THE SYNERGY BETWEEN MANIPULATIVE AND DIGITAL ARTEFACTS IN A MATHEMATICS TEACHING ACTIVITY: A CODISCIPLINARY PERSPECTIVE

Eleonora Faggiano¹ Antonella Montone¹ Pier Giuseppe Rossi²

¹Università di Bari Aldo Moro eleonora.faggiano@uniba.it; antonella.montone@uniba.it ²Università di Macerata - pgrossi.unimc@gmail.com

Keywords: Synergy between artefacts, Digital artefacts, Co-disciplinarity, Didactic Mediation, Theory of Semiotic Mediation.

This paper presents a teaching experiment aiming at constructing the meaning of axial symmetry through the mediation of a "duo of artefacts", made up by a digital artefact and a manipulative one. The meaning of the term "mediation" is described and used from a dual perspective, joining General Didactics and Mathematics Education. Herein, we describe an interactive book, created in a Dynamic Geometry Environment and a teaching sequence, based on the use of such a digital artefact, combined with a manipulative one. The main potential of the interactive book is based on the possibility to drag geometric objects and observe the effects of the dragging. The sequence has been experimented with a 4th grade class and the activities have been videotaped and analysed. Results have been analysed through the cited dual perspective and reveal how the mediation of the duo of artefacts can foster the construction of the mathematical meaning. In this paper we show how the digital artefact, acting in synergy



with the manipulative artefact, seems to exploit the potential of the sequence in terms of embodied involvement of the pupils in their cognitive process.

1 Introduction

Many researches in the latest years have investigated on the potentialities of the use of technologies in the teaching and learning processes. Definitive answers have not yet been given, however a research result seems to be confirmed, namely that successful outcomes depend on the consistency of the artefact, seen as an instrument (Rabardel, 1995), its potential and its affordances, as the models and the teaching strategies used. Crucial is also the educational environment that takes into account: the activities carried out with the artefacts by the students, individually or in groups; the interactions and the collective class discussions, in which experience is organized and structured.

The analysis of the research was done with a dual research perspective, conceived by the dialogue/interaction between General Didactics and Mathematics Education. The development of this dual perspective has required a special focus on the polysemic term "mediation", and related to it, on the term "artefact".

The research consists on the design, implementation and analysis of a teaching experiment, framed by the Theory of Semiotic Mediation (TSM) (Bartolini, Bussi & Mariotti, 2008) and the Didactic Mediation (Damiano, 2013; Rossi, 2016a), concerning the construction/conceptualization of axial symmetry at Primary School. It has been designed with the purpose of exploiting the potential of the synergic use of a "duo" (Maschietto & Soury-Lavergne, 2013) of artefacts/mediators, which is made up by: a digital artefact, developed in a Dynamic Geometry Environment (DGE); and a manipulative artefact, whose components are a sheet of paper and a pin.

The research discussed in this paper, was done comparing individual analysis made by each of the researchers on video collected during class interactions. In particular, a multimodal approach was used and the attention was focused on to both verbal aspects and gestures.

2 Theoretical framework

In order to make General Didactics and Mathematics Education interact, we have adopted co-disciplinarity (Blanchard-Laville, 2000). The reference framework used for Mathematics Education is the Theory of Semiotic Mediation (TSM) elaborated by Bartolini Bussi and Mariotti (2008) from a Vygotskian point of view, while the reference framework for General Didactics is the Didactic Mediation (DM) approach (Damiano, 2013) and the interactional

approach (Altet, 2012; Laurillard, 2014; Rossi, 2016b).

According to the TSM, there is an evolution from the artefact signs to the mathematical signs and during this evolution "pivot signs" play a key role. "Pivot signs" allow to bridge (see also Laurillard, 2014) common sense meanings and mathematical meanings, facilitating the transition from the context of the artefact to the mathematics context.

According to the DM theory, learning is a process that belongs to pupils, but it can occur only with teaching mediation. During the learning process, pupils organize and conceptualize their own experience and this is possible thanks to the interaction with didactic mediators that facilitate the transition from the specific experience to the generalization of it. In accordance with Damiano, every learning activity has a system of didactic mediators, that is the educational action makes use of functional multiples mediators that follow each other. In the TSM, the artefact and its affordance play a key role, while in the DM theory, the key role is played by the mediators.

