitiveness ; it makes him more intelligent, more ready, more trustworthy in his ordinary work ; it raises the tone of his life in working hours and out of working hours; it is thus an important

> means towards the production of material wealth ; at the same time that, regarded as an end in itself, it is inferior to none of those which the production of material wealth can be made to subserve.(Marshall, 2013, pp. 175 - 176)

Hence, as in Mill, he calls for a strong state support in education in order to offer to the masses more possibilities than what they would find on the market and attributes a decisive role to parents.[15]

Government support is justified also by parent's unavailability of the resources necessary to provide an education for their children. In addition, greater difficulty is generated by a lag process; labour supply depends on the resources of parents, who support the education of their children, often willing to sacrifice themselves with no own benefit. In this sense, the supply in a certain generation is determined by (the income of) the previous generation and, therefore, skills development lags behind labour demand required by a highly dynamic technical change.[16]

Among the authors analysed so far, Marshall seems to be the one that mainly stresses the role of public expenditure on education and on this aspect he shows confidence and coherence in his thought.

> To that end we need to move in the same direction as in recent years, but more strenuously. Education must be made more thorough. The schoolmaster must learn that his main duty is not to impart knowledge, for a few shillings will buy more printed knowledge than a man's brain can hold. It is to educate character, faculties and activities; so that the children even of those parents who are not thoughtful themselves, may have a better chance of being trained up to become thoughtful parents of the next generation. To this end public money must flow freely. And it must flow freely to provide fresh air and space for wholesome play for the children in all working class quarters.(Marshall, 2013, p. 597)

What Marshall had not the audacity to properly formalise has been done by Fisher, which explicitly includes human beings in the definition of capital:

> The best example of the choice between those uses of capital instruments affording immediate and those affording remote returns is found in the case of human capital, commonly called labor. Man is the most versatile of all forms of capital, and among the wide range of choices as to the disposition of his energies is the choice between using them for immediate

[15]"Most parents are willing enough to do for their children what own parents did for them; and perhaps even to go a little beyond it if they find themselves among neighbours who happen to have a rather higher standard. But to do more than this requires, in addition to the moral qualities of unselfishness and a warmth of affection that are perhaps not rare, a certain habit of mind which is as yet not very common. It requires the habit of distinctly realising the future, of regarding a distant event as of nearly the same importance as if it were close at hand (discounting the future at a low rate of interest); this habit is at once a chief product and a chief cause of civilization, and is seldom fully developed except among the middle and upper classes of the more cultivated nations." (Marshall, 2013, p. 180)

[16]It is curious to notice how the perception of current times by the authors can change completely when moving chronologically. For instance, Smith observed that technical change was very slow.

> or for remote returns. This choice usually carries with it a choice between corresponding uses of other instruments than man, such as land or machines. But the existence of optional employments of labor, however inextricably bound up with optional employments of other instruments, deserves separate mention here both because of its importance and because it usually supplies the basis for the optional employments of other forms of capital. (Fisher, 1930, p. 200)

Here Fisher breaks a historical ethical hostility by cancelling out the Marshallian resistance to market human beings.[17] Therefore, not only the skills owned by an individual are part of the capital, but the skilled individual herself becomes a human resource which can be included in the notion of capital.

> It is true that freemen are not ordinarily counted as wealth; and, indeed, they are a very peculiar form of wealth, for various reasons: first, because they are not, like ordinary wealth, bought and sold; secondly, because the owner usually estimates his own importance so much more highly than any one else; and finally, because the owner and the thing owned in this case coincide. Yet they are, like other wealth, "material" and "owned". (Fisher, 1906, p. 5)

In this way, Fisher makes a key simplification that will allow the typical marginalistic approach to characterise the concept of human capital.

1.5 The emergence of human capital theory

After Marshall and Fisher we have to wait at least fifty years to reach a turning point in the evolution of human capital theory. To Nicholson (1891, 1892) is attributed the first estimation of human capital and to Walsh (1935) the clear formalisation of education as an investment. In Friedman and Kuznets (1954) we can find the measure of returns to human capital and, finally, the human capital theory is formalised through the contribution of those defined as the *founding fathers* of the economics of education: Theodore Schultz, Jacob Mincer and Gary Becker.

This breakthrough was certainly fostered by the development and the thriving literature on growth theory in the late Fifties and Sixties. From the Solow's residual, the idea that capital and labour are not the only factors of production starts to develop; Solow's technological progress becomes the "measure of our ignorance" in Abramovitz (1956), until the expression "human capital" becomes popular and frequent in the works appeared in the decade 1950-1960, in particular among the economists coming from the University of Chicago. The contribution of Solow (1957)

[17] With respect to Blandy (1967), Kiker (1966) does not go beyond the most superficial interpretation of Marshall's words and maintains a point of view strictly connected to the definitions we find in Marshall:

> he [Marshall] disregarded the notion [of human capital] as "unrealistic" since human beings are not marketable

and Abramovitz (1956) is consecrated by Arrow (1971), as a starting point to hypothesise that technical change can be ascribed to experience or, in other words, to learning-by-doing.

> Though doubtless no economist would ever have denied the role of technological change in economic growth, its overwhelming importance relative to capital formation has perhaps only been fully realised with the important empirical studies of Abramovitz and Solow. (Arrow, 1971)

Building on Arrow (1971), Lucas (1998) focuses "on various aspects of what economists, using the term very broadly, call the *technology*", considering human capital accumulation both through schooling and learning-by-doing, and Romer (1986) presents a long-run growth model that takes into account externalities. Hence, approaching the Nineties, a whole strand of literature based on the affirmed growth models of the Sixties starts to develop. This line of research studies economic growth focusing on human capital accumulation, despite the contradictions and scepticism which characterise the economics of education in this decade.[18] In fact, these recent theories of growth attribute a central role to the government, in the sense that public expenditure for education could be a key to avoid poverty traps and promote mobility of workers, bringing positive effects in long-term growth. Examples of this primary role assigned to public authorities in literature can be found in Barro (1990) which incorporates a public sector inside a model of economic growth, by focusing on government spending on education, and in Andersson and Konrad (2003) who study how globalisation can affect government's incentives to provide education subsidies. It is a fact that by the Nineties, there is no more uncertainty on the belief that human capital is a major determinant of growth:

> In a country that cannot draw on major reserves of natural resources or other types of rent income, income from human-capital investment is really the major source of national income. (Andersson & Konrad, 2003)

In this framework, the transition from the macroeconomic to the microeconomic point of view is due to the *founding fathers*.

It is Schultz (1960) who first defines in a clear way education as a proper investment in human beings and its result as a form of capital:

> I propose to treat education as an investment in man and to treat its consequences as a form of capital. Since education becomes a part of the person receiving it, I shall refer to it as a human capital. Since it becomes a integral part of a person, it cannot be bought or sold or treated as property under our institutions. Nevertheless, it is a form of capital if it renders a productive service of value to the economy. (Schultz, 1960)

[18]Starting from the Seventies the beliefs on education have a fluctuating character. See Teixeira (2000) for a detailed discussion on the renewed trust in the economics of education during the Nineties.

As an agricultural economist, Schultz first studied agriculture in the United States and then in developing countries. He understood "how investments in education can affect productivity in agriculture as well as the economy as a whole," according to his 1979 Nobel citation. At a time when other economists were having trouble explaining how the economies of such nations as Germany and Japan grew so quickly after World War II and attributed the improvements in those nations to "technical change", Schultz identified people as the source of the economic growth, a long time before Solow's model.

Schultz himself acknowledges that among his predecessors, a few economists have given a remarkable contribution to human capital and those are Adam Smith, von Thunen and Fisher. He also provides an evaluation of the thought of Marshall and Mill, discussing their limitations and stating that "surely Mill was wrong" because he "insisted that the people of a country should not be looked upon as a wealth because wealth existed only for the sake of people.":

> Economists have long known that people are an important part of the wealth of nations. [...] What economists have not stressed is the simple truth that people invest in themselves and that these investments are very large. [...] The mere thought of investment in human beings is offensive to some among us. [...] By investing in themselves, people can enlarge the range of choice available to them. It is one way free men can enhance their welfare. (Schultz, 1961)

The work of Schultz soon becomes known to Mincer, who had fruitful relationships both with Schultz and Becker. If Schultz focused on aspects connected with growth, Mincer finally opens the possibility for the economics of education to find applications in labour economics, by studying the impact of human capital on earnings and empirically estimating the rate of returns of education, which can, at last, be compared with the returns from other types of investment. Mincer (1958) was able to give to a concept linked with macroeconomics and growth, a microeconomic and individual status.

The official affirmation of economics of education as a branch of political economics is definitive with the contribution of Becker, which first with his article of 1962 and then his famous book "Investment in Human Capital: a Theoretical and Empirical Analysis with Special Reference to Education" (Becker, 1962, 1964) gives to the economics of education its typical neoclassical approach, based on the rational behaviour of agents and micro foundation on stable preferences.

1.6 The human capability approach

Despite the appellation of "founding fathers" to more recent authors and the belief that a "revolution" brought us the notion of human capital, Adam Smith is the author Amartya Sen has a preference for when analysing the concept of human capital. In many of his works, Sen quotes Smith and takes him as a reference to build his *human capability* approach in the 1980s. "Adam Smith's belief in the power of education and learning was peculiarly strong", Sen (1997) points out, and "the development of human capability in leading a worthwhile life as well as in being more productive is quite central to Smith's analysis of *The Wealth of Nations*." And again he states:

> There are so many ways in which Smith's ideas have insights to offer to the world today. There are a great many departures that were proposed by Smith that have not been fully taken up yet, despite the frequency with which Smith has been quoted in the literature over the last two centuries and more. (Sen, 1997)

When dealing with human capital theory, Sen seems to be the only author to fully acknowledge the several doors that Smith's thought opened for building future research and, most of all, he appears to be the only one to have thoroughly taken up the road marked by Smith.

Perhaps, this is the real revolution. Sen finally manages to abandon the merely "productivistic" view of human capital and by recalling Smith's idea he somewhat closes a circle that nobody before him was able to complete. Sen's concept of human capability is much broader than the one of human capital, such that it includes the latter. They both place humanity at the centre of their analysis, therefore cannot but be related. Yet, human capability relates to the substantial freedom that allows people "to lead the lives they have reason to value and to enhance the real choices they have." With respect to human capital, the human capability approach covers both *direct* and *indirect* consequences of human skills, while the former only refers to the indirect effects of human abilities, i.e. in terms of increased productivity. In a broader sense, the capability approach includes the *realised functioning*, that is what a person is really able to do, and the *capability set* of alternatives she can choose from, that is what she is actually free to do. Here is the overcoming of the "productivistic" view tied to the concept of human capital. While human capital only focuses on one part of the picture and on human beings as means of production, human capability sees them as the ends of a process. To be precise, Sen (2001) distinguishes three effects to take into account when studying human capital:

1. the direct influence on people's well-being;

2. the indirect effect through the influence on social change;

3. the indirect effect through the influence on economic production.

Sen (1997) argues that human capital theory only concentrates to the last role and thus this perspective needs to be integrated and enriched. And with his human capability approach, he finally enlarges the human capital theory by including those positive externalities that the human capital approach as we know it only formally incorporated, but that substantially neglected.

> Consider an example. If education makes a person more efficient in commodity production, then this is clearly an enhancement of human capital. This can add to the value of production in the economy and also to the income of the person who has been educated. But even with the same

> level of income, a person may benefit from education, in reading, communicating, arguing, in being able to choose in a more informed way, in being taken more seriously by others and so on. The benefits of education, thus, exceed its role as human capital in commodity production. The broader human-capability perspective would note-and value-these additional roles as well. The two perspectives are, thus, closely related but distinct. (Sen, 1997)[19]

Ultimately, education allows people to freely and fully express their personality and private initiative, and to be able to value choices in life and lead the type of life they wish to live, as well as to consciously shape desires and ambitions. The result cannot be other than an orderly society able to coexist in each other's respect, as Smith explained two centuries before. Sen (1997) does not renounce to analyse Smith's thought also by going through the debate on ”nature” and ”nurture” and recognises in Smith the role of a ”nurturist” as he believes that it is not nature, and thus some kind of innate talent that makes a difference in the skills of human beings but that this is due to other factors, such as education.[20]

> It is not my purpose here to examine whether Smith's emphatically “nurturist” views are right, but it is useful to see how closely he links the productive abilities to the ability to lead different types of lives. That connection is quite central in seeing human capital in the broader context of the human- capability perspective. (Sen, 1997)

Within the capability approach, Sen (2001) identifies five types of “instrumental freedoms that contribute to the overall freedom people have to live the way they would like to live”, and among them, the freedoms he calls *social opportunities* include the availability of services such as education and health care.[21] As these instrumental freedoms are interconnected and complement each other, the facilities considered as social opportunities not only directly help live better but also allow for greater economic and political participation. Sen (2001) provides an example considering education:

[19]In this quotation, Sen is clearly developing from Adam Smith. In *Development as freedom*, he makes a direct reference to him by recalling his example from *The Wealth of Nation* explaining the ability to speak and show ourselves in public without shame:

> to be able to “appear in public without shame” may require standards of clothing and other visible consumption in a richer society than in a poorer one (as Adam Smith noted more than two centuries ago). (Sen, 2001)

[20]”The difference of natural talents in different men is, in reality, much less than we are aware of; and the very different genius which appears to distinguish men of different professions, when grown up to maturity, is not upon many occasions so much the cause, as the effect of division of labour. The difference between the most dissimilar characters, between a philosopher and a common street porter, for example, seems to arise not so much from nature, as from habit, custom, and education. When they came into the world, and for the first six or eight years of their existence, they were, perhaps, very much alike, and neither their parents nor playfellows could perceive any remarkable difference.”(Sen, 1997)

[21]”I shall consider the following types of instrumental freedoms: (1) political freedoms, (2) economic facilities, (3) social opportunities, (4) transparency guarantees and (5) protective security.” (Sen, 2001)

> For example, illiteracy can be a major barrier to participation in economic activities that require production according to specification or demand strict quality control (as globalized trade increasingly does). Similarly, political participation may be hindered by the inability to read newspapers or to communicate in writing with others involved in political activities. (Sen, 2001)

What Sen is doing here is not only transforming and revolutionising the human capital theory, but he is also overcoming the modern forms of utilitarianism, which see utility "as the fulfilment of desire, or as some kind of representation of a person's choice behaviour". Sen adopts a view of utility that is strictly connected with happiness, that is not new in the history of economic thought, but goes back to William Petty[22] and other major economic authors[23], who were already concerned with people's living conditions. In this sense, the limitations of the utilitarian perspective that are abandoned include also adaptation and mental conditioning: people with any kind of constant deprivation may adapt or not be able to acknowledge their condition and thus actively shape their own destiny and ambitions, in a sort of "adaptation to unhappiness". This aspect has been already anticipated by Mill as we recall from previous paragraphs. Basically, Sen is reshaping the major interest points of economists. Do we need to earn more or to live better? What are the benefits of having more wealth? He states:

> we generally have excellent reasons for wanting more income or wealth. This is not because income and wealth are desirable for their own sake, but because, typically, they are admirable general-purpose means for having more freedom to lead the kind of lives we have reason to value. (Sen, 2001)

This key view that, as he stresses, needs to be applied to the general approach of economics certainly he has been able to adopt it in the approach to education and enlarge it to the point to finally lose the limitation of a vision still too much anchored to productivity and earnings.

1.7 Final remarks

The historical perspective of this work is necessary to set together the fragmented puzzle of points of view in the history of human capital theory and let emerge some critical points that in a way have been forgotten. It is worth here to use a quotation from the Italian economist Piero Sraffa. Even though referred to the theory of value, it is relevant for every field one wants to consider.

> In order to understand the modern theory of value it is necessary to have some knowledge of its history. This in the sense that the history is not only necessary in order to understand the origin of the theory, which

[22] See Petty (1986).

[23] We have undoubtedly observed this interest in the economists studied in our *excursus*.

> is obvious; but it is also necessary in order to understand its significance, that is, the nature of the problems which it proposes to solve. (D2/4, p. 1)[24]

As Sraffa says, a historical analysis allows not only to know about the origin of a theory but also to know the issues and aspects that theory deals with, that is essentially what we have done in the present work. By rebuilding when and how the concept of human capital is developed for the first time, we sounded all the possible aspects that the theory focused on during its development, but also the aspects linked to education which have been left behind during this process.

What we have seen so far is a pluralistic view on the role of education that struggled to get affirmed. Incoherence and contradictions are present in the work of many authors, mainly in Smith and Marshall. In Smith, we can see the attempt to collect many ideas together when a general theory of economics does not even exist. Smith shows the indecision of who is moving the first steps into something and trying to explain a thought still in a phase of development. We find a great hesitation in Marshall as well, but with some differences. Marshall seems to have a very clear line of thought in mind, but he only hesitates in "using the right words". His trouble is providing a definition, rather than defining a concept as it was for Smith. In all the other authors we went through, we can only find a taste of a complete theory, pieces that if combined together will not be able to come up with an integrated scheme.

Moreover, the many points of view never manage to completely agree with each other. The contributors here analysed have made progress from the theory of some precedent authors, have sometimes tried to complete previous elaborations, have recalled and manipulated notions by adding something new, but as a result, mostly contrasts and differences can be identified.

To the founding fathers of the economics of education we certainly must reckon the ability of coherently developing a formalised and unitary theory; nevertheless, on the path conducting to the theory of human capital, it seems that interest in people's well-being and living condition has lost consideration and the focus became more and more confined only to the aspects of investment and labour productivity. Some extremely important implications to the concept of education, such as positive externalities, were abandoned in favour of a merely "productivistic" and utilitarian view. In the process of stating a theory and simplifying it, the multidimensional approach went missing and we due to Sen the renovated focus on the questions which were forgotten.

