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ESSAY IN MACROECONOMIC, MONETARY AND FISCAL DYNAMICS

RELATORE
Chiar.ma **Prof. Cristiana Mammana**
Chiar.ma **Prof. Elisabetta Michetti**

DOTTORANDO
Dott. Lorenzo Di Giorgio

COORDINATORE
Chiar.mo **Prof. Luca De Benedictis**

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Introduction

The research here exposed aims to analyze the macroeconomic dynamics and to make a comparative analysis of the different fiscal rules within the European economic and monetary territory within the broader Theory of Optimal Currency Areas (AVO) introduced by Robert Mundell in 1961 through the use of reference macroeconomic modeling with respect to models of dynamic stochastic general economic equilibrium (DSGE). Specifically, the research is articulated in the I Chapter in a critical analysis of the reference literature, especially with respect to the theoretical-economic evolution in reference to the optimality -or otherwise- nature of a given economic area with respect to the regulatory principles of the European Monetary Union and then address and hinge the principles in the main models of economic dynamics in the reference literature with a comparison of possible and potential novelties. Specifically, starting from the most pioneering researches introduced by the Nobel Prize in 1999, in the Chapter I will be traveled a path of introductory discovery of what appear to be the fundamental requirements in the identification of a given OCA according to economic theory with respect to the regulatory evolution of the European economic area and then complete a complete analysis with respect to the theoretical modeling of reference in Open Economy, analyzing the main reference models introduced since the development of the first stochastic dynamics models up to the latest experiments with respect to the introduction of fiscal dynamics models and endogenization of fiscal policies. The exposition is then articulated in chapters II and III where, through a rigorous analytical-mathematical exposition, on the one hand of a theoretical nature (through calibration techniques), and on the other of an empirical nature (through Bayesian estimation methods), the systemic logic of the European and Italian economies combined with the arduous task of capturing and comparing the nature and effects of distortionary-behavioral taxation, nomi-

nal rigidities, international investment mobility and endogenous fiscal policy will be exposed in four different scenarios accompanied by nominal rigidities and different sensitivities of fiscal policy to output growth and public debt. Each scenario will therefore be characterized by the multiple fiscal rules present in the European reference section. Specifically, as can be easily seen from the work, there will be four computational simulation experiments in which these rules will be embedded. Starting from the first scenario characterized by the respect of the limits of 3% on the Deficit/GDP ratios and followed by the second through the separation of Public Investments from the calculation of the Total Deficit (i.e: Golden Rule), we will arrive at the third and fourth scenario through the inclusion of the rules imposed by the Stability and Growth Pact (SGP) with the relative cyclical-adjusted balance-budget (CABB) for the purpose of the calculation of the respective public accounting ratios. The technique used will be that of macroeconomic modelling of reference with respect to the Dynamic and Stochastic Models of General Economic Equilibrium (DSGE), the results of which derive from the computational elaboration of impulse response functions (IRFs) capable of providing important information and results with reference to the response and volatility of the principal macroeconomic aggregates with respect to demand shocks (on Current Public Expenditure) and Supply shocks (on Total Productivity of Productive Factors) detected intersystemically, and at the same time allowing a comparison between the different outputs obtainable by subjecting the Eurozone system to the four exogenous shocks in a broader framework with the respective endogenous variables of reference. The result will therefore be a comparison in different scenarios characterized by budget limits and public spending accompanied by a more complete endogenization of policy variables capable of clarifying on the one hand the volatility of the main macroeconomic aggregates to the different intersystem shocks and on the other which policy complex would be better to adopt in light of the theoretical and empirical results thus obtained.

Chapter 1

A Review of Macroeconomic Dynamics and Macroeconomic Modelling of Fiscal Rules in Optimal Monetary Unions and Eurozone

1.1 Introduction

Following the crisis of 2008 and immediately after the sovereign debt crisis of 2011, the situation of the world economy has undergone considerable changes, especially within Europe and Eurozone, as a result of the various measures taken by international institutions, central banks and individual countries. From this point of view, particular doubts have begun to arise concerning, on the one hand, the suitability of certain fiscal rules in the European treaties to adapt to and be in line with certain periods of economic recession and, on the other, the real effectiveness and efficiency of the European system as a whole as an Optimal Monetary Area. Indeed, while the numerous interventions over the last decade have allowed the Eurozone to recover, albeit slowly and, as we shall see, with many limitations, from the period of recession, it is also true that numerous delays have been observed in the implementation of the same economic policies. The delayed and often non-coordinated joint intervention of monetary policy, centralised in the hands of the European Central Bank (ECB), with individual fiscal policies in the hands of individual member countries, has often had the effect of delaying economic recovery and at the same time limiting the anti-cyclical intervention hoped for by individual fiscal policies. In fact, the

numerous fiscal constraints in the European treaties have often prevented individual national economies from implementing expansionary fiscal policies in coordination with, and in line with, the monetary interventions of the European Central Bank (albeit sometimes limited and with delays compared to some of its counterparts in the main advanced economies (e.g. FED, BOJ)).(Orphanides (2019)).

All this has given rise to two particular types of needs at an academic and regulatory level: the first, undoubtedly inherent in new doubts about the real optimality of the European system, and especially of its operating rules, as an Optimal Monetary Area (with respect to what should be, as we shall see, its characteristics within macroeconomic theory), the second with respect to the possibility of forecasting and describing on the one hand, and implementing economic policies on the other, in line with its own internal dynamics.

In the light of the negative effects of the crises, at times even persistent in the European economy, numerous studies have therefore sought to shed light on the dynamics of the main macroeconomic variables in order to identify the most critical aspects of the Eurozone's economic system as well as to allow a better examination of the possible solutions that can be implemented at the level of economic policies (both fiscal and monetary).

Specifically, there is a need, not only in Europe but also worldwide, to provide new and additional answers to the ongoing debate on policies to be implemented in particular recessionary moments, as well as to create and build forecasting models capable of better capturing the main fiscal and macro-structural dynamics. Numerous examples come in the constant research to expand the possible responses that can be obtained from, in particular, the stochastic dynamic general equilibrium macroeconomic modelling (DSGE) used by central banks, and an invitation, as we shall see, for it to be expanded into new forms and extensions. Indeed, the review analyses carried out in this regard by, among others, Lindé (2018) and Chrstiano, Eichenbaum and Trabandt (2018) are clear references of how such extensions can come to light and be implemented in such models in order to allow for better predictions on the functioning and forecasting of the economy, moving forward and seeking to implement new forms of macro-structural relations. Starting, indeed from the first models developed by Chrstiano, Eichenbaum and Evans (2005)

and later with the new forecasting Bayesian techniques implemented by Smets and Wouters (2007) with the better examination of the economic dynamics and rigidities present in the system (nominal stickiness, real stickiness etc...) it is true that new answers and new relationship are coming up following a period of strong moderation of macroeconomic volatilities (*"Great Moderation"*) Lindè (2018).

Following indeed the economic crisis of 2008-2011 (and the ongoing of COVID-2019) and the difficulties in dealing with these recessionary dynamics, new important developements have arisen for a stronger description of economic reality and especially for the endogenisation of variables that were previously less considered (e.g. heterogeneity, financial frictions, public sector and fiscal rules, etc.). If, therefore, the current debate takes shape on these developements, this survey focuses precisely on one of them - that relating to fiscal rules and public spending policy - imposed in the European territory as an optimal monetary area in order to highlight its dynamics and provide possible future new responses in the light of both the institutional aspects and the evolution of the Eurosystem, and with respect to the reference literature within the broader panorama of the theory of Optimal Monetary Areas.

Reminder of this review is therefore as follows: in section II we will first highlight, at an introductory level, the concept of Optimal Monetary Area within macroeconomic theory and then provide a careful examination of what are the main critical and institutional aspects highlighted in the European reference framework and its fiscal rules in the light of empirical data and the ongoing policy debate, and then, in section III, we focus more on the studies that have taken place in the last decade with respect to those same rules within the new models of fiscal dynamics and on the new responses obtained precisely through new forms of implementation of new macroeconomic variables and dynamics (e.g. public expenditure rules, presence of public authorities, presence of the fiscal distorsions and comparison of different european fiscal scenarios etc.). In fact, if it is possible to analyse the euro system and try to frame its suitability as an optimal monetary area, this analysis must be based, on the one hand, on a correct examination of macroeconomic theory and, on the other, on a correct functioning of the models of economic dynamics that are useful for its refutation in order to achieve new responses to the many policies that can be implemented. In fact, it is not possible to frame this economic system as suitable

for ex-ante or ex-post optimality without first being aware of and able to provide answers, today in continuous evolution, on its individual internal dynamics and on the possible improvements that can be implemented, also thanks to the reference modeling, both at the positive and normative level. Section IV finally close the analysis.

1.2 Open Economies, Optimal Monetary Unions and Fiscal Rules

1.2.1 The concept of Monetary Unions: an historical and theoretical preface

Before introducing in depth the critique of macroeconomic modelling following the 2008 financial crisis, mainly on what turn out to be the fundamental critical aspects of the economic dynamics models that emerged following the inability to forecast it and on what turn out to be the main shortcomings in the literature of a more complete modelling implementation of fiscal rules and of their comparison within the European Monetary Union, it is necessary, at least in an introductory perspective, to dwell on and deepen what is the main subject of this review and to make the right and necessary in-depth analysis with the same economic theory. Indeed, if one of the main problems identified in the following work is the lack of a well-structured fiscal and public sector within the DSGE model framework for the Eurozone, especially in light of the effects of the financial crisis and the subsequent implementation of debt sustainability rules in the European treaties, it is necessary to provide a theoretical justification for why and how such a system can be improved and implemented in the macro-structural model system. The introduction to the concept of the optimal currency area and the institutional principles of the European Monetary Union plays a key role in this review.

In fact, it turns out to be the main starting point of economic theory for the construction of macro-structural models for the study of areas such as the Eurozone and the USA where the adoption of a single currency and, therefore, the absence of exchange rates, is accompanied by a fiscal policy of various kinds. The lack of a greater involvement of the public sector and taxation, together with the study of the dynamics arising from the main public budget items to the various idiosyncratic

shocks to which the economic system may be exposed, is in fact addressed in this framework especially from the point of view of open economies and in particular of those that turn out to be the Optimal Monetary Areas of which the Eurozone should be part of, at least in theoretical principle (we will see below – ss.2-s.3 - numerous critical aspects identified by the literature in this review on the real optimality of the Eurozone as an optimal currency area). In fact, the effects of the 2008 financial crisis (as well as the current one underway from 2019 following the COVID-19 pandemic) have important repercussions not only on the individual economies considered, but in an extremely interconnected manner, on the economic system as a whole, considered in the broader framework of international interdependencies and connections. Preliminarily, therefore, it is necessary to emphasise and dwell briefly on the concept of OCA and on the main characteristics listed by R. Mundell's economic theory. This clarification is necessary in order to understand from the outset how these aspects will later be linked both to the procedures for coordinating international monetary and fiscal policy, and especially to the rules of fiscal harmonisation within Europe and their correct implementation in macroeconomic analysis models. Introducing the concept and what have been the projects in the creation of currency areas and monetary unions in open economy systems, and especially in the Euro area, is obviously not a new concept in the world of economic theory. Introduced by Robert Mundell's pioneering studies in 1961, it began to take shape with the first Bretton Woods trade agreements in 1944 and was given concrete form in 1999 with the European Monetary Union project. In short, the question of Mundell's theory was based on whether and how a particular geo-economic area possessed certain characteristics that made it optimal enough to adopt a shared currency within it (Torti (2015)). The theory in its introductory form, as expressed by Mundell (1961), focuses therefore, through some initial assumptions on the coexistence of unemployment and idiosyncratic shocks, on identifying an area in which high levels of employment, balance of payments sustainability and low intertemporal inflation rates can be obtained through a minimisation of costs, allowed by monetary centralisation. Initially, therefore, the main possibility of success in creating monetary harmonisation and unification is identified as that of the Mobility of Productive Factors with consequent perfect elasticity and automatic adjustments

at international level. Thus allowing any asymmetric shocks - e.g. on the aggregate demand or production side - to be immediately reabsorbed in the regions subject to the Union, guaranteeing the absence of any peaks in unemployment and at the same time reducing potentially inflationary pressures. There is no doubt that the author intended to provide a strong differentiation between possible cooperation and strategies implemented by the central banks of states in situations of decentralisation and centralisation, foreseeing undoubted macroeconomic advantages in the latter case. The difficulty of establishing continuous harmonisation and strong monetary alignments between individual central banks, if any, gives way to the possibility of finding easier harmonisation in the creation of a single authority capable of automatically rebalancing any systemic shocks without the need for mixed strategies.[1]

Obviously, and as previously pointed out, all this starts from simplifying hypotheses on the mobility of productive factors - and more specifically of the labour factor itself - which is capable of triggering adjustments in the nominal exchange rate so as to absorb any asymmetric shocks. However, in addition to the mobility of labour, Mundell stresses that, for a region to be truly an optimal monetary area, it may at the same time present other characteristics of an economic nature beyond the only mobility of the labour factor. Specifically, characteristics such as:

- Trade integration: so as to favour synchronisation of the business cycle in the presence of excess demand and supply favoured by total price and wage flexibility
- Independence between asymmetric shocks: which in the absence of labour mobility or full price and wage flexibility would still be a key feature for the induction of a OCA
- Monetary efficiency: inversely proportional in its nature with the increase of currencies at flexible rates

The possibility of the creation of a currency area and thus of the adoption of a single exchange rate is therefore outlined by Mundell with respect to the essential features for its creation as well as with respect to the alternative of setting exchange rates

[1] An introductory reference and a stylized model of strategies with a Game Theory approach can be found in in Montalbano and Triulzi (2017) and, among others, Canzoneri and Gray (1985) and Hamada (1974) in Dornbusch and Frenkel (2015)

among countries in cooperative strategies. However, as Cesaratto (2018) points out, the possibility of the existence of exchange rate agreements between countries and cooperative strategies should always be respected by each country in an exchange rate agreement with another. On the contrary, this could create a phenomenon of deresponsibility in the revaluations of individual national currencies in the presence of imbalances in their respective trade balances. It is precisely in such cases that the establishment, not of exchange rate agreements, but of the adoption of a single common monetary policy between the two countries sharing a single currency finds its pivotal principles in the features listed by Mundell. The end of Mundell's work therefore underlines how the fundamental difference between cooperation in exchange rate agreements and the adoption of a single currency implies, starting with the associated wages levels, substantial differences at the aggregate level on deflationary tendencies on the one hand and inflationary tendencies on the other, favouring however a possible and easier rebalancing in the second case and a possible negative systemic and knock-on effect in the first.

1.2.2 From Optimal Monetary Unions to the Eurozone

The hypotheses put forward in Mundell's theory gradually came to life starting in 1944 with the Bretton Woods agreements and arriving, on 1 January 1999, in Europe where eleven member countries belonging to the European Union (EU) gave rise to the concrete phenomenon that in pure macroeconomic theory of the past had simply been so hypothesised. The Economic and Monetary Union (EMU), created in this sense, has in fact given rise to a currency area composed of hundreds of millions of individuals and fixed exchange rates through the adoption and creation of a true single currency whose trends and exchange activities are completely linked and harmonised by all adopters under the backing of a centralised banking system as the sole monetary authority: the European System of Central Banks (ESCB). The birth of the euro has postulated in its adoption in this way the particularity of the renunciation of the monetary discretion of individual states and consequently the mere process of agreement on the simple fixing of fixed exchange rates. The birth of the single currency thus represented an "extreme solution" towards the creation of a true currency area with the consequent application of the three fundamental

lemmas listed in Mundell's monetary theory (absolute stability of exchange rates, complete freedom in the mobilisation of the labour-capital factor and the abolition of monetary decentralisation).(Mundell (1961)) From this premise, the question arises spontaneously with respect to the evolution and birth, in the first analysis of such a system with the consequent theoretical and empirical answers on the Euro area and on the monetary and then fiscal dynamics that arise through elements of analysis in both institutional and macroeconomic terms. In fact, the European project came into being as a result of the long experimental process inaugurated with the Bretton-Woods trade agreements in which the member countries cooperatively accepted the adoption of exchange rates fixed against the US dollar. Consequently and logically, this fixation was also extended to individual foreign currency pairs so as to harmonise their performance with respect to the $n-th$ currency in question. The collapse of these agreements therefore frustrated the possibility of cooperative exchange rate setting strategies, as outlined above (Cesaratto (2018)), and meant that the participating countries had to carefully control exchange rate fluctuations within the stabilization of their trade balances. It is precisely this kind of continuous attempt to control currency fluctuations that has led some countries to adopt a single currency such as the euro with pure international exchange rate setting strategies. Of course, it is neither possible nor institutionally appropriate to attribute the creation of the European system to the only objective of minimising monetary fluctuations. In fact, the European system has seen the concomitance of two main objectives and activities of a political nature, which are, however, beyond the scope of this scientific work. Mainly these are to be recognised:

- A strengthening of the European monetary system: the need for which became apparent following the difficult cooperation with the US currency during its periods of economic stabilization, which naturally tended to close outwards in the absence of a more stringent constraint.

- The creation of a single and unified market: certainly already inaugurated by the Treaty of Rome in 1957 with the birth of the European Economic Community, but in need of further openings beyond the mere removal of customs barriers (such as e.g. the setting of exchange rates in order to standardise and unify trade flows within Europe and avoid natural fluctuations in regional

prices of an anti-competitive nature)

. The European system thus began its evolutionary path, starting with the institutional constitution of the European Monetary System (EMS) in 1979 with which eight of the first twelve participants inaugurated a form of bilateral fixing of their exchange rates through the so-called "Exchange Rate Mechanism" (ERM). This system provided for the creation of quantitative limits on the fluctuation of monetary exchange rates in order to limit their respective monetary fluctuations.[2]

The situation in 1993 was subsequently severely disrupted by strong speculative attacks on currencies, which led on the one hand to currency crises leading to strong political disagreements (as in the case of Italy and the UK) and on the other to the setting of larger fluctuation bands of up to 15%. However, it is necessary to point out that the ERM system was not totally devoid of certain rescue instruments in the event of monetary shocks; it is essential to recall how the agreements provided, among other measures, for forms of credit extension in the event of strong bilateral depreciation between currencies with the possibility of soft loans in order to rebalance trade balances and respective economic competitiveness (Basel-Nyborg Agreements, 1987).

However, the process of setting exchange rates and the resulting monetary alignments on a purely cooperative basis is a sticky one with respect to the performance and functioning of the economic system; From this conceptual proposition it is easy to understand why in 1993, following the German unification process and the consequent economic expansion of Germany, the high internal inflationary trends saw the Bundesbank react by raising interest rates abruptly in an anti-inflationary manner, leading to an attempt at systemic alignment towards the same rates in order to compensate for trade balances, but at the same time doing nothing but increasingly favouring a possible recession in all states.

However, attention should be paid to this historical process, which has seen a strong alignment of European currencies with the German currency. As analysed in the literature, the anchorage to an actor with particular anti-inflationary tendencies, as

[2]This system was, at least initially, quite successful in aligning the correlations between the monetary trends of the member states, although in the early 1990s the agreements were broken following the exit of Italy and the UK, allowing a return to the wider pre-agreement fluctuations. Specifically, however, fluctuations had long been contained within minima and ceilings of 2.25%, and in some circumstances and for some members, possible limits of even around 6%.