So, the question is: what does change if in the mediators' system, or with the use of artefacts, there are also digital tools?

In the field of Mathematics Education, there have been many studies about the use of manipulative artefacts with regard to gestures, sensorimotor experiences and embodied cognition (Edwards *et al.*, 2009). Moreover, today scholars generally agree that digital artefacts can play a crucial role in the processes of teaching and learning (Monaghan *et al.*, 2016; Faggiano *et al.*, 2014). However, if an artefact, being manipulative or digital, is used only as an auxiliary tool to generate and show images, expand human memory or increase the turnaround in feedback, it would be unable to become an instrument and foster the progressive construction of mathematical knowledge, skills and attitudes. It is extremely important that teachers understand and become aware of the affordances, constraints, and mediating role of them as educational resources

Thanks to the possibilities provided by the use of technology, for instance, it is possible to shift from using static representations to experimenting with dynamic and interactive modes of visualization and exploration (Hoyles & Lagrange, 2010). In particular, research has underlined the role of Dynamic Geometry Environment. A DGE is a computational microworld, embedding Euclidean Geometry, in which it is possible to construct geometric figures and interact with them, dragging the independent elements of the construction and observing relationships remain intact (confirmatory dragging) or whether any properties of the figure remain invariant (exploratory dragging). This typical characteristic, usually called the "dragging function", appears to be particularly important, as it can be instrumental in helping students to solve



construction problems, to explore geometrical situations and to formulate conjectures. Dragging allows to visualize the subsequent states of the same system and this can also be described as morphing (Rossi, 2016b). As Leung (2008) underlines, DGE is an experimental ground that enables the generation of various qualitatively different ways of seeing a geometrical phenomenon in action. Mathematical concepts can be naturally given visual dynamic forms, subject to our actions. Leung suggests that, the conceptualization process in DGE can be studied taking in consideration the theory of variation (Marton & Tsui, 2004). According to this theory «learning in terms of changes in or widening in our way of seeing the world can be understood in terms of discernment, simultaneity and variation» (Bowden & Marton, 1998, p.7).

Among the studies concerning digital artefacts, some researchers have created digital artefacts reproducing existing manipulative artefacts, aiming to understand the difference between the manipulative and digital versions based on the same concept. Other studies analyse the potentialities of using a duo of artefacts (Maschietto & Soury-Lavergne, 2013), intended as a couple of artefacts, a manipulative artefact and its digital equivalent, being used simultaneously during the same activity.

However, what does happen if the two artefacts, used in the same experience, although during different phases, differ in both structure and role?

In order to answer, we have to focus on the learning results, on the activated cognitive processes and on the role of the body in these processes (Sibilio, 2014; Rizzolatti & Sinigaglia, 2006). According to neuroscientific studies, the body actively participates in learning processes and this is connected to the centrality of the action in knowledge processes (Caruana & Borghi, 2016; Rivoltella, 2012; Rossi, 2011). Therefore, processes activated by different artefacts should be analysed while discourse and body language during manipulative and digital processes should be used. So the question is, if the role of action in knowledge processes is central, how should the action and role of the body be considered when digital artefacts are involved? Semiotic bundle (Arzarello, 2006) and Kress' (2015) multimodality concepts allow us both to understand the synergy between gestures and discourse in conceptualization and to explore the synergy of different types of communication, actions and simulations with manipulative and digital artefacts.

3 The duo of artefacts and the design of the teaching sequence

In order to analyse the impact of technology on the process, we first need to describe the teaching sequence. In this study, differently from what proposed by Maschietto and Soury-Lavergne, the digital artefact is not a counterpart of the manipulative one but it has different, whilst complementary, characteristic.

The manipulative artefact consists of a sheet of paper and a pin to be used to pierce the paper. This artefact allows an axial symmetry to be created in a direct fashion. The digital artefact is embedded in an Interactive Book (IB) created within the authoring environment of New Cabri (Cabrilog). The IB appears as a sequence of pages including the designed tasks, together with some specific tools. In particular, the tools are: those that allow the construction of some geometric objects (point, straight line, segment, middle point, perpendicular line, intersection point), the "Symmetry" and "Compass" artefacts and the "Trace" tool. A fundamental role is also played by the drag function, boosted by the tracing tool, that allows to observe the invariance of the properties characterizing the figures.