As a social science, economics is made of contradictory approaches which are historically and socially anchored; nevertheless, some social aspects linked to the role of education and developed along history have not flown into a unitary theory, and therefore they do not find application in models and methodologies.

As this work shows, approaching education in terms of rate of returns and cost-benefit ratio over lifetime is restricted only to those concepts included in the human

[24]The file D2/4 refers to Sraffa's *Lectures on advanced theory of value* given to his students undertaking the economics tripos from 1928 to 1931. The manuscript is conserved in the Wren Library of Trinity College, Cambridge.

capital theory. Individuals can consider or not education as an investment and make the choice to undertake this investment on the basis of multiple factors which change with society over time, and that such a restrictive view cannot include. The empirical literature on economics of education has tried to include multiple factors in its analysis, like for instance individuals' socio-economic background,[25] yet this is always realised through the estimation of a Mincerian wage equation, relying on those confining approach of human capital theory, thus it is not enough to account for the multiple features of education highlighted in this work. New methodologies should allow the possibility to model behaviours that so far have been ignored and capture those dimensions that along the centuries did not find space in the human capital theory. The evolution of social phenomena calls for an evolution in theories and methodologies able to explain behaviours and changes in the society and to capture more than one dimension to a concept.

[25] See for instance Cohn and Kiker (1986) and Papanicolaou and Psacharopoulos (1979).

Chapter 2

An Agent-Based Model for tertiary educational choices in Italy

2.1 Introduction

Although the low level of tuition fees and absence of other access barriers, Italy is characterised by low educational attainments at the university level.

Within the 10-year European strategy Europe 2020, the European Commission elaborated five targets to be reached by 2020, among which the goal concerning education includes reaching a share of 40% of 30-34-year-old completing a third level education. These common goals translate into national targets for member states. Italy, in particular, aims at reaching 26-27% of people aged between 30 and 34, having attained a tertiary education degree. Indeed, according to Eurostat, in 2017 the 26.5% of Italians aged between 30 and 34 has obtained a higher education degree, placing Italy in line with national targets. Nevertheless, Italy performs badly when compared to the other EU countries: a lower share is registered only in Romania (26.3%).[1]

The OECD report Education at a glance 2016 states that the Italian tertiary education system is not attractive to potential students. This is demonstrated by the low entry rate to tertiary education and the high share of so-called NEETs (i.e., people who are Not in Education, Employment or Training). The first-time bachelor's entry rate in Italy is 37%, which is much lower than in most OECD countries. Discussing these data,Viesti (2018) suggests that low university enrolment rates in Italy could be explained by a growing distrust of Italian families towards investment in higher education, which may be affected, among other factors, by low earning levels of young graduates when entering the job market.

Moreover, Naticchioni, Raitano, and Vittori (2016) who investigated earnings gaps across generations in Italy, find that the generation born between 1975 and 1979 suffered from a remarkable earnings loss from the first entry in the job market, with respect to the previous generations. In particular, high-skilled workers of the

[1]See OECD (2016) for an overview of the tertiary education attainment in the EU countries, as well as of national targets.

1975-1979 generation, who completed a tertiary educational level, have been affected by a decrease in their earnings in a much larger measure than low-educated workers, without a catch up of this gap in the following years.

This paper introduces a model of how individual educational preferences form, in order to analyse the determinants behind university enrolment decisions and explore whether these determinants could explain the low educational attainment characterising Italy. The model implies that the individual preference to enrol at university depends upon

(i) economic motivations, represented by the expectations on future income;

(ii) influence from peers;

(iii) the effort of obtaining a university degree.

The novelty of this contribution mainly resides in the methodology; the model is explored by means of agent-based computational simulations.

Agent-based models allow to take into account heterogeneity of agents, social interaction and exchange of information, and through their "bottom-up" approach they permit to explain macro-consequences starting from micro-processes; in brief, Agent-Based Modelling (ABM) is particularly suited to study complex phenomena, where emergent results are not simply the aggregation of single agents behaviours.

As a consequence of the methodology adopted, the originality of this contribution can be seen also from a theoretical point of view. In this context, in fact, ABM allows for modelling the decision-making process concerning tertiary education, without the constraints of the standard human capital theory, that in the *mainstream* literature would be the basic foundation of any study related to education. Therefore, it becomes possible to analyse educational decisions with a multidimensional approach, rather than limited to a productivistic/utilitarian view.

The model is rigorously calibrated and set up using data from different sources. It is then simulated for 100 periods of time, over 100 Monte Carlo simulations, with the purpose to give robustness to our results.

The aim is to verify if, simulated over t periods of time, the model is able to provide a realistic representation of the socio-economic phenomenon investigated. We are interested in a relative measure of the number of individuals that, after completing secondary education, are willing to continue their studies. Therefore, the primary output of interest produced by the model is the rate of enrolment to the university.

Computational results obtained through simulations are compared with aggregate data on the rate of transition from secondary school to university in Italy, obtained from the Italian National Institute of Statistics (Istat). This information is available for 6 years, from 2008 to 2013; the rate of transfer ranges from 65.8% to 55.7%, thus decreasing over time.

The comparison shows that the model is able to realistically represent the phenomenon of transition from secondary school to tertiary education, with an enrolment rate of about 56%.

The majority of those deciding to enrol to university, even if slight, comes from an educated family, with a proportion of about 56% over the 43% of individuals enrolling coming from unskilled parents. Among the agents faced with the education decision, half of them maintain the same education level than their parents, while the rest is evenly composed by individuals moving upward or downward in the *ladder* of levels of education, with respect to their parents.

The model is characterised by the presence of only one parameter, on which a sensitivity experiment is carried out, showing that the enrolment rate increases when the parameter ruling the concavity of the relation between ability and study effort increases.

The Chapter is organised as follows. In Section 2.2 we describe the methodology and review the literature on ABM, in general and in the field of economics of education. In Section 2.3 we describe the theoretical model and in Section 2.4 we explain the process of calibration, imputation and the simulation algorithm. Section 2.5 presents the computational results obtained through simulations and Section 2.66 reports the results of the sensitivity experiment. Finally, conclusions are drawn in Section 2.7.

2.2 Literature review

2.2.1 Methodology in brief

This work contributes to the existing literature in an original way by means of the methodology adopted, that is agent-based modelling.

Based on the definitions provided by Caiani, Russo, Palestrini, and Gallegati (2016) and Gilbert (2008), agent-based modelling is a computational method that enables to create, analyse, and experiment models populated by (heterogeneous) agents interacting with each other from the bottom up, that is without a central coordinator, and within an environment. Thus, an ABM is typically characterised by three elements:[2]

1. A set of *agents*, owning attributes and rules of behaviour.

2. *Interaction* among agents and between agents and the environment, defined by a set of relationships and methods on how and with whom interacting.

3. The *environment*: agents interact with the environment in addition to other agents.

Agents are characterised by the ability to be autonomous, that is to act on their own in response to the events happening and to take decisions independently. ABM also allows agents to be heterogeneous: agent characteristics and behaviours may vary in extent and specification. Heterogeneity can exist at the set-up of the model but it may also result from the process of interaction, further differentiating the agents with respect to the model initialisation.

Just as in the real world, agents interact with other agents, but not all agents interact directly with all the other agents all the time. They typically interact

[2]See Macal and North (2010) for a brief introduction to ABM, and its main concepts and foundations.

with a subset of other agents, named *neighbours*, and with a certain location in the environment (not from any part of the entire environment). Through these interactions, agents obtain local information.

As already mentioned, agent-based models are decentralised systems, there is no central coordinator making the information globally available to the agents. This is anchored to the concept of *bounded rationality*, proposed for the first time by Simon (1957). He criticised as unrealistic the assumptions based on rational choice theory, by suggesting to model agents with limited cognitive abilities, which translates in limited ability to optimise their utility.

The way agents are connected to each other is generally called *topology*. An ABM topology could be represented for instance by a spatial grid or network of nodes (agents) and links (relationships).

The environment can be solely a social environment, that is a map of the relationships among agents, or could provide information about the spatial location of agents, even in a rich information set as in a GIS.

Agent-based modelling is employed in a variety of fields, which have in common the aim to study phenomena in their complexity: from biology (see for example Alber, Kiskowski, Glazier, & Jiang, 2003; and Troisi, Wong, & Ratner, 2005) and ecology (see Taylor, Koyuk, Coyle, Waggoner, & Newman, 2007), to social sciences and economics. In particular, in the social sciences, ABM is particularly suited for studying large-scale outcomes deriving from the interaction among individuals. These interactions produce emergent results, that cannot be deduced by simply aggregating the properties of the agents and that is different from what if would be if agents were isolated from each other.

Starting with rules and assumptions about agents and their interactions, ABM uses computer simulation to generate ”histories” that can reveal the dynamic consequences of these assumptions, allowing to investigate how emergent properties arise from the micro-processes among agents. In social sciences and economics, agents can represent people (for example consumers or voters) or social groups, like households, firms, government agencies, etc.

Together with deduction and induction, simulation can be considered as a third scientific method. Deduction is used to derive theorems from assumptions; induction allows to find patterns in empirical data. Simulation, like deduction, starts with a rigorously specified set of assumptions describing the system, but, unlike deduction, it generates data suitable for analysis by induction. In contrast to typical induction, however, the simulated data comes from controlled experiments rather than from direct measurements of the real world. Consequently, simulation differs from standard deduction and induction in both its implementation and its goals. Simulation permits increased understanding of systems through controlled computational experiments. (see Axelrod, 1997; and Tesfatsion, 2006)

2.2.2 State of the art

The application of ABM in the field of economics is relatively new. We can find examples in macroeconomics, where in contrast with standard theory, an “equilibrium” is not assumed a priori.

Riccetti, Russo, and Gallegati (2015) present a macroeconomic micro-founded framework with heterogeneous agents, represented by households, firms and banks, which interact across four different markets – goods, labour, credit and deposit. Through computer simulation, the model highlights that extended crises can endogenously emerge, leading to a large unemployment status, without the possibility to quickly recover unless an exogenous intervention.

Westerhoff (2010) proposes an agent-based macroeconomic model in which firms hold heterogeneous sales expectations: optimistic or pessimistic. The type of expectations depends on macroeconomic conditions and on the average mood prevailing within its local neighbourhood. The model shows that such an economy may give rise to co-evolving dynamics between the business cycle and the firms' average sentiment.

We can find examples of applications of ABM also in labour economics. Fagiolo, Dosi, and Gabriele (2005) develop an ABM of a decentralised market economy consisting of a labour and a product market. Their aim is to provide possible "bottom-up" micro explanations for well-known macro empirical regularities, such as the Beveridge curve, the Okun's Law, and wage data patterns.

Dawid et al. (2008) study the growth and labour market effects of various policy types that promote workers' general skill levels by means of an agent-based macroeconomic model that is then integrated within the larger simulation platform for European policy-making, Eurace.
Eurace is a simulator characterised by a complete set of interrelated markets and different types of interacting agents, modelled according to a rigorous balance-sheet approach. Eurace is also used by Cincotti, Raberto, and Teglio (2010). They investigate the interplay between monetary aggregates and the dynamics and variability of output and prices by considering both the money supplied by commercial banks as credit to firms and the fiat money created by the central bank through the quantitative easing monetary policy.

The adoption of ABM is particularly useful also for studying microeconomic policy issues, by describing a society, an industry or a market from a micro perspective.

Yet, moving to the economics of education and to the analysis of schooling decisions, the application of ABM is quite scarce. Grow and Van Bavel (2015) study assortative mating and gender inequality in education in Europe through an agent-based model. Starting from the fact that there are more highly educated women than men reaching the reproductive ages and looking for a partner, the authors develop an ABM that explicates the mechanisms that may have linked the reversal of gender inequality in education with observed changes in educational assortative mating. They demonstrate that the observed changes in educational assortative mating can be explained without any change in male or female preferences.

Friant (2015) uses simulation in order to study school decisions in French-speaking Belgium, where pupils and their parents are free to choose the school they prefer and where high levels of socio-economic segregation can be observed; his aim is to assess the impact of school choice on school segregation.

The study most connected to this work can be found instead in Manzo (2013). Using empirical data for France from the year 2003, Manzo (2013) proves that among the determinants of the distribution of educational choices across social groups, social influence among agents of a network cannot be ignored. Ability and subjective perceptions of education benefits are not sufficient on their own to generate the actual stratification of educational choices across educational backgrounds existing in France at the beginning of the twenty-first century.

This work has many points in common with the study of Manzo (2013), however, it provides a different model for the Italian context relying on survey data for Italy. To the best of my knowledge, no other work adopting ABM for studying education or educational choices is available, thus, by adopting an agent-based computational model, this analysis contributes to enrich the literature on the economics of education.

2.3 The theoretical model

2.3.1 Agents

In this model, agents' population is constituted by two generational groups: *juniors* and *seniors*. *Juniors* have completed secondary education and have to decide whether to enrol or not at university. *Seniors* are in the labour market and can be skilled or unskilled, depending on whether they attended university in their previous period. *Juniors* are provided with a monetary endowment deriving from their parental income, which can be spent in education or in the transition from school to the job market. Agents also own a certain level of *ability*, capturing innate talent and personal skills. Although it is an unobservable variable, regardless of the measure adopted, the empirical literature has proved that it affects educational decisions (see, for instance, Stocké, 2007; Cheadle, 2008). Given a certain distribution for ability, the paper assumes that this is common for all the agents in the population.[3]

2.3.2 Environment

The agents set is organised in a social environment constituted by N neighbourhoods representing agent's social relations: family, friends, acquaintances. Agents' population is modelled as a social network structured in social circles. This type of social network structure is proposed by Hamill and Gilbert (2009). The authors argue that this network model is able to embed the key characteristics of real social networks, such as low density, high clustering and assortativity of the degree of connectivity, which other network types fail to reflect.[4]

The idea of social circles dates back to Simmel (1902). The circle is a powerful tool in this context; its circumference will contain all those points within a distance set by a radius and creates a cut-off, limiting the size of personal networks; thus, the agents included in one's circle given a certain radius will be the agent's neighbours. An example is provided in Figure 2.1,

[3]Researchers interested in differences among social groups could consider social differentiation of ability and attribute different distributions or let the parameters of the distribution vary across social groups.

[4]The authors compare their network model with a regular lattice, a random network, a small-world network and a preferential attachment (scale-free) network.

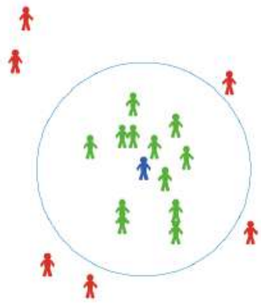


Figure 2.1: An example of social circle.

In this picture, drawn from NetLogo, the circumference defines the social circle of the blue agent. The agents included in the circumference, coloured in green, represent the blue agent's neighbours, while the agents outside the circumference, coloured in red, are excluded from her personal network.

As in Hamill and Gilbert (2009), let us call the radius *social reach*. Small values of social reach are of course associated with small personal networks, let us say to replicate a network of relatives and close friends; while larger values of social reach will correspond to bigger personal networks, including for example acquaintances.

According to the formalisation of Hamill and Gilbert (2009), agents are only permitted to link with agents who can reciprocate; in other words, others whose reach includes ego.

This situation is represented in Figure 2.2.

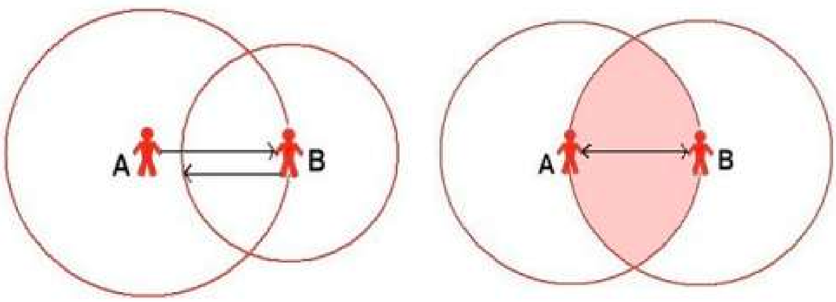


Figure 2.2: Reciprocity in social circles. Source: Hamill and Gilbert (2009)

In the left panel of Figure 2.2, there is difference in social reaches and, therefore, no reciprocity. A has a bigger reach than B then B can be in A's neighbourhood but not vice-versa, implying that A knows B but B does not know A. In the right panel, corresponding to our case, all agents have the same social reach, thus reciprocity is easily reached. An idea for future research would be to include a certain level of "sociability" in the model, that could be reflected by different levels of social reach, allowing to distinguish sociable agents with bigger personal networks and less sociable agents, with less populated personal networks.

In order to consider family effect and model the influence that parents have on their children educational choices, we assume that agents are linked to their parents with a stronger tie than with the rest of their neighbours.

Therefore, we assign a weight w to connections with parents that can account up to double a standard unweighted connection.

2.3.3 Budget constraint

Juniors will ponder the choice to enrol at university only if the following budget constraint is satisfied:

$$X_{i,t} - CostEdu > 0 \tag{2.1}$$

where $X_{i,t}$ represents the endowment agents are provided with; $CostEdu$ is the cost of education, which is assumed to be exogenous and fixed according to the educational system taken into account.[5] If the budget constraint is not satisfied, the *junior* agent will directly enter the job market as unskilled, without evaluating the possibility to enrol at the university.