Germany was, allowed some states to take advantage in practice of the theorisation carried out by Tavlas (1993) and then studied in its European realisation by Rogoff (1985), Giavazzi and Pagano (1988) (in Krugman, Obstfeld and Melitz (2018)). In fact, the theory of the credibility of the EMS makes it possible to read in the light of the theory itself precisely that anchoring of certain economies to the trends of currencies and monetary authorities naturally tending towards the preservation of inflationary trends. The confirmation of these premises comes when we look at the trends from 1978 to 2020 in the European area, allowing us to see a progressive phenomenon of convergence starting from the values initially recorded in the 1990s in Germany. (Fig.2.1):

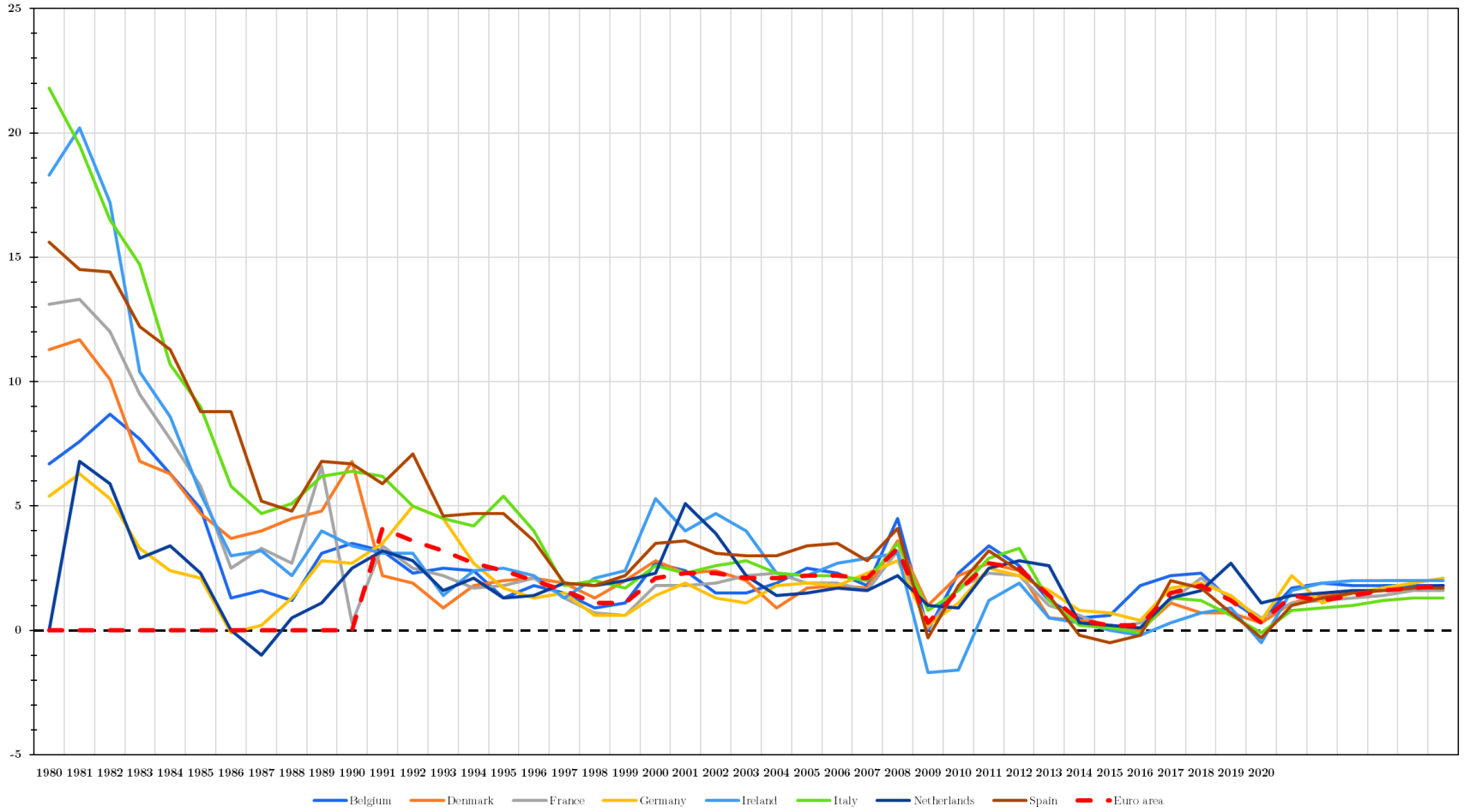

Fig.1.1: Convergence of inflation rates in percent change (ACP) in Eurozone (period: 1980-2020). Source International Monetary Found (IMF, 2020)

1.2.3 Fiscal and Monetary Rules in Eurozone: overview and issues of the institutional framework

Thus, on 10 December 1991, as a result of the above-mentioned economic pressure and political will, the member countries of the Economic and Monetary Union met in Maastricht with a view to making radical changes to the Treaty of Rome and to the functioning of the Monetary Union itself. This was the instrument to steer the continent towards a gradual and profound change in its economic structure and functioning. It was here that the Maastricht Treaty came into being, with the creation of a single currency and banking centralisation, leading to the birth of the Euro system in 1999. Although the instrument may appear to be an extreme departure from the mere agreements to fix international exchange rates, the objectives of the members were clear:

- To ensure greater economic integration through the elimination of continuous currency realignments with consequent minimisation of transaction costs on international trade
- To untie the Western economies from the "Contribution" found towards Germany and its macroeconomic objectives rather than towards those of the Community (through this process, in fact, the Bundesbank would be replaced in its role of anchor by the more centralised and shared European Central Bank (ECB) with a broader decisive panorama towards the individual situations of member countries and common goals).
- Harmonising common objectives with respect to the circulation of capital flows through the creation of a single monetary reference system (rather than through a system of multiple exchange rates)
- Achieve a form of greater political and social cooperation to foster relations between states and national governments

If these were the main objectives to be achieved, in their rather broad and theoretical form, the instruments needed to implement them in practice were, however, certainly more stringent and binding; indeed, the Treaty did not provide just for simple political or normative wording but also stressed the requirements and economic systematisation needed for entry and contribution to the new monetary cause.

In fact, the 1992 treaty came into being following the presentation of the famous "Delors Report" of 1988 as a result of research and prospective studies carried out by the Committee for the Study of Economic and Monetary Union (CSUEM). In this report it is possible to see a threefold time division of the gradual approach that would be made to monetary harmonisation and banking centralisation. Specifically, in the "First Phase" a form of harmonisation would be sought on individual markets with the removal of potential obstacles to financial integration and the search for international convergence between the economic policies of each member country. The "Second" and "Third Phases" would then have ensured the creation of unifying institutional bodies and then gradually established the fixed exchange rate regime with respect to the flexibilities of the individual national currencies. However in addition to the above-mentioned stages provided for in the Treaty particular attention should be paid here to the fiscal and financial rules included in the European institution itself.
In this case explicit reference is made to the fiscal convergence criteria as follows:

- Maintenance of average inflation rates of no more than 1.5% (always within certain temporary inflationary bands)
- Interest rate not more than 2 percentage points above the rates observed in countries with more stable prices
- Absence of exchange rate depreciation in the two years preceding entry into the Union
- Public budget deficit of more than 3% of GDP
- Public debt at more than 60% of GDP

Specifically, monetary union therefore set itself the main objective, at least initially, of aligning and stabilising prices at around 2% per year through forms of contingent foreign exchange reserves for the purpose of financing in the form of working capital and stabilising the respective exchange rates and interest rates in order to avoid forms of surplus or deficit in the respective current account.
Particularly significant, however, are the institutions on budget limits in the respective articles concerning the impossibility of certain forms, on the one hand of

quantitative extension of government deficit limits, and on the other hand of its financing. Considering in fact - in a sample form - the accounting deficit as the difference between public expenditure (goods, services, interest on debt securities) and taxation (revenue) according to:

$$D_t = (G_t + r_t B_{t-1}) - T_t$$

it is possible to derive potential financing activities by imposing equality according to the following three possible activities:

- Increasing taxation
- Issuing debt securities on the financial markets
- Injection of money into the economic system

From which follows:

$$G_t + r_t B_{t-1} - T_t = \alpha \Delta B_t + (1 - \alpha) \Delta M_t$$

where: G: Government Expenditure, r: Interest rate on securities, B: Bond, T: Taxation, M: Monetary quantity, α: share of deficit financed through issuance of debt securities (Canale (1998)).

However, through these public finance events on the issue of securities, an inflationary process would be created on the debt, with consequent increases in the interest "tranches" connected to it and a causal extension of the deficit values (under the assumption of proportional growth of national income). This activity could lead to unsustainability of the debt itself in the long run, as stated by Mankiw and Taylor (2011)'s on *"debt sustainability theory"* (in Ieluzzi (2015)), thus making it necessary to inject new money into circulation in order to finance the liabilities created.

Obviously, such a procedure is not possible in the light of the aforementioned European treaties, of which the regulation on the conduct of monetary policy is the centralisation of the ECB.

Since monetary and movable assets are no longer in the hands of national economic autonomy, the reduction of the recorded debt levels could only be achieved through fiscal activities in order to provide a fair compromise between economic growth and compliance with the imposed budgetary constraints. On this aspect, as we shall see, part of the literature has expressed itself with reference to the consequent reduction in aggregate demand, employment and income following restrictive effects of fiscal

policies aimed at forms of bond coverage. (Orphanides (2019))

It is precisely from this conflicting aspect and quantitative incompatibility between taxation, growth and debt financing that the problem of bank centralisation arose with respect to the objectives of monetary and inflationary stability versus income and employment stability. Kyndland and Prescott (1977) pointed out in this connection that, in the "time inconsistency" dilemma, if such a procedure were to be conducted without rebalancing, the monetary authority alone would be unable to cope with certain long-run asymmetric shocks. These prerogatives later came to the fore, as previously introduced and specified among others by Orphanides (2019), with the financial crisis of 2008 after which the various European institutions suffered important recessive influences on the realisation of an optimal system in light of the budgetary and fiscal constraints imposed precisely because of, on the one hand, the reduced intervention of the ECB to stabilise markets and national debts for a sufficiently long time, and on the other hand, the misalignment in the coordination of fiscal policies in the hands of individual states, especially when compared with the fiscal interventions of the main advanced economies, due to the limits imposed by the treaties themselves. As carried out by the analysis of Orphanides (2019) it is possible to notice, some of these differences with respect to the expenditure intervention after the year 2008 in Fig.2.2 with respect to the ratio of General Government primary net lending/borrowing to GDP.

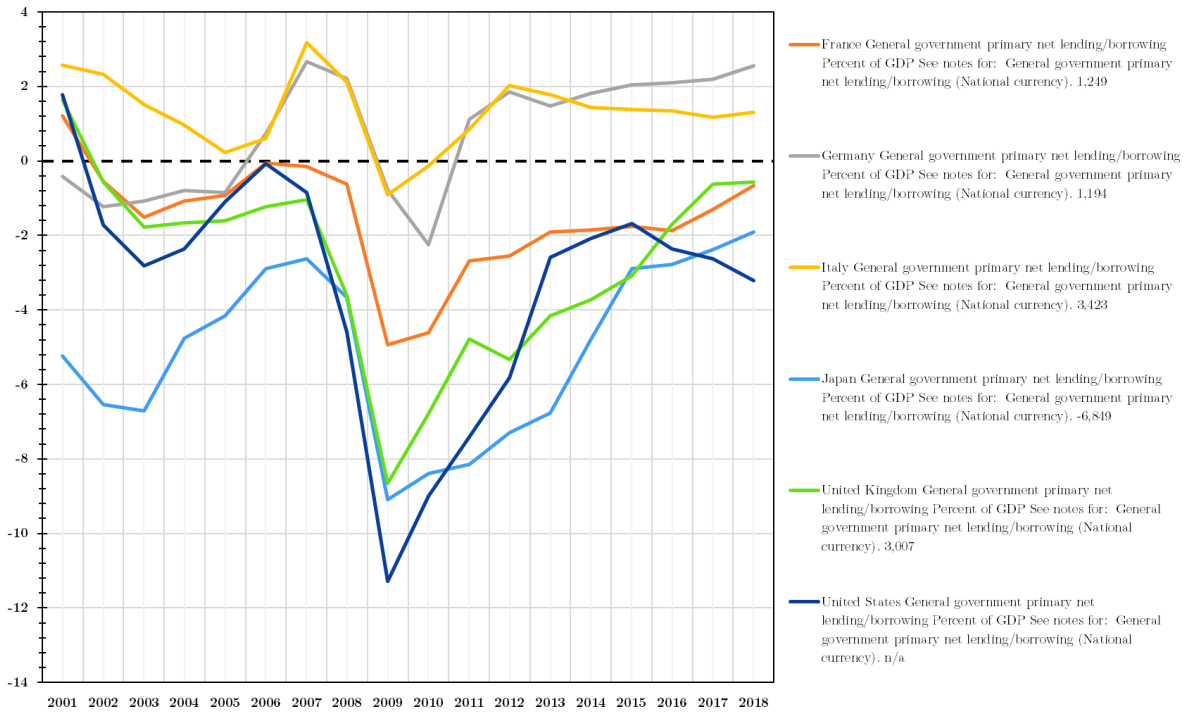

Fig.1.2: General Government primary net lending/borrowing as a percentage of GDP. Specifically, it can be seen that in the USA there was an increase of around 10% of GDP in the period 2008-2009 with a slow flattening in the following years to around 2%. Likewise Japan, with an initial increase of almost 9% and a subsequent gradual reduction of 3% until 2015. In the EU territory, on the other hand, the increase was certainly of smaller proportions, starting from an initial increase of only 4% (in the area as a whole) and then falling back in 2011-2012 to a greater control of the budget aggregates in the individual states (in the figure, Germany, France and Italy). Own revision and elaboration from Orphanides (2019)

In response to this, institutionally the process of evolving the Eurozone's own fiscal rules in this sense is further expanded, precisely for these purposes, as of 2011 with the introduction of the new monitoring rules for fiscal assessment procedures. Specifically, explicit reference is made to the 'Treaty on Stability, Coordination and Governance' (TSCG) with the regulations included in the 'Six-Pack' and 'Two-Pack' plans. These instruments formed part of the broad framework of the so-called "Economic Adjustment Programs" (EAP) as a rescue vehicle for the countries hardest hit by the rise in sovereign debt following the 2008 recessionary cycle. Specifically, further strict conditions were introduced regarding structural reforms, labour, public borrowing and privatisation procedures. The result was an even stricter control on the so-called "Deficit Spending" possible in times of recession in order to favour a potential economic recovery. (Torti (2015)).

Thus, first in 2012 with the European Fiscal Compact (2012) and then in 2013, in the wake of the aforementioned budgetary controls, the so-called "European Semester" was introduced as a procedure for joint planning of individual national economic programmes at EU level in order to provide principles and guidelines ex ante for the implementation of certain fiscal policies.

It makes use of two particular instruments: 1) Annual Growth Survey (AGS) and 2) Alert Mechanism Report (AMR). The AMR is particularly important given the requirement for Member States to provide forward-looking information on their public budgets one year after the event, on a programmatic and precautionary basis, and then to analyse, at the EU level and under the Commission's recommendations, any subsequent consolidation plans and corrective actions in compliance with the deficit and debt parameters of the SGP.

Finally, this procedure takes the form of the annual publication of the "Macroeconomic Imbalance Procedure", which identifies macroeconomic imbalances (and specifically employment and fiscal imbalances) with respect to the proper functioning of the currency area. As a result, its objective becomes that of redefining medium-term budgetary objectives (MTO) in order to ensure a structural improvement of at least 0.5 per cent of GDP per year.

In quantitative terms, this value should have and should have maintained a certain "safety band" with respect to the previously introduced 3% deficit limit, allowing

countries with particularly high debt ratios ($> 60\%$) to operate in a freely more expansive way in positive economic cycles and vice-versa in negative ones.[3]

Certainly, the forms of justification, at least in a treaty-based way, of the 2012 "Fiscal Compact" derive almost autonomously from the assumptions set out above, which, through the provisions of the Article 3, introduces national regulatory obligations with respect to the aforementioned budget principles in order to ensure a definition of the deficit and debt levels set by the previous European limits, albeit with some forms of objective limitation.

In this sense, the provisions subjected to the stipulation of the 2011 "Europlus" pact aimed at the transposition and subsequent introduction at a legislative-constitutional level of the rules of the "Fiscal Compact" through interventions decided at the discretion of the individual national member states, however subject to the maximum deficit limits and fluctuations provided by Directive 85/2011/EU. The result is an extension of the subsequent European control of fiscal policy with respect to the provisions of the "Two-Pack", as set out in Commission Resolution 821/2011 (EU Commission (2014)), as an additional surveillance instrument with respect to the economic-financial rebalancing programmes through the "Post-Programme" surveillance phase, with active monitoring of any financial amounts received and used for the economies most in difficulty. The balanced budget rule and the regulatory obligation to respect it is therefore heavily inserted in the abstract factuality of fiscal autonomy with respect to monetary autonomy, making the process of macroeconomic rebalancing even more viscous with respect to the aforementioned idiosyncratic shocks.

Such an interpretation, certainly endowed with a particular elasticity, derives from the same provisions of Article 109-J of the Maastricht Treaty itself on the entry into the currency area according to the parameters of sustainability of the financial positions of individual governments, as well as in Article 104-C with respect to the possibility of excess of the rules provided for although compensated by prospective trends if adequately included in a positive framework. An example of this could be the provisions relating to the possibility of extending the structural deficit with

[3] In this sense it is understood that economically the dynamic growth rate of the "G" value (public expenditure) will never be able to follow a faster growth (in relative terms) of the medium-term GDP if it is not compensated by more than proportional forms of revenue. Ieluzzi (2015)

possible surpluses of up to 1 per cent of GDP (compared to the mandatory 0.5 per cent), but only for economies with a debt ratio significantly below 60 per cent of GDP.

This temporary nature of possible surpluses with respect to the deficit/GDP and debt/GDP ratios would thus seem to have allowed some form of fiscal elasticity. Nonetheless, a restrictive form is reached with reference to states with larger debts and to the related activity of the ECB with respect to possible interventions on the granting of overdrafts or credit facilities to favour these forms of internal rebalancing (Art. 123). The result is that the monetary authority is unable to finance any excess debt values, and the possibility of accommodative monetary policies in the case of intervention with expansionary fiscal policies is no longer possible. (at least in principle and according to the conventionality of the ECB's monetary policies).[4] In this context, the amendment to the Stability and Growth Pact (SGP) with respect to budget policies has been introduced, in turn making such interventionist relations even more restrictive, as confirmed later by the European Commission (2016) itself (*"Towards a Positive Fiscal Stance for the Euro Area"*), leaving open questions on the actual definition of commonly agreed rules, and of possible equitable advantages, rather than with reference to forms of economic reassurance for the more stable economies with respect to those with inflationary natures and higher financial imbalances. (Masera (2002)). This is probably a form of risk hedging towards the monetary structure which, however, in the absence of the regulatory and fiscal federalism typical of certain currency areas, it risks making it even more difficult to achieve symmetries in the economic structures while generating, as underlined by the research that has emerged in recent years - and as we shall see in the following paragraphs - forms of negative transmissions with respect to systemic interdependencies in terms of both monetary and fiscal policies.(Orphanides (2019))

[4] Although as specified in the Treaties, following the sovereign debt crisis and the ongoing pandemic crisis (COVID-19), activities of a non-conventional monetary nature were then implemented by the Central Bank. For an examination of their role with respect to European fiscal rules, see among others, the analysis and review by Orphanides (2019)

1.3 Empirical Evidences on Fiscal Dynamics

1.3.1 Critical aspects and results in policy debate from a comparative model-based litterature review

Once the regulatory and institutional aspects on which the Eurozone system is based have been introduced in the broader framework of the Optimal Currency Area Theory (OCA), the research of this chapter then requires a careful analysis of the European situation in the light of the aforementioned theories and of the main results obtained, both from empirical analyses and from the reference model-based literature and the resulting debate on feasible economic policies. As previously pointed out, in fact, it is necessary to go into the specifics of both the fiscal aspects within the broader panorama of the policies of individual states and the harmonisation and integration of economic policy. More specifically, in the light of the arguments set out above, the question arises as to whether Europe can constitute an optimal currency area ex ante, on the basis of classical theoretical assumptions, while at the same time guaranteeing an adequate cost/benefit trade-off with respect to the sole fixing of exchange rates. As previously stated, some of the possible triggering factors for the identification of an optimum monetary area are those related ex-ante to:

- The intensity and integration of international trade flows
- The mobility of the production factors
- The presence of asymmetrical macroeconomic shocks
- Convergences between nominal interest rates
- Similarities in individual production subsystems
- Similarities in individual fiscal subsystems

In such a panorama it is possible to extract adequate data and results to become aware of the nature of the European area and to draw, at least preliminarily, conclusions regarding its monetary suitability.
As regards the first aspects relating to trade integration and factor mobility, it is possible to quickly analyse the intensity of trade between individual countries in order to gain a better understanding of integration in the product market on the

one hand, and integration in the factor market itself on the other. Specifically, since 1992, European foreign trade between member states (Intra-EU) amounted to a flourishing range of 13-20%/GDP. Foreign trade has also increased as a share of trade with major world economies and foreign direct investment (FDI) in the European area has increased.(Carril-Caccia and Pavlova (2018) in Draghi (2018)).
Considering these data in order to analyse the degree of economic integration, we can see that the harmonisation of fluctuations prior to the introduction of the euro has thus favoured a growth in European intra-system trade. This fact, however it may suggest a certain appropriate economic policy through the old ERM system, is certainly not enough on its own to label Europe as an optimal territory right from the start, nor does it automatically justify the systematisation of a single currency. In fact, although the Single European Act (SEA) of 1986 had made it possible to harmonise price trends, in some markets no particular differences were initially observed between the pre- and post-adoption period, at least as regards aspects linked to price divergences. (Engel and Rogers (2004)). It was only later that it was possible to notice that these effects were not the same with regard to trade intensity.
Indeed, through such a simple European analysis, it is possible to see how the trade system has seen a gradual integrative increase in intra-European flows following the very introduction of the euro. Thus, if the continental territory did not show any particular characteristics that could define its suitability for internal price alignments, the creation of a currency area has certainly allowed, ex-post, a certain form of self-adaptation. In the opposite direction, however, are on the one hand the studies carried out with respect to the mobilisation factor of production factors, which, following empirical econometric studies, show that there is a certain limitation due to internal regulations in individual states, but especially the differences in the alignment, coordination and implementation of individual economic policies in the light of the ECB's broader monetary interventions. (Orphanides (2019))
In this respect, empirical research on asymmetric shocks and divergence in inflationary trends among member states is worthy of attention. In fact, it specifies how, through banking centralisation and harmonisation in continental trends, the pre- and post-euro phase, thanks to the anticipatory adaptation of markets to exchange rate stabilization, led nominal interest rates towards initial forms of convergence.

Nevertheless, the same phenomenon could not be observed for real interest rates due to the inflationary divergences of countries such as those belonging to "peripheral" Europe giving rise to a certain form of "monetary instability" (among others, Krugman and Obstfeld and Melitz (2018) and Ferrero, Raffo and Eggertsson (2014) (Fig.2.3).

The appreciation of the real exchange rates of these countries therefore led to an initial rather marked form of deficit in their current account balances (Fig. 1.4).