The expression didactic cycle refers to the organization of teaching in activities. These consist of using the artefact, individually producing signs and then in the end collectively producing and absorbing signs through Mathematical Discussion activities (Bartolini Bussi, 1998). In accordance with the TSM, the design of the teaching sequence follows the general scheme of (six) successive "didactic cycles". The use of one or the other artefact has been alternated throughout the sequence.

The sequence begins with a task to be accomplished with the use of the manipulative artefact. Given a black figure (convex quadrilateral) and a red line drawn on a sheet, the pupils are asked to draw in red a symmetrical figure to the black one, with respect to the red line, by folding the sheet along the line and using the pin to mark the necessary symmetrical points by piercing the paper. After completing this task, on the same paper they are asked to draw a blue symmetrical figure to the black one, employing a new blue line (Fig. 1). Finally, the pupils are asked to write an explanation of why and how they drew the red and blue figures and what looks the same and what looks different about them.

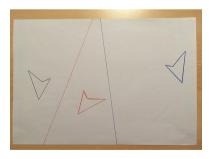


Figure 1: The manipulative artefact as it should appear at the end of the first cycle's tasks

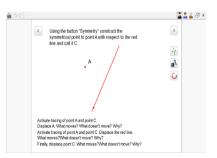


Figure 2: A screenshot of the first page of the digital artefact



The task of the second cycle focuses on the dual dependence of the symmetric point from the point of origin and from the axis, exploiting the potential of the dragging function and the tracing tool within the DGE¹. The pupil is asked to build the symmetric point of a point A with respect to a given line, using the button/tool "Symmetry" and call it C. The second step is to activate the "Trace" on point A and point C, drag A, drag C, and drag the line and see, in each case, what moves and what doesn't, and explain why (Fig. 2).

The task of the third cycle requires to construct the symmetric point without the use of the pin with the aim to: observe that the line joining two symmetrical points is perpendicular to the axis and that the two points are equidistant from the axis; recognise that these two properties are reversible and that they characterise axial symmetry.

In the fourth cycle pupils are asked to construct the symmetric point of a point A, with respect to a given line, without the use of the button/tool "Symmetry". To do this, it is necessary to use in a right way the two properties already emerged during the third cycle, that is: to draw the perpendicular line to the axis, passing through the point A, and to use the "Compass" to look for the point on the perpendicular line which has the same distance from the axis that A has.

In the fifth and sixth cycle the order of use of the artefacts is inverted but the task is the same: a couple of two points, A and C, is given; it has to be interpreted as a couple of symmetrical points with respect to a hidden line; it is required to find and to draw the line; finally, pupils are asked to verify if the symmetric point of A with respect to the line drawn is C and to describe the procedure used, justifying it.

4 Research methodology

The research is based on a teaching experiment concerning the sequence described above (Montone *et al.*, 2017) developed in a fourth grade class, composed of 20 students. The teaching experiment was conducted during the normal school timesheet with biweekly meetings for three weeks. In the alternation of didactic cycles, students worked in randomly chosen pairs. The activities with the manipulative artefact were carried out in class, while activities with the digital artefact were conducted in the laboratory (in two shifts of ten children, working on five computers). At the end of each didactic cycle collective discussions were carried out. In the case of activities with digital artefact, an IWB has been used during the discussions. Activities have been videotaped and the video have been analysed by each of the researchers,

¹ The use of the tracing tool gives back the sequence of the various positions taken by both the points, while the point A is dragged by the student.

working independently. Notes on transcription and gestures have then been compared and discussed. In some cases, a shared vision emerged, sometimes a different meaning (but never opposite) was assigned to the same video fragment due to the different perspectives of researchers. For the video analysis the reference is the plural analysis (Altet, 2012, Vinatier & Altet, 2008). To analyse video fragments we also refer to Santagata and Guarino (2012) and to Scherin and van Es (2009). For the classification of gestures, we refer to the Semiotic Bundle (Arzarello, 2006).

5 The teaching experiment: results analysis and discussion

With the analysis of the results of teaching experiment we tried to figure out how the use of these two artefacts and their synergy are involved in the construction of the mathematical meanings and the interactions throughout the activities.