2.3.4 Consumption

If the budget constraint (2.1) is satisfied, then real expected consumption for skilled workers (s) at time $t+1$ will be:

$$C^e_{is,t+1} = X_{i,t} - CostEdu + Y^e_{is,t+1} \tag{2.2}$$

while real consumption for unskilled workers (u) will be:

$$C^e_{iu,t+1} = X_{i,t} + Y^e_{iu,t+1} \tag{2.3}$$

where $Y^e_{is,t+1}$ and $Y^e_{iu,t+1}$ are respectively the average expected income for skilled and unskilled workers. These expectations on future income are modelled as naive expectations based on the information set of *senior* neighbours:

$$Y^e_{i,t+1} = E_t(Y_{t+1}|\Omega_{n,t}) = \frac{\sum_{i=1}^{nw} wY_{nsen,t}}{nw} \tag{2.4}$$

[5] To replicate real features of the Italian university system, a possible extension of the constraint would be to add the possibility to receive a full scholarship from the government when parental income $Y_{pi,t}$ does not exceed a certain threshold c:

$$Y_{pi,t} < c$$

In this case, the cost of education would be totally funded by the government:

$$Costedu = 0$$

Scholarships to fund tertiary education studies are provided by regional bodies devoted to study rights and they can include up to total exemption from tuition fees, daily meals at the university restaurant, free accommodation in a student dorm or a contribution for rent payment. At the entrance at university, the scholarship awarding is generally only income-based and requirements are usually related to the level of ISEE, an indicator for the economic situation of households. Reductions or total exemption from tuition fees could also be offered by the university itself depending on the level of ISEE as well as on the final grade obtained with a secondary school diploma.

Basically, *junior* agents observe their *senior* neighbours' income and simply take the average. Given (2.2) and (2.3), the agent will compare the expected consumption in the two cases by taking the natural logarithm of their ratio:

$$ln\left(\frac{C^e_{is,t+1}}{C^e_{iu,t+1}}\right) = ln\left(\frac{X_{i,t} - CostEdu + Y^e_{is,t+1}}{X_{i,t} + Y^e_{iu,t+1}}\right) \tag{2.5}$$

2.3.5 Preference for enrolling

Juniors elaborate their preference for enrolling or utility as an additive function depending on three factors: (i) a *materialistic* term represented by expectations on consumption, which represent the economic motivation; (ii) a *social* term SI_{it} and (iii) an *intangible* term, given by the disutility for the effort of education:

$$P_{it} = ln\left(\frac{C^e_{is,t+1}}{C^e_{iu,t+1}}\right) + SI_{it} - EF \tag{2.6}$$

The first term, specified in (2.5), represents the economic motivation driving the preference for enrolment. Taking the natural logarithm of the ratio between the two type of consumption allows to approximately parametrise the resulting number in the range $(-1, 1)$.

SI_{it} represents *social influence*, given by the fraction of the number of peers within one's neighbourhood deciding to enrol over the total number of peers in the neighbour. Peers are assumed to be the agent's classmates and schoolmates, which are simultaneously faced with the choice to enrol or not at the university:

$$SI_{it} = \frac{Npeersenr}{N_{npeers}}, \tag{2.7}$$

The formalisation of the *SI* term implies that the higher the proportion of choices for continuing studying at tertiary level, the larger the impact on the agent's probability of also enrolling at University. By construction, values of SI will range from 0 to 1. SI can be considered as a measure of "educational conformism" which reflects a merely imitative behaviour. As explained in Manzo (2013), this type of behaviour has three sides: (i) on a cognitive level, the larger SI, the more predominant in cognitive terms the tertiary educational level becomes (Harding, Gennetian, Winship, Sanbonmatsu, & Kling, 2010); (ii) from the normative point of view, the larger the proportion of the agent's peers deciding to enrol at University, the higher the psychological costs in terms of individual and social identity the agent would have to bear in case of different choice (Akerlof, 1997; Akerlof & Kranton, 2002); (iii) in relation to opportunity, the larger SI, the higher the probability that the agent will have access to information and resources, such as study materials and notes, housing and transportation, during her university career, if the choice is the same.

The third term of equation (2.6), EF, stands for the effort necessary to obtain a university degree. It is assumed to depend only on individual ability through the following function:

$$EF = (1 - a_{it})^{\gamma} \tag{2.8}$$

where a_{it} measures individual ability and $\gamma > 0$ measures the concavity of the function: the larger is one's ability, the lower is the effort necessary to obtain a university degree. In particular, depending on whether γ takes values lower, equal or larger than 1, this parameter signals the presence of *returns to scale*:

- when $\gamma < 1$ returns to scale are *decreasing*, meaning that with larger levels of ability, the effort decreases less than proportionally;
- when $\gamma = 1$ returns to scale are *constant*, which means that when ability increases, the effort proportionally decreases;
- when $\gamma > 1$ returns to scale are *increasing*, i.e. the larger the value of ability, the lower the effort, with a more than proportional decrease.

As in Staffolani and Valentini (2007), a_{it} is assumed to be included between 0 and 1; therefore, the term EF will assume values in the range $(0, 1)$.

By construction, the function for the preference for enrolling allows considering solely the economic motivations guiding the enrolment decision and/or the intangible motivations identified in the rest of the equation.

Based on the values assumed by the terms defining this function, the preference for enrolment can assume values in the range $(-2, 2)$.

Junior agents enrol at university with a probability increasing in the level of preference P,

$$Pr_{enr} = f(P_{it}). \tag{2.9}$$

Building on Manzo (2013), this could be written as:

$$Pr_{it}(enr) = \frac{exp(P_{it})}{1 + exp(P_{it})} \tag{2.10}$$

meaning that the decision to enrol at university is assumed to be a monotonically non linear increasing probabilistic function in the level of preference.

2.4 Simulating the model

2.4.1 Initialisation and calibration procedure

The model has been implemented in NetLogo. NetLogo is an open-source programming environment for simulating natural and social phenomena. It was authored by Uri Wilensky in 1999 and it has been in continuous development at the Center for Connected Learning and Computer-Based Modelling (CCL) at Northwestern University. It is particularly suitable for modelling complex systems which evolve over time and for applications of agent-based models.

Its interface is provided with an area called *world*, that is a two-dimensional grid of "patches"; each patch is a square piece of "ground" over which agents can move. Thus, the NetLogo world allows visualising agents and their movements in the space. Figure 2.3 provides a snapshot of the NetLogo world of the model at its initialisation.

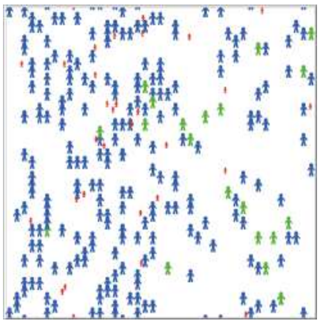


Figure 2.3: The NetLogo world of the model at its set-up.

At the set-up 250 senior agents are created, each one with an age assigned randomly starting from 21. Agents aged between 30 and 40 will *hatch* a child, that is a junior agent, according to a probability named *birth rate* fixed to 0.4. The size identifies the generational group: agents with a bigger size are seniors, while the small agents are juniors. The colour identifies whether the agent has undertaken a university degree course or not: green agents are skilled, blue agents are unskilled; juniors are all red.

The model is initialised with a proportion of skilled agents over unskilled ones based on the Survey on Household Income and Wealth (SHIW provided by the Bank of Italy and published every two years. We have considered waves from 2002 to 2016 and obtained an average proportion of 8.99% individuals owning a university degree on the total number of observations, that has been approximated to 9% for the model set-up.

To take into account the social context, agents are not randomly located in the world space, but they are placed according to a certain level of initial *segregation*, so to reflect the possibility that individuals with similar educational level will tend to have friends and acquaintances with their same educational level. This is modelled by exploiting the construction of the world. The NetLogo world is in fact built on a system of coordinates; exactly in the middle, we find the (0,0), meaning that we can imagine splitting the space vertically in two halves, separated by the y-axis.

Given this property of segregation, and assuming as a convention that the right half is the one where skilled agents are segregated, and the other way around for the left half, let us define a segregation index $p_s \in [0, 1]$.

At the initialisation of the model, a random number x_i is assigned to each senior agent i, with x_i drawn from a standard uniform distribution.

Each agent i will be located according to the following rule: if x_i is lower than p_s, agent i is assigned to her area of segregation, if x_i is higher than p_s, agent i tosses a coin to decide to which side she will be assigned.
Formally we have:

$$\begin{cases} p_i = 1 & \text{if } x_i < p_s \\ p_i = \frac{1}{2} & \text{otherwise} \end{cases}, \tag{2.11}$$

where p_i is the probability that agent i is assigned to her area of segregation, i.e. the probability to be assigned to the right half if i is skilled, or the probability to be assigned to the left half if i is unskilled.

Therefore the probability that agent i is assigned to her area of segregation is equal to:[6]

$$p_i = \frac{1}{2} + \frac{1}{2}p_s. \tag{2.12}$$

Thus p_s can be considered as a measure of the level of segregation in society. The higher p_s, the higher the probability that a skilled agent is placed on the right side, and that an unskilled agent is placed on the left side.[7]

As mentioned in par 2.3.2, agents have all the same level of social reach, that we fix to 10. Distance is measured from the centre of the patch where the agent is located and its units represent the number of patches. This means that a social reach equal to 10 implies that all the agents located within a radius of 10 patches from *ego* will be her neighbours. Therefore, just as in real life, the size of the personal network will be different for every agent.

When juniors are *hatched*, by default they inherit the same location of their parent, thus when they are just born, they are placed on the same patch than their parent. In order to differentiate the neighbours of parent and child, we let the juniors move away from their parent agent of a certain number of steps that we fix to 7. In this case, as well, the spatial units are represented by patches of the NetLogo world.

As mentioned in Section 2.3, we assigned a weight w to the relationships that agents have with parents. This weight is built by adding a number drawn from $U(0, 1)$ to a unit, meaning that parental effect can account more than a unit, to a maximum of 2, i.e. double than a standard relationship.

The cost of education is set on the basis of data from the 2017 report of Federconsumatori on the expenses linked to Italian universities.[8] The average estimates for Italian resident and non-resident university students per year are listed in the following table.[9]

Values reported in table 2.1 include costs for accommodation (only for non-resident students), bills, transportation, tuition fees, handbooks and study material. Starting from these values, the cost of education has been approximated to 5000€ per year and multiplied by 3 to consider the full length in years of an undergraduate degree.

As briefly touched in section 2.3, the ability has a common distribution for all the agents populating the model. We assume it to be distributed as a normal, as in

[6]Before the extraction of x_i the probability that agent i is assigned to her area of segregation is equal to

$$p_i = 1 \cdot prob(x < p_s) + \frac{1}{2} \cdot prob(x_i > p_s),$$

given that $x_i \sim U(0, 1)$, it follows that $prob(x < p_s) = p_s$ and $prob(x > p_s) = 1 - p_s$ from which equation (2.12) follows.

[7]Notice that if $p_s = 1$ then $p_i = 1$, meaning that all skilled agents are on the right, and all unskilled agents are on the left, therefore there is complete segregation. If $p_s = 0$ then $p_i = \frac{1}{2}$ meaning that each agent has the same probability of being on the left or on the right, therefore there is no segregation.

[8]Federconsumatori is a non-profit association aimed at informing and protecting consumers.

[9]Income ranges are defined on the basis of the ISEE level. II range includes ISEE equal or less than 10000€ while the III range includes ISEE up to 20000€.

	Non-resident students	Resident students
II income range		1425.63
in single room	9415.7	
in shared room	7944.1	
III income range		1668.82
in single room	9658.89	
in shared room	8187.29	

Table 2.1: Average yearly expenditure for Italian university students, for different income ranges and, if non-resident, for renting a single or a shared room. Source: Federconsumatori.

Breen and Goldthorpe (1997), with $\mu = 0.5$ and $\sigma^2 = 0.1$, obtaining values included between 0 and 1.

To each senior agent is assigned a certain level of income, randomly drawn from a distribution that we assume to be lognormal.[10] The lognormal distribution is a typical example of heavy-tailed distribution.

Figure 2.4 shows the profile of the personal income distribution for the year 2012, displayed on a log-log plot: we take the horizontal axis as the logarithm of the income in euro and the vertical axis as the logarithm of the complementary cumulative distribution function (*ccdf*).

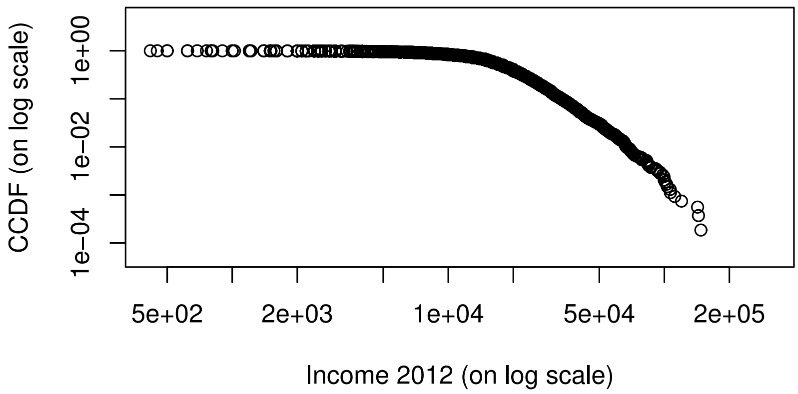


Figure 2.4: Log-log scaled CCDF of the Italian personal income in 2012. Data source: SHIW 2012, Bank of Italy.

[10]The probability density function (pdf) of a lognormal distribution is the following:

$$p(x) = \frac{1}{x\sigma\sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{logx - \mu}{\sigma}\right)^2\right], x > 0$$

where the location parameter $\mu \in \mathbb{R}$ and the shape parameter $\sigma > 0$ are the mean and the standard deviation of the normal distribution.

The complementary cumulative distribution function is the probability to find a person with an income greater than or equal to x:

$$P(X \geq x) = \int_{x}^{\infty} p(t)dt \tag{2.13}$$

The lognormal distribution has been widely employed in economics. For instance, Gibrat (1931) identified a lognormal distribution in the distribution of the size of French manufacturing firms.

Aitchison and Brown (1954, 1957) observed that the lognormal type of distribution is particularly appropriate for the distribution of incomes, especially for the bulk of earnings from a homogeneous section of the workforce. Clementi and Gallegati (2005), using micro-data from SHIW for the years 1977–2002, find that the central body of Italian incomes distribution is consistent with a lognormal model.

Using annual data from SHIW, for the years 2002-2016, we have split each dataset in two, separately analysing the income distribution for skilled individuals, identified as those individuals owning a university degree of any kind, and unskilled individuals, those not holding a tertiary education degree.

In order to be consistent with what observed by Aitchison and Brown (1954, 1957), we only restricted the fit to earnings from employees,[11] dropping observation for self-employed.[12] To avoid biased estimates, we took into account the sampling weights provided by the survey and whose adoption is recommended by the Bank of Italy.

For each dataset and for each year considered, we have fitted the 2 parameters lognormal distribution by maximum likelihood (ML), obtaining estimates for the two parameters μ and σ for the distribution of skilled and the distribution of unskilled workers, over the 8 years considered.

Given that incomes are compared across years, in order to remove the inflation effect, data are converted to 2010 prices using the Istat household consumption deflator included in the SHIW documentation of the historical archive. All the estimates can be found in Appendix 2.A.

To account for changes over time, we then averaged the parameters, obtaining the following results:

Therefore, a certain level of income is randomly extracted for skilled and unskilled workers, from two distributions of the same type, identified by different parameters.

The individual income is then multiplied by the average number of expected years of working life, that we fix to 32 according to Eurostat.[13]

Finally, on the basis of the income attributed to senior agents, the endowment $X_{i,t}$ available to their children is computed as a portion of that income; this portion is proxied by the average propensity to save of Italian households computed by Istat and fixed to 9%.[14] This choice comes from the nature of the variable endowment; it

[11] Earnings are net of tax and social security contributions.

[12] According to Brandolini and Cannari (1994), the SHIW underestimates self-employment income by about 50%.

[13] The value is derived from the estimated duration of working life in years for a person who is 15 years old in 2017, by country, provided by Eurostat.

[14] This information is available for every quarter. We approximately averaged the propensity to save over the last two years.

	$\hat{\mu}$		$\hat{\sigma}$	
	S	U	S	U
2002	10.27	9.67	0.79	0.84
2004	10.20	9.62	0.89	0.90
2006	10.13	9.63	0.89	0.83
2008	10.04	9.53	0.88	0.87
2010	10.09	9.48	0.79	0.99
2012	9.51	9.18	0.79	0.95
2014	9.59	9.24	0.96	1.00
2016	9.90	9.34	0.84	1.01
Mean	9.97	9.46	0.85	0.92

Table 2.2: Average parameters estimates for a lognormal distribution of Italian incomes for skilled and unskilled individuals over the years 2002-2016; data provided by the Bank of Italy (SHIW).

can be considered as a sort of bequest, rather than a consumption entry.

Table 2.3 summarises the inputs used for the model set-up.