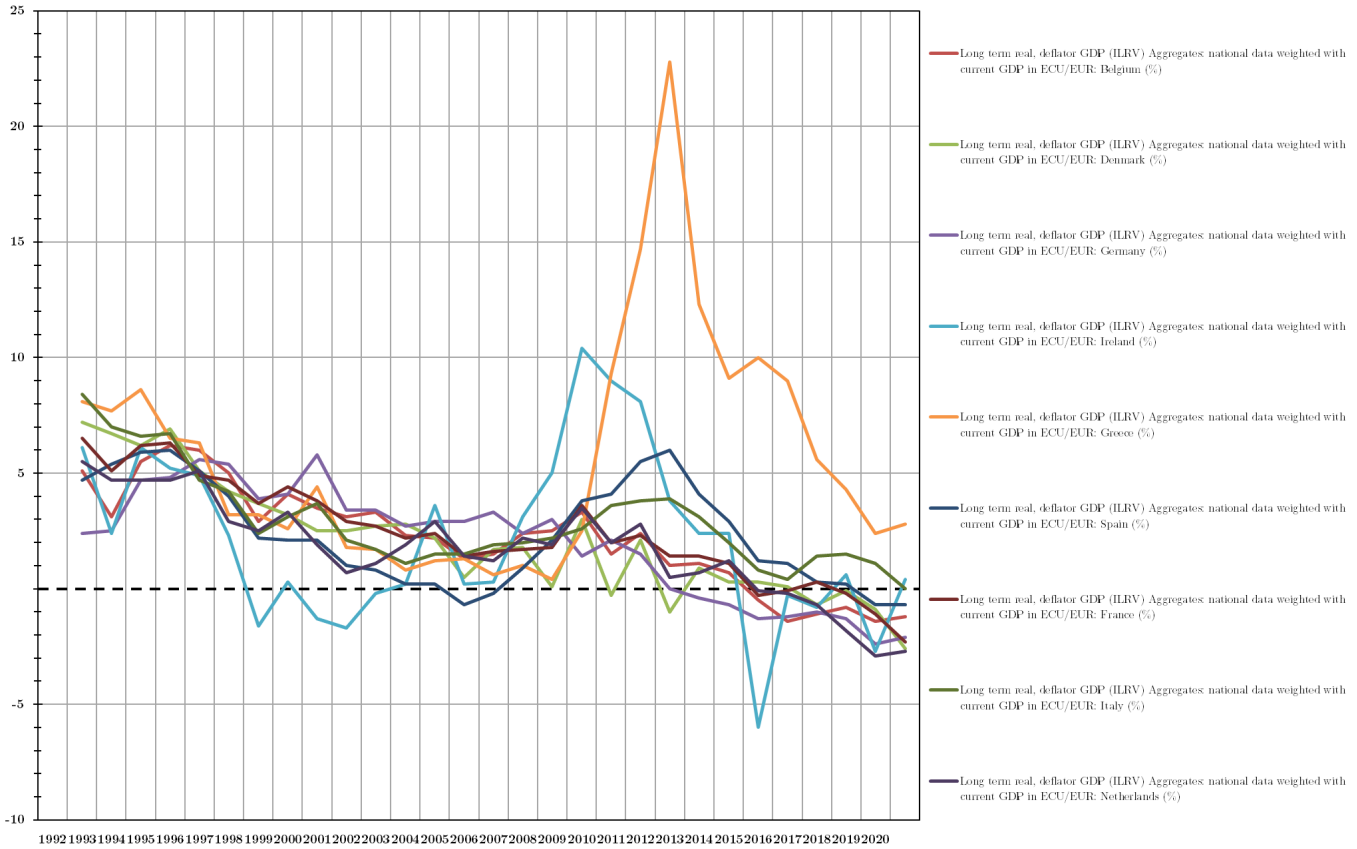

Fig.1.3: Euro area real interest rate divergence. Own elaboration from 1992-2020. Source: AMECO database

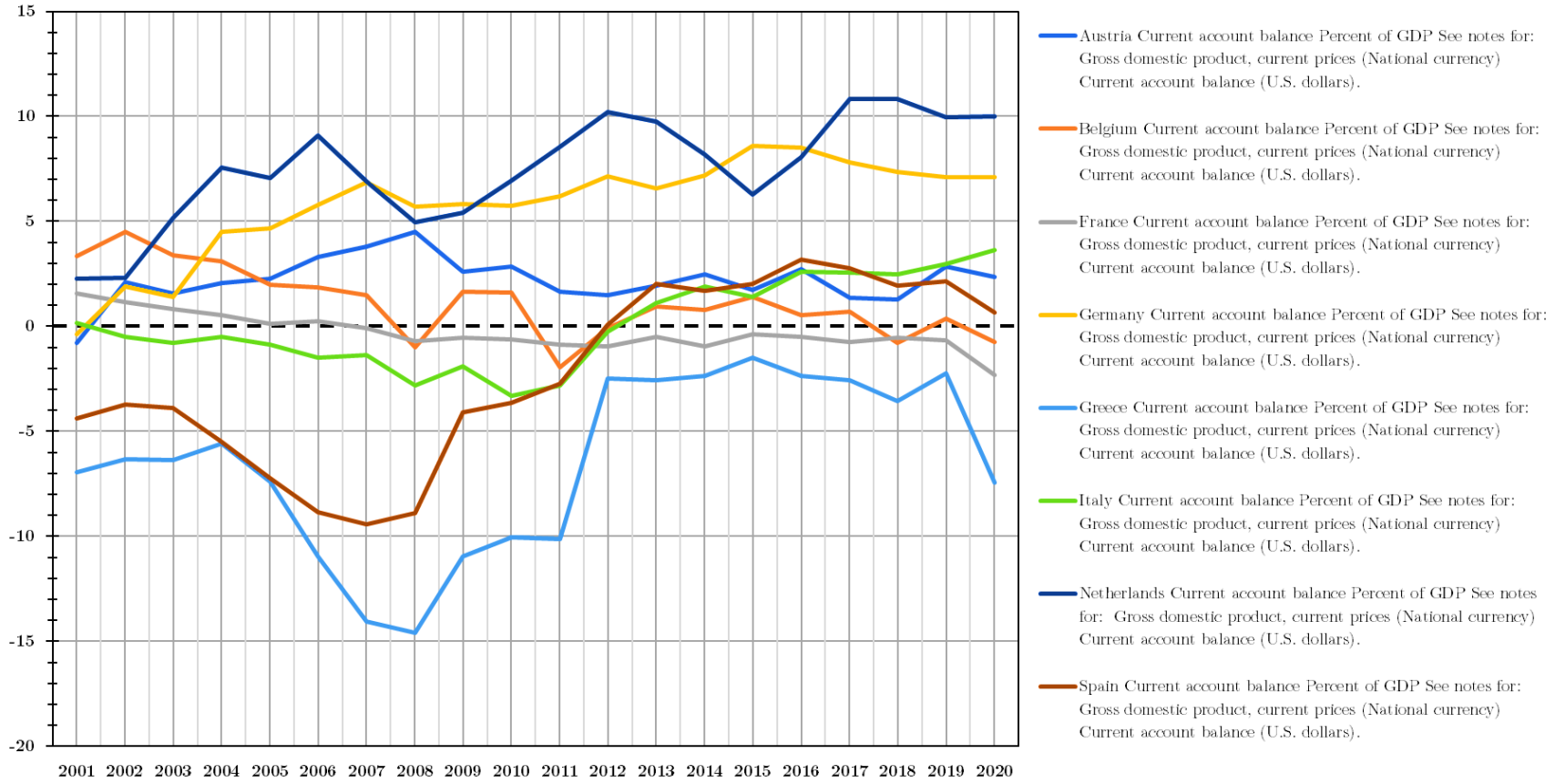

Fig.1.4: Current account balances in Euro area countries as % of GDP from 2001-2020. Own elaboration. Source: IMF, World Economic Outlook database

In addition, the differences in the individual industrial structures and fiscal stickiness should also be highlighted; these factors could suggest a causal relationship with the sub-optimal economic situations in the European area. However, as the studies have shown, monetary or trade analyses alone are not sufficient, albeit necessary, empirical evidence to refute macroeconomic theory.

While on the one hand, there are facilitations in the mobilisation of capital flows with consequences on financial markets, on the other hand there are strong stickiness and frictions on the labour factor side, with high unemployment rates and wage differentials, leading to stickiness and frictions on possible asymmetric shocks arising therefrom. (see, among others, World Economic Forum (2012) in Ferrero, Raffo and Eggertsson (2014)). This is certainly due, on the one hand, to the impossibility of adjusting different domestic interest rates as could happen in situations of flexible exchange rate regimes, and on the other hand, to the lack of centralisation of fiscal policy which, as we shall see, is the main lever, in accommodation with monetary policy (Orphanides (2019)), to rebalance economic growth and consequently the balances of systemic interdependence. The fiscal rules set by the treaties of the last decades (SGP, Maastricht, Lisbon, Fiscal Compact, etc.) do not allow, as is evident, total fiscal flexibility and its consequent systemic harmonisation, making their implementation often dependent on forms of "creative accounting" aimed at a form of circumvention of the imposed system of rules (Krugman (2010)).

In this sense, it is easy to see how the Euro system should be analysed from further perspectives, trying to understand both the "second-best" dynamics of the respective markets for productive factors, as well as the fiscal and financial aspects relating to the forms of risk coverage, in order to understand the sub-optimalities noted in the previous analyses. If, in fact, the theoretical perspective developed by economists regarding the ex-ante or ex-post optimality of certain economies makes it possible to read the Eurozone system as lacking some of the fundamental requirements, such proof must come from a careful modelling and reference analysis with respect to the data regimes and economic scenarios with annexed systemic consequences in terms of positive or negative margins created through the respective interventions of the central, fiscal and monetary authorities.

In fact, given this necessary conceptual premise, express reference is made to the eco-

nomic interdependencies existing between European countries and their respective financial markets in order to highlight the most critical issues existing with respect to the European Treaties of reference. To this end, express reference is made to the results obtained from the reference literature through which an examination of the marginalities arising from the same budgetary and monetary policies can be provided and possible developments understood, on the one hand for a correct examination and study of the European situation from a theoretical and empirical point of view, and on the other for a clearer understanding in the ongoing policy debate.

In accordance with the recent literature of reference (Montalbano and Triulzi (2017)) it is in fact possible to become easily aware of the growing international economic interdependence in terms of transmission of the effects of economic and financial policies with respect to the connected economies in the growing feeding of a "*circular causation*" system.

In the specifics of the various cases where the so-called "*macroeconomic externalities*" are present, there can certainly be opportunities of a positive sign if carried out from the viewpoint of a broader cooperative panorama, rather than of a negative sign if certain basic assumptions are not respected as such. In this sense, therefore, it is advisable to be aware of these direct and indirect repercussions in order to coordinate individual economic policies as well as possible so as to positively marginalise these transmission effects.

In this sense, starting from the first analyses carried out in the 70s and 90s by Niehans (1968), Oudiz and Sachs (1985), Canzonieri and Gray (1985), Hamada (1985), among the numerous researches that have emerged in recent years with respect to this structural interdependence, those related to the analyses carried out in Stochastic Dynamic General Equilibrium (DSGE) modelling are of particular economic importance here, starting from open economy models for an analysis of the dynamic transmission effects of international economic policies in the presence of a first form of distortionary effects moved by the taxation of individual European states (Schmitt-Grohe and Uribe (2004, 2006, 2007)) up to models directly applicable to the Optimal Monetary Areas and in particular to the criticalities exposed in the Eurozone.

In this sense, Gali and Monacelli (2005) and especially Gali and Monacelli (2008),

accept the request for a greater involvement in the study of European dynamics of further theoretical elements in order to link them to the above mentioned volatilities and dynamics of the Eurozone. In fact, by including nominal rigidities and fiscal policy in a *continuum* of small economies, they are among the first to starting to point out that the presence of those strong constraints seen so far (especially in the presence of nominal rigidities) has started to call into question the ex-ante European optimality specifically in terms of the ability and possibility to make national fiscal policies anti-cyclical while allowing fiscal policy to play its own stabilising role in the absence of the possibility of nominal exchange rate adjustment on individual currencies. As specified in the reference literature, international economic-policy coordination under fixed exchange rate regimes requires greater attention (including at the modelling and forecasting level) than in similar situations under flexible monetary regimes. While nominal exchange rate fluctuations are capable of rebalancing any short-term financial and trade imbalances in such a way as to act as automatic stabilisers, this is not possible in fixed exchange rate systems, making the relationship between taxation and money particularly delicate for the purposes of inter-system economic rebalancing. The work of Ferrero (2009), where the impossibility of nominal exchange rate adjustment is replaced by the role of individual tax rates in the task of rebalancing the impact of shocks on international competitiveness among the interconnected economies of the Euro-system, makes a fundamental contribution in this respect. Like the results previously achieved by Beetsma and Jensen (2015), and evolving from the work of Gali and Monacelli (2005) and Benigno (2004), Ferrero also shows the possibility in the European system characterised by the presence of nominal rigidities and a greater disaggregation then of distortionary taxes with the inclusion (with annexed dynamic effects) also of national debts in order to analyse their optimality with respect to the targets of the monetary and fiscal rules themselves as seen above. In this way, part of the answers on the coordination between fiscal and monetary policy and on the cost-benefit effects of the previous paragraph are analysed with respect to the joint conduct of discretionary fiscal and monetary policy at the European Central Bank (ECB), allowing an analysis of the effects on the real values of debt. The presence of European price stickiness, according to Ferrero, leads in this sense to the creation of real effects by monetary policy,

while distortionary taxation introduces non ricardian effects of fiscal policy giving rise to results related to the influence that changes in the nominal interest rate (and the equilibrium inflation rate) have on fiscal decisions through the real burden of debt.(Ferrero (2009)) A specific extension to this landscape and on possible effects on the aggregate demand is also inaugurated in Ferrero's research developements, precisely through the further extension of the above-mentioned fiscal disaggregation, for instance through, and as we will see in the following chapters, the implementation of distortionary taxation on consumption and on the same returns to private capital and financial assets. An example of this is the possibility of the inclusion of consumption taxation given by the formulation: $(1 + \tau_c)P_tC_t$ as well as for Capital and Bonds as: $(1 - \tau_K)R_t^K K_{t-1}$, $(1 - \tau_B)R_tB_{t-1}$.

Finally, in its normative implementation, the policy debate is then routed on the right implementation of an optimal fiscal policy plan, desirable as more flexible on debt, with respect to a greater policy rigidity of monetary targets (fixed in maintaining price stability) in the light of European budget rules. In the current debate, the best example of this is specifically provided when, in contrast to the assumptions of traditional economic policy theory regarding equality between available instruments and achievable objectives, a problem of constrained optimisation with respect to the economic policies to be adopted arises. Today, this term is expressed in the concept of "flexible objectives" as the undisputed protagonists of the European economic debate and discretion in the implementation of given optimal policies. It is indeed within the Eurozone that the trade-off between maximisation of objective functions and economic constraints comes to life most in the decentralised management of fiscal policies, entrusted to individual national governments under specific contractual constraints (Stability and Growth Pact, "Fiscal Compact", Two-Pack, Six-Pack, European Semester) as opposed to monetary centralisation entrusted discretionally (or at least partially) to the ECB's banking autonomy. This denotes the phenomenon of the fiduciary crisis in the validity of the single currency, starting from the financial crisis of 2008, which takes on, precisely in the light of these phenomena, greater positively related meanings. (Matheron, Mojon and Sahuc (2012)).

Particular attention should therefore be paid to the factual activities available to individual economic authorities towards systemic stabilization in light of the impos-

sibility that has emerged towards the implementation of dynamic adjustment policies in public budgets by the banking authority in question. (as recognised by Art. 105(1) TEU and Art. 123 Lisbon Treaty). (This concept also returns in Orphanides (2019) in the case of discretionary intervention and policies of an unconventional nature in the presence of Zero-Lower Bound (ZLB) by the Central Bank).
This leads to the search for the aforementioned fiscal flexibility on the part of individual states in order to alleviate the burden of national public debts on a completely discretionary basis but still subject to EU budgetary constraints. (Montalbano and Triulzi (2017)).
The situation of flexibility, and therefore constraint, makes fiscal policy choices in this sense subject to differentiated expected values and risks causing at the same time misaligned procedures on the values of interest rates applied to any given national security ("spread") useful for forms of financing on the main financial market. One example could be the possible new model-based implementation of this differentiation between the returns of individual government bonds through the endogenisation of risk premia by differentiating these interest rates with respect to the union-wide interest rate set by the European Central Bank. Attempts of this kind in the literature have been made in recent years by various authors, in an attempt to show the differences between different country-specific rates of return on financial assets. Beginning with the work of Furceri and Mourougane (2009) and Albonico et al. (2019), through the implementation of a measure of risk premiums over the central bank interest rate, as well as the implementations of term premiums in Christoffel et al. (2011) with respect to short-term and long-term bonds, as well as implementations of perpetual bonds and debt-elastic bonds as in Woodford (2001) and Schmitt-Grohe (2003), respectively[5]. Obviously, as stated by Orphanides (2019) such economic inhomogeneity only makes expectations and the conduct of respective fiscal policies asymmetrical to such an extent that individual countries are more likely to default than other partner economies.

[5]For a review of different methodologies to implement risky bonds see, among others, Rudebusch and Swanson (2012)

1.3.2 The key role of Public Sector Endogenizations, Feed-back rules and new results from the fiscal dynamics analysis

If what we have seen so far allows us to read the criticalities and characteristics of the Euro area and its fiscal rules with respect to the results obtained from the reference literature on optimal monetary areas, within the framework of dynamic and stochastic reference modelling, following the 2011 sovereign debt crisis other fundamental aspects have come to light with respect to the possibility, both of analysing further operating dynamics within the European territory and assessing their optimality with respect to the characteristics initially listed in Mundell's theory, and of searching for further tools to implement correct economic policies within the EU territory and study the dynamics of transmission between individual states.
In the light of this, in fact, a further fundamental aspect linked to the analyses carried out by macroeconomic studies with respect to "flexibility" and coordination between different policies within the Eurozone territory, which then makes it possible to analyse the dynamics relating not only to individual fiscal policies in coordination with monetary policies, as well as to study one of the fundamental characteristics listed in traditional theory but at the centre of strong contrasts with respect to the institutional rules in the Eurozone, especially after the crisis triggered by sovereign debt in 2011, is that more generally concerning the issues related to the mobility of production factors and specifically to the presence and dynamics of public capital and related public investments as well as their evolution as a further element of criticality and financial intervention.
If flexibility in fiscal targets is required and identified as a fundamental characteristic for the proper functioning of the EU apparatus, it is also clear from studies that this flexibility can be partly curbed by the budgetary rules laid down in the Treaties.
In fact, by bringing to light the negative effects of the financial crisis of 2008, which was later channelled into the sovereign debt crisis of 2011, public investment and capital stock items have also played an increasingly important role in relaunching the individual European economies. Indeed, if the previous analyses allow us to understand the stabilising role of tax rates in a single currency system (coupled with the greater realism of a present nominal stickiness on european prices and wages), in the same way extensions and contributions on the role of public investment also

allow us to read the nature of individual national financial policies within a monetary centralisation system. In fact, the presence of fiscal rules such as those present within the Eurozone territory (referring to the limits imposed on the ratios between Deficit/GDP and Debt/GDP, as well as the constraints on the structural balance (CABB)) require a correct implementation, not only of those that result to be the regulatory dynamics and coordination with the ECB monetary policy but, as seen above, also with respect to the correct implementation of variables driving volatility and different transmissions on the economic cycle of individual countries. (Ghironi (2017))

On one side so in the following research, new exemples of this kind of implementation will be done according to the guidelines of the International Monetary Found (IMF (2018)) and MEF (2013), introducing the Cycle Adjusted Balance according to the following rule:

$$CABB_t = d_t - \varsigma OG_t$$

where $CABB_t$ Indicates the (public) cycle-adjusted balance budget and ς is the Sensitivity parameter of the deficit d_t to the Output Gap (usually estimated for EA= $0,48$). This parameter, obtainable through the difference between the sensitivity of the inputs to the Output Gap and the Outputs such that: $\varsigma = \varsigma_e - \varsigma_u$, It allows to consider which types of income and expenses are sensitive to the Economic Cycle. Specifically, as specified by the OECD and the ECB, we obtain a sensitivity of ς_e with respect to: Taxes on personal income, Taxes on firms income, Indirect taxes, Social Contributions; on the contrary for ς_u the most significant item is that relating to Unemployment benefits. The elasticity of these items to the OG will change the total sensitivity value ς.

On the other side new studies with implementation of capital and public investment are fundamental in order to frame them in this broader framework of European fiscal rules. It is Pappa (2004), Kumbof and Laxton (2007) and then in extension for the European Monetary Union in Straub and Tchakarov (2007) who first presented the possibility of including capital and public investment within a model of economy in an optimal currency area through an extension of the more famous ECB-NAWM ('European Central Bank-New Area-Wide Model') as in Coenen, McAdam and Straub (2007). If, as seen above, the mobility of productive factors, and in par-

ticular of capital, is one of the fundamental requirements for the proper functioning of the monetary area as well as, with their accumulation process, for economic growth in general and despite the exhortations to increase public investment in order to safeguard the growth of the european economy by the European Commission in 2003 (European Commission (2003), in Straub and Tchakarov (2007)) and (IMF (2014-2015) in Burlon and D'Imperio (2019) and Burlon et al. (2017)), the actual expenditure for the same and the accumulation of public capital stock, due to the stringent dynamics on Deficit/GDP and SGP ratios, have seen just in the continental territory a constant reduction in the last forty years (specifically Straub and Tchakarov (2007) underline how in the EU-12 since 1970 the reduction has been 4. 2% in relation to GDP and to just under 3% in 2005, especially in Austria, Belgium and Germany, while in Italy there has been a small evolution from 1995, when they were around 2% in relation to GDP, to a rise again around 3% (an example of these dynamics can be seen in Fig.1.5 for the period from 1992-2022) on the contrary of a public consumption always in greater expansion (the authors record levels of public consumption increasing from 1970 to 2005 going from 14.8% to 20% of GDP (an example of such dynamics can be seen in Fig.1.6 for the period from 1992-2022).