We will report a series of interactions in which the pupil refers to the digital artefact with both verbal communication and gestures in order to describe and explain a series of conceptual steps.

The first episode refers to the discussion held with the class at the end of the second cycle. During this discussion one of the children had constructed on the IWB the symmetric point of a given point with respect to a line. Pupils are asked to move the objects on the screen for the functional dependence between those objects to be perceived.

When the teacher focuses on why, when we move A, the symmetric point C moves too, S. states «if you move point A, point C has to move too because there must remain the symmetry», matching her verbal expressions with some gestures which will soon be caught and repeated by other pupils: by simulating the dragging of point A on the desk surface with her left hand, she moves it away from an imaginary line and simultaneously moves her right hand in the opposite direction; in particular, she opens her hands with the palms facing each other and puts them symmetrically ahead of her when she says, "there must remain the symmetry". Here, the pupil simulates with the gestures what she has visualized on the IWB and reproduces those movements on the desk, trying to explain what she observed and identify the existing relationship between the points and the line.

The immediately following discourse of another pupil (M.), shown in the Tab.1, underlines how the interpretation is changing and the digital artefact is becoming the mediator in the construction of the interpretation of what it is happening.



Table 1
M's discourse description

Her words		Her gestures		
If you move point A only, point C has to move with point A because they must be symmetrical (a)		M. has her elbows on the desk and moves her hands ahead of her while speaking		
like, if you move point A higher (b)		She raises her left hand to indicate point A moving higher and looks towards her left hand		
point C moves lower so it is the same (c)		She puts her hands in front of her face, to simulate, with the thumb and index of each hand, two identical segments, then she moves her right hand lower to show that, in this case, point C moves lower and looks towards her right hand		
because there must be the same space be- tween the two points (d)		With a fast coordinated movement of her hands, she simulates two segments having the same length, using the thumb and index and bending the other fingers		
			THE REAL PROPERTY.	
a	b	c d		

This episode shows how the elements used by pupils to support their claims refer to the dragging process visualized in the digital artefact. The manipulative artefact gives a static vision because, for instance, after finding a symmetric point of a given point, making a hole in a sheet of paper by piercing it with a pin, the two points cannot move at all. Instead, in the previous transcription, the pupil refers to the dynamic process visualized with the digital artefact: "if you move it", "it moves" and matches words with hand gestures that simulate what she saw on the computer.

The role of the synergy needs also to be underlined: in order to indicate what a symmetric point is, pupils refer to the activity carried out with the paper and the pin and their initial conceptualization depends on the direct experience made by folding and piercing. In other words, the pupils refer to the manipulative artefact for the concept of symmetry and to the digital artefact when they want to describe the properties of the symmetric point. In this second case, it becomes essential to understand how point C moves when point A and the symmetry axis change and the dragging plays a key role in understanding this relationship. Thus, we talk about synergy: if the pupil action carried out with the manipulative artefact is essential to acquire the concept of symmetry,

the immersion in the digital simplifies the understanding of the relationships. In any case, one artefact refers to the other, since it is possible for the pupil to understand the relationships in the digital artefact only by referring to the previous experience with the sheet of paper and the pin. The same happens when going back to the manipulative artefact, the immersion in the digital supports the development of conceptualization.

The discussion continues and it seems interesting to report another episode (involving two other pupils: G. and V.) in which it is possible to underline the need to mentally go back to the digital artefact as for G. and the reference in synergy of both artefacts as for V. The teacher restarts and asks again how they know that the distance is always the same, and G. says: «We figured it out because when [he] moved point A, point C moved too, but when they were very far away from the red line it was always the distance from the red line... from point C to the red line there was the same distance as... from point A to the red line».

G. matches his discourse gesticulating in the space ahead of him. In fact, he looks towards the IWB screen, points his finger towards a hypothetical point A in front of him, with his right hand, while he symmetrically raises his left hand at the same height. He leans back with his body and spreads his arms outwards simulating the two points moving and keeping the same distance from the axis. Here, it shows how the interaction with the digital artefact allowed G. to perceive the invariant element, the distance, thanks to the variation on the screen of the position of point A and consequently of point C, which depends on A. He visually perceives and anticipates the generalization of the invariance of the distance of these two points from the line. In other words, it is as if the pupil visually analysed the variation of an aspect of the whole configuration, keeping another aspect constant, hence anticipating the surfacing of invariant schemes.