Variable	Inputs and calibration
N. senior agents	250
N. steps	7
Social reach	10
Probability of segregation p_s	0.5
Proportion skilled/unskilled	9% (SHIW Bank of Italy)
Endowment	9% (Istat)
Cost of education	5000€ per year (Federconsumatori 2017)
Average working life	32 years (Eurostat 2016)
Ability	$\sim N(0.5, 0.1)$
Income distribution	Skilled $\sim Lognormal(9.97, 0.85)$
	Unskilled $\sim Lognormal(9.46, 0.92)$

Table 2.3: Variables' values used in the model set-up

2.4.2 Model dynamic

What has been described in the previous section is what happens to the model at its initialisation, that is at $t = 0$, where t represents the model time units. The model is then run for 100 periods of time. Starting from $t = 1$, junior agents aged 20,[15] which satisfy the budget constraint (2.1), observe their neighbours' income and shape their expectations on future earnings as in equation (2.4). Consequently, they form their expectations on future consumption according to (2.5) and eventually for each of them, their preference for enrolling is computed. These agents will then formalise

[15] This age approximately corresponds to the age in which Italian students complete secondary education and potentially start their university studies.

their decision to enrol or not at University, following (2.10). At that point, they ”grow up”: they turn their generational status into *senior*, they change size into the larger size characterising senior agents, and they turn their colour to green or blue, depending on their educational choice, becoming green if educated and blue if entering immediately the job market.[16] Together with that, they are assigned with a level of income drawn from the corresponding distribution; the income distribution for skilled if they decided to enrol at university; the income distribution for unskilled, if they decided not to study at university. These two distributions are those described in the previous sections and identified by the parameters reported in table 2.2. This process happens at each t.

In order to smooth and ease the population dynamic, We assume that each model's unit of time corresponds to 5 years; thus, passing from t to $t+1$, the age of agents is updated consequently. At each t, junior agents are born following the rule explained in Section 2.4.1, that is according to a certain probability called *birth rate* and fixed to 0.4. Like what happens in the model initialisation, juniors move 7 steps far away from their parent in order to differentiate their personal network. In addition, at each t, senior agents aged more than 50 die with a probability named *death rate* and fixed to 0.9.[17] When agents reach the age of 65 or more, they certainly die and leave the model. This dynamic of the population does not aim at mirroring the processes related to population happening in real life, yet in this model it is necessary to account for factors such as parental effect and social network influence.

In order to understand the model behaviour and give robustness to its results, we perform a Monte Carlo experiment, by simulating the model 100 times.[18] Box 2.1 summarises the steps of the simulation algorithm programmed to collect the outputs of interest through simulations.

[16]We assume that there is no drop-out, so that enrolling at the university certainly means obtaining the degree.

[17]The birth rate and death rate used in the model have the mere purpose to smooth the population dynamic; they are not meant to represent observable demographic variables.

[18]The number of periods of times and Monte Carlo experiments for which the model is simulated are set on the basis of the computational capacity of the computer employed for performing the simulations.

Box 2.1. The simulation algorithm

1. INITIALISATION
   (a) Population and agents' attributes
   - Create 250 *senior* agents
   - Set location according to segregation
   - Set seniors' size, colour, age, income, educational status
   - Let seniors *hatch* a junior according to birth rate
   - Set juniors' size, colour, age, endowment
   - Let juniors move 7 steps away from parents

   (b) Environment
   - Set social reach 10 to identify neighbours
   - Set a weight w for parents

2. MODEL DYNAMICS

   Iterate the model 100 t.
   For each t:

   (a) juniors aged 20 and satisfying the budget constraint:
   - observe income of neighbours and compute:
     * expectations on future income
     * expectations on future consumption
     * social influence
     * effort
     * preference for enrolling
   - make a decision:
     * enrol/do not enrol according to Pr_{enrol}
     * change consequently their education status
     * change generational status into senior
     * change size and colour
     * obtain a skilled/unskilled income

   (b) seniors *hatch* and die according to birth rate and death rate

   (c) set-up new juniors as in the initialisation procedure

   (d) update agents' age by adding 5 years

   (e) compute and update output variables

3. MONTE CARLO EXPERIMENT

   (a) Define output variables of which results must be collected

   (b) Run 100 experiments using the Behavior Space tool in NetLogo

2.5 Computational results

As anticipated in Section 2.1, the main output we are interested in is the enrolling rate, that is computed as the percentage of juniors aged 20 deciding to enrol over the total number of juniors with the same age.

In the second place, we will look at how many among the agents enrolling at university come from an educated or uneducated family, thus how many have a skilled or unskilled parent, in relative terms.

The model also allows to analyse outputs related to educational mobility, that is how many agents stay immobile with respect to their family educational status, how many go downward, that is they choose a lower educational level with respect to their parents, or upward, i.e. having an uneducated family, they decide to advance in the educational *ladder*.

This is reported in Box 2.2, which summarises the model mechanism and gives a simple and general overview of it.

Box 2.2. Overview of the model

Input	Income Cost of education	Proportion S/U Personal Network	Segregation Family effect
Model	Economic motivation	Social Influence	Effort
Output	% enrolling	% Enrol. from S % Enrol. from U	% Immobile % Upward % Downward

2.5.1 A single Simulation

In order to have a first impression of the model dynamics, the following graphs display the outputs of interest over $t = 100$ for one simulation ($MC = 1$).

We can already observe that the enrolling rate stabilises around slightly less than 60% and that among the agents enrolling at the university, the majority comes from an educated family, with a ratio of about 60/40 with respect to those deciding to enrol and coming from an unskilled family. Most of the juniors facing the educational choice eventually remain immobile with respect to their family of origin (around 50%); the rest is mainly moving downward in the educational levels with respect to their parents (around 30%) and the smallest percentage is choosing a higher educational level than their parents.

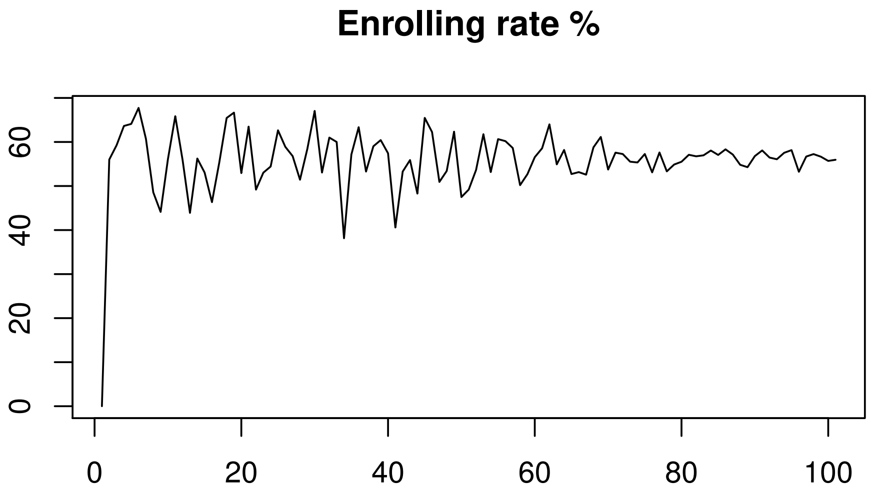


Figure 2.5: Enrolling rate for a single simulation.

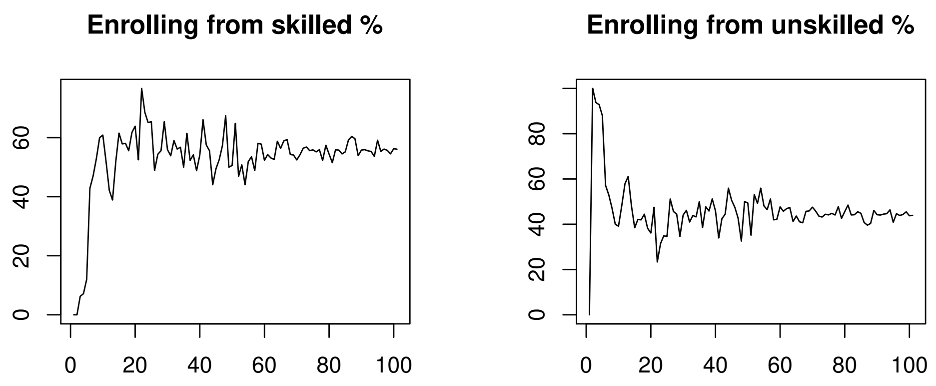


Figure 2.6: Enrolling rate given family background for a single simulation.

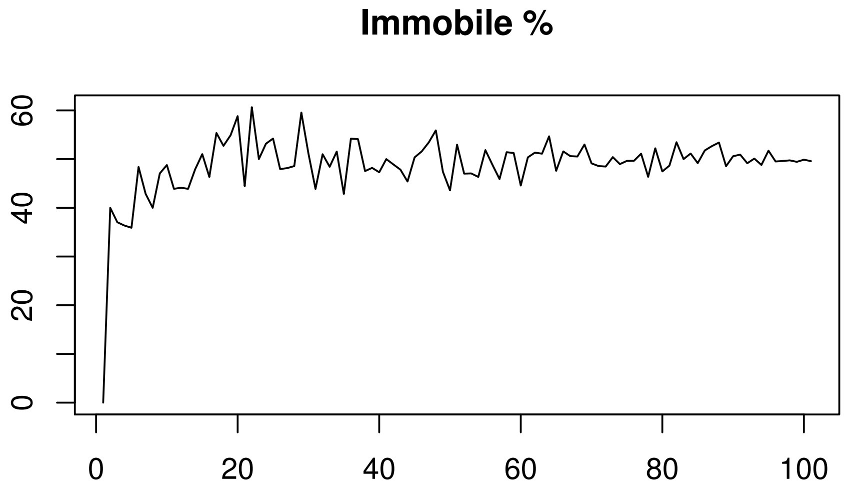


Figure 2.7: Social mobility for a single simulation: immobile.

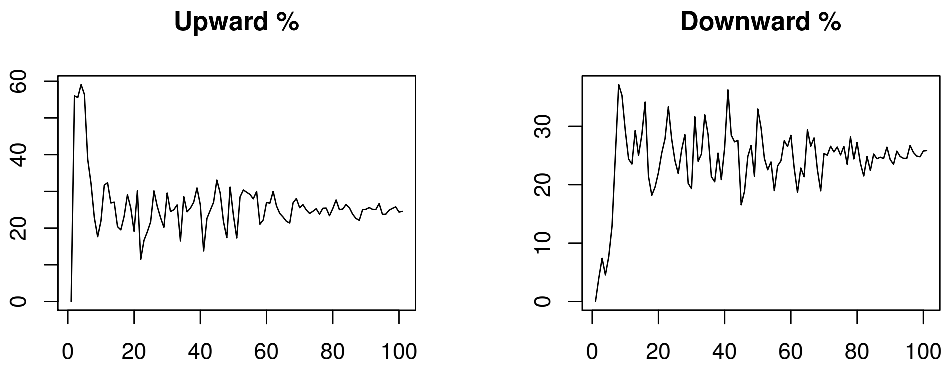


Figure 2.8: Social mobility for a single simulation: upward and downward.

2.5.2 Monte Carlo simulations

Moving to the analysis of the simulation experiment, table 2.4 lists the mean values and corresponding standard deviation of the variables of interest across the Monte Carlo simulations performed.[19]

Variable	Mean	SD
enrolling rate %	56.61	0.51
enrolling from skilled %	56.19	0.74
enrolling from unskilled %	43.81	0.74
Immobile %	50.44	0.42
Upward %	24.80	0.28
Downward %	24.76	0.30

Table 2.4: Average values and corresponding standard deviations across 100 simulations, calculated over the time span 20-100

The values obtained from the Monte Carlo experiment seem to confirm the preliminary overview given by a single simulation. In addition, the average rate of enrolment obtained from our computations is included in the range displayed in Table 2.5. This table reports the rate of transfer from secondary education to university of Italian students who have just obtained their secondary school diploma, for 6 years from 2008 to 2013. This index is provided by Istat and calculated using information from the Italian Ministry of Education, Universities and Research (MIUR). Its value has consistently decreased over the time span reported, that is the one for which this information is available.

Results deriving from our computations seem to better represent the index of the latest years, thus providing an updated representation of the transfer from secondary to tertiary education.

Year	Rate of transfer from secondary education %
2008	65.8
2009	63
2010	63.3
2011	61.3
2012	58.2
2013	55.7

Table 2.5: Rate of transfer of Italian students from secondary school to university, in percentage. Source: Istat

Values for the variables we are interested in are also displayed over the simulations time span, in order to compare Monte Carlo simulations and track their

[19]The initial period of the simulations (20 time units) is neglected as it represents a transient phase.

dynamics over the time span.

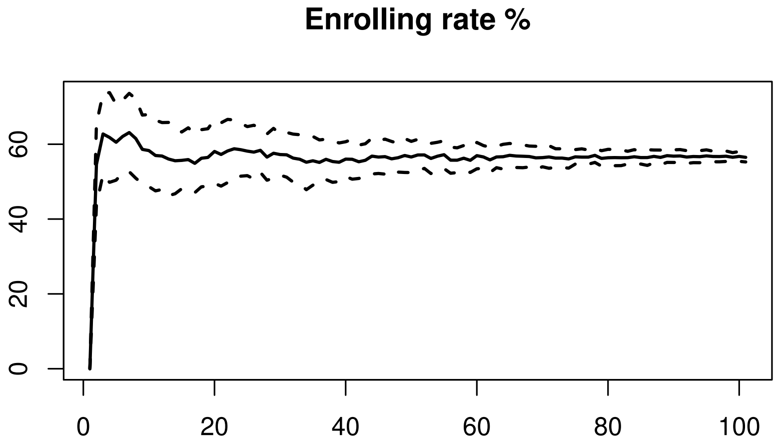


Figure 2.9: Average enrolling rate in percentage (*continuous line*) and standard deviation (*dashed line*) over the simulations time span

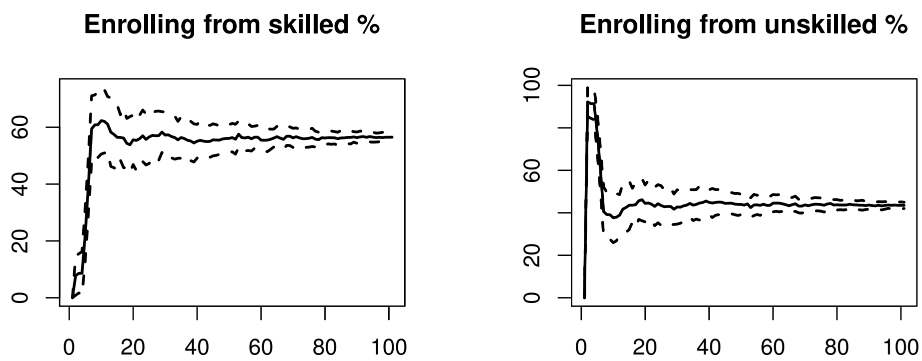


Figure 2.10: Average rate (%) of agents enrolling to university, coming from a skilled/unskilled family (*continuous line*) and corresponding standard deviation (*dashed line*) over the simulations time span

Again, we observe an enrolling rate between 50 and 60%, mostly composed of individuals with an educated family of origin. Looking at all the young agents at the moment of their educational choice, the half of them maintains the same educational level with respect to their parents, while the other half is equally composed by individuals improving or worsening the educational level in relation to their family of origin.

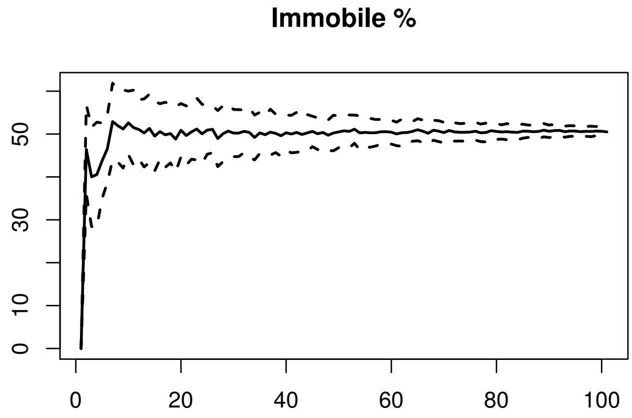


Figure 2.11: Average rate (%) of agents remaining immobile, across the education levels (*continuous line*) and corresponding standard deviation (*dashed line*) over the simulations time span

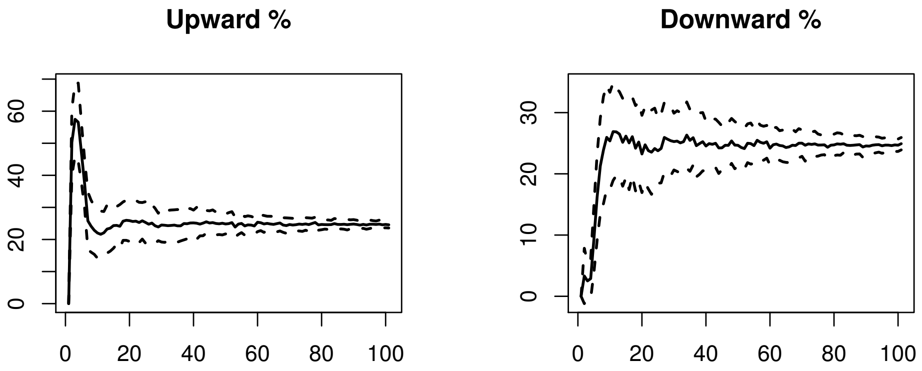


Figure 2.12: Average rate (%) of agents moving upward or downward across the education levels (*continuous line*) and corresponding standard deviation (*dashed line*) over the simulations time span

2.6 Sensitivity analysis

The model is characterised by the presence of only one parameter, $\gamma > 0$, that, as discussed in Section 2.3, rules the concavity of the relationship between individuals' ability and effort and indicates the presence of *returns to scale*.