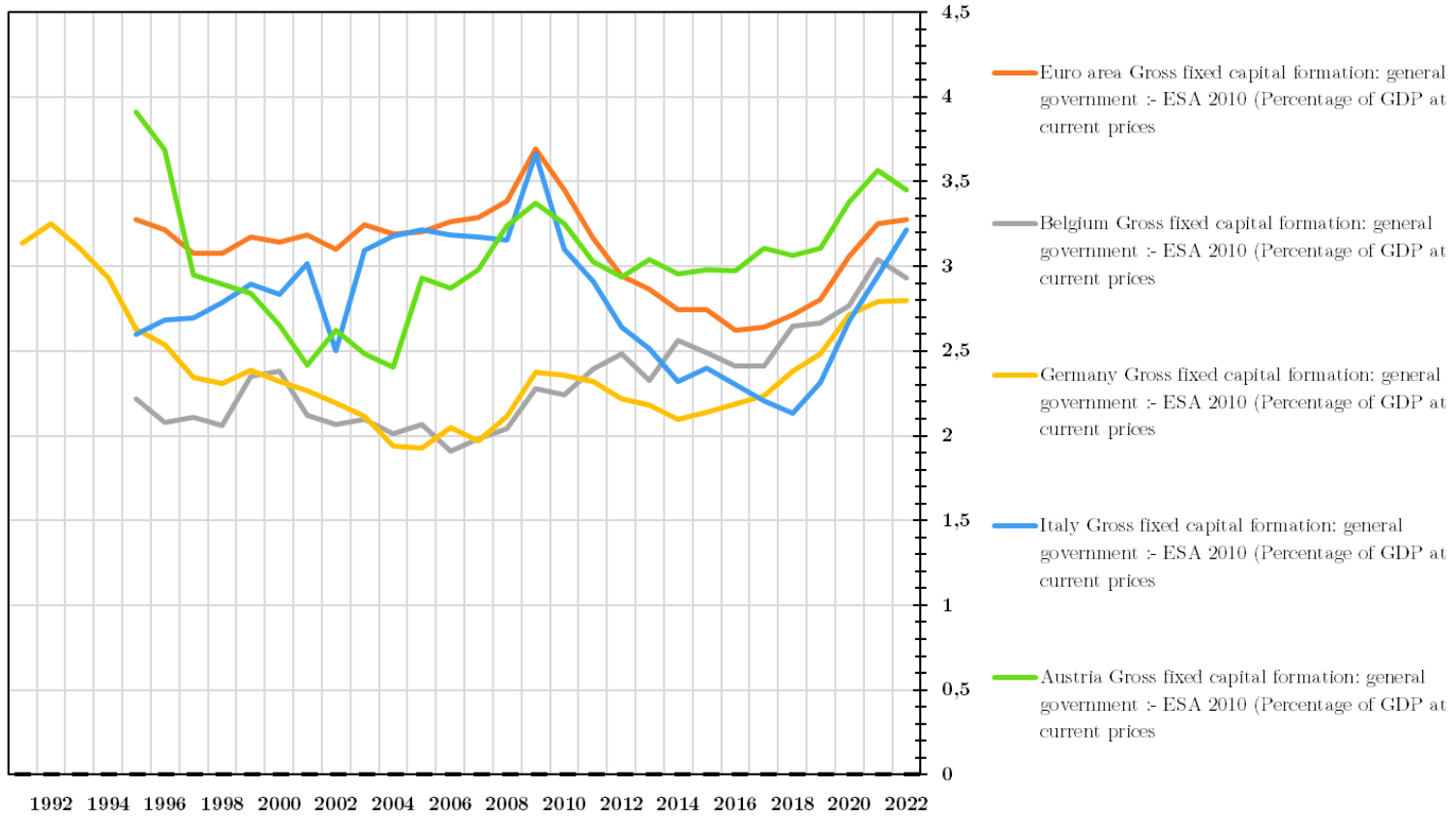

Fig.1.5: Example of trend in public capital stock formation as a percentage of GDP in Euro area 1992-2022. Own elaboration. Source: IMF, World Economic Outlook database

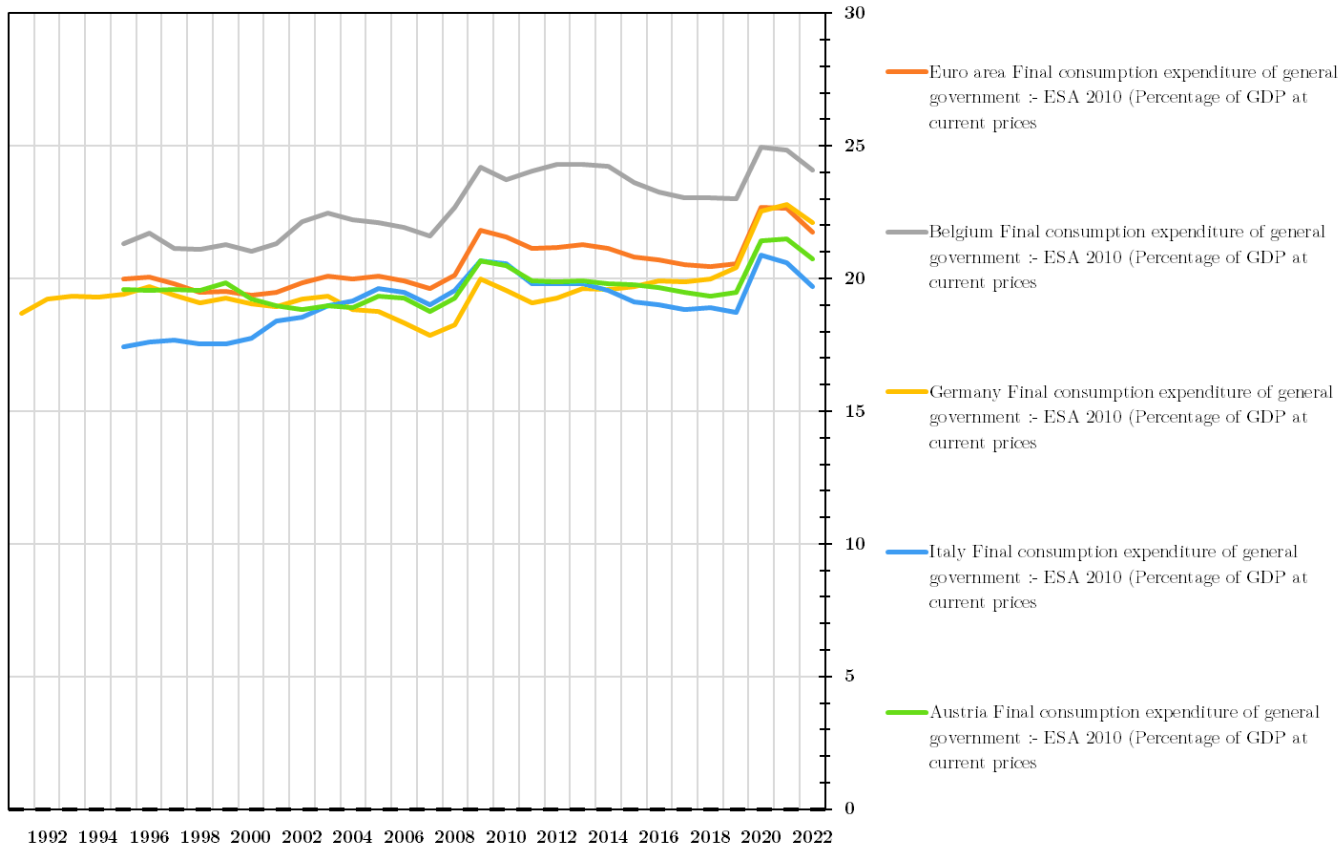

Fig.1.6: Example of trend in public consumption as a percentage of GDP in the Euro area 1992-2022. Own elaboration. Source: IMF, World Economic Outlook database

Through their analysis, Straub and Tchakarov (2007) were among first to identify such improvements in EU economic growth with respect to a possible trend reversal. Specifically, they found higher rates of return than public consumption from public investment, with respect to both short and long-term dynamics, through both a greater increase in aggregate demand (with a greater adjustment path, and through the same effect on supply through a transmission effect on greater marginal productivity of the labour factor and private capital, and consequently through the generation of higher dynamic fiscal multipliers than fiscal policies aimed only at increasing public consumption).

The Straub and Tchakarov (2007) results also keep open the debate on the actual effectiveness of policies implemented in compliance with European fiscal rules especially after the crisis phenomena, as seen above. The composition of total public spending has indeed an effect on the fiscal multipliers generated, if there are differentials between investments and public spending, calling for a greater safeguard of these investments with respect to the above-mentioned rules for limiting total spending. An input possibility for noticing such differentiated fiscal multiplier dynamics can be obtained for instance by implementing public capital within the production function in the form:

$$Y_t = A_t N_t^{\alpha_1} K_t^{\alpha_2} Z_t^{\alpha_4}$$

where the pubblic capital Z_t is assumed following a typical law of motion in the form $Z_t = (1 - \delta_Z)Z_{t-1} + I_t^Z$ and including the same public investments in the overall public expenditure function (in the following research this implementation is done according to the equality $G_t^{\omega} = G_t + I_t^Z$, noting also the possible effects in a symmetrical international capital-exchange system K_i, K_f [6] with a law of motion as: $K_{i,t}^i = (1 - \delta_{K_i^i})K_{i,t-1}^i + I_{K_{i,t}^i}^i ; K_{i,t}^f = (1 - \delta_{K_i^f})K_{i,t-1}^f + I_{K_{i,t}^f}^i K_{f,t}^f = (1-\delta_{K_f^f})K_{f,t-1}^f + I_{K_{f,t}^f}^f ; K_{f,t}^i = (1-\delta_{K_f^i})K_{f,t-1}^i + I_{K_{f,t}^i}^f$ in the same domestic (foreign) production function $Y_{i,t} = A_{i,t}N_{i,t}^{\alpha_1} K_{i,t}^{\alpha_2} K_{f,t}^{\alpha_3} Z_{i,t}^{\alpha_4}$).

Similar results have been subsequently highlighted by Ganelli and Tervala (2020) (with annexed welfare implications) and by Burlon and D'Imperio (2019) who per-

[6] This implementation makes it possible to see how the exchange of capital on international markets affects private investment by households (i.e. because of capital in the private constraints and Euler equations) and how certain economic policies have different impacts on the choice and accumulation of capital by firms, resulting in different effects on investment dynamics on international markets

form in extension to the addition of capital and public investments a (positive) empirical Bayesian analysis of the model applied to the European economy, implementing at the same time estimates of the Output Gap (OG) and its evolution and dynamics. The continuation of the work in the research on the application of public sector and European fiscal rules is articulated with increasing involvement of different scenarios and in the comparison, in some cases of public expenditure and public investment rules in relation to monetary rules, in the broader spectrum of coordination between monetary and fiscal policy.

Examples of this are Elmgrem (2018) and Batini et al. (2020) where the role of fiscal policy is focused on respecting European budget balance parameters with respect to its role, as seen earlier in Ferrero's (2009) work, of stabilising the economic system in the absence of exchange rate adjustment possibilities. To this end, Elmgrem (2018) presents a scenario of implementation of the SGP within a broader dynamic model in the presence of distortionary taxes on wages and private consumption. The use of expenditure rules, taking up and extending Leeper (1991), Leeper, Plante and Traum (2010) and Forni, Monteforte and Sessa (2009) and performing a normative analysis with respect to welfare effects in the presence of different uses of taxation on wages compared to that on consumption in order to stabilise the debt/GDP ratio.

Batini et al. (2020), then, through new forms of endogenization of the feed-back rules of public spending in the pre-announcement form (with the parameter ϑ): $\log(G_t/G) = \rho^g log(G_{t-1}/G) - \rho^{gy} log(Y_t/Y) - \rho^{gb} log(B_{t-1}/B) + (1-\vartheta)\varepsilon_t^g + \vartheta\varepsilon_{t-1}^g$ and taxation itself $\log(T_t/T) = \rho^t log(T_{t-1}/T) + \rho^{ty} log(Y_t/Y) + \rho^{tb} log(B_{t-1}/B) + (1-\vartheta)\varepsilon_t^t + \vartheta\varepsilon_{t-1}^t$ (in extension to Cantore, Levine, Melina and Pearlman (2017) and Leeper, Walker and Yang (2013) and in application to the European territory) show instead the asymmetric response between Germany, France and Italy (due to different fiscal orientations) just after the 2008 global financial crisis (GFC) by first comparing the fiscal stimulus implemented, in conjunction with monetary policy, in the US, and then presenting results on the lack of anti-cyclicality of possible fiscal interventions in the EU in times of economic recession due to different fiscal budget rules and at the same time looking for a level of balance and coexistence between the same fiscal interventions (hoped for as more expansive) at the regional level in coordination with monetary centralisation at the ECB and the new accommodative

policies implemented after the 2008 and 2011 crises.[7] The use of fiscal policy implementation models of this type therefore allows not only, of course, to understand the dynamics of the main macroeconomic variables in the presence of possible idiosyncratic shocks, but also to study the actual effectiveness of pro and anti-cyclicality of policies and different fiscal rules in cases of economic growth or recession. Similarly, given the legal structure in which these rules operate in the Eurozone, it is also important to allow a different comparison in order to analyse their possible dynamics on the business cycle. (Ghironi (2017))

The last contributions in this sense, of extreme importance for this research, for the nature of the implementations made and for the implications they have on the side of the present debate on European policies and on the reference modelling, are finally those of the implementation of alternative fiscal rules to those already mentioned and included within the European economic treaties. The suggestion and application of fiscal policies complementary or alternative to those already present in Europe is an expanding field in the macroeconomic modelling scenario (Ghironi (2017)) in order, as already specified above, to note possible alternative dynamics to those present with respect to the European fiscal rules in place and to compare different scenarios. In this sense, Zeyneloglu (2018) presents a model of application and scenario comparison, in a stochastic dynamic framework of general equilibrium, of the *"Golden Rule"* on public investment with respect to the dynamics of public deficit and debt. Zeyneloglu's model starts from the assumption, by taking the dynamics of reduction of public investment in the European territory, and extending it to the OECD territory, between the period 1970-2015 of Straub and Tchakatov (2007), and noting in this extension also a reduction in terms of ratio to GDP from 4.2% to 3.2% and how public investment and the accumulation of public capital stock can have positive multiplication effects on economic growth.

Specifically, through the possibility of using public investment as an instrument of fiscal policy without necessarily incurring the limits of the European fiscal pacts and rules in force (by separating them from the calculation of budget balances),

[7] A similar implementation of this kind of policy rule, in steady-state log-deviation form, will be used in this research but for all the monetary-union and just for the current public spending policy rule in the anti-cyclical form: $\hat{g}_{i,t} = \rho_i^g \hat{g}_{i,t-1} - \rho_i^{gy} \hat{y}_{i,t} - \rho_i^{gb} \hat{b}_{i,t-1} + \upsilon_{i,t}^g$ and simmetrically for the foreign block as: $\hat{g}_{f,t} = \rho_f^g \hat{g}_{f,t-1} - \rho_f^{gy} \hat{y}_{f,t} - \rho_f^{gb} \hat{b}_{f,t-1} + u_{f,t}^g$ noting different effects in different fiscal scenarios with respect to public investments dynamics

a comparison between the *"Crowding out"* effect on consumption and private investment is demonstrated, showing how, in conjunction with monetary policy, the system allows for an initial *"Crowding-out"* effect but with a gradual adjustment in the medium term, allowing for an attribution to the application of the *"Golden Rule"* of a smoothing of the fiscal burden in the medium term with respect to the short term and a greater control of the public deficit and total output growth, while respecting the fiscal discipline of the Eurozone. (Fig. 2.7)

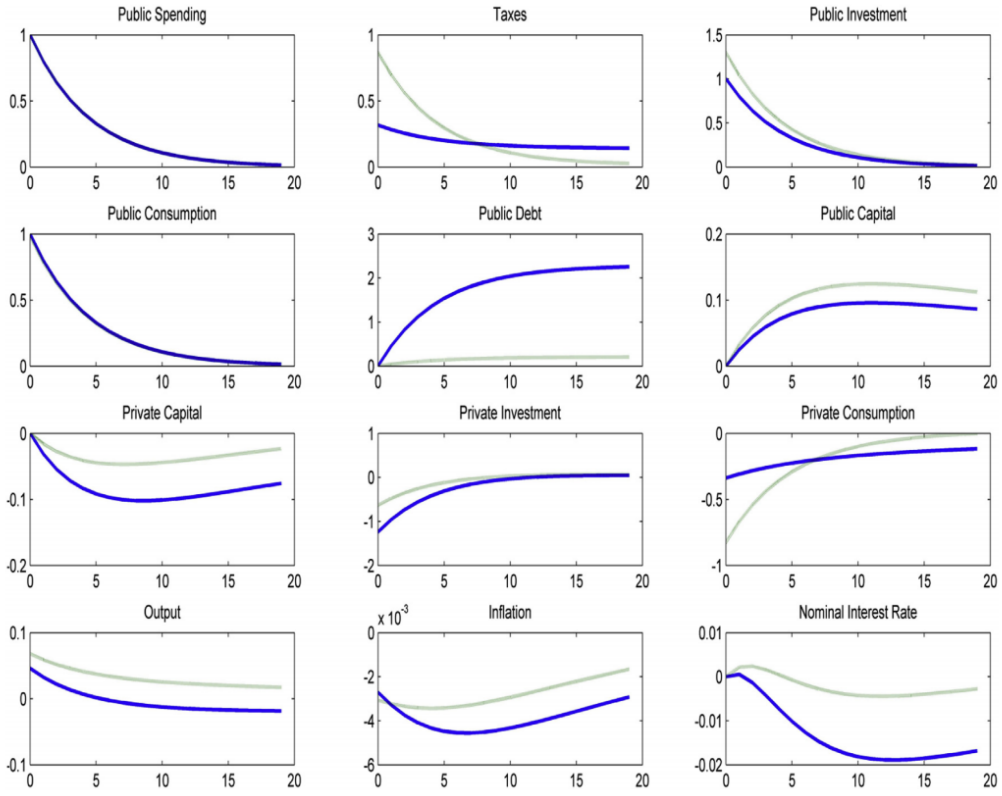

Fig.1.7: Results of the Zeyneloglu (2018) Analysis. Note the IRFs with respect to the effects of a public spending shock in the presence or absence of a *"Golden Rule"* application (green line) and the *"Crowding-out"* dynamics on both investment and private consumption with greater smoothing in the medium term of taxation in case of *"Golden Rule"* application

Zeyneloglu's paper concludes in this sense by presenting as a possible future extension also a more realistic and disaggregated distortionary tax presence in this landscape and thus allowing to note possible cost effects and different degrees of substitution (driven by tax and return differentials) then between public and private

capital always in the light of European fiscal rules.

1.4 Discussion, further developments and Conclusive Remarks

If what has been said so far is therefore an overview of the institutional situation and of the dynamics that have emerged from macroeconomic studies of the European economy and of the functioning of its fiscal rules in the broader panorama of the theory of Monetary Areas, in conclusion this review can only ask whether it is possible to consider Europe as an optimal currency area in the light of the theoretical framework and of the empirical results that have so far matured. As the studies that have been carried out so far have shown, the dynamic analyses alone do not provide sufficient, albeit necessary, evidence for a refutation or otherwise of macroeconomic theory. In fact, there are many steps forward to be taken for a correct examination of the dynamics of European fiscal rules.

While, on the one hand, there are facilitations in the mobilisation of capital flows with consequences on the financial markets, on the other hand there are strong stickinesses and frictions on the labour and capital factors side, with high unemployment rates, wage differentials and accumulation dynamics, leading to stickinesses and frictions on the possible resulting asymmetric shocks. This is undoubtedly caused, as we have seen, on the one hand by the impossibility of adjusting the different internal interest rates as might occur in situations of flexible exchange rate regimes, and on the other by the lack of centralisation of fiscal policy, which, as we have seen, is the main lever, in accommodation with monetary policy if necessary, for rebalancing economic growth and hence the balances of systemic interdependence. The fiscal rules laid down by the treaties of recent decades (SGP, Maastricht treaty, Lisbon treaty, Fiscal Compact, etc.) do not yet allow for total fiscal flexibility and its consequent systemic harmonisation. While, as seen above, the data suggest in some cases significant divergences between the acceding countries and trends in macroeconomic fundamentals, it is also true that the studies carried out still require numerous developments and evolutions in their correct implementation.

These current combinations of factors leave room for a great deal of new research and models that must make explicit reference to fiscal variables and the dynamic

functioning of the economic structure so as to be able to carry out an adequate theoretical analysis of their possible alignment in order to ensure a more realistic profile with respect to the description of European dynamics.