Then V., in order to analyse the relationships and after carrying out the activity with the digital artefact, asks to and receives from the teacher a sheet of paper and a pin. She obtains a symmetric point with respect to a fold piercing the paper with the pin, reopens the paper, looks at it, and, simultaneously looking at the IWB adds: «It is more visible there and it is easier... because there you can move the point and so I easily realize that if I move the point... the already created figure... it is easier to realize that there is the same distance because just by moving, you can understand, especially when we distance a lot from the line, that also point C moves... and so there is always the same distance. But I was able to understand it on the paper, also».

V.'s discourse confirms the hypothesis that the digital artefact is acting in synergy with the manipulative one. However, it is also clear that the modality with which these two artefacts operate is different. The manipulative artefact



allows the direct action of the pupil. The pupil's body learns while acting and this emerges in other situations in which pupils simulate the folding and piercing, in order to describe what a symmetric point is. In other words, when they refer to the digital artefact pupils describe and simulate the actions that they perform with their own hands. When they refer to the digital artefact, the procedure to find the symmetric point seems to become less important and objects movements caused by dragging become essential instead. However, in this case, pupils do not refer to their action to describe what they made but they identify themselves with what they observed and simulate the movements of the points and lines as seen on the screen. Here they move their arms as lines and their hands as points drawing in the air those movements seen on the computer. The dragging function, together with the tracing, after allowed pupils to mentally move the objects and the previous visualization of what happened made explicit the implicit dynamism of thinking mathematical objects.

The next steps show the difference in the way pupils perceive that the distance between A and C from the line is always the same: with the manipulative artefact, folding the sheet of paper and observing the superimposition of the two holes; with the digital artefact, animating/moving point A and observing how consequently point C moves. The role of the animation seems to be more effective than the static analysis. The underlined difference is at the base of the synergic use of the two artefacts since they operate on cognitive processes and with different operative and non-superimposable modalities.

We must point out that besides the importance of the activities conducted with the two artefacts, the role of the elaboration of concepts and the debate after each activity is fundamental. Actually, according to the TSM, at the end of each activity pupils were asked to describe what they made, explain what happened and give interpretations. Neither direct action nor the one with the digital artefact alone allow pupils to conceptualize, but it is the subsequent collective Mathematical Discussion (Bartolini Bussi, 1998) at the end of each activity that allows them to construct the mathematical meaning starting from the carried out experience. During the discussion, pupils reorganize their knowledge thanks to the teacher guide and here the references to the performed actions are central. In fact, exchanges are both verbal and non-verbal.

Conclusions

This paper presents a teaching experiment investigating the synergy between manipulative and digital artefacts. The collective discussions have reified the construction of meanings. Artefacts have acted as pivot between experience and mathematical knowledge and as mediator between experience and conceptualization.

The experimentation has shown the different relationship of the body and representation when pupils work with the two artefacts. When they work with manipulative artefacts, the action is the focus, that is, in the specific case, folding, piercing, and manipulating the sheet of paper. When pupils work with the IB, they create the points and lines using the digital artefact and when they describe what they did using discourse and gestures, they seem to have mainly absorbed the effect of the action, not the action alone. They simulate movements with their hands and arms and use words that refer to the visualization of the objects on the screen. In fact, pupils say "it moves", "trace", "movement". In the second case it seems that pupils are immersed in the process and that they have identified themselves with what they saw and have absorbed not their own action but the objects movements.

The results obtained have to be confirmed by other experiments. It is necessary to verify how much the results depend on the specific digital artefact and on the specific sequence. The study could be developed in two directions: realizing different math teaching sequences to verify the synergy between digital and manipulative artefacts, and realizing sequences in other subjects to verify further synergies.