In order to understand whether and to what extent this parameter affects the results of the model, we carry out a sensitivity analysis by performing a "parameter sweep" of γ, that is by repeating the Monte Carlo experiment for different values of γ. In addition to the value of 1.2 assigned to the parameter in the *benchmark* model, we perform a sensitivity experiment on the following 4 values: 0.5, 1, 1.6, 1.8.

The results of the sensitivity analysis are displayed in Figure 2.13 for the outputs investigated so far. The figures show the mean and standard deviation of the variables of interest in the various sensitivity scenarios. These values are reported in Table 2.6.

		Values of γ				
		0.5	1	1.2	1.6	1.8
enrolling rate%	Mean	48.95	54.70	56.61	59.36	60.38
	SD	4.59	4.07	4.67	3.89	3.74
Enr. from skilled %	Mean	47.66	53.08	56.19	57.80	58.91
	SD	7.23	6.51	5.25	5.83	5.64
Enr. from unskilled %	Mean	52.34	46.92	43.81	42.20	41.09
	SD	7.23	6.51	5.25	5.83	5.64
Immobile %	Mean	49.37	49.42	50.44	50.54	50.90
	SD	3.92	3.91	3.22	4.24	4.22
Upward %	Mean	25.63	25.67	24.80	25.07	24.83
	SD	4.41	4.16	3.86	4.00	3.93
Downward %	Mean	25.00	24.90	24.76	24.39	24.27
	SD	3.16	3.00	3.43	3.26	3.34

Table 2.6: Sensitivity analysis on γ

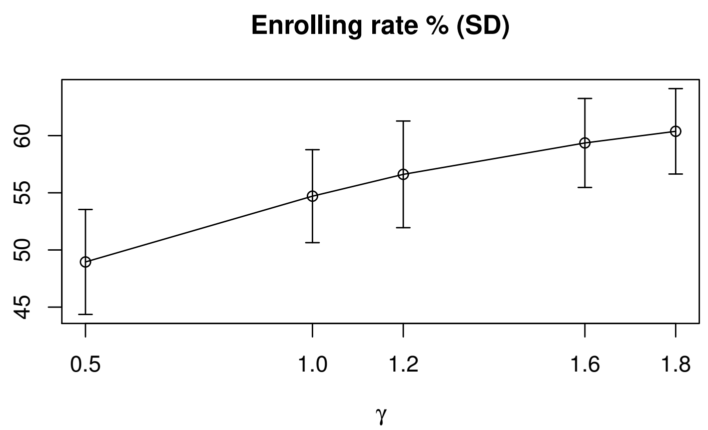


Figure 2.13: Synthetic statistics for the sensitivity experiment performed on γ: enrolling rate

The previous pictures show that the model appears to be sensitive to changes of the parameter γ. The enrolling rate seems to be in an increasing relation with the parameter: increasing the value of γ causes higher levels of the enrolling rate, reaching an average value of 60.38% for $\gamma = 1.8$, while its volatility is not particularly affected. This is consistent with the theoretical predictions of the model, given that for $\gamma > 1$ increasing returns to scale reduce more than proportionally the disutility

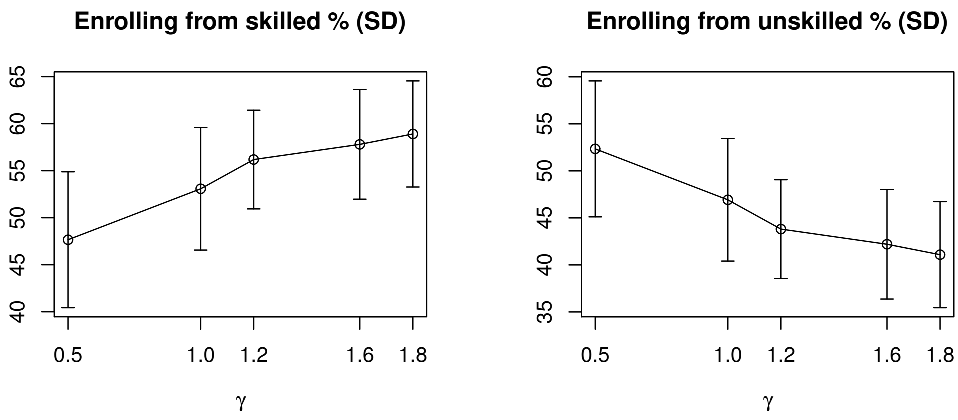


Figure 2.14: Synthetic statistics for the sensitivity experiment performed on γ: enrolling rate from skilled/unskilled

of pursuing a university career in term of effort, thus stimulating the decision to enrol at the university. Consistently, when γ takes values lower than 1, the enrolling rate shows lower levels (47.66%), as for higher levels of ability the effort decreases less than proportionally, thus producing a weaker stimulus on the preference for enrolling.

Looking at the composition of individuals deciding to continue their studies, it seems to be affected by changes in the parameter γ. Lower levels of the parameter cause a reduction in the percentage of agents deciding to enrol coming from a skilled family, while larger values of γ seem to push enrolment from individuals with an educated family. This behaviour follows the trend of what we have observed for the enrolment rate. Obviously, the contrary happens for the children of unskilled parents. We can attribute this effect to the bigger (smaller) proportion of educated (uneducated) individuals populating the model, and thus having children which in turn will grow up and have children again, and so on. This reinforces the social influence coming from peers and, in addition, the bigger (smaller) proportion of educated (uneducated) agents, which will become parents, produce an influence through the family effect mechanism, modelled by the heavier weight attached to parental relationships.

Curiously, the volatility of these two variables is higher for $\gamma < 1$ (7.23), decreasing when the parameter takes larger values, up to the level of 1.2 used in the benchmark model, starting from which the system volatility becomes more stable. The trend of the volatility in these variables obviously evolves in the same way, since the variables are complementary.

Adjustments in the level of γ do not affect much transfers across education levels with respect to the family of origin. The same applies to the volatility of the system, given that the standard deviation of immobile, upward and downward agents remains more or less unaffected.

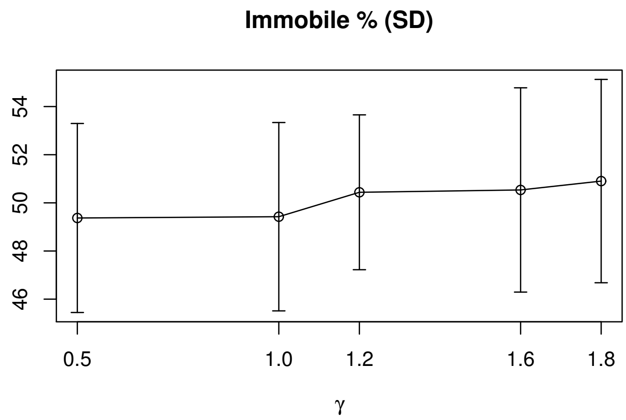


Figure 2.15: Synthetic statistics for the sensitivity experiment performed on γ: immobile

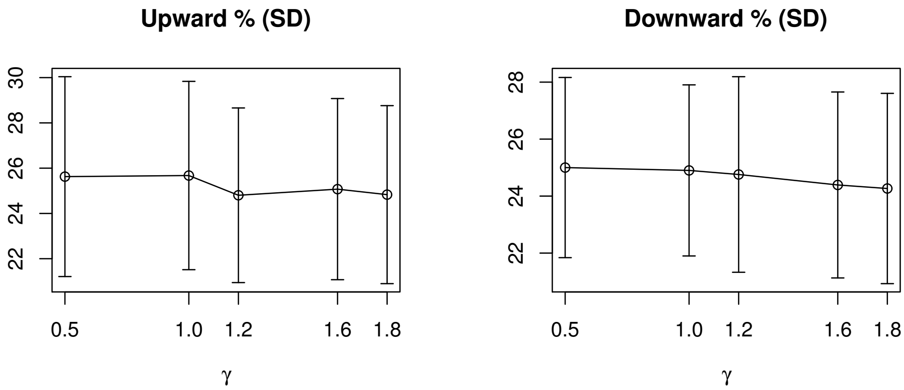


Figure 2.16: Synthetic statistics for the sensitivity experiment performed on γ: upward/downward

2.7 Discussion

The model developed in this work explores the factors influencing educational choices at the tertiary level. It takes into account both economic and social motivations, approaching the economics of education with an innovative methodology and a multidimensional perspective.

Junior and *senior* agents populate the model, where they interact with each other. Juniors who have just obtained their secondary school diploma, are faced with the decision to continue their studies at the university or drop out and immediately enter the labour market. The probability to enrol is increasing in the level of the preference for enrolling, which depends on three main elements: economic reasons, social influence from peers and immaterial cost of obtaining a university degree. These terms take into account the educational segregation of agents at the initialisation of the model, family effect, level of ability and, most importantly interaction. Juniors' expectations on future income are modelled as naive expectations based on the information that they obtain from their neighbours, that is their level of income. The simulation shows that the average enrolling rate in percentage terms is about 56%; when compared with actual data on the transfer rate of Italian students from secondary to tertiary education, the model seems to be able to reasonably reproduce the system. Monte Carlo simulations confirm the robustness of the results. The model also shows that among those deciding to continue their studies, the majority, although slight (56.19%) comes from parents owing a university degree. Considering all the individuals faced with the educational decision, about half of them will maintain the same education level than their parents, while the rest will evenly advance or remain behind the education level of their family.

A sensitivity analysis has been carried out on the parameter governing the concavity of the relationship between individual ability and effort to obtain a university degree. Results are consistent with the theoretical prediction of the model, showing the presence of increasing returns to scale for values of the parameter larger than 1. In this case, when the level of ability increases, the effort of studying diminishes more than proportionally, and in this way it triggers the preference for enrolling, pushing the enrolling rate up to 60%. This effect is then reflected also in the composition of those enrolling. A larger (lower) proportion of educated (uneducated) individuals populating the model stimulate the family effect and the social influence from peers, resulting in an increasing proportion of agents coming from skilled families rather than unskilled, for larger levels of the parameter, and vice-versa for lower levels of the parameter.

Our modelling framework showed that by taking into account the ability of agents to interact and adapt their choices to their neighbours, it is possible to reproduce features mirroring reality. The model provides a useful framework for studying the decision-making process about education and it can be used as a basic tool for further investigating this socio-economic phenomenon. Future developments could enrich the model introducing, for example, heterogeneity in the level of ability or by adding modules such as universities receiving students, and a geographical environment, allowing for a spatial analysis. Another possibility would be to relax the assumption on drop-out and also consider the option of abandoning tertiary education, without completion of a degree. In this case, the focus could move on the analysis of drop-out rates, which also represents a challenge in the Italian education system. Another possibility about further developments includes a calibration experiment that, for instance, could provide an alternative distribution for income. The framework presented could also be employed as a tool for understanding the effect of policies, such as the reduction of tuition fees or the introduction of scholarships. Finally, starting from this scheme, one could adapt the model to different educational systems, so to have the possibility to analyse the decision-making process concerning education in other countries.

Appendix

2.A Estimations results

This section provides the estimations results of the fit of a 2 parameter lognormal distribution to Italian personal income, as reported in Table 2.2.[20] Estimations have been performed in Stata 14 using the package `lognfit`.

Table 2.A.1 reports the estimation of the parameters μ and σ for all the waves of SHIW considered for the calibration of the income distribution, specifically the years 2002, 2004, 2006, 2008, 2010, 2012, 2014 and 2016. Given the necessity to refer to individuals as the unit of observation, rather than households, we have used data provided in the annual databases. Each database has been split in two based on the education status of individuals, by means of the variable *studio*, which measures with progressive numbers (from 1 to 8) the level of the degree held by the individual, starting with no education up to a postgraduate qualification.

	Skilled			Unskilled		
	μ	σ	N	μ	σ	N
2002	10.27*** (0.0228)	0.791*** (0.0161)	1205	9.671*** (0.00739)	0.837*** (0.00523)	12281
2004	10.20*** (0.0248)	0.889*** (0.0176)	1283	9.625*** (0.00802)	0.901*** (0.00567)	12634
2006	10.13*** (0.0243)	0.894*** (0.0172)	1356	9.626*** (0.00752)	0.826*** (0.00531)	12063
2008	10.04*** (0.0231)	0.881*** (0.0163)	1457	9.533*** (0.00789)	0.872*** (0.00558)	12220
2010	10.09*** (0.0193)	0.788*** (0.0137)	1664	9.479*** (0.00907)	0.995*** (0.00641)	12040
2012	9.513*** (0.0188)	0.787*** (0.0133)	1750	9.184*** (0.00873)	0.951*** (0.00617)	11880
2014	9.591*** (0.0230)	0.964*** (0.0163)	1752	9.245*** (0.00921)	0.998*** (0.00651)	11759
2016	9.896*** (0.0217)	0.843*** (0.0154)	1506	9.344*** (0.00998)	1.015*** (0.00706)	10340

Table 2.A.1: Estimates of the parameters μ and σ of a lognormal distribution

[20] Significance levels are indicated as follows: $^{*}p < 0.05,^{**} p < 0.01,^{***} p < 0.001$; standard errors are reported in parentheses.

Chapter 3

A calibration experiment of an Agent-Based model: fitting the Italian income distribution

3.1 Introduction

Agent-Based Models (ABM) have been widely adopted in recent years. Their applications can be found in many fields, including economics and the social sciences (see for example Gatti, Gallegati, Giulioni, & Palestrini, 2003; Gatti, Desiderio, Gaffeo, Cirillo, & Gallegati, 2011; and Bravo, Squazzoni, & Boero, 2012).

Their purpose is to study the behaviour of several heterogeneous interacting agents. ABMs are characterised by a bottom-up approach, that is by establishing behavioural rules at the single agent level, emergent behaviours arise through interactions so that they could not be inferred from the individual level.

ABMs have proven to be able to reproduce economic and social features, observable in the real world (see Cont, 2001; Manzo, 2013). Given this useful property, calibration and validation can play a major role, yet issues related to these phases of the modelling process have only started to be considered recently (Bianchi, Cirillo, Gallegati, & Vagliasindi, 2007; Sargent, 2009; Fabretti, 2013).

Calibration and validation are strictly linked to each other so that they often overlap. Tesfatsion (2006) defines empirical validation as the process of ensuring that an agent-based model is consistent with empirical data. She identifies four aspects related to empirical validation:

1. Input validation
2. Process validation
3. Descriptive output validation
4. Predictive output validation

This work deals with the input validation, which requires that the exogenous inputs of the model are empirically meaningful and appropriate for the purpose of the investigation. With exogenous model inputs, we mean for examples parameter estimates, functional forms, random shock realisations.

Richiardi, Leombruni, Saam, and Sonnessa (2006) instead refer to parameter estimation indifferently simply with the term *estimation* or calibration. They define this phase as the process of choosing the values of the parameters that maximise the accordance of the model's behaviour with the real-world system.

This Chapter performs a calibration experiment on the ABM developed in Chapter 2. The model has been used to explore the determinants behind university enrolment decisions in Italy and has proved to be able to realistically represent the Italian rate of transfer from secondary to tertiary education. In this model, the income distribution of agents has been assumed to be a two-parameter lognormal. Two distributions of this kind have been used to account for income of educated individuals and for uneducated ones. The parameters identifying the two distributions have been estimated by fitting the distribution of personal income derived from the Survey on Household Income and Wealth (SHIW) provided by the Bank of Italy, for the time span 2002-2016, including 8 waves in total .

The aim of this work is to provide a more accurate representation of Italian personal income in the calibration of the ABM developed in Chapter 2. With this purpose, using the distribution of personal income from the same datasets, for all the waves considered, we fit to the data three Beta-type distributional forms: the generalised beta distribution of the second kind (hereafter referred to as GB2), the Singh–Maddala distribution and the Dagum distribution. We then compare the estimation results by testing for the best fit by means of the likelihood ration (LR) test and the AIC and BIC measures for comparing nested models.

With the exception of the wave 2016, for which estimation results show some anomaly, the GB2 is the best fit in terms of likelihood in all the other cases. These distributions are nested distributional forms belonging to the family of Beta distributions, where the Singh–Maddala and the Dagum distributions can be considered sub-cases of the GB2.

Therefore, we assume that the income distribution in our model takes the form of a GB2, identified by four parameters. As in Chapter 2, we average the estimates for the parameters identifying the distribution for skilled and unskilled agents. We then run the model for 100 periods of time and perform Monte Carlo simulations.

Results are slightly different than those obtained from the benchmark model in Chapter 2, with an enrolling rate of 3 percentage points lower than the previous result (53.61%). This is in line with the decreasing trend shown by the aggregate data on the rate of transfer of Italian students from secondary school to tertiary education, provided by the Italian Institute of Statistics (Istat) . This rate is available for 6 years from 2008 to 2013; it decreases year by year, starting with a level of 65.8% and diminishing to 55.7% in 2013.

The composition of individuals enrolling at university changes of about 2% with respect to results from the reference model, with a lower rate for those coming from skilled parents and vice-versa for individuals from uneducated families.

Finally, educational mobility is rather invariant to changes in the calibration of income distribution.

The remainder of the work is organised as follows. In Section 3.2 we briefly review the three distributions adopted in the work and explore the literature that employed these functional forms. Section 3.3 describes the process of fitting the distributional models to the survey data and testing their goodness-of-fit. Section 3.4 reports the results obtained from the simulation of our reference model, simulated under the new conditions. Finally, Section 3.5 draws some final remarks and leaves input for future research.

3.2 Beta-type distributions

The distributions used in this work belong to the family of the Beta-type distributions. An essential distribution in this group is the generalised beta of the second kind (GB2). The GB2 distribution has been developed as an income distribution starting from the contribution of McDonald (1984).