As indeed Ghironi (2017) points out, it is easy to see how the Euro system should be analysed from further angles, both with regard to the critical nature of fiscal rules and to recent developments in the European policy debate. While the analysis of the optimality or otherwise of the Union is still under discussion, there is no doubt that an evolution of macroeconomic study models in this sense could lead to new answers. In this sense, Del Negro and Giannoni (2017)) point out that dynamic and stochastic general equilibrium models are now solid tools for policy analysis and the search for such responses, combining at the same time structural relations of the economy, based on strong theoretical postulates, with statistical methods of estimation and forecasting, together with the possibility of analysing the dynamic response of the main aggregates to different shocks. Thus, in this sense, it makes it possible to obtain answers on the evolution of the economy or of some sectors of it in the coming periods as well as its response to the intervention with certain economic policies (monetary and fiscal). It is precisely on this aspect, however, that the policy debate becomes active. The inclusion, as we have seen, of certain sectors, or the new heterogeneous agent dynamics or other possible endogenizations in a model play a fundamental role in this sense in order to underline its realism in providing responses precisely to the intervention of certain policies rather than others. Del Negro and Giannoni (2017) underline how it is precisely the scenario of interest, and at the same time its correct construction and modelling implementation, that plays a fundamental role in this. Examples are inherent in scenarios such as those characterised by constraints such as the liquidity constraint (ZLB)[8] where the use of conventional monetary policies or simple interest-rate rules may not provide the desired effects or provide the right information on the responses of the reference aggregates (examples are the inclusion of the new forward-guidance rule (FG) - i.e. forms of early deviations from the policy rule – as in Cambpell et al. (2012) and

[8]Examples of models for forecasting and analysing economic policies in the Eurozone are varied and growing in recent years. For a discussion of non-conventional monetary policies, see, among others, the review on the role of QE in ZLB by Orphanides (2019). For an example of policy reforms (e.g. structural) in the event of an economic recession following the 2011 crisis, see for example among others, Ferrero, Raffo, Eggertsson (2013), and for soil effects of fiscal and public spending policies in ZLB in recessionary periods Bouakez, Guillard and Roulleau-Pasdeloup (2019)

Del Negro (2013) in Gürkaynak and Tille (2017) or average-inflation targeting rules as presentend by Ferrero (2021)). Therefore, it would also automatically result in the need for the proper implementation of sometimes differentiated and sometimes non-conventional policies in order to analyse the dynamics and subsequent responses of the system in coordination with the economic policies of the individual regional economies that are part of the European system. In this sense, models that include fiscal and public spending policies are beginning to play a fundamental role, as opposed to monetary policy analyses alone, in order to analyse both the transmission and coordination effects and the dynamics of the main public spending items within the various economic areas. The research and debate on the correct implementation of economic policies and the policymaking process has certainly taken on increasing importance since the 2008-2009 economic crisis. Ghironi (2018) in Gürkaynak and Tille (2017) points out that speeches issued by Draghi (2016a, 2016b, 2016c) G20 (2016), Lagarde (2016a, 2016b) and Praet (2016) are strong testimonies of the urgency of addressing this debate. Specifically, the G20 (2016) and Lagarde (2016) refer to a greater coordination of monetary and fiscal policies, while Draghi's (2016) speeches are extremely important in that it calls for a search for answers on new tests and on the need to improve modelling and ongoing research for a possible better communication and implementation in forecasting models not only of monetary policy but also of its interactions in modelling environments with other economic policies related to the fiscal scenarios under consideration. Moreover, in his 2016 speech, Draghi makes explicit reference to *"interdependence within interdependence"* referring to the close link between the activities of central banks and the economic system in which they operate. In this sense, and referring to the Eurozone territory, there is no doubt that this process requires a greater involvement of fiscal policy and European fiscal rules within the reference macrostructural models in order to better understand their possible optimality both ex-ante and ex-post (through e.g.: new counterfactual analyses or implementation of new policy and scenario rules). Ghironi (2018) and Lledo, Dudine, Eyraud, Peralta-Alva (2018) also underline how the insertion process is not easy to implement but requires considering a series of key characteristics of each policy in order to capture the effects on macroeconomic aggregates and the interdependencies between them. (An example of this is the

dynamics seen above and the results first with Straub and Tchakatov (2007) then in extension with Zeyneloglu (2018) with respect to the *"Crowding-out"* effects of public investment and public consumption). The level of microfounding allowed by stochastic dynamics models in this sense is certainly a major advantage. In fact, the possibility of disaggregation makes it possible to analyse specifically different aspects relating especially to fiscal policies.

An example of this is the possibility of including distortionary taxation and its possible complete disaggregation into the various tax rates applied to the various consumption, wealth or income variables. The new developments underlined are also covered by the possibility of noting, thanks to the dynamics of the models themselves, possible long-term and short-term effects given by the implementation of different and interacting monetary and fiscal rules, and thanks to stochasticity, thus capturing the realism of business cycles, in which uncertainty and the perceived risk of the agents operate, and where each individual and given policy can modify the trend of those same dynamics. (An example in this sense is provided by the latest developments of the models such as Batini et al. (2020) with the insertion of different parameters of anticipation of the feedback rules - *"pre-announcement effects"* - of public spending and taxation). Flexibility in the inclusion of such features, different shocks, empirical estimates and different policy rules would certainly allow for an improvement of the modelling and forecasting perspectives going against that *"interdependence within interdependence"* of Draghi (2016) in the subsequent implementation of fiscal policies with respect to the activity of the central bank itself. This point was also addressed in April 2016 by the World Economic Outlook (IMF (2016) in Ghironi (2018)), which underlined the connection both with possible reforms implemented in individual economies and their consequent interconnections, and with the economic policies adopted as a consequence and especially with the scenarios in which those policies are implemented. In this way, the economic reference conditions, as we will see in the next chapters comparing different fiscal scenarios (Chapter 2), always play a fundamental role in achieving the objective of greater realism and descriptive capacity of macroeconomic models in positive and normative sense, in possible forecasting of the macro-fundamentals and in providing additional answers on the dynamics, possible improvements and potential optimality

of European Monetary Union.

1.5 References

1. Albonico, A., Calés, L., Cardani, R., Croitorov, O., Ferroni, F., Giovannini, M., Vogel, L. "Comparing post-crisis dynamics across Euro Area countries with the Global Multi-country model", Economic Modelling, 2019

2. Batini, N., Cantelmo, A., Melina, G., Villa, S., "How loose, how tight? A measure of monetary and fiscal stance for the euro area", Bank of Italy, Temi di Discussione (Working Paper) N. 1295, 2020

3. Burlon, L., D'Imperio, P., "The euro-area output gap through the lens of a DSGE model", Bank of Italy Occasional Paper, (477), 2019

4. Burlon, L., Locarno, A., Notarpietro, A., Pisani, M., "Public investment and monetary policy stance in the euro area", Bank of Italy, Temi di Discussione (Working Paper) No, 1150, 2017

5. Campbell, J. R., Evans, C. L., Fisher, J. D., Justiniano, A., Calomiris, C. W., Woodford, M., "Macroeconomic effects of federal reserve forward guidance", Brookings papers on economic activity, 1-80., 2012

6. Canale R., "L'Unione monetaria europea fra integralismo monetario e dipendenza della politica fiscale: Alcune considerazioni", Studi e note di economia, n°1, 1998

7. Cantore, C., Levine, P., Melina, G., Pearlman, J., "Optimal fiscal and monetary policy, debt crisis, and management", Macroeconomic Dynamics, 23(3), 1166-1204, 2019

8. Canzoneri M., Gray J., "Monetary Policy Games and the Consequences of Non-Cooperative Behavior", International Economic Review, 1985

9. Carril-Caccia, F., Pavlova, E., "Foreign direct investment and its drivers: a global and EU perspective", Economic Bulletin Articles, 4, 2018

10. Cesaratto S., "Politica Monetaria e Fiscale dell'Unione Monetaria Europea", Lecture Notes, 2018

11. Christiano, L. J., Eichenbaum, M. S., Trabandt, M., "On DSGE models", Journal of Economic Perspectives, 32(3), 113-40, 2018

12. Christiano, L. J., Eichenbaum, M., Evans, C. L., "Nominal rigidities and the dynamic effects of a shock to monetary policy", Journal of political Economy, 113(1), 1-45, 2005

13. Christoffel, K. P., Jaccard I., Kilponen, J..,"Government bond risk premia and

the cyclicality of fiscal policy.", European Central Bank Working Paper Series, 2011
14. Coenen, G., McAdam, P., Straub, R., "Tax reform and labour-market performance in the euro area: A simulation-based analysis using the New Area-Wide Model", Journal of economic Dynamics and control, 32(8), 2543-2583., 2008
15. Del Negro, M., Eusepi, S., Giannoni, M. P., Sbordone, A. M., Tambalotti, A., Cocci, M., Linder, M., "The FRNBY DSGE model" FRB of New York Staff Report, (647), 2013
16. Del Negro, M., Giannoni, M., "Using dynamic stochastic general equilibrium models at the New York Fed", DSGE Models in the Conduct of Policy: Use as intended, 61. 2017
17. Draghi M., "Europe and the euro 20 years on", speech by Mario Draghi, President of the ECB, at Laurea Honoris Causa in Economics by University of Sant'Anna, Pisa, 2018
18. Draghi M., "Hearing of the Committee on Economic and Monetary Affairs of thee European Parliament", Brussels, 2016b
19. Draghi M., "On the importance of Policy Alignment to fulfil our economic potential", Brussels, Burssels, 2016a
20. Draghi M., "Researching the Macroeconomic Policies of the Future", Frankfurt, 2016c
21. Eggertsson, G., Ferrero, A., Raffo, A., "Can structural reforms help Europe?", Journal of Monetary Economics, 61, 2-22., 2014
22. Elmgren, P. G. V., "The effects of debt stabilising fiscal rules in a macroeconomic (DSGE) model", Hanken School of Economics, Helsinki, 2018 23. Engel C., Rogers J., "European Product Market Integration after the Euro", Economic Policy 39, 2004
24. European Commission, "Public Finances in EMU", European Commission, DG for Economic and Financial Affairs Publication (Brussels, Belgium), 2003 25. European Commission, "The corrective arm" and "Fact Sheet: The EU's economic governance explained", 2014
26. European Commission, "Towards A Positive Fiscal Stance For The Euro Area", Communication From The Commission, Brussels, 2016
27. Ferrero A., "Monetary Policy during the Covid Crisis (and Beyond)", ECO-

NOMICS OF POST-CRISIS RECOVERIES, OUBEP, University of Oxford, 2021
28. Ferrero, A., “Fiscal and monetary rules for a currency union”, Journal of international Economics, 77(1), 1-10, 2009
29. Furceri, D, Mourougane, A., “The effects of fiscal policy on output: A DSGE analysis”, OECD, 2010
30. G20, “G20 Leaders Summit” 2016
31. Gali, J., Monacelli, T., “Monetary policy and exchange rate volatility in a small open economy”, The Review of Economic Studies, 72(3), 707-734, 2005
32. Gali, J., Monacelli, T., “Optimal monetary and fiscal policy in a currency union”, Journal of international economics, 76(1), 116-132., 2008 33. Ganelli, G., Tervala, J., “Welfare Multiplier of Public Investment”, IMF Economic Review, 68, 390-420, 2020
34. Ghironi, F., “Policy Packages: Challenge and Opportunity for DSGE Research”, DSGE Models in the Conduct of Policy: Use as intended, 61. 2017
35. Giavazzi F., Pagano M., “The Advantage of Tying One’s Hands: EMS Discipline and Central Bank Credibility”, European Economic Review 32, 1988
36. Gürkaynak, R. S., Tille, C., “DSGE models in the conduct of policy: Use as intended”, CEPR Press, 2017
37. Hamada K., “Alternative Exchange Rate Systems and the Interdependence of Monetary Policies”, in R. Aliber, “National Monetary Policies and the International System”, University of Chicago Press, 1974
38. Hamada K., “Macroeconomic Strategy coordination under alternative exchange rates”, in R. Dornbusch, J. Frenkel, “International Economic Policy”, John Hopkins University Press, Baltimore, 2979
39. Ieluzzi G., “Politica Fiscale e Monetaria nell’Unione Europea: Necessità di Coordinamento in vista di un Obiettivo Comune”, Luiss University, 2015
40. IMF, “Time for a Supply-Side Boost? Macroeconomic effects of labour and product market reforms in advanced economies”, World Economic Outlook, 2016
41. International Monetary Fund, “Making public investment more efficient”, IMF Policy Papers, 2015
42. International Monetary Fund, World Economic Outlook, Chapter 3, April 2014
43. International Monetary Fund, World Economic Outlook, Chapter 3, October.

2014

44. Kenen P.B., “The Optimum Currency Area: An Eclectic View”, Monetary Problems of the International Economy, Chicago University Press, 1969

45. Krugman P., “The Making of a Euromess”, New York Times, 2010

46. Krugman P., Obstfeld M., Melitz M.J., “International Economics: Theory and Policy”, Pearson, 2015

47. Krugman P., Obstfeld M., Melitz M.J., “International Economics 2 , International Monetary Economics”, Pearson, 2015

48. Kumhof, M. and Laxton D., ”A PartyWithout A Hangover? On the E§ects of U.S. Government DeÖcits ”, IMF, 2007

49. Kydland, F. E., Prescott, E. C., Rules rather than discretion: The inconsistency of optimal plans. Journal of political economy, 85(3), 473-491, 1977

50. Lagarde C., “Making Glbalization work for All”, Sylvia Ostry Lecture, Toronto, 2016a

51. Lagarde C., “The Managing Director’s Global Policy Agenda”, IMF-World Bank Annual Meeting, 2016b

52. Leeper, E. M., Walker, T. B., Yang, S. C. S., “Fiscal foresight and information flows”. Econometrica, 81(3), 1115-1145, 2013

53. Linde, J., “DSGE models: still useful in policy analysis?”, Oxford Review of Economic Policy, 34(1-2), 269-286, 2018

54. Lledo, V. D., Dudine, P., Eyraud, L., Peralta-Alva, A., “How to Select Fiscal Rules; A Primer” (No. 2018/001). International Monetary Fund, 2018

55. Mankiw N.G., Taylor M., “Macroeconomics”, 2011

56. Masera R., “Il patto di stabilità e di crescita: investimenti in infrastrutture e Golden Rule”, Mondo Bancario, 2002

57. Matheron J., Mojon B., Sahuc J.G, “The sovereign debt crisis and monetary policy”, Financial Stability Review, Banque de France, 2011

58. Matheron, J., Mojon, B., Sahuc, J. G., “The sovereign debt crisis and monetary policy”. FSR FINANCIAL, 155., 2012

59. Montalbano P., U. Triulzi, “La Politica Economica Internazionale”, Utet, 2017

60. Mundell R.A. (1961), “A theory of Optimum Currency Areas”, The American Economic Review 51, 1961

61. Niehans J., “Monetary and Fiscal Policies in Open Economies under fixed exchange rates: an Optimizing Approach”, Journal of Political Economy, n°76, 1968

62. Niehans, J, “Monetary and fiscal policies in open economies under fixed exchange rates: An optimizing approach”, Journal of Political Economy, 76(4, Part 2), 893-920, 1968

63. Orphanides A., “The Fiscal-Mone ary Policy mix in the Euro Area: Challenges At The Zero Lower Bound, Economic Policy, 70th Economic Policy Panel Meeting, 2019

64. Oudiz G., J. Sachs, “Macroeconomic Policy Coordination among the Industrial Countries”, Brooking papers on Economic Activity, 1984

65. Oudiz, G., Sachs, J., “International policy coordination in dynamic macroeconomic models”, In International economic policy coordination (pp. 274-330). Cambridge University Press, 1985

66. Pappa. E., ”New Keynesian or RBC Transmission? The Effects of Fiscal Policy in Labor Markets,” Working Papers 293, IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University. 2004

67. Praet P., “Beyond Monetary Policy: on the importance of a proper alignment of Economic Policies in EMU”, 2016

68. Rogoff K., “The optimal Degree of Commitment to an Intermediate Monetary Target”, Quarterly journal of Economics 100, 1985

69. Rudebusch, G. D., Swanson E. T., ”The bond premium in a DSGE model with long-run real and nominal risks”, American Economic Journal: Macroeconomics, 2012

70. Schmitt-Grohé, S., Uribe, M., “Optimal fiscal and monetary policy under sticky prices”. Journal of economic Theory, 114(2), 198-230, 2004

71. Schmitt-Grohe, S., Uribe, M., “Optimal inflation stabilization in a medium-scale macroeconomic model”. Monetary policy under inflation targeting, 125-186, 2007

72. Schmitt-Grohé, S., Uribe, M., “Optimal simple and implementable monetary and fiscal rules”: Expanded version (No. w12402). National Bureau of Economic Research, 2006

73. Schmitt-Grohé, S., Uribe, M., “Closing small open economy models”, Journal of international Economics, 2003

74. Smets, F., Wouters, R., “Shocks and frictions in US business cycles: A Bayesian DSGE approach”, American economic review, 97(3), 586-606, 2007

75. Tavlas G.S., “The New Theory of Optimum Currency Areas”, The World Economy 16, 1993

76. Torti G., “L’Unione Europea e la teoria delle aree valutarie ottimali: criticità e obiettivi futuri, Luiss Università, 2015

77. Woodford, M., ”Fiscal Requirements For Price Stability,” Journal of Money, Credit and Banking, 2001

78. World Economic Forum, “Global Competitiveness Report”, 2012

79. Zeyneloglu, I., “Fiscal policy effectiveness and the golden rule of public finance”, Central Bank Review, 18(3), 85-93, 2018

Chapter 2

Macroeconomic Dynamics of Fiscal Rules in a Monetary-union DSGE

2.1 Introduction

The global financial and debt crisis of the past years and the growing economic and financial instability derived from have highlighted the economic difficulties of many countries. As seen in Chapter I, especially within the european territory these difficulties have led to continuous adjustment policies aimed at a possible resolution and harmonization of the main european economies. By these assumptions, the present chapter sets the goal of a comparative analysis of the various fiscal rules and economic policies within the European economic and monetary union leading to a double result. On the one hand, it provides an explanation of european economic and equilibrium dynamics with respect to different shocks and public investments dynamics and, on the other, it provides a new answer to the request for completeness with respect to the DSGE (Dynamic Stochastic General Equilibrium) reference literature. Specifically, through the modelling exposition, the systemic dynamics of the european scenarios is displayed combining all the arduous task of capturing and comparing the nature and effects of distortive tax impositions in the behaviors of four different scenarios, each of them characterized by the fiscal rules proper of the european reference treaties. Indeed, as will be easily deduced in next sessions there will be 4 different scenarios for economic and financial simulations in which these rules will be based. It will start from the I scenario which it will be described by the compliance with the 3 percent threshold on the Deficit/GDP ratio (Art. 126

Stability and Growth Pact (ex Art. 104 Treaty on the Functioning of the European Union)) and followed by the II one through the splitting of the Public Investments from the calculation of the overall Public Deficit (e.g: "Golden Rule"). It will come therefore end up to the III and IV simulation scenarios through the insertion of the rules proper of the Fiscal Compact of 2013 (with the presence of Cycle Adjust Balance (CABB) values as previously introduced) and with the related possibility of application of the fiscal "Golden Rule". The modelling technique used will be, as said above, the Dynamic and Stochastic models of General Economic Equilibrium (DSGE) whose results derive from computational processing of Impulse Response Functions (IRFs) letting to interpretate important information and results with respect to the response of the main macroeconomic aggregates to different exogenous shocks. The basic structure is then quite common in literature. Inspired by the most cited Smets and Wouters (2003) and fiscal policy models of Leeper, Plante and Traum (2009) and Leeper, Walker and Yang (2010) the structure of the model allows the inclusion of prices rigidity a là Calvo (Calvo (1983)) and the presence of monopolistic competition. As presented before by Gali and Monacelli (2008) and then as in Ferrero (2009) the model is articulated as an open-economy with symmetrical effects for two European territorial areas (e.g. Italy and Europe). Later, as a natural extension of the Straub and Tchakarov (2007) and Zeyneloglu (2018) models, it will be the inclusion of a more disaggregated taxes apparatus and capital and public investment structure. The result is therefore a complex elaborate on economic dynamics able to catch partly the european economic realism and to provide perhaps useful answers to the purposes of a theoretical-comparative analysis and a greater understanding of the dynamics underlying the same treaties of community foundation. The assumptions on fiscal policies differentiations and monetary centralization with ECB, proper of the European Monetary Union (EMU), leads in fact to important questions about the best interventions to be carried out with respect to the Euro countries and their national public accounting in harmony with the rules imposed by the European treaties. The results obtained allow so to understand different potential interventions about possible effects of the Current Spending on the interactions between economic variables.

Therefore, starting from the results obtained by this model, their correct identifi-

cation would let to reach new answers on the policy measures to be adopted and on the potential levels of general economic equilibrium that can be reached in order of a possible and desirable systemic harmonization and upon reaching such values through cooperative alignment procedures and greater compliance with respect to the requirements of European fiscal policies.

2.2 Model Structure

2.2.1 Households

Let's consider the behavior of representative households with identical preferences through an individual Utility function in Isoelastic form (CRRA):

$$E\left\{\sum_{t=0}^{\infty}\beta_i\left[q_1^i\frac{C_{i,t}^{1-\eta_i}}{1-\eta_i}-\frac{N_{i,t}^{1+\theta_i}}{1+\theta_i}+(1-q_1^i)(G_{i,t})\right]\right\} \tag{2.1}$$

where $\beta_i \in (0,1)$ will correspond to the discount factor on future utility and η_i and θ_i respectively to parameters of risk adversion and inverse of Frisch elasticity of labour supply.