This experience has shown that we need to understand the different impact of the two artefacts in the incorporation and conceptualization of the experience itself and the importance of synergy. This refers to one of the hypothesis we started with: technologies today cannot be considered as a specific sector and a separated field of research in mathematics education as well as in general didactics

REFERENCES

- Altet, M. (2012). L'apporto dell'analisi plurale dalle pratiche didattiche alla coformazione degli insegnanti. In P.C. Rivoltella, P.G. Rossi (eds.). L'agire didattico. Brescia: La Scuola.
- Arzarello, F. (2006). Semiosis as a multimodal process, *Relime*, Numero Especial, 267-299.
- Bartolini Bussi, M. G. (1998). *Verbal interaction in mathematics classroom: A Vygotskian analysis.* In H. Steinbring, M. G. Bartolini Bussi, & A. Sierpinska (Eds.), Language and communication in mathematics classroom (pp. 65–84). Reston, VA: NCTM
- Bartolini Bussi, M. G. & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artifacts and signs after a Vygotskian perspective, in L. English (ed.), *Handbook of International Research in Mathematics Education*, (second edition), Routledge, 746-783.



- Blanchard-Laville, C. (2000) De la co-disciplinarité en sciences de l'éducation. Revue française de pédagogie. 132, pp. 55-66.
- Boweden, J. and Marton, F. (1998). The University of Learning. London: Kogan Page.
- Caruana, F., & Borghi, A. (2016) Il cervello in azione. Bologna. Il Mulino.
- Damiano E. (2013) La mediazione didattica. Franco Angeli. Milano.
- Edwards, L., Radford, L., Arzarello, F. (Eds.). (2009). Gestures and multimodality in the construction of mathematical meaning. *Educational Studies in Mathematics*, 70(2).
- Faggiano, E., Ferrara, F., Montone, A. (2014), Special issue: Papers from the 11th International Conference for Technology in Mathematics Teaching (ICTMT11), *Teaching Mathematics and its Applications*, Oxford University Press, Volume 33, Issue 1, pp. 1-2
- Hoyles, C., & Lagrange, J.-B. (Eds.). (2010). *Mathematics education and technology Rethinking the terrain*. New York, NY/Berlin, Germany: Springer.
- Kress, G. (2015) Multimodalità. Un approccio socio-semiotico alla comunicazione contemporanea. Progedit. Bari.
- Laurillard, D. (2014) Insegnamento come scienza della progettazione. Franco Angeli. Milano.
- Leung, A. (2008) Dragging in a Dynamic Geometry Environment Through the Lens of Variation, *International Journal of Computers for Mathematical Learning* (2008) 13:135–157.
- Marton, F., Tsui, A.B.M. (2004) *Classroom discourse and the space of learning*. Mahwah, NJ: Lawrence Erlbaum Associates.
- Maschietto, M., Soury-Lavergne, S. (2013) Designing a Duo of Material and Digital Artifacts: the Pascaline and Cabri Elem e-books in Primary School Mathematics. *ZDM Mathematics Education* 45(7), 959-971.
- Monaghan, J., Trouche, L., & Borwein, J. (2016). *Tools and mathematics*. Dordrecht: Springer International Publishing
- Montone, A., Faggiano, E., Mariotti, M. A. (2017) *The design of a teaching sequence on axial symmetry, involving a duo of artefacts and exploiting the synergy resulting from alternate use of these artefacts.* Proceedings of CERME 10, to appear
- Rabardel, P. (1995). Les hommes et les technologies; approche cognitive des instruments contemporains. Paris: Armand Colin
- Rivoltella, P.C., (2012) *Neurodidattica. Insegnare al cervello che apprende*, Raffaello Cortina Editore. Milano.
- Rizzolatti, G. & Sinigaglia, C. (2006) So quel che fai. Il cervello che agisce e I neuroni specchio. Raffaello Cortina: Milano
- Rossi, P.G. (2011). Didattica enattiva. Franco Angeli. Milano.
- Rossi, P.G. (2016a). Gli artefatti digitali e la mediazione didattica. *Pedagogia Oggi*. 2, 41-64.
- Rossi, P.G. (2016b). Allignment. ESS. 7, 2, 33-60.
- Santagata, R. & Guarino, J. (2011). Using video to teach future teachers to learn from teaching. ZDM The International Journal of Mathematics Education, 43, 1, 133– 145.

Sherin M.G. & van Es E.A. (2009). Effects of Video Club Participation on Teachers' Professional Vision. Journal of Teacher Education. 60. 20-37.

Sibilio, M. (2014). La didattica semplessa. Napoli. Liguori.

Vinatier, I. & Altet, M. (2008). *Analyser et comprendre le pratique enseignante*. PUR. Paris.