It is a four-parameter distribution defined over the support $(0, \infty)$, and obtained by transforming a standard beta random variable defined on $(0, 1)$. Its probability density function (pdf) is given by:

$$f(x|a, b, p, q) = \frac{ax^{ap-1}}{b^{ap}B(p, q)\left(1 + \left(\frac{x}{b}\right)^a\right)^{p+q}}, x > 0 \tag{3.1}$$

where $B(p, q) = \int_0^1 t^{p-1}(1-t)^{q-1}dt$ is the beta function. $a > 0$, $b > 0$, $p > 0$, $q > 0$ are the four parameters identifying the distribution, where b is a *scale* parameter, which stretches and squeezes the distribution, and a, p, q are the *shape* parameters, affecting the shape of the distribution.

The corresponding cumulative distribution function (cdf) is the following:

$$f(x|a, b, p, q) = \frac{1}{B(p, q)} \int_0 wt^{p-1}(1 - t)^{q-1}dt \tag{3.2}$$

where

$$w = \frac{\left(\frac{x}{b}\right)^a}{\left[1 + \left(\frac{x}{b}\right)^a\right]} \tag{3.3}$$

Equation (3.2) equals $B = (w|p, q)$, which is the cdf for the normalised beta distribution, defined on the $(0, 1)$ interval, with parameters p and q, and evaluated at w.

An extensive overview of the properties of the GB2 distribution can be found in Kleiber and Kotz (2003), which devote an accurate description to its special cases. Among those, we will use two sub-cases that have been popular in literature: the three-parameter distributions Singh-Maddala and Dagum.

The Singh–Maddala distribution corresponds to the case of the GB2 distribution, with $p = 1$. It has been introduced Singh and Maddala (1975, 1976), thus before the GB2 distribution.[1] Nevertheless, as mentioned, it can be considered as a special case of the GB2. Its pdf is:

$$f(x) = \frac{aqx^{a-1}}{b^a[1+(x/b)^a]^{1+q}}, x > 0 \tag{3.4}$$

with a, b, q all positive.

The other case, the Dagum distribution, has been introduced as an income distribution only one year after the Singh-Maddala by Dagum (1975, 1977). It is a GB2 distribution with the shape parameter $q = 1$. Therefore its density is:

$$f(x) = \frac{apx^{ap-1}}{b^{ap}[1+(x/b)^a]^{p+1}}, x > 0 \tag{3.5}$$

Figure 3.1 displays a parametric distributions tree for Beta-type distributions and their interrelations, including the GB2 and its nested models Singh-Maddala and Dagum.

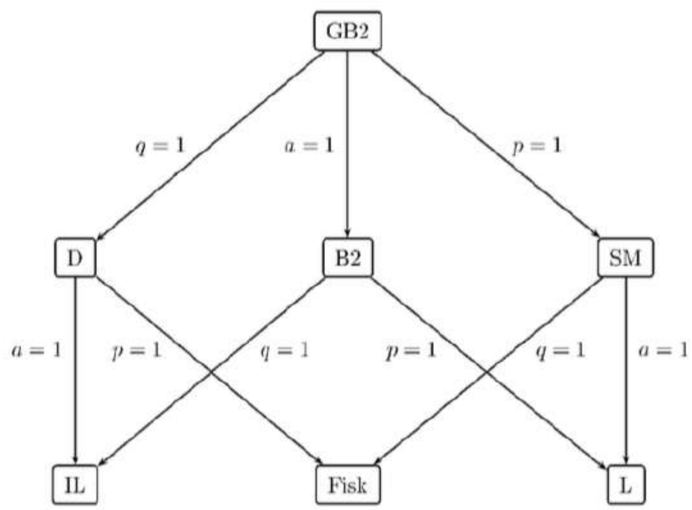


Figure 3.1: Parametric distributions tree of Beta-type distributions and their connections. In addition to the distributions reviewed in this Chapter -generalised beta distribution of the second kind (GB2), Dagum distribution (D), Singh–Maddala distribution (SM)- the figure also displays the beta distribution of the second kind (B2), the inverse Lomax distribution (IL), the Fisk distribution (Fisk) and the Lomax distribution (L). Source: Kleiber and Kotz (2003, p.188)

Since their discovery, these distributions have been widely employed as income distributions.

McDonald and Xu (1995) studied 1985 U.S. family incomes, considering 11 distributions, mainly of the gamma and beta type, and the GB2 ranked first in terms of likelihood. Brachmann, Stich, and Trede (1996) fit the GB2 and some of its subcases

[1]This distribution has been independently rediscovered in a variety of different field; therefore it is known under many different names. Apparently, it was first considered by Burr (1942). See Kleiber and Kotz (2003) for details about the origin of the Singh-Maddala distribution.

to individual household incomes from the German Socio-Economic Panel (SOEP) for the years 1984–1993. They found that only the GB2 and the Singh–Maddala distribution are satisfactory. Chotikapanich, Griffiths, Hajargasht, Karunarathne, and Prasada Rao (2018) review estimation methodology for the GB2 and summarise expressions for inequality, poverty, and pro-poor growth from GB2 parameter estimates. They provide an application to data from China and Indonesia. Botargues and Petrecolla (1997) estimated the Singh–Maddala model for the income distribution in the Buenos Aires region, for the years 1990–1996. They found that the Dagum distribution outperform the Singh-Maddala. Other applications include Graf and Nedyalkova (n.d.), which evaluate the performance of different fitting methods for a GB2 using the European Union Statistics on Income and Living Conditions (EU-SILC) survey. Quintano and D'Agostino (2006) model income distribution using a Dagum distribution; they focus on the inequality of income distribution among single-person households in France, Germany, Italy and the U.K., and study poverty differences starting from the results of Biewen and Jenkins (2005). The latter analyse poverty differences using the Singh-Maddala and Dagum distributions, with parameters as functions of personal household characteristics.

Finally, using data from ten countries, Hajargasht and Griffiths (2013) compare the goodness-of-fit of the double Pareto-lognormal distribution with the GB2, finding a better performance of the GB2 in 6 cases out of 10.

3.3 Fitting the Italian income distribution

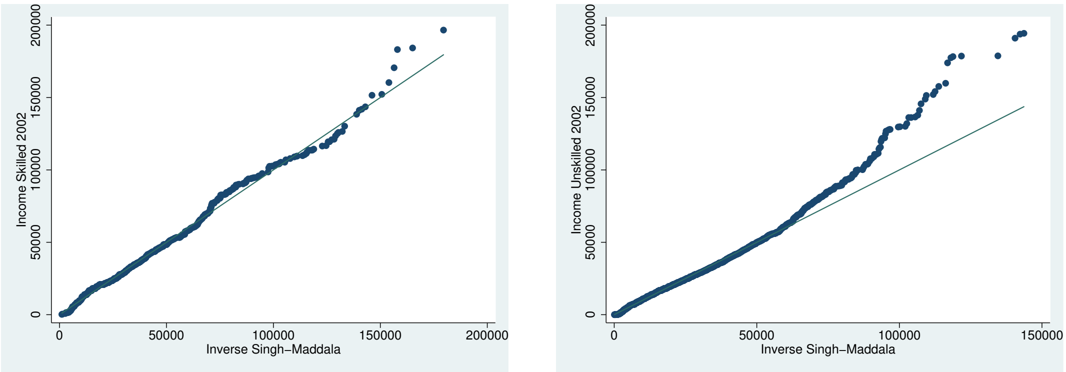
We have employed data on personal income[2] provided by the Survey on Household Income and Wealth, published by the Bank of Italy, for 8 waves, from 2002 to 2016. As in Chapter 2, the annual archive has been preferred to the historical dataset in order to obtain information on each individual observed. Each dataset has then been divided in two new datasets, based on the education degree held by the individuals; thus we defined a dataset for skilled individuals and another one for those unskilled for each year considered. Values have been converted to 2010 prices using the Istat household consumption deflator included in the documentation included with the SHIW historical archive. Contrary to the work developed in Chapter 2, we employed the total amount of observations in the dataset, including information for self-employed. Sampling weights provided by the survey have been employed to ensure unbiased estimates.

The fit of the distributions has been performed with the statistical software Stata, using the dedicated packages `smfit`, `dmfit`, `gb2lfit`, which respectively fit by Maximum Likelihood a three-parameter Singh-Maddala distribution, a three-parameter Dagum distribution and a four-parameter GB2 distribution to sample observations on the variable of interest. ML estimation consists in finding the parameter values that maximise the likelihood function.

The following figures show the quantiles of income against the quantiles of the estimated Singh-Maddala, Dagum and GB2 distributions, respectively for the skilled and unskilled individuals for the year 2002. The same plots are available for each

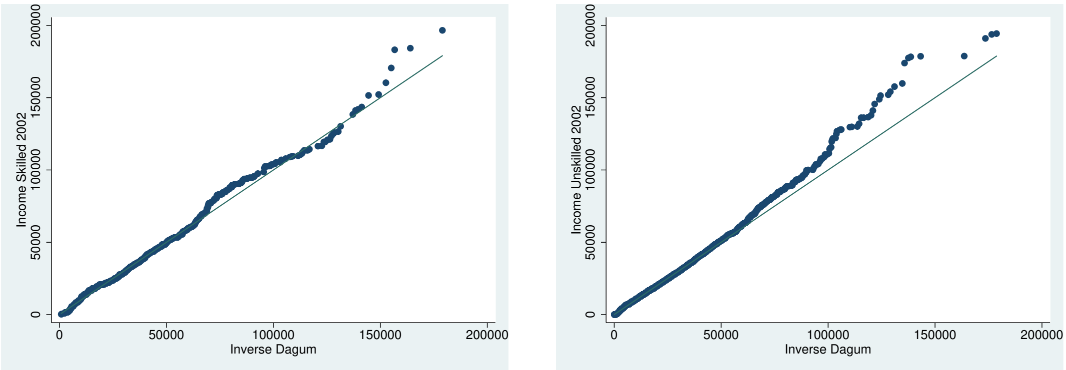
[2] Income is considered net of taxes and social security contributions.

wave used in the analysis in Appendix 3.A.



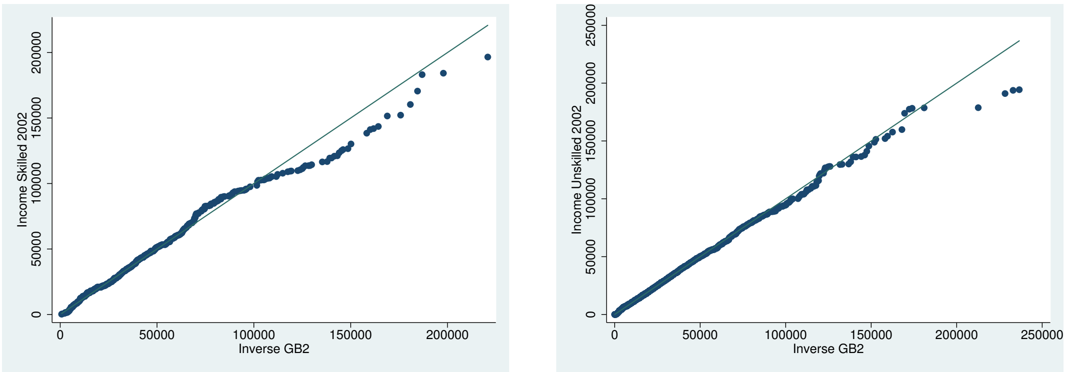
(a) Skilled (b) Skilled

Figure 3.2: Fit for the Singh-Maddala distribution, year 2002



(a) Skilled (b) Skilled

Figure 3.3: Fit for the Dagum distribution, year 2002



(a) Skilled (b) Skilled

Figure 3.4: Fit for the GB2 distribution, year 2002

After estimation, a likelihood ratio test (LR test) has been used for comparing the goodness of fit of the nested models. The LR test is based on the likelihood ratio, which expresses how many times more likely the data are under one model than the other. In general, it tests the null hypothesis that the parameter vector of a statistical model satisfies some smooth constraint. Thus, in our case, we test the nested model against the GB2 model.

Together with the LR statistics and its corresponding p-value, the test also shows the values of the Akaike's Information Criterion (AIC) and Bayesian Information Criterion (BIC). These are two popular measures for comparing maximum likelihood models. They are respectively defined as

$$AIC = -2 * ln(L) + 2 * k$$
$$BIC = -2 * ln(L) + ln(N) * k ,$$

where L is the maximised value of the likelihood function, k corresponds to the number of parameters estimated and N is the number of observations.

They both measure the fit negatively by $-2 * ln(L)$: the larger the value, the worse the fit. Thus, given two models fitted on the same data, the model associated with the smaller value of these two measures is considered to be better.[3]

Table 3.1 shows the estimates for the fit of data for skilled and unskilled individuals for the wave 2002, while table 3.2 reports the LR test for both the Singh-Maddala and the Dagum against the GB2.

In both the datasets (skilled and unskilled), the LR test rejects the null hypothesis of a Dagum or Singh-Maddala distribution fitting better than a GB2. In addition, lower AIC and BIC are measured for the GB2 model in each case.

The same applies to the rest of the waves analysed, for which parameters estimates and LR tests are displayed in Appendix 3.A.

The only exception is represented by the year 2016, where estimates for the shape parameters a, p, q of the GB2 for the data related to skilled individuals are

[3] There is substantial literature on these measures (for further references see for example Akaike, 1974; Schwarz, 1978; Sakamoto, Ishiguro, & Kitagawa, 1986; and Raftery, 1995).

	Skilled			Unskilled		
	S-M	Dagum	GB2	S-M	Dagum	GB2
a	2.247	2.832***	4.515***	1.967***	3.616***	5.415***
	(0.0936)	(0.117)	(0.7905)	(0.0217)	(0.0542)	(0.3629)
b	35528.5***	36019.8***	31949.39***	31541.1***	25195.3***	22825.45***
	(2721.2)	(1673.0)	(1581.98)	(1028.0)	(283.6)	(333.04)
p		0.702***	0.417***		0.446***	0.2844***
		(0.0572)	(0.083)		(0.0109)	(.0213)
q	1.337***		0.496***	2.604***		0.551***
	(0.151)		(0.1164)	(0.117)		(0.048)
N		1205			12821	

Table 3.1: Parameters estimates for the fit on the year 2002

	Skilled			Unskilled		
	AIC	BIC	LR	AIC	BIC	LR
Dagum	27432.09	27447.37	8.57**	275969.1	275991.5	50.05***
GB2	27425.51	27445.89		275921.0	275950.9	
S-M	27442.92	27458.2	19.41***	276288.6	276311	369.58***
GB2	27425.51	27445.89		275921.0	275950.9	
N		1205			12821	

Table 3.2: AIC, BIC and LR test for the year 2002

not statistically significant. However, the LR test together with the AIC and BIC measures still suggest the better fit of the GB2 distributional model. We attribute this incongruity to a possible difficulty in the process of maximisation of the likelihood function by the ML algorithm integrated with the packages adopted. Given this partial contradiction in the estimation results, as a precaution, we decided to discard the year 2016 from our analysis and base our fit on the time span 2002-2014.

Table 3.3 summarises the parameters estimates obtained for the years 2002-2014. From these results, we took the average for each parameter and then used the values as inputs in our ABM.

	$\hat{a}$		$\hat{b}$		$\hat{p}$		$\hat{q}$	
	S	U	S	U	S	U	S	U
2002	4.52	5.42	27156.89	19401.60	0.42	0.28	0.50	0.55
2004	5.04	6.81	30816.64	22427.11	0.32	0.21	0.40	0.44
2006	6.82	5.44	28294.48	22608.41	0.24	0.28	0.29	0.60
2008	4.03	6.95	27393.31	20348.92	0.40	0.21	0.55	0.44
2010	4.91	8.57	26742.75	21210	0.37	0.15	0.45	0.36
2012	5.38	9.86	17183.32	16280.64	0.32	0.13	0.45	0.32
2014	5.65	12.57	23415.56	19892.36	0.26	0.10	0.44	0.27
Mean	5.19	7.94	25857.56	20159.84	0.33	0.20	0.44	0.43

Table 3.3: Average parameters estimates for the GB2 distribution of Italian income for skilled and unskilled individuals over the years 2002-2014; data provided by the Bank of Italy (SHIW).

3.4 Model simulations

We simulate the ABM described in Chapter 2, leaving everything unchanged except for the attribution of income to agents, for which in this experiment the value is extracted from a GB2 distribution identified by the average parameters displayed in table 3.3.

The model has been run in NetLogo using the R extension. With this extension, it is possible to load and use the statistical software R within a NetLogo model. This choice results from the necessity to use the R package *GB2*, which allows the random number generation from a GB2 distribution, defined by the four parameters estimated as described in Section 3.3.

3.4.1 A single simulation

Again, we are primarily interested in the measure of the enrolment to university. Secondly, we will look at the composition of agents deciding to undertake tertiary education, that is whether they come from a skilled or unskilled family, and finally, at educational mobility with respect to agents' parents.

A first flavour of the model dynamics is provided in the following graphs, which display the variables of interest over t = 100 for one simulation (let us consider the first simulation, MC = 1). Results do not seem to vary with respect to those shown in Chapter 2. Figures 3.5 and 3.6 show the trend of the enrolment rate.

Figure 3.5 presents the aggregate rate, while Figure 3.6 the disaggregate levels according to family background.