Where $C_{i,t}, N_{i,t}, G_{i,t}$ correspond to private consumption, working hours, public current spending respectively. Following Gali (2008) and Gali and Monacelli (2008) precisely $C_{i,t}$ represents the composite private consumption index between domestic and foreign goods defined as:

$$C_{i,t}=\frac{(C_{i,t}^i)^{1-\nu}(C_{F,t}^i)^{\nu}}{(1-\nu)^{(1-\nu_i)}\nu^{\nu}} \tag{2.2}$$

Where the term $\nu \in (0,1)$ indicates the weight of goods imported on the utility function compared to private consumption. This is traduced into an index of openness to consumption of foreign products.

While $C_{i,t}^i$ indicates the index of consumption of domestic goods produced in the domestic country expressed by the following CES (Constant Elasticity of Substitution) function:

$$C_{i,t}^i=\left(\int_o^1 C_{i,t}^i(j)^{\frac{x-1}{x}}dj\right)^{\frac{x}{x-1}} \tag{2.3}$$

Where the term $j(0,1)$ indicates the variety of goods (within the set of goods produced in the domestic country $(i-th)$.

Symmetrically the variable $C_{F,t}^i$ indicates the consumption, always in the domestic

country i but of the goods imported from the foreign country f according to the following function:

$$C^i_{f,t} = \left(\int_o^1 C^i_{f,t}(j)^{\frac{x-1}{x}} dj \right)^{\frac{x}{x-1}} \tag{2.4}$$

where the term $C^i_{F,t}$ is the the consumption index of the total imported goods given by the following aggregator:

$$C^i_{F,t} = exp \int_0^1 c^i_{f,t} df \tag{2.5}$$

where therefore $c^i_{f,t} = logC^i_{f,t}$ is then the logarithm of the index of the quantity of goods consumed by the country i produced and imported by the foreign country.
In these functions the term $x > 1$ indicates the elasticity of substitution among the varieties of goods produced within each given economy regardless of the country of origin. This parameter will therefore differ for the domestic country and foreign countries (of which the subsequent symmetrical indexes i, f).
Not to be confused with the term η as an index of intertemporal preference of consumption in the function of utility, however, always indexed and differentiated by domestic country and foreign countries i, f.
Extending Ferrero (2009), in this setup characterized by distortionary taxation for consumption, wages, capital and government bond renting, we are therefore going to indicate the constrained maximization subject to the following budget constraint for the households of the country i:

$$\begin{aligned} &\left[(1+\tau^c_i)\left(\int_0^1 P^i_t(j)C^i_{i,t}(j)dj + \int_0^1\int_0^1 P^f_t(j)C^i_{f,t}(j)djdf\right)\right] + K^i_{i,t} + K^f_{i,t} + [(\phi)B_{i,t}] + [(1-\phi)B_{f,t}] = \\ &\left[(1-\tau^\ell_i)W_{i,t}N_{i,t}\right] + [(1-\tau_i^{R^K_i})(R^K_{t,i}K^i_{i,t-1})] + [(1-\tau_i^{R^K_f})(R^K_{t,f}K^f_{i,t-1})] + [(1-\tau_i^{R^B_i})(\phi)(R^{B_i}_t B_{i,t-1})] + \\ &[(1-\tau_i^{R^B_f})(1-\phi)(R^{B_f}_t B_{f,t-1})] + \psi^Z_{i,t} \end{aligned} \tag{2.6}$$

for every $t = 0, 1, 2...\infty$, and where the term $P^f_t(j)$ indicates the price of the good j produced in the foreign country. The term ϕ indicate respectively the shares of Domestic government bonds and Foreign government bonds held by domestic households (and simmetrically for the foreign households). Assuming the existence of the Single Price Law we can indicate the optimal allocation of any given expenditure on the goods produced in each economy (in this case we indicate that of the domestic

country) with respect to the demand function given by:

$$C^i_{i,t}(j) = \left(\frac{P^i_t(j)}{P^i_t}\right)^{-x} C^i_{i,t}; \quad C^i_{f,t}(j) = \left(\frac{P^f_t(j)}{P^f_t}\right)^{-x} C^i_{f,t} \quad \forall i, f, j \in [0,1] \tag{2.7}$$

We denote with $P^i_t = \left(\int_0^1 P^i_t(j)^{(1-x)}dj\right)^{\frac{1}{1-x}}$ the price index of the country i (i.e. the price index of the goods produced in domestic country). Following the Single Price Law we will therefore have symmetrically: $P^f_t = \left(\int_0^1 P^f_t(j)^{(1-x)}dj\right)^{\frac{1}{1-x}}$ as the price index for the set of goods produced and imported from abroad.
By that we have $\int_0^1 P^i_t(j)C^i_{i,t}(j)dj = P^i_t C^i_{i,t}$ as well as $\int_0^1 P^f_t(j)C^i_{f,t}(j)dj = P^f_t C^i_{f,t}$
We therefore know how the allocation of expenditure on goods imported from the country i implies that:

$$P^f_t C^i_{f,t} = P^*_t C^i_{F,t} \tag{2.8}$$

where the term $P^*_t = exp\int_0^1 p^f_t\, df$ indicates the consumer price index of the Monetary Union. Following Gali and Monacelli (2008), from the point of view of the single country, P^*_t also indicates the price index of imported goods. From this we deduce that the total expenditure on imported goods is equal to the following equality:

$$\int_0^1 P^f_t C^i_{f,t} df = P^*_t C^i_{F,t} \tag{2.9}$$

Considering the consumer price index (CPI) for the country i as:

$$P^i_{c,t} = (P^i_t)^{1-\nu}(P^*_t)^{\nu} \tag{2.10}$$

then we will denote the optimal expenditure allocation between domestic and foreign goods imported into the domestic country i such as:

$$P^i_t C^i_{i,t} = (1-\nu)P^i_{c,t}C^i_t; \quad P^*_t C^i_{F,t} = \nu P^i_{c,t}C^i_t \tag{2.11}$$

and therefore by combining the results obtained we can rewrite the total consumption expenditure of the country i as:

$$P^i_t C^i_{i,t} + P^*_t C^i_{F,t} = P^i_{c,t}C_{i,t} \tag{2.12}$$

Substituting everything in our budget constraint of the households of the country i we get:

$$\begin{aligned}&\left[(1+\tau^c_i)P^i_{c,t}C_{i,t}\right] + K^i_{i,t} + K^f_{i,t} + [(\phi)B_{i,t}] + [(1-\phi)B_{f,t}] = \\ &[(1-\tau^{\ell}_i)W_{i,t}N_{i,t}] + [(1-\tau_i^{R^K_i})(R^K_{t,i}K^i_{i,t-1})] + [(1-\tau_i^{R^K_f})(R^K_{t,f}K^f_{i,t-1})] + [(1-\tau_i^{R^B_i})(\phi)(R^{B_i}_t B_{i,t-1})] + \\ &[(1-\tau_i^{R^B_f})(1-\phi)(R^{B_f}_t B_{f,t-1})] + \psi^Z_{i,t}\end{aligned} \tag{2.13}$$

where:

1)$\tau_i^c, \tau_i^\ell, \tau_i^{R_i^K}, \tau_i^{R_f^K}, \tau_i^{R_i^B}, \tau_i^{R_f^B}$: are the distorsive taxation on consumption, income from work, income from both domestic and foreign physical capital and bond returns from both domestic and foreign countries respectively.

2)$R_{t,i}^K$= Domestic Physical Capital Rent| $R_{t,f}^K$ Foreign Physical Capital Rent | with $R_t^{B_i}$= Domestic Bond Rent | $R_t^{B_f}$= Foreign Bond Rent

3)$(\phi)B_{i,t}$ and $(1-\phi)B_{f,t}$: Domestic and foreign government bonds accumulated in the country i

4)$\psi_{i,t}^Z$: Profits for the households (discussed later in the next block)

5)$W_{i,t}$: Wages

6)$N_{i,t}$: Hours of Work

7)$K_{i,t}^i$ and $K_{i,t}^f$: Domestic Physical Capital used by domestic companies and Domestic used by Foreign companies accumulated through Investments in Domestic and Foreign Capital: $I_{K_{i,t}^i}$ | $I_{K_{i,t}^f}$

8)$P_{c,t}^i(C_{i,t})$= Private consumption expenditure in the country $i-th$

Defining therefore the equation of motion of domestic and foreign physical capital where $K_{i,t}$ means the physical capital of the domestic country both lended to domestic and foreign companies and symmetrically for foreign countries with $K_{f,t}$ for foreign capital both used by foreign firms and lendend to firms in the country i:

$$\begin{aligned} K_{i,t}^i &= (1-\delta_{K_i^i})K_{i,t-1}^i + I_{K_{i,t}^i}^i; \quad K_{i,t}^f = (1-\delta_{K_i^f})K_{i,t-1}^f + I_{K_{i,t}^f}^i \\ K_{f,t}^f &= (1-\delta_{K_f^f})K_{f,t-1}^f + I_{K_{f,t}^f}^f; \quad K_{f,t}^i = (1-\delta_{K_f^i})K_{f,t-1}^i + I_{K_{f,t}^i}^f \end{aligned} \tag{2.14}$$

with total investments for domestic and foreign countries:

$$I_{i,t} = I_{K_{i,t}^i}^i + I_{K_{i,t}^f}^i + I_{Z_{i,t}}^i; \quad I_{f,t} = I_{K_{f,t}^f}^f + I_{K_{f,t}^i}^f + I_{Z_{f,t}}^f \tag{2.15}$$

such that it results with the total mobility of the Physical Capital in the union wide area. From here it is now possible to impose the problem of maximization of households belonging to the country i according to the Lagrangian:

$$\begin{aligned} &\mathcal{L} = \max_{(C_{i,t},N_{i,t},K_{i,t}^i,K_{i,t}^f,B_{i,t},B_{f,t})_{t=0}^\infty} E\left\{\sum_{t=0}^\infty \beta_i^t \left[q_1^i \frac{C_{i,t}^{1-\eta_i}}{1-\eta_i} - \frac{N_{i,t}^{1+\theta_i}}{1+\theta_i} + (1-q_1^i)G_{i,t}\right]\right\} \\ &- \lambda_t\left[(1+\tau_i^c)P_{c,t}^i C_{i,t}\right] + K_{i,t}^i + K_{i,t}^f + [(\phi)B_{i,t}] + [(1-\phi)B_{f,t}] = \\ &[(1-\tau_i^\ell)W_{i,t}N_{i,t}] + [(1-\tau_i^{R_i^K})(R_{t,i}^K K_{i,t-1}^i)] + [(1-\tau_i^{R_f^K})(R_{t,f}^K K_{i,t-1}^f)] + [(1-\tau_i^{R_i^B})(\phi)(R_t^{B_i}B_{i,t-1})] + \\ &[(1-\tau_i^{R_f^B})(1-\phi)(R_t^{B_f}B_{f,t-1})] + \psi_{i,t}^Z \end{aligned} \tag{2.16}$$

We therefore impose the FOC (First Order Conditions):

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} : \left[(1+\tau_i^c)P_{c,t}^i C_{i,t}\right] + K_{i,t}^i + K_{i,t}^f + [(\phi)B_{i,t}] + [(1-\phi)B_{f,t}] =$$
$$[(1-\tau_i^\ell)W_{i,t}N_{i,t}] + [(1-\tau_i^{R_i^K})(R_{t,i}^K K_{i,t-1}^i)] + [(1-\tau_i^{R_f^K})(R_{t,f}^K K_{i,t-1}^f)] + [(1-\tau_i^{R_i^B})(\phi)(R_t^{B_i} B_{i,t-1})] +$$
$$[(1-\tau_i^{R_f^B})(1-\phi)(R_t^{B_f} B_{f,t-1})] + \psi_{i,t}^Z \tag{2.17}$$

$$\frac{\partial \mathcal{L}}{\partial C_{i,t}} : 0 = q_1^i C_{i,t}^{\eta_i} - \lambda_t (1+\tau_i^c) P_{c,t}^i \tag{2.18}$$

$$\frac{\partial \mathcal{L}}{\partial N_{i,t}} : 0 = -L_{i,t}^{\theta_i} - \lambda_t (1-\tau_i^\ell) W_{i,t} \tag{2.19}$$

$$\frac{\partial \mathcal{L}}{\partial K_{i,t}^i} : 0 = -\lambda_t + \beta_i E_t \left[(1-\tau_i^{R_i^K})[(\lambda_{t+1} R_{t+1,i}^K)]\right] \tag{2.20}$$

$$\frac{\partial \mathcal{L}}{\partial K_{i,t}^f} : 0 = -\lambda_t + \beta_i E_t \left[(1-\tau_i^{R_f^K})(\lambda_{t+1} R_{t+1,f}^K)]\right] \tag{2.21}$$

$$\frac{\partial \mathcal{L}}{\partial B_{i,t}} : 0 = -\lambda_t + \beta_i E_t \left[(1-\tau_i^{R_i^B})(\phi)(\lambda_{t+1} R_{t+1}^{B_i})\right] \tag{2.22}$$

$$\frac{\partial \mathcal{L}}{\partial B_{f,t}} : 0 = -\lambda_t + \beta_i E_t \left[(1-\tau_i^{R_f^B})(1-\phi)(\lambda_{t+1} R_{t+1}^{B_f})\right] \tag{2.23}$$

From here, combining and solving with respect to the Lagrangian we obtain the following MRS (Marginal Rate of Substitution) between Work and Consumption with the labour supply equation and the Euler Equations on Endogenous State variables: Physical Private Capital lended to Domestic companies, Physical Private Capital lended to Foreign companies, Domestic Bonds Purchased by domestic households and Foreign Bonds always purchased by domestic households:

$$C_{i,t}^{\eta_i}(N_{i,t}^{\theta_i}) = q_1^i \frac{(1-\tau_i^\ell)W_{i,t}}{(1+\tau_i^c)P_{c,t}^i} \tag{2.24}$$

$$1 = E_t \left[\beta_i \left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{\eta_i} \left(\frac{P_{c,t}^i}{P_{c,t+1}^i}\right)(1-\tau_i^{R_i^K})(R_{t+1,i}^K)\right] \tag{2.25}$$

$$1 = E_t \left[\beta_i \left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{\eta_i} \left(\frac{P_{c,t}^i}{P_{c,t+1}^i}\right)(1-\tau_i^{R_f^K})(R_{t+1,f}^K)\right] \tag{2.26}$$

$$1 = E_t \left[\beta_i \left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{\eta_i} \left(\frac{P_{c,t}^i}{P_{c,t+1}^i}\right)(1-\tau_i^{R_i^B})(\phi)(R_{t+1}^{B_i})\right] \tag{2.27}$$

$$1 = E_t \left[\beta_i \left(\frac{C_{i,t}}{C_{i,t+1}}\right)^{\eta_i} \left(\frac{P_{c,t}^i}{P_{c,t+1}^i}\right)(1-\tau_i^{R_f^B})(1-\phi)(R_{t+1}^{B_f})\right] \tag{2.28}$$

2.2.2 Firms

The firms in the economy i compose an aggregator for the aggregate output according to the following CES (Constant Elasticity of Substitution) type Technological Function:

$$Y_{i,t} = \left(\int_0^1 Y_{i,t}(j)^{\frac{x-1}{x}} dj \right)^{\frac{x}{x-1}} \tag{2.29}$$

where we have a production function for each firm in Cobb-Douglas version:

$$Y_{i,t}(j) = A_{i,t} N_{i,t}(j)^{\zeta_1^i} K_{i,t-1}^i(j)^{\zeta_2^i} K_{f,t-1}^i(j)^{\zeta_3^i} Z_{i,t-1}(j)^{\zeta_4^i} \tag{2.30}$$

Where we denote by $Y_{i,t}$ the production of the firms of the domestic country i of the j-th domestic good according to the factors Work, Domestic Private Physical Capital, Foreign Private Physical Capital and Public Physical Capital and the country-specific productivity shifter $A_{i,t}$ which it is assumed to follow an $AR_{(1)}$ process such that:

$$\hat{a}_{i,t} = \rho_a \hat{a}_{i,t-1} + \epsilon_{i,t}^a, \qquad \varepsilon_{i,t}^a \sim N(0,1) \tag{2.31}$$

Finally, assume that exist CRS (Constant Returns to Scale) such that: $\zeta_1^i + \zeta_2^i + \zeta_3^i + \zeta_4^i = 1$.

Given the above definitions and $(W_{i,t}, R_{t,i}^K, R_{t,f}^K, R_{t,i}^Z)_{t=0}^{\infty}$ each firms will face the following problem:

$$\begin{aligned} \min_{[N_{i,t}(j), K_{i,t-1}^i(j), K_{f,t-1}^i(j), Z_{i,t-1}^i(j)]_{t=0}^{\infty}} & -(W_{i,t} N_{i,t}(j) + R_{t,i}^K K_{i,t-1}^i(j) + R_{t,i}^K K_{f,t-1}^i(j) + R_t^{Z_i} Z_{i,t-1}(j)) + \\ & \varphi(j)\{A_{i,t}[N_{i,t}(j)]^{\zeta_1^i}[K_{i,t-1}^i(j)]^{\zeta_2^i}[K_{f,t-1}^i(j)]^{\zeta_3^i}[Z_{i,t-1}(j)]^{\zeta_4^i} + (1-\delta_{K_i^i}^i)K_{i,t-1}^i(j) + (1-\delta_{K_f^i}^i)K_{f,t-1}^i(j) + \\ & (1-\delta_{Z_i}^i)Z_{i,t-1}(j) - \frac{P_t^i(j)}{P_t^i}^{-x} Y_{i,t}\} \end{aligned} \tag{2.32}$$

where the Lagrangian $\varphi(j)$ is associated with the marginal costs. The problem therefore leads to the following FOCs:

$$W_{i,t} = \varphi(j) \left[\zeta_1^i A_{i,t} [N_{i,t}(j)]^{\zeta_1^i - 1} [K_{i,t-1}^i(j)]^{\zeta_2^i} [K_{f,t-1}^i(j)]^{\zeta_3^i} [Z_{i,t-1}(j)]^{\zeta_4^i} \right] \tag{2.33}$$

$$R_{t,i}^K = \varphi(j) \left[\zeta_2^i A_{i,t} [N_{i,t}(j)]^{\zeta_1^i} [K_{i,t-1}^i(j)]^{\zeta_2^i - 1} [K_{f,t-1}^i(j)]^{\zeta_3^i} [Z_{i,t-1}^i(j)]^{\zeta_4^i} + (1-\delta_{K_i}^i) \right] \tag{2.34}$$

$$R_{t,f}^K = \varphi(j) \left[\zeta_3^i A_{i,t} [N_{i,t}(j)]^{\zeta_1^i} [K_{i,t-1}^i(j)]^{\zeta_2^i} [K_{f,t-1}^i(j)]^{\zeta_3^i - 1} [Z_{i,t-1}^i(j)]^{\zeta_4^i} + (1-\delta_{K_f}^i) \right] \tag{2.35}$$

Regarding Public Capital considering the maximization problem above we obtain an income generated by public capital claimed by nobody of the magnitude $\zeta_4^i Y_{i,t}$. Following Gou and Lansing (1997), Cassou and Lansing (1998) and Torres (2016) we assume that the public does not charge a price in order to cover the cost of the public investment therefore an extra profit will be generated considering this positive profit equal to the difference between the value of the output and the cost of the production factor. Finally, under the assumption that households are the owner of the firms they will receive this positive earning $\psi_{i,t}^Z$

This profit will be given by the following formulation inserted in the budget constraint of domestic families in a similar way of a public transfer.

$$\psi_{i,t}^Z = \zeta_4^i Y_{i,t} + (1 - \delta_{Z^i}^i) \tag{2.36}$$

Hence the derived equation of marginal costs for companies operating in the domestic country (and symmetrically for foreign ones) becomes:

$$CM_{i,t} = \zeta_1^{i,-\zeta_1^i} \zeta_2^{i,-\zeta_2^i} \zeta_3^{i,-\zeta_3^i} \zeta_4^{i\,-\zeta_4^i} (W_{i,t})^{\zeta_1^i} (R_{i,t}^K)^{\zeta_2^i} (R_{f,t}^K)^{\zeta_3^i} (R_{i,t}^{Z_i})^{\zeta_4^i} \frac{1}{A_{i,t}} \tag{2.37}$$

Given the above Price equations, firms will have a given probability $(1 - \alpha_i) \forall \alpha \in [0; 1]$ to reset them in any given period. This probability is independent among the companies. Taking these prices "a là Calvo" (1983), the term α_i will therefore indicate the index of price stickiness in any given country.

The dynamics of aggregate prices will therefore follow the following equation:

$$P_t^i = \left[\alpha_i (P_{t-1}^i)^{1-x} + (1 - \alpha_i)(P_t^{i,Res})^{1-x} \right]^{\frac{1}{1-x}} \tag{2.38}$$

Where the term $P_t^{i,Res}$ indicates the prices reset in the period t. Dividing each member of the same by the domestic prices P_{t-1}^i we obtain:

$$\Pi_{i,t}^{1-x} = \alpha_i + (1 - \alpha_i) \left(\frac{P_t^{i,Res}}{P_{t-1}^i} \right)^{1-x} \tag{2.39}$$

Equally, the following equalities for the domestic country (and therefore symmetrically for abroad) can be obtained from the log-linearization around the steady state of inflation equal to zero:

$$\pi_{i,t} = (1 - \alpha_i)(p_t^{i,Res} - p_{t-1}^i); \quad p_t^i = \alpha_i p_{t-1}^i + (1 - \alpha_i) p_t^{i,Res} \tag{2.40}$$

Each firm in the period t will therefore choose a given price $P_t^{i,Res}$ which maximizes its value of profits $\Upsilon_{i,t}$ such that:

$$\max_{P_t^{i,Res}} \sum_{n=0}^{\infty} \alpha^n E_t\{H_{t,t+n}^i(P_t^{i,Res} Y_{i,t+n|t} - TC_{t+n}^i(Y_{i,t+n|t})\} \tag{2.41}$$

subject to the sequence of demand constraints:

$$Y_{i,t+n|t} = \left(\frac{P_t^{i,Res}}{P_{t+n}^i}\right)^{-x} C_{i,t+n} \tag{2.42}$$

$\forall\ n = (0,1,2,...)$ and where $H_{t,t+n}^i = \beta_i^n \left(\frac{C_{i,t+n}}{C_{i,t}}\right)\left(\frac{P_t^i}{P_{t+n}^i}\right)$ indicates the discount factor, $TC^i(\cdot)$ indicates the total cost function of the company belonging to the country i and $Y_{i,t+n|t}$ indicates the production in the period $t+n$ for each given firm that resets its prices in the given period t.

We can therefore indicate the FOC associated with the maximization problem as:

$$\sum_{n=0}^{\infty} \alpha_i^n E_t\{H_{t,t+n}^i Y_{i,t+n|t}(P_i^{i,Res} - M^i NMC_{t+n|t}^i)\} = 0 \tag{2.43}$$

where the term $NMC_{i,t+n|t} = TC'^i_{t+n}(Y_{i,t+n|t})$ indicates the nominal marginal cost in the period $t+n$ for each firm that resets its prices in the period t and where $M = \frac{x}{x-1}$ indicates the markup desired by the firm itself in the absence of friction on the frequencies of price adjustment.

In the absence therefore of price rigidity equal to $\alpha_i = 0$ (in the example of the domestic country) the above equation refers to the optimal setting of perfectly flexible prices such that:

$$P_t^{i,Res} = M^i NMC_{t|t}^i \tag{2.44}$$

Therefore the optimal price setting becomes a mark-up over the marginal costs. By comparing instead the terms of (42) with respect to P_{t-1}^i we obtain:

$$\sum_{n=0}^{\infty} \alpha_i^n E_t \left\{ H_{t,t+n}^i Y_{i,t+n|t} \left(\frac{P_t^{i,Res}}{P_{t-1}^i} - M^i * MC_{t+n|t}^i \Pi_{t-1,t+n}^i \right) \right\} = 0 \tag{2.45}$$

Where the term $MC_{t+n|t}^i = \frac{NMC_{t+n|t}^i}{P_{t+n}^i}$ indicates the real marginal cost for the period $t+n$ for the firms whose last prices were fixed in the period t.

Log-linearizing with respect to the steady state at zero inflation through an expansion in Taylor-series at the first order we thus obtain that:

$$p_t^{i,Res} - p_{t-1}^i = (1-\beta_i\alpha_i) \sum_{n=0}^{\infty} (\beta_i\alpha_i)^n E_t[\widehat{mc}_{t+n|t}^i + p_{t+n}^i - p_{t-1}^i)] \tag{2.46}$$

where $\widehat{mc}^i_{t+n|t} = mc^i_{t+n|t} - \overline{mc}^i$ indicates the log- marginal cost deviation from its steady-state value.

The optimal price strategy for the firm that resets its price in the given period t can finally be derived from the previous one so as to obtain:

$$p_t^{i,Res} = \mu^i + (1 - \beta_i\alpha_i) \sum_{n=0}^{\infty} (\beta_i\alpha_i)^n E_t[mc^i_{t+n|t} + p^i_{t+n}] \tag{2.47}$$

with $\mu^i = log\frac{x}{x-1}$ as the optimal markup in the absence of friction in the price adjustment frequency ($\alpha_i = 0$).

Assuming CRS and therefore that the marginal costs are independent of the level of production so $mc^i_{t+n|t} = mc^i_{t+n}$ and therefore equal between the firms, the previous condition can be rewritten as:

$$p_t^{i,Res} - p^i_{t-1} = (1 - \beta_i\alpha_i) \sum_{n=0}^{\infty} (\beta_i\alpha_i)^n E_t[mc^i_{t+n} + \sum_{n=0}^{\infty} (\beta_i\alpha_i)^n E_t[p^i_{t+n}] \tag{2.48}$$

which expressed in recursive form, it becomes:

$$p_t^{i,Res} - p^i_{t-1} = 1 - \beta_i\alpha_i E_t[p^{i,Res}_{t+1}] - (1 - \beta_i\alpha_i)p^i_t + (1 - \beta_i\alpha_i)\widehat{mc}^i_t \tag{2.49}$$

which combined with log-linear version (39) allows us to obtain the final equation of domestic inflation (NKPC). Therefore, from this equation it is possible to positively derive the inflation function in relation to the expected value of inflation in $t + 1$ with respect to the log-deviation of the marginal cost according to the degree of rigidity in the price adjustment captured by the parameter α_i such that:

$$\pi_{i,t} = \beta_i\pi^e_{i,t+1} + \frac{(1 - \alpha_i)(1 - \alpha_i\beta_i)}{\alpha_i}\widehat{mc}^i_t \tag{2.50}$$

with $\pi^e_{i,t+1} = E_t[\pi_{i,t+1}]$.

From the above definitions, we can finally therefore now define the Market Clearing Condition given by:

$$Y_{i,t} = C_{i,t} + G^{\omega}_{i,t} + \left(I_{i,t} - I^i_{K^f_{i,t}} + I^f_{K^i_{f,t}} - I^i_{Z_{i,t}} \right) \tag{2.51}$$

2.2.3 Public Sector

Imposed the equation of the overall deficit on GDP:

$$d_{i,t} = \frac{R^{B_i}_t B_{i,t-1} + G^{\omega}_{i,t} - T_{i,t}}{Y_{i,t}} \tag{2.52}$$

where the overall public deficit will be equal to:

1) $B_{i,t-1}$: The national public debt at the end of the previous year $t-1$

2) $R_t^{B_i}$: The return on the bond

3) $R_t^{B_i} B_{i,t-1}$: Interest rates paid on outstanding securities

4) $G_{i,t}^{\omega}$: Total Public expenditure on goods and services at the time t

5) $T_{i,t}$: Indicates the total fiscal revenues on Consumption, Wages and Income from Capital and Income from Bond both domestic and foreign $\left(\tau_i^c, \tau_i^{\ell}, \tau_i^{R_i^K}, \tau_i^{R_f^K}, \tau_i^{R_t^{B_f}}, \tau_i^{R_t^{B_i}}\right)$

We therefore, imposed the possibility of the existence of public investments and Golden Rule, as seen previously:

$$G_{i,t}^{\omega} = G_{i,t} + I_{Z_i,t} \tag{2.53}$$

where:

1) I_z: Public Investments

We assume that these investments follow this Public Capital Accumulation Law:

$$Z_{i,t} = (1 - \delta_{Z_i^i}^i) Z_{i,t-1} + I_{Z_{i,t}} \tag{2.54}$$

where:

1) δ_Z: Public capital depreciation rate

2) $I_{Z,t}$: Investments in public capital

Following then Batini et al. (2020), Leeper et al. (2010, 2013, 2016) and Muscatelli and Tirelli (2005) we therefore allow the presence of an endogenous current public spending policy rule of countercyclical nature with an output component (y) in order to catch an "automatic stabiliser" component and a backward-looking component of response to public debt (b) from its steady state (SS) (being in line with the SGP guidelines). In the following function the parameters that allow the policy to follow an anti-cyclical rule are ρ_t^{gy} and ρ_t^{gb}. Following then Leeper and Nora (2010), we also allow for the presence of an $AR_{(1)}$ current government expenditure shocks thus assuming some persistence shocks according to a given correlation parameter ρ_{ug}. Hence the log-linear with respect to Steady-State feed-back rule for the domestic and foreign block is:

$$\hat{g}_{i,t} = \rho_i^g \hat{g}_{i,t-1} - \rho_i^{gy} \hat{y}_{i,t} - \rho_i^{gb} \hat{b}_{i,t-1} + \upsilon_t^G \tag{2.55}$$

Symmetrically for the foreign block:

$$\hat{g}_{f,t} = \rho_f^g \hat{g}_{f,t-1} - \rho_f^{gy} \hat{y}_{f,t} - \rho_f^{gb} \hat{b}_{f,t-1} + \upsilon_t^G \tag{2.56}$$

with:

$$\upsilon_t^G = \rho_{ug} \upsilon_{t-1}^G + \varepsilon_t^G, \quad \varepsilon_t^G \sim N(0,1) \tag{2.57}$$

where the $\hat{x}$ variables denote log-deviations of the variables from their Steady-State values and where:

ρ^g= Current expenditure persistence parameter

ρ^{gy}= Responsiveness of current expenditure to changes in output.

ρ^{gb}= Responsiveness of current expenditure to the public debt component

ε_t^G= shock on current government expenditure

We will then analyze, following Sims (2017) and Isaac (2009), the part of the debt with respect to the additional "Rule" according to the impossibility of monetary financing. The public budget constraint therefore stresses that the change in debt in the year t must be equivalent to the sum between the interest accrued on the debt of the previous year $t-1$ and the primary deficit (or surplus) per year t ensuring a tie. Debt will therefore evolve over time according to the following equation:

$$B_{i,t} = (1 + r_t^{B_i}) B_{i,t-1} + G_{i,t}^{\omega} - T_{i,t} \tag{2.58}$$

We also require that the debt grows at a rate equal to the interest rate such that:

$$B_{i,t-1} = (1 + r_t^{B_i})^{t-1} \tag{2.59}$$

We end by imposing the Transversality Condition on the debt such that:

$$\lim_{t \to \infty} E_t \left(R_t^{B_i} B_{i,t} \right) = 0, \quad \lim_{t \to \infty} E_t \left(R_t^{B_f} B_{f,t} \right) = 0 \tag{2.60}$$

We therefore set the dynamics of the public debt with respect to the GDP (multiplying the terms referring to $t-1$ with Y_{t-1}/Y_{t-1}

$$\frac{B_{i,t}}{Y_{i,t}} - \frac{B_{i,t-1}}{Y_{i,t}} \frac{Y_{i,t-1}}{Y_{i,t-1}} = r_t^{B_i} \frac{B_{i,t-1}}{Y_{i,t}} \frac{Y_{i,t-1}}{Y_{i,t-1}} + \frac{G_{i,t}^{\omega} - T_{i,t}}{Y_{i,t}} \tag{2.61}$$

We express in lowercase the relationships between variables with respect to the GDP and its growth rate indicated by Ψ:

$$\Psi_i = \frac{Y_{i,t} - Y_{i,t-1}}{Y_{i,t-1}} \rightarrow \frac{1}{1 + \Psi_i} = \frac{Y_{i,t-1}}{Y_{i,t}} \tag{2.62}$$

So solving the equation We get the final intertemporal public budget constraint as a function of:

1) Primary requirement

2) Interest rate

3) GDP growth rate

4) Public debt outstanding

$$b_{i,t} = (g^{\omega}_{i,t} - t_{i,t}) + \left(\frac{1 + r^{B_i}_t}{1 + \Psi_i}\right) b_{i,t-1} \tag{2.63}$$

Finally, We impose this cycle-adjusted balance budget constraint according to the CABB equation:

$$CABB_{i,t} = d_{i,t} - \varsigma_{bi} OG_{i,t} \tag{2.64}$$

where:

1) $CABB_{i,t}$: Indicates the (public) cycle-adjusted balance budget

2) ς_{bi} = Sensitivity parameter of the deficit to the Output Gap (usually estimated for EA=0,48)

This parameter, obtainable endogenously through the difference between the sensitivity of the inputs to the Output Gap and the Outputs such that:

$$\varsigma_{bi} = \varsigma_e - \varsigma_u \tag{2.65}$$

It allows to consider which types of income and expenses are sensitive to the Economic Cycle. Specifically, as specified by the OECD and the ECB, we obtain a sensitivity of ς_e with respect to: 1) Taxes on personal income 2) Taxes on firms income 3) Indirect taxes 4) Social Contributions; on the contrary for ς_u the most significant item is that relating to 1) Unemployment benefits.

The elasticity of these items to the OG will change the total sensitivity value ς_{bi}.

Finally we obtain the Equation of the Output Gap (OG):

$$OG_{i,t} = \frac{Y_{i,t} - Y_{P,i,t}}{Y_{P,i,t}} \tag{2.66}$$

where:

$Y_{P,i,t}$: corresponds to the Potential GDP (in fully flexible prices)

2.2.4 ECB

Closing the model the ECB fixes an interest rate centrally for the Union Wide according to a log-linar Taylor Rule and therefore according to:

1) Union Wide Inflation $\pi_{f,t}$

2) Union Wide Output Gap $OG_{f,t}$

multiplied by their respective sensitivity parameters ρ_π, ρ_{OG}, such that:

$$r_t = \bar{r} + \rho_\pi(\pi_{f,t}) + \rho_{OG}(og_{f,t}) \tag{2.67}$$

By definition we can from here endogenize the risk premium[1] on domestic and foreign government bonds according to the following equalities with respect to the ECB interest rate $r_t^B = RP_t^B + r_t$ such that:

$$r_t^{B_i} = (R_t^{B_i} - 1 + \delta_{B_i}^i) \tag{2.68}$$

$$RP_t^{B_i} = r_t^{B_i} - r_t \tag{2.69}$$

$$r_t^{B_f} = (R_t^{B_f} - 1 + \delta_{B_f}^f) \tag{2.70}$$

$$RP_t^{B_f} = r_t^{B_f} - r_t \tag{2.71}$$

Fiscal Rules Scenarios Comparison

The simulation as follows presents 4 different fiscal rules scenarios according the eurozone fiscal rules and implementable ones. Specifically:

I SCENARIO: It presents an open economic structure characterised by the presence of nominal rigidities (in the form of Calvo Prices) and a limit to the Deficit/GDP ratio at 3 per cent according to the EMU fiscal rule (art. 104 PSC).

II SCENARIO: It presents the same structure but with the removal of public investments from the calculation of the ratio between Deficit/GDP ("*GoldenRule*").

III SCENARIO: It starts from the assumption of the first scenario but with the addition of the Cyclically Adjusted Budget Balance (CABB) as stated by Fiscal Compact treaty with an initialisation value equal to 0.5 %/GDP, incorporating the objective of the MTO (Medium Term Budget).

IV SCENARIO: Reconciles the analysis by separating public investments (*GoldenRule*)

[1] A similar specification of the risk premium is used in Furceri and Mourougane (2009), Rudebusch and Swanson (2012), Basu and Wada (2018) and Albonico et al. (2019). See Chapter I for similar and alternative references in risk-premium endogenization methodologies

from the deficit/GDP computation (with the 3% rule) and noting the possible effects in light of the presence of the CABB.

Each scenario thus presents its own and current features on what are the main debates in the reference literature on the adoption and harmony or not of certain fiscal rules within the aforementioned treaties. In fact, fiscal decentralisation in eurozone does not impose the same forms of uni-directionality as in the monetary case, although it still leads to the respect of the same parameters by differentiating the fiscal instruments of implementation. The following analysis examines the case of a domestic country (i.e. Italy) and of foreign countries (i.e. Eurozone) in order to show how different effects emerge and in order to analyze possible readings in the reciprocal systemic interconnections.

2.3 Methodological aspects: Calibration, Log-Linearization and Policy Comparison

The calibration process presented in the simulation consists of a set of 61 equations and 61 endogenous variables (in the total of four fiscal scenarios simulated). The simulation process included the necessary but complete linerarization of the non-linear stochastic dynamic system seen in the previous section. The need for log-linearisation is an established technique within the macroeconomic environment, insofar as, within the theoretical-mathematical modelling of Real Business Cycle Models (later evolved into DSGE), the presence of non-linear equations entails the impossibility of solving the system in closed form and consequently identifying the respective values of the single steady-states. To this end, the main principle on which the log-linearization technique is based hinges on the use of a first-order Taylor series development around the steady-state value in order to obtain approximations as linear functions in log-deviation of the given endogenous variables Uhlig (1997). Formally define a variable X_t as a vector of variables and define $\overline{X}$ as its steady state value. We obtain then:

$$\hat{x}_t = \log X_t - \log \overline{X}$$

as a vector of log-deviation. Following Uhlig (1997) "the product of such a value $\hat{x}_t * 100$ allows one to notice the deviations % of the multiple variables at given exogenous shocks from their steady state level in any given period t". In most situations the nonlinear system will require the following algebraic transformation procedure on the sum chains (where the algebraic constants are simplified at the end of the procedure) obtaining:

$$\begin{aligned} Z_t &= X_t + Y_t \\ \rightarrow \overline{Z}e^{(\hat{z}_t)} &\approx \overline{X}e^{(\hat{x}_t)} + \overline{Y}e^{(\hat{y}_t)} \\ \rightarrow \overline{Z}(1+\hat{z}_t) &\approx \overline{X}(1+\hat{x}_t) + \overline{Y}(1+\hat{y}_t) \\ \rightarrow \overline{Z}\hat{z}_t &\approx \overline{X}\hat{x}_t + \overline{Y}\hat{y}_t \end{aligned}$$

For product chains the procedure is not an approximation but returns a result equivalent to the starting function as linearized. An example of this is the classical Cobb-

Douglas production function which, after some algebraic simplification, becomes:

$$Y_{i,t} = A_t L_{i,t}^{\zeta_1^i} K_{i,t-1}^{i,\zeta_2^i} K_{f,t-1}^{i,\zeta_3^i} Z_{i,t-1}^{\zeta_4^i}$$

$$\rightarrow \hat{y}_{i,t} = \hat{a}_{i,t} + \zeta_1^i \hat{\ell}_{i,t} + \zeta_2^i \hat{k}_{i,t-1}^i + \zeta_3^i \hat{k}_{f,t-1}^i + \zeta_4^i \hat{z}_{i,t-1}$$

The equations thus log-linearized are codified in order to obtain possible computational predictions in order to understand the complex trend in response to the exogenous shocks of each model under examination. In the following paragraph it will be possible to notice the deviations in log-deviation with respect to the steady state of the 61 endogenous variables subjected to the symmetrical exogenous shocks of supply and demand, respectively on Total Factor Productivity (TFP) and Current Public Expenditure, both domestic and foreign. Since the closed-form solution of the dynamic system is not possible, given its nonlinearity, the present computation bases the values of the behavioural parameters, some of which on endogenously detected computations, and others extracted from the reference literature. The values detected instead for the remaining parameters relative to the variables in steady state refer directly to the creation of the so-called "Great Ratio" (e.g. $\frac{\overline{C}}{\overline{Y}}$) in order to note eventual deviations and simplify at the same time the computational values, bringing them all back to states of ratios. In particular, the calibration of the model parameters mainly refers to values calibrated in the literature in line with those of Smets and Wouters (2003, 2007), Straub and Tchakarov (2007) regarding the behavioural ones as well innovations ones. In Chapter III the a priori calibrated parameters will be discussed, together with the estimates derived from the Bayesian procedure. While the steady-state ratios are calibrated from Italian and European data from AMECO Database and FRED (Federal Reserve Economic Data). Specifically, the results of the simulations are based on different comparisons and policy experiments. Following the same methodology used in literature of policy comparison (see among others for details the similar studies and methodology of McManus (2015) regarding the comparison and policy experiments of Austerity and Fiscal Stimulus policies and, among others, the Federal Reserve studies and policy comparisons of Cwik (2012)) and Vieira, Machado and Ribeiro (2016) for different effect on stabilization on output and debt of fiscal policy sensitivity coefficients, different specifications of fiscal policy response to systemic variables will be included. Assuming the counter-cyclical nature of governement expenditure in log-deviation

form with respect to output and public debt as specified before and in line with the role of fiscal policy as guidelines of the SGP with respect to public debt dynamics and its stabilization, it is possible to specify different responses of the same policy function with respect to the volatility of the variables considered so as to provide a stronger response with respect to fluctuations in the business cycle or public debt and to their active stabilization. In this sense, the value of the parameters used reflects the estimates presented by the Bank of Italy in Batini et al. (2020). The estimated European values for the response coefficients will therefore be assumed to be stronger in the case of a scenario of higher output stabilization (henceforth OSFP) and debt stabilization (DSFP). Specifically in the OSFP scenarios we will then obtain a more active response of government policy to domestic and foreign output fluctuations as for their stabilization by imposing the respective response coefficients as $\rho^{gy} > \rho^{gb}$ and respectively with $\rho^{gy} = 0.14$ and $\rho^{gb} = 0.08$ as well as in the case of DSFP $\rho^{gb} > \rho^{gy}$ and therefore $\rho^{gy} = 0.08$ and $\rho^{gb} = 0.14$ for debt stabilization as for european SGP. Each simulation is then conducted in the different scenarios, baseline ones and with golden rule application, analysed above with respect to public investment dynamics in the light of european fiscal rules. Finally are presented the 4 exogenous shocks that follow a first-order autoregressive process $AR_{(1)}$ for which: $ln\varepsilon_i^X = (1-\rho_X)\ln\overline{X} + \rho_X lnX_{t-1} + e_i^X$ with the relative: $[\rho_G, \rho_A]$ In the following, therefore, it is possible to analyse the system's responses to different shocks in the model economy.

2.4 Results and IRFs Analysis

Domestic Productivity Shock

An initial impact of the domestic productivity shock shows results in line with the literature on simulation scenarios characterised by nominal rigidities (e.g. Calvo nominal rigidities). In fact, a positive response of the productivity increase brought about by the TFP shock is immediately noticeable. This increase in supply leads to an upward response of household consumption around 0.02 in terms of log-deviation and to a general increase in the respective remunerations of production factors for recorded values of 0.05 and 0.06 in terms of log-deviation.