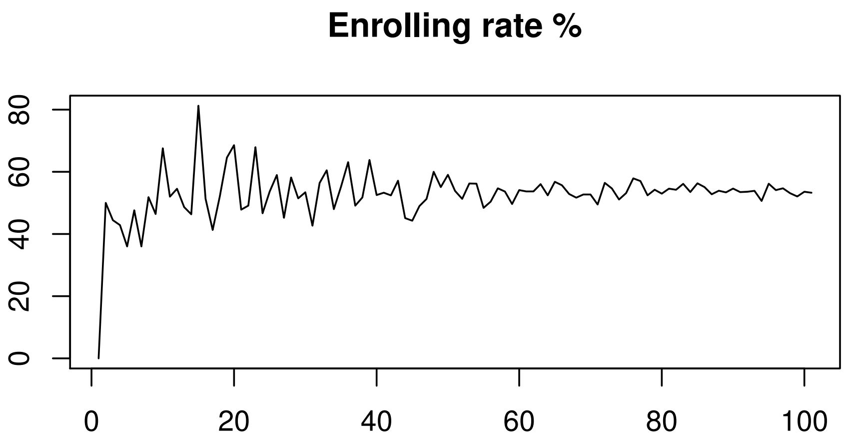


Figure 3.5: Enrolling rate for a single simulation

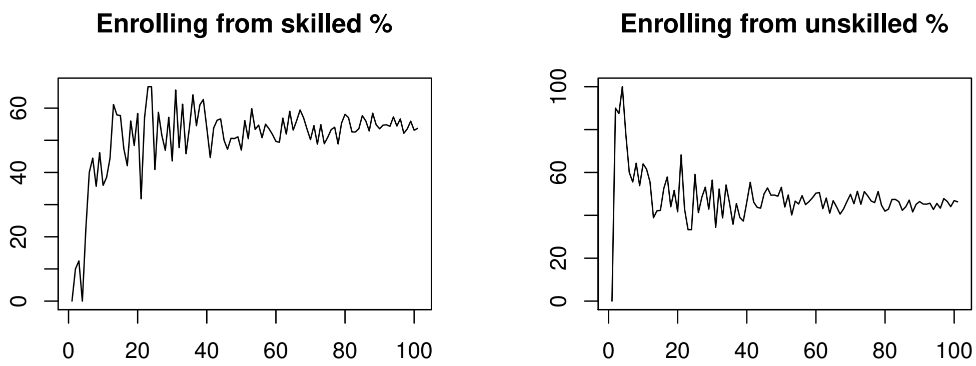


Figure 3.6: Enrolling rate given family background for a single simulation

Figures 3.7 and 3.8 show the trend of social mobility. Figure 3.7 displays the share of agents that does not change the educational status with respect to their family, while Figure 3.8 presents the share of agents that improve (worsen) their educational level with respect to their family background.

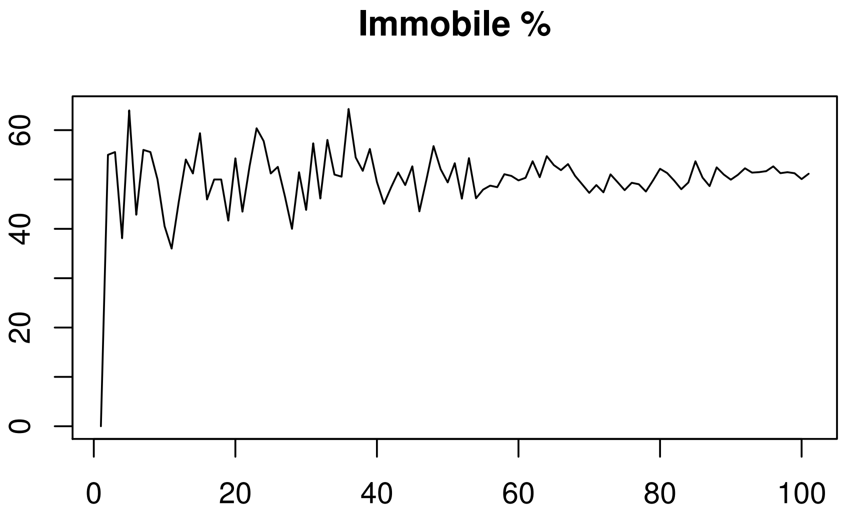


Figure 3.7: Social mobility for a single simulation run: immobile

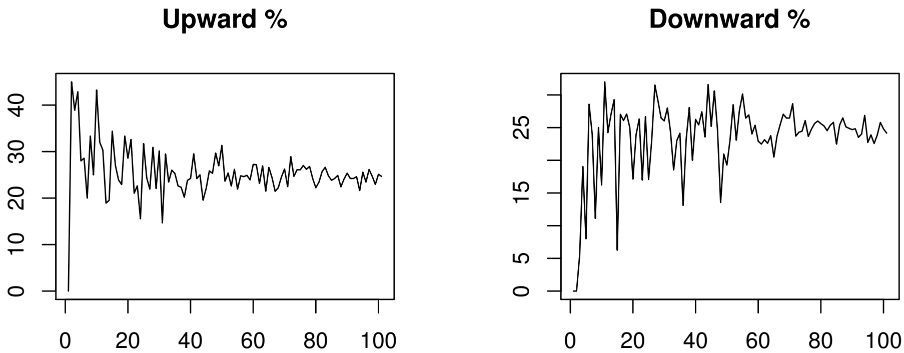


Figure 3.8: Social mobility for a single simulation: upward and downward

3.4.2 Monte Carlo Simulations

We ran a simulation experiment of 30 Monte Carlo simulations.[4] Synthetic statistics computed across the simulations are reported in table 3.4.

The Monte Carlo experiment gives robustness to the preliminary results observed for a single simulation, with an enrolment rate of 53.61%. In addition, outcomes are in line with the decreasing tendency observed in the rate of transfer from secondary

[4]Because of the architecture of R, both software (R and NetLogo) share one system process and therefore the memory given to NetLogo. When using too much memory, it happens that NetLogo closes abruptly. Therefore, the number of Monte Carlo experiments is limited to the memory capacity of the computer employed for running the simulations.

Variable	Mean	SD
Enrolling rate %	53.61	0.60
Enrolling from skilled %	54.51	0.94
Enrolling from unskilled %	45.49	0.94
Immobile %	51.27	0.39
Upward %	24.38	0.31
Downward %	24.35	0.25

Table 3.4: Mean value and SD of the outcome variables computed across the simulations run.

to tertiary education showed in table 2.5 in Chapter 2 and used as comparison for understanding the capacity of the model to represent real figures in a satisfactory way. The rate of transfer ranges from 65.8% to 55.7%, from 2008 to 2013, decreasing year by year.

The output values are also graphically displayed over the simulations time span, in order to compare the simulations and still follow their dynamics over time.

Figure 3.9 present the aggregate rate, while Figure 3.10 the disaggregate levels according to family background.

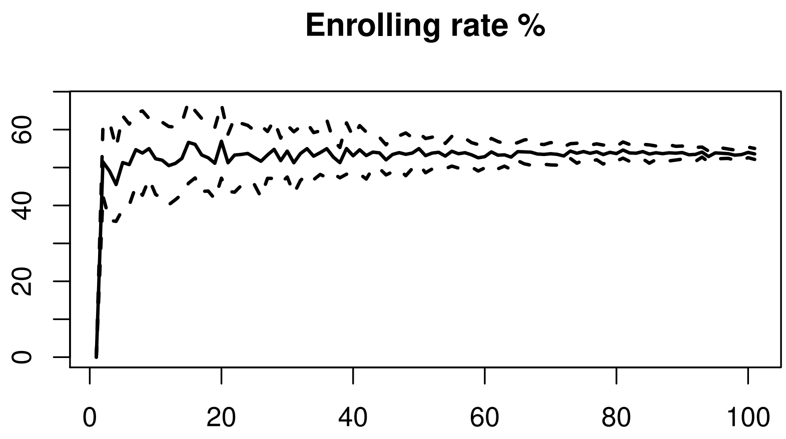


Figure 3.9: Monte Carlo simulations: enrolling rate

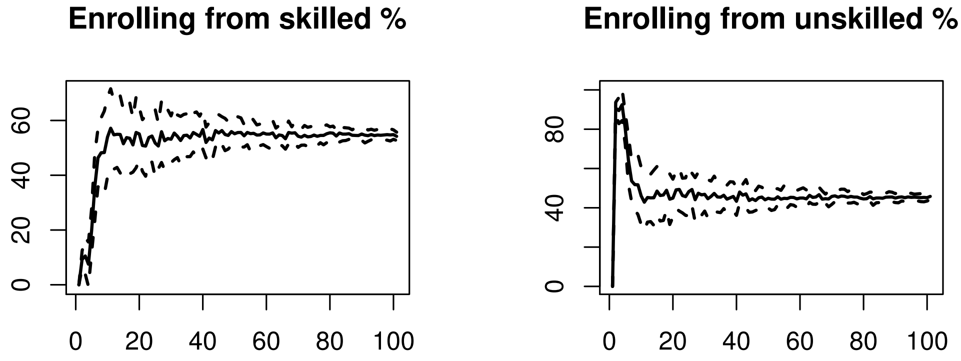


Figure 3.10: Monte Carlo simulations: enrolling rate given family background

Figures 3.11 and 3.12 show the trend of the social mobility.

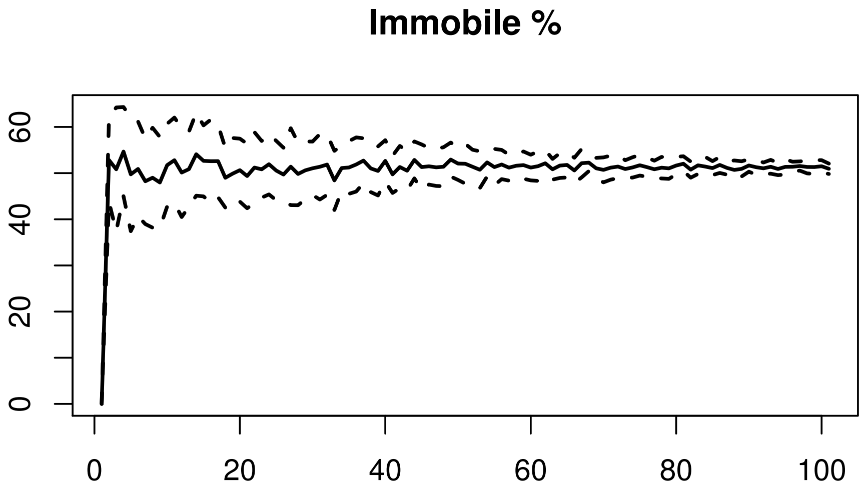


Figure 3.11: Monte Carlo simulations, educational mobility: immobile

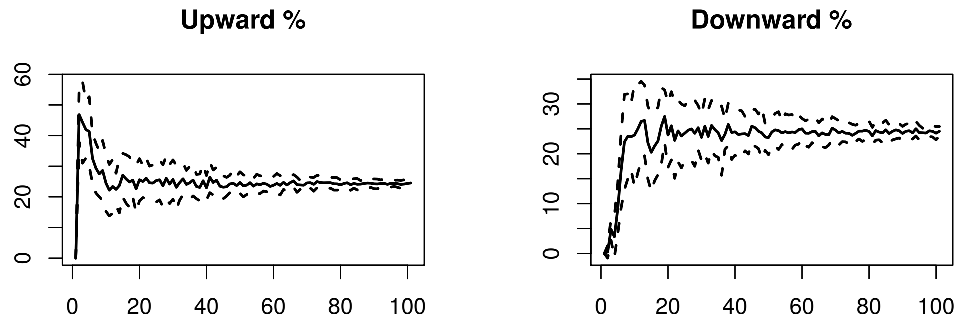


Figure 3.12: Monte Carlo simulations, educational mobility: upward and downward

3.5 Final remarks

The literature on Agent-Based Modelling is characterised by a debate on the issue of estimation. Given the increasing number of applications of ABMs and their capacity to reproduce regularities and emergent behaviours observable in the real world, we decide to perform a calibration experiment on the ABM developed in Chapter 2 in order to provide a more accurate representation of reality.

Using data on personal income from SHIW for 8 years, from 2002 to 2016, we fit three distributional models of the Beta family of distributions: the three-parameter Singh-Maddala distribution, the three-parameter Dagum distribution and the four-parameter GB2 distribution. The former two distributions can be considered as special cases of the GB2. We then compare the models by means of likelihood ratio test and using the measures AIC and BIC for comparing nested models. Results confirm a better fit of the GB2 over all the year considered, with the only exception of the year 2016, for which the parameters estimates show some anomaly. Therefore, we decide to drop the observations for the wave 2016 and consider only the time span 2002-2014. By taking the average of the parameters estimates, we obtain inputs for identifying the income distribution of skilled and unskilled individuals to be adopted in our model. We want to check whether there could be significant changes in the results when modifying the model inputs.

The model is run under the new set-up showing a slight difference with respect to the benchmark model. The average enrolling rate is 53.61%, 3 percentage points smaller than the results obtained using the reference model. However, the result seems to be in line with the decreasing trend of the rate of transfer from secondary to tertiary education, provided by Istat. While the educational mobility is not affected by the different model specification, the composition of individuals deciding to enrol is subjected to a modest adjustment. About the 54% of agents continuing their studies comes from an educated family, with respect to the 56% observed in the reference model.

The main result highlights the potential of both distributions (lognormal and GB2) to be a good candidate to describe the income distribution in our model. However, the calibration experiment set in this Chapter allows for a more rigorous imputation of parameter values, that attributes more robustness to the outcomes of the model and gives solidity to the parameters estimation.

The main drawback to applying the described procedure consists of long execution times required for computation and abrupt interruption of the computations given to the high amount of memory required for running together the NetLogo and R software. Our intention for future development of this work is to simplify the programming code in order to lighten the computational load required by the software and, possibly, re-program the ABM using another programming environment; in this way, a larger number of Monte Carlo experiments could be carried out, donating more robustness to our results, and allowing to perform a sensitivity analysis on the model parameters. More reliable results could also be obtained by improving the specification of the model, and defining new parameters to be calibrated.

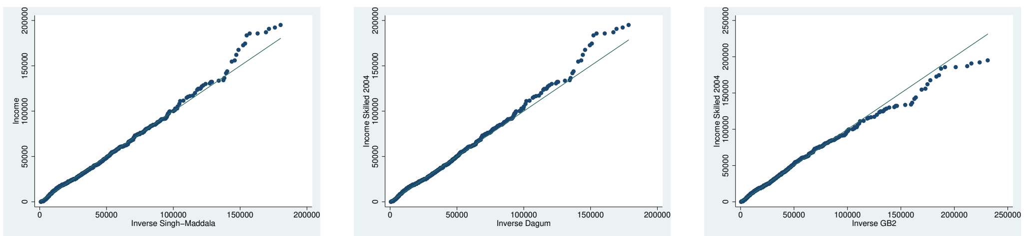
Appendix

3.A Estimations results

In this section, we report the parameter estimates for a Singh-Maddala distribution, a Dagum distribution and a GB2 distribution obtained for the years 2004, 2006, 2008, 2010, 2012, 2014 and 2016. We also report quantile plots for the fitted data against the three distributional models, for the years 2004, 2006, 2008, 2010, 2012 and 2014, since, eventually, we decided to discard the wave 2016. The tables report significance levels as follows: $^{*}p < 0.05,^{**}p < 0.01,^{***}p < 0.001$; standard errors are reported in parentheses.

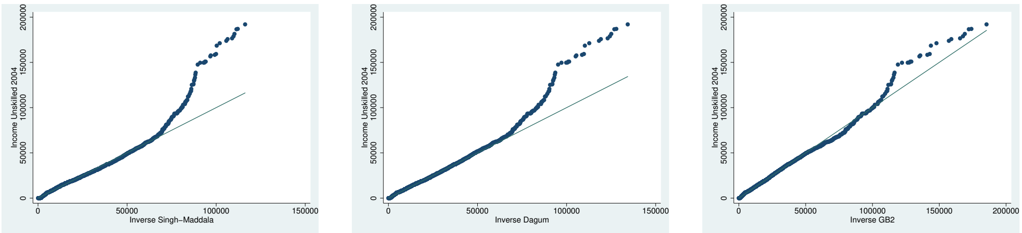
Notice that for the dataset related to skilled individuals, for the year 2008, the AIC and BIC measures are discordant; even though for a very small difference, the BIC signals that the Dagum distribution would be the best model. Nevertheless, the LR is significant at 5% level; estimates for the GB2 parameters also show that the confidence interval for the estimate of parameter p is [0.27618980.5928889], not including the value 1, thus it let us discard the hypothesis that $p = 1$. Therefore, we take the GB2 as the best fit for the year 2008.

2004



(a) Singh-Maddala (b) Dagum (c) GB2

Figure 3.A.1: Fit for skilled agents, year 2004



(a) Singh-Maddala (b) Dagum (c) GB2

Figure 3.A.2: Fit for unskilled agents, year 2004

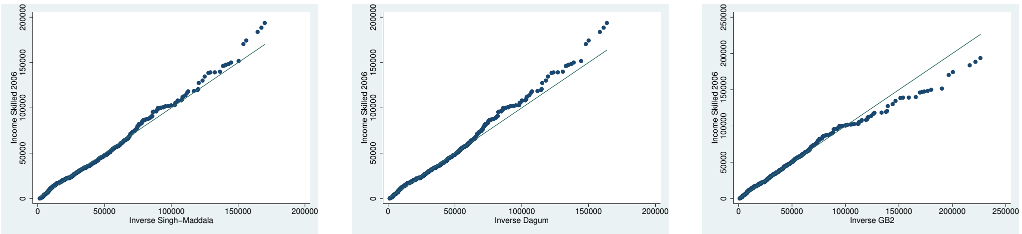
	Skilled			Unskilled		
	S-M	Dagum	GB2	S-M	Dagum	GB2
a	1.999***	2.685***	5.042***	1.898***	3.837***	6.810***
	(0.0775)	(0.109)	(1.029)	(0.0200)	(0.0584)	(0.5172)
b	36225.3***	36283.8***	30816.6 ***	33919.5***	25603.9***	22427.1***
	(2983.7)	(1676.6)	(1489.5)	(1138.2)	(265.9)	(304.3)
p		0.644***	.3225***		0.394***	0.212***
		(0.0491)	(0.0719)		(0.00939)	(0.0173)
q	1.457***		0.3999***	3.000***		0.4364***
	(0.159)		(0.1036)	(0.136)		(0.0414)
N		1283			12634	

Table 3.A.1: Parameters estimates for the fit on the year 2004

	Skilled			Unskilled		
	AIC	BIC	LR	AIC	BIC	LR
Dagum	29271.54	29287.02	14.02***	271137.3	271159.6	93.97***
GB2	29259.52	29280.15		271045.3	271075.1	
S-M	29289.77	29305.24	32.25***	271648.6	271670.9	605.27***
GB2	29259.52	29280.15		271045.3	271075.1	
N		1283			12634	

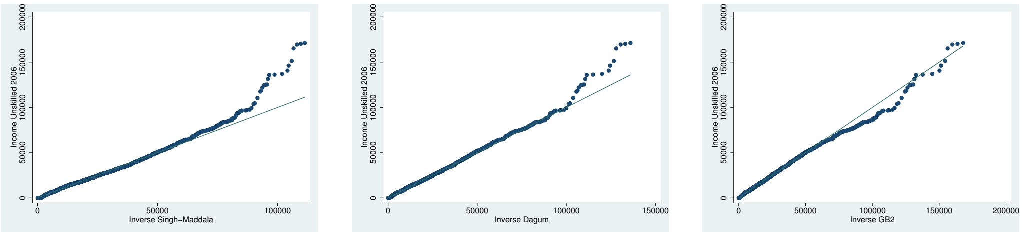
Table 3.A.2: AIC, BIC and LR test for year 2004

2006



(a) Singh-Maddala (b) Dagum (c) GB2

Figure 3.A.3: Fit for skilled agents, year 2006



(a) Singh-Maddala (b) Dagum (c) GB2

Figure 3.A.4: Fit for unskilled agents, year 2006

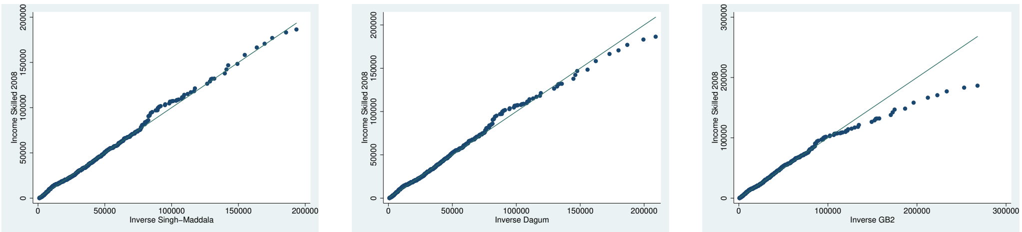
	Skilled			Unskilled		
	S-M	Dagum	GB2	S-M	Dagum	GB2
a	2.038***	2.743***	6.818***	1.973***	3.887***	5.437***
	(0.0758)	(0.106)	(1.4402)	(0.0217)	(0.0618)	(0.3576)
b	33062.3***	33971.3***	28294.48 ***	32887.4***	24700.9***	22608.4***
	(2471.8)	(1434.6)	(1078.5)	(1148.9)	(268.8)	(357.63)
p		0.633***	0.236***		0.411***	0.284***
		(0.0453)	(0.0534)		(0.0104)	(0.0208)
q	1.419***		0.287***	3.016***		0.601***
	(0.143)		(0.0710)	(0.149)		(0.0537)
N		1356			12063	

Table 3.A.3: Parameters estimates or the fit on the year 2006

	Skilled			Unskilled		
	AIC	BIC	LR	AIC	BIC	LR
Dagum	30722.82	30738.45	29.75***	257902.7	257924.9	33.58***
GB2	30695.07	30715.92		257871.1	257900.7	
S-M	30747.87	30763.50	54.80***	258230.1	258252.3	360.97***
GB2	30695.07	30715.92		257871.1	257900.7	
N	1356			12063		

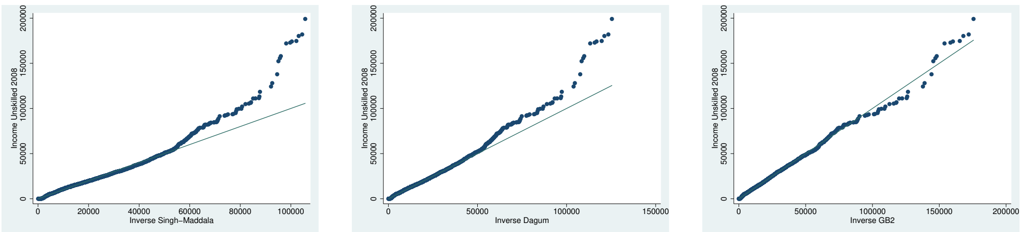
Table 3.A.4: AIC, BIC and LR test for the year 2006

2008



(a) Singh-Maddala (b) Dagum (c) GB2

Figure 3.A.5: Fit for skilled agents, year 2008



(a) Singh-Maddala (b) Dagum (c) GB2

Figure 3.A.6: Fit for unskilled agents, year 2008

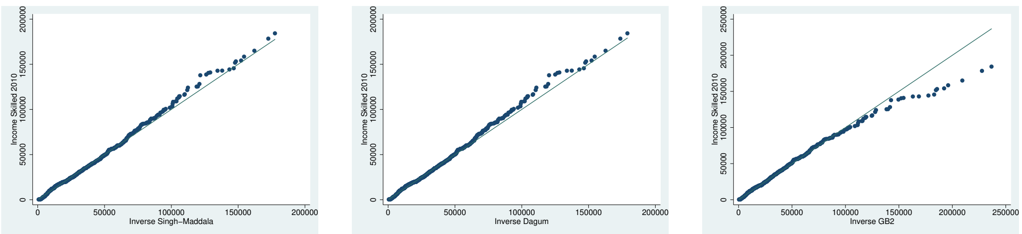
	Skilled			Unskilled		
	S-M	Dagum	GB2	S-M	Dagum	GB2
a	1.930***	2.722***	4.0254***	1.925***	3.934***	6.949***
	(0.0718)	(0.106)	(0.702)	(0.0208)	(0.0608)	(0.516)
b	34030.3***	31502.4***	27682.3***	31275.3***	23177.6***	20348.9***
	(3061.7)	(1385.9)	(1545.6)	(1090.6)	(237.9)	(270.7)
p		0.623***	0405***		0.392***	0.212***
		(0.0456)	(0.0789)		(0.00944)	(0.0169)
q	1.661***		0.546***	3.112***		0.438***
	(0.192)		(0.132)	(0.150)		(0.0407)
N		1457			12220	

Table 3.A.5: Parameters estimates for the fit on the year 2008

	Skilled			Unskilled		
	AIC	BIC	LR	AIC	BIC	LR
Dagum	32751.96	32767.82	6.16*	259505.3	259527.6	94.21***
GB2	32747.81	32768.94		259413.1	259442.8	
S-M	32768.18	32784.03	22.38***	260002.0	260024.3	590.90***
GB2	32747.81	32768.94		259413.1	259442.8	
N		1457			12220	

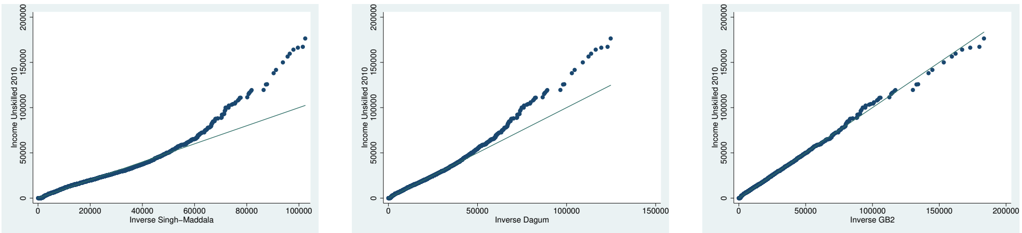
Table 3.A.6: AIC, BIC and LR test for year 2008

2010



(a) Singh-Maddala (b) Dagum (c) GB2

Figure 3.A.7: Fit for skilled agents, year 2010



(a) Singh-Maddala (b) Dagum (c) GB2

Figure 3.A.8: Fit for unskilled agents, year 2010

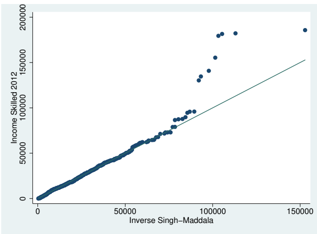
	Skilled			Unskilled		
	S-M	Dagum	GB2	S-M	Dagum	GB2
a	2.217***	2.847***	4.906***	1.724***	4.161***	8.568***
	(0.0777)	(0.101)	(0.827)	(0.0180)	(0.0678)	(0.756)
b	30154.2***	30374.1***	26742.7***	42239.2***	24647.6***	21210***
	(1990.6)	(1193.0)	(1087.9)	(1963.8)	(239.1)	(264.1)
p		0.687***	0.374***		0.325***	0.151***
		(0.0475)	(0.0704)		(0.00775)	(0.0140)
q	1.376***		0.450***	4.568***		0.363***
	(0.132)		(0.099)	(0.274)		(0.0381)
N		1664			12040	

Table 3.A.7: Parameters estimates for the fit on the year 2010

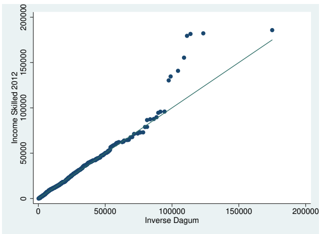
	Skilled			Unskilled		
	AIC	BIC	LR	AIC	BIC	LR
Dagum	37278.52	37294.77	13.41***	255860.4	255882.5	126.89***
GB2	37267.11	37288.78		255735.5	255765	
S-M	37294.35	37310.6	29.24***	256569.5	256591.7	836.04***
GB2	37267.11	37288.78		255735.5	255765.0	
N		1664			12040	

Table 3.A.8: AIC, BIC and LR test for the year 2010

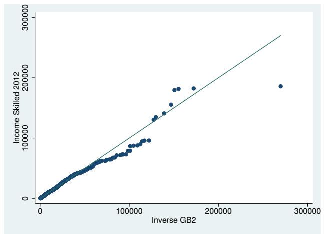
2012



(a) Singh-Maddala

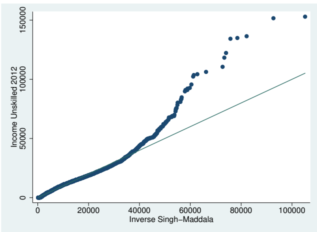


(b) Dagum

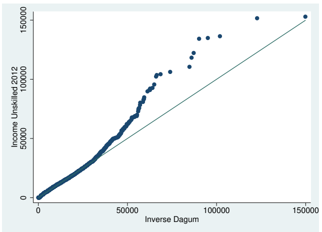


(c) GB2

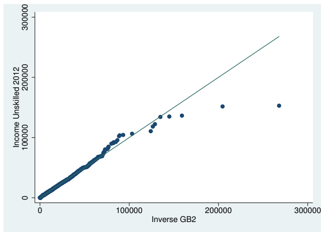
Figure 3.A.9: Fit for skilled agents, year 2012



(a) Singh-Maddala



(b) Dagum



(c) GB2

Figure 3.A.10: Fit for unskilled agents, year 2012

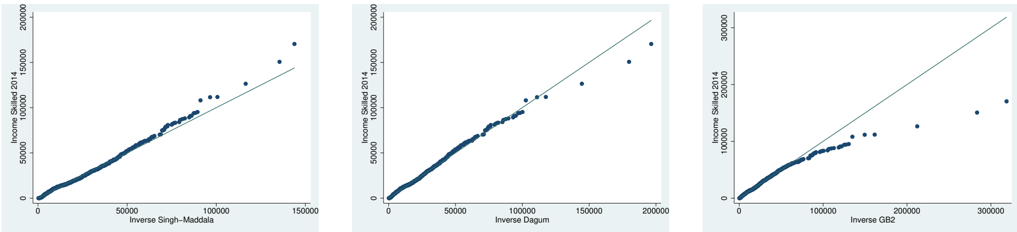
	Skilled			Unskilled		
	S-M	Dagum	GB2	S-M	Dagum	GB2
a	2.128***	3.092***	5.378***	1.785***	4.212***	9.858***
	(0.0692)	(0.113)	(1.122)	(0.0188)	(0.0691)	(1.0341)
b	19792.6***	18536.7***	16257.1***	28321.8***	17902.4***	15403.1***
	(1418.9)	(657.0)	(679.8)	(1201.9)	(172.4)	(175.4)
p		0.596***	0.323***		0.331***	0.134***
		(0.0400)	(0.0734)		(0.00799)	(0.0147)
q	1.709***		0.446***	4.131***		0.3160***
	(0.175)		(0.120)	(0.231)		(0.0380)
N		1750			11880	

Table 3.A.9: Parameters estimates for the fit on the year 2012

	Skilled			Unskilled		
	AIC	BIC	LR	AIC	BIC	LR
Dagum	37138.20	37154.60	10.13**	244857.6	244879.8	145.23***
GB2	37130.07	37151.93		244714.4	244743.9	
S-M	37159.58	37175.98	31.51***	245525.5	245547.7	813.13***
GB2	37130.07	37151.93		244714.4	244743.9	
N	1750			11880		

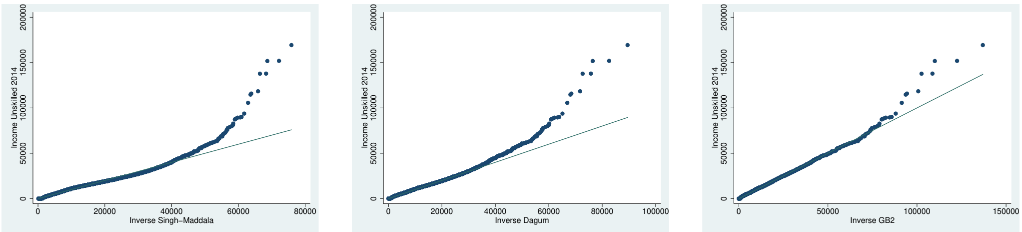
Table 3.A.10: AIC, BIC and LR test for year 2012

2014



(a) Singh-Maddala (b) Dagum (c) GB2

Figure 3.A.11: Fit for skilled agents, year 2014



(a) Singh-Maddala (b) Dagum (c) GB2

Figure 3.A.12: Fit for unskilled agents, year 2014

	Skilled			Unskilled		
	S-M	Dagum	GB2	S-M	Dagum	GB2
a	1.766***	3.328***	5.653***	1.734***	4.736***	12.574***
	(0.0546)	(0.126)	(1.079)	(0.0173)	(0.0815)	(1.525)
b	34763.4***	24218.6***	20361.4***	38528.1***	20251.7***	17297.8***
	(3895.2)	(762.0)	(973.3)	(1868.4)	(175.0)	(184.6)
p		0.445***	0.255***		0.280***	0.101***
		(0.0277)	(0.0513)		(0.00673)	(0.0126)
q	3.006***		0.443***	5.754***		0.271***
	(0.423)		(0.116)	(0.372)		(0.037)
N		1752			11759	

Table 3.A.11: Parameters estimates for the fit on the year 2014

	Skilled			Unskilled		
	AIC	BIC	LR	AIC	BIC	LR
Dagum	37734.22	37750.62	11.64***	243663.3	243685.4	166.36***
GB2	37724.58	37746.45		243498.9	243528.4	
S-M	37787.72	37804.12	65.14***	244538.3	244560.5	1041.45***
GB2	37724.58	37746.45		243498.9	243528.4	
N		1752			11759	

Table 3.A.12: AIC, BIC and LR test for the year 2014

2016

	Skilled			Unskilled		
	S-M	Dagum	GB2	S-M	Dagum	GB2
a	2.103***	3.018***	56.682	1.766***	4.956***	11.487***
	(0.0741)	(0.114)	(100.366)	(0.0189)	(0.0917)	(1.2167)
b	28276.1***	27294.9***	21959.7***	45406.7***	22125.6***	19216.0***
	(2114.9)	(1012.6)	(479.5)	(2576.0)	(195.8)	(211.8)
p		0.597***	0.0290		0.273***	0.113***
		(0.0410)	(0.0514)		(0.00702)	(0.0124)
q	1.622***		0.035	6.644***		0.317***
	(0.170)		(0.0616)	(0.519)		(0.0384)
N		1506			10340	

Table 3.A.13: Parameters estimates for the fit on the year 2016

	Skilled			Unskilled		
	AIC	BIC	LR	AIC	BIC	LR
Dagum	33199.79	33215.74	34.38***	215698.5	215720.2	130.25***
GB2	33167.4	33188.67		215570.2	215599.2	
S-M	33226.72	33242.67	61.32***	216489.7	216511.5	921.51***
GB2	33167.4	33188.67		215570.2	215599.2	
N		1506			10340	

Table 3.A.14: AIC, BIC and LR test for year 2016

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