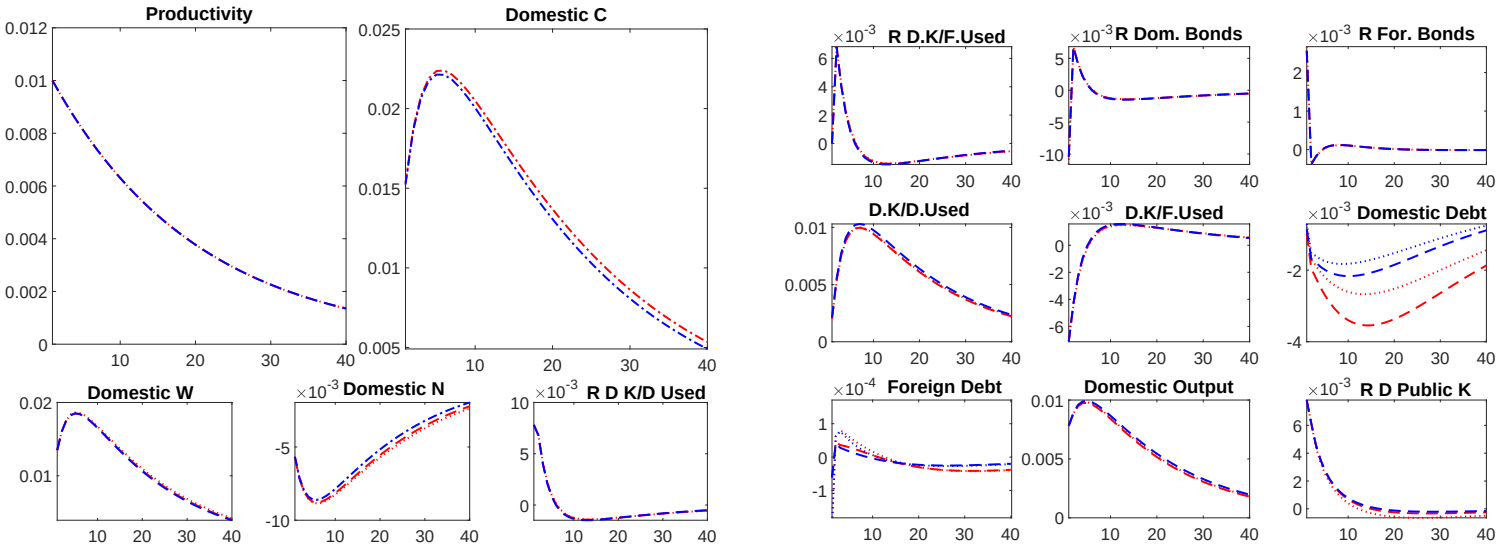

Fig.2.1: *System responses to a 1% innovation in domestic productivity. The red and blue lines represent respectively the fiscal policy scenarios of ouput and debt stabilization. The dotted line shows the benchmark scenarios (I-III) and the dash line the golden rule scenarios (II-IV).*

In this sense, in fact, we note a general increase in wage levels and a general reduction in working hours. This phenomenon, as evidenced by the literature and empirical evidence (see among others Gali and Rabanal (2005) and QUEST III Model of Ratto et al. (2008)) is due precisely in the presence of the same nominal

rigidities and the creation of demand externalities. In fact, since firms lower prices insufficiently as a response to a cost reduction shock and there is a momentary lack of aggregate demand, it makes it optimal for individual firms to reduce employment (Gali (1999)). An increase in the respective remuneration of the returns on domestic capital used both domestically and abroad can be noted as analyzed. At the same time it is also possible to notice increases in the quantities of the same capital used by firms in the production function. In this research it is important to underline that there is also a general increase in the return recorded by public capital around the values of 0.06, also subjected to a general increase in productivity with small increases recorded in scenarios II-IV compared to benchmark scenarios (I-III). A reduction in domestic debt is subsequently confirmed, greater in scenarios II-IV and greater in cases of output stabilization policies, as well as an almost instantaneous increase in the foreign debt presumably purchased by the rapidly growing domestic country. which then sees in the following a phase of re-stabilization probably due to the stabilization effects of the foreign fiscal policies themselves. This process leads to responses in line with the results achieved in the literature Badarau et al. (2014) as regards both domestic and foreign bond returns, with the associated risk premiums, which respond to domestic (foreign) impact in a negative (positive) way. A rapid increase in domestic output is subsequently confirmed for values of 0.01 in terms of log-deviation, slightly greater in scenarios II-IV than in the benchmark scenarios (I-III) for all the periods considered. There is also a general increase in public investments and therefore a general increase in total domestic investments driven by the greater quantities of capital used. For the purposes of this work, it is also interesting and in line with the results achieved in the literature with respect to economic theory, to note a reduction in the Deficit/Output ratio, greater in scenarios II-IV as noticeable from the respective points of inflection, due to the rapid growth of the domestic economy. In this sense, in fact, the stabilization of the output leads to a greater reduction of this ratio around the thirtieth period with respect to the equivalent benchmark scenario relating to the fiscal policy of stabilization of the debt. In this dynamic it is also interesting to note how an output stabilization policy leads to greater effects of this reduction than a debt stabilization policy associated with a total factor productivity innovation. Finally, it is interest-

ing to dwell on the responses of the Domestic Capital purchased Externally: well, the supply shock, combined with the presence of nominal rigidities on prices, leads to a response with a negative impact on the stocks of this capital due, with extreme probability, precisely of that excess supply due to the increase in TFP but which does not find levels of conjunction in the foreign demand for the asset itself, seeing a decrease as confirmed, moreover, by the negative response of Investments in Domestic Capital used internationally and by the increase in domestic investments in domestic capital used domestically.

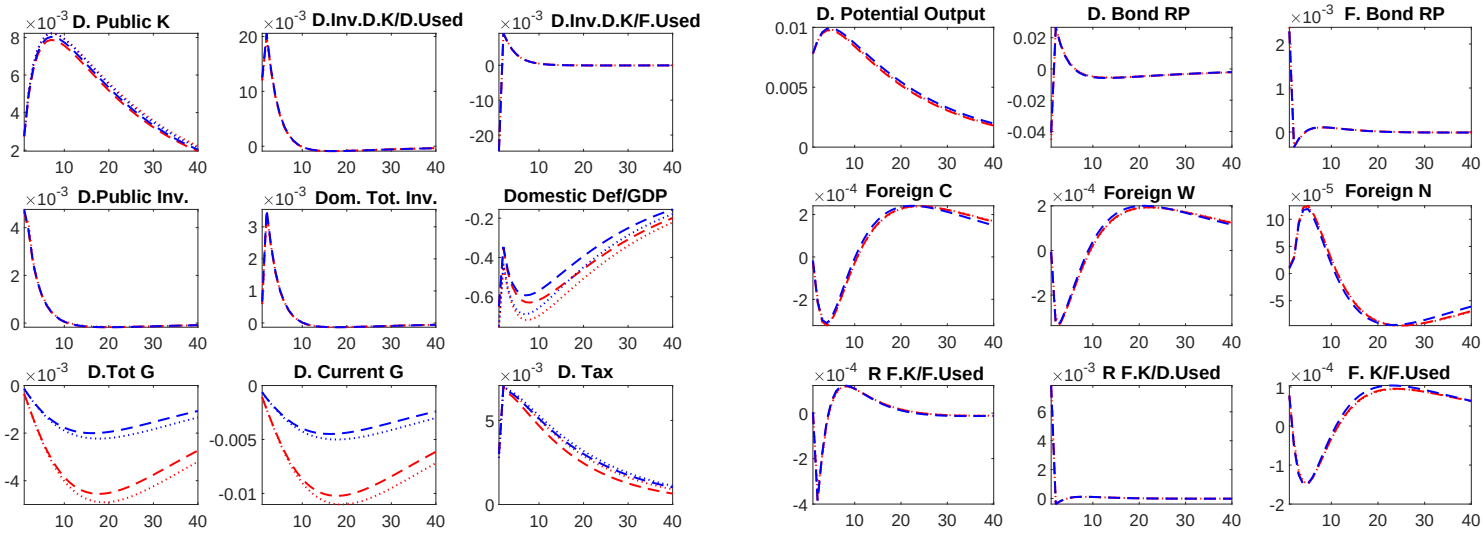

Fig.2.2: *System responses to a 1% innovation in domestic productivity. The red and blue lines represent respectively the fiscal policy scenarios of ouput and debt stabilization. The dotted line shows the benchmark scenarios (I-III) and the dash line the golden rule scenarios (II-IV).*

The countercyclical nature of public spending is therefore confirmed (higher in the case of output stabilization policies due to the rapid growth of the domestic GDP recorded and a reduction in the respective debt), both in capital and current accounts, which sees a reduction driven by the rapid growth of the domestic economy as well as the pro-cyclical nature of taxation which is rapidly increasing. In line with the increase in the potential capacities of the economy, there is also an increase in potential output, also in this case greater in the case of unbundling of

investments from European fiscal rules. Finally, we note that abroad the high level of domestic productivity associated with the dynamics of costs has repercussions with negative effects almost everywhere in relation to the demand and returns of production factors except for investments from the domestic country as well as for the higher returns recorded for foreign capital used domestically (where this increase in productivity is recorded).

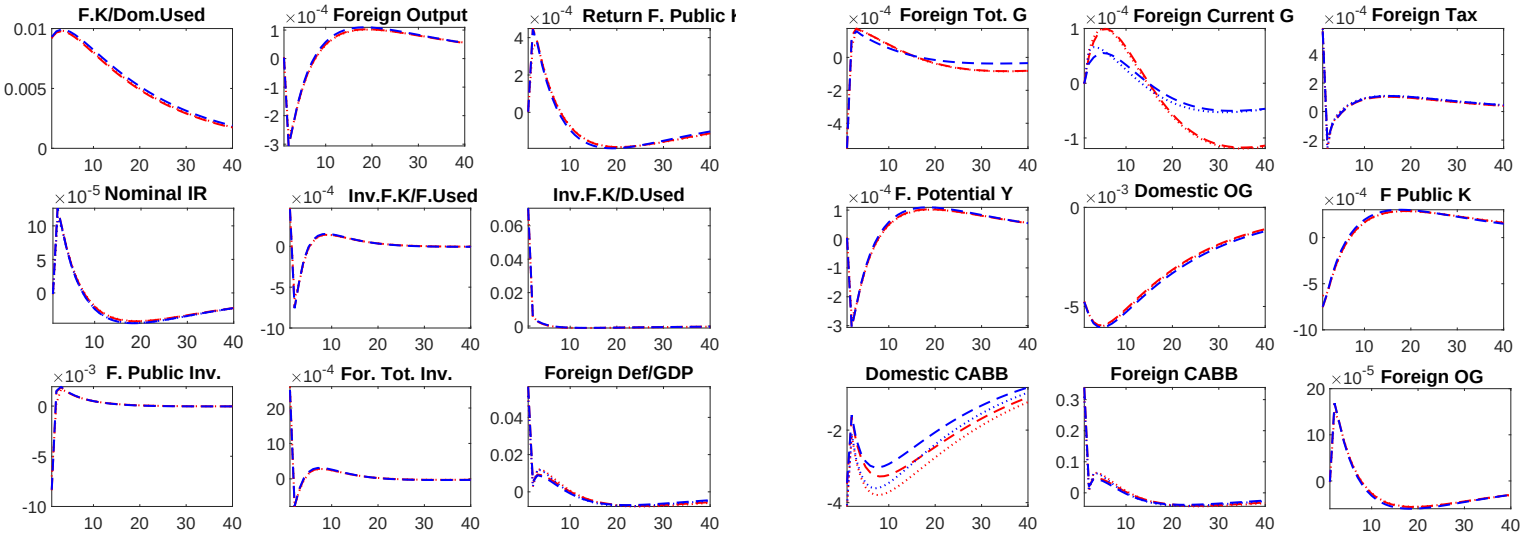

Fig.2.3: *System responses to a 1% innovation in domestic productivity. The red and blue lines represent respectively the fiscal policy scenarios of ouput and debt stabilization. The dotted line shows the benchmark scenarios (I-III) and the dash line the golden rule scenarios (II-IV).*

The counter-cyclical responses of foreign public spending, both in current and capital accounts (although of a smaller magnitude), in response to the short and medium-term decline in foreign ouput, recorded as greater in cases of greater stabilization of the same output, are also confirmed rather than on debt levels as well as higher in the investment spin-off scenarios (II-IV). In line with economic theory and with the literature (Gali (2006)), in the presence of this supply shock, there is also an increase response in the ratio between foreign deficit/GDP as well as a reduction of the domestic Output Gap of values around 0.05 in terms of log-deviation. Finally, it is interesting to note a bullish response from the European central bank,

albeit with low levels of persistence aimed at re-stabilizing the equilibrium levels as well as on the negative (positive) responses of the domestic and foreign structural deficit also in line with the reduction (increase) of the deficit itself and with the same dynamics recorded in relation to scenarios II-IV and the greater stabilization effect of the fiscal policy of stabilization of the output recorded around the thirtieth period compared to the benchmark scenario I-III of the debt stabilization policy. In these terms, it is finally possible to note how this trend abroad is more offset by the values of the foreign output gap around the fifth period in scenarios II-IV compared to the benchamark. By these results in the presence of increases in domestic productivity, a policy at the domestic level of greater stabilization of the ouput leads to greater levels of debt reduction and domestic deficit/output ratios, as well as to the recorded levels of structural deficit in the medium-long term, if at the same time combined with a fiscal rule for the unbundling of public investments capable of encouraging greater growth accompanied by a lower tax burden. These reduction (increase) effects can also be seen on foreign budget balances in relation to the application of a golden rule, both in relation to increases in deficit levels and in relation to adjustments with respect to structural deficit levels, more in the short term if accompanied by a foreign debt stabilization policy. These results therefore confirm, in their innovations with respect to both the structure in an open economy and the presence of distorting taxation, nominal rigidities a la Calvo and endogenous fiscal policy as previously stated in the reference literature (see among others: Gali and Monacelli (2005) and Gali (2006)).

Foreign Productivity Shock

In the presence of a foreign productivity shock, many of the same dynamics are observed as in the presence of a domestic shock, in line with previous empirical studies (Gali and Rabanal (2005) and Ratto et. al (2008)) although characterised by different magnitudes due to the different parameterization of Europe compared to Italy and to the different weight of your economies in their respective comparisons. It is also interesting to dwell on how, at the aggregate EU level, qualitatively and quantitatively interesting effects can be found in the different fiscal policy scenarios.

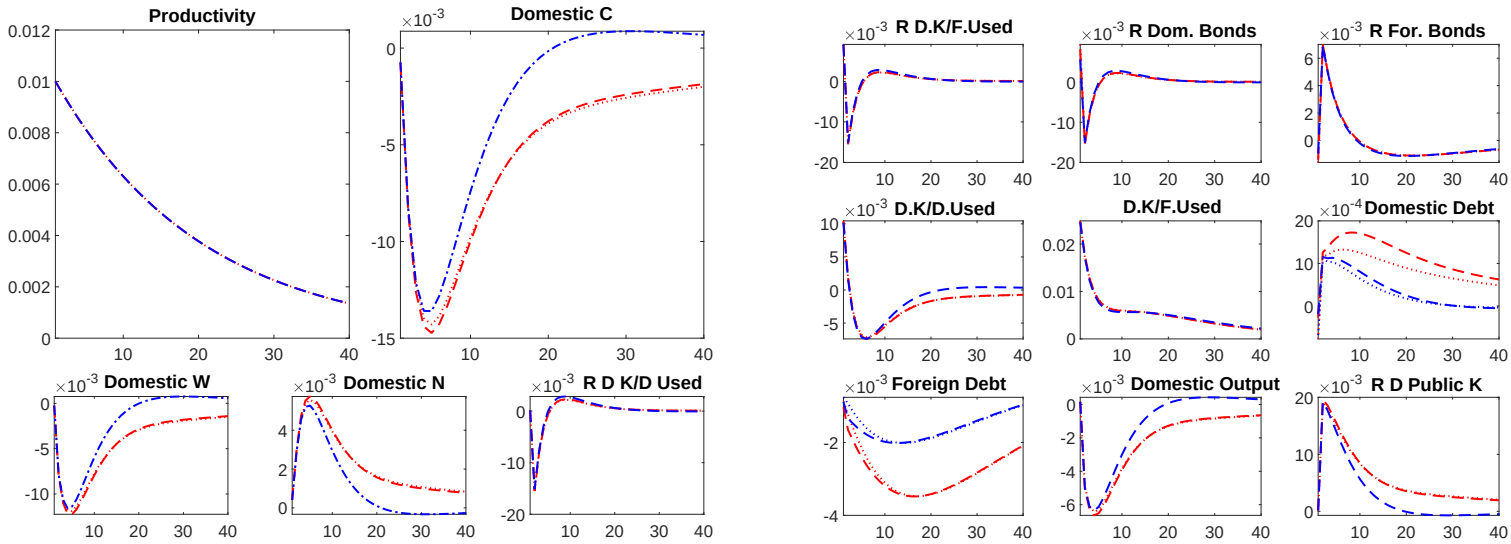

Fig.2.4: *System responses to a 1% innovation in foreign productivity. The red and blue lines represent respectively the fiscal policy scenarios of ouput and debt stabilization. The dotted line shows the benchmark scenarios (I-III) and the dash line the golden rule scenarios (II-IV).*

We note in this sense, at first, a confirmed increase in foreign TFP with a negative effect almost everywhere referred to domestic variables, as happened in the previous simulation of the TFP shock in Italy, while there is a general increase in foreign consumption for values around 0.02 in terms of log-deviation due to the greater supply generated by firms in the presence of increased productivity, present in this case with greater increases in scenarios II-IV together with a policy of output stabilization rather than debt. Similarly, there is an increase in foreign capital

investment and an increase in international investment by the EA as well as an immediate reduction (increase) in higher levels of foreign (domestic) public debt, again in the presence of output (debt) stabilization policies and the application of the golden rule (II-IV) compared to the benchmark scenarios (I-III).

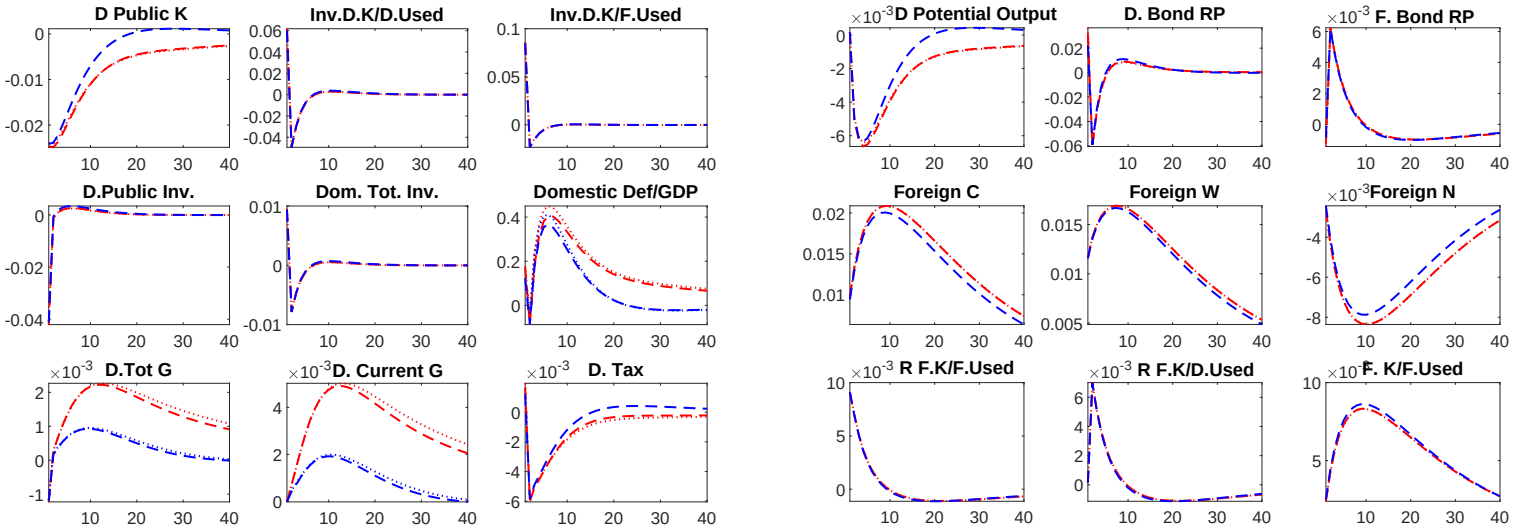

Fig.2.5: *System responses to a 1% innovation in foreign productivity. The red and blue lines represent respectively the fiscal policy scenarios of ouput and debt stabilization. The dotted line shows the benchmark scenarios (I-III) and the dash line the golden rule scenarios (II-IV).*

Thus, returns on all foreign production factors are rising, both in terms of wages and the return on foreign capital used by domestic and European companies, and there has also been a general increase in their capacity utilisation in the production function. The result of increased private and public investment is also confirmed, with a general increase in public capital in the production function. This supply shock and increased productivity also leads in this case to an increase in total output (0.01 in log-deviation terms) and potential output and to a reduction (increase) in the Deficit/GDP ratios which is also higher in the presence of output stabilization policies at European level and debt stabilization policies at domestic level in conjunction with scenarios II-IV. There is thus a response of the countercyclical nature of foreign capital and current government expenditure as well as a pro-cyclical increase in foreign tax revenues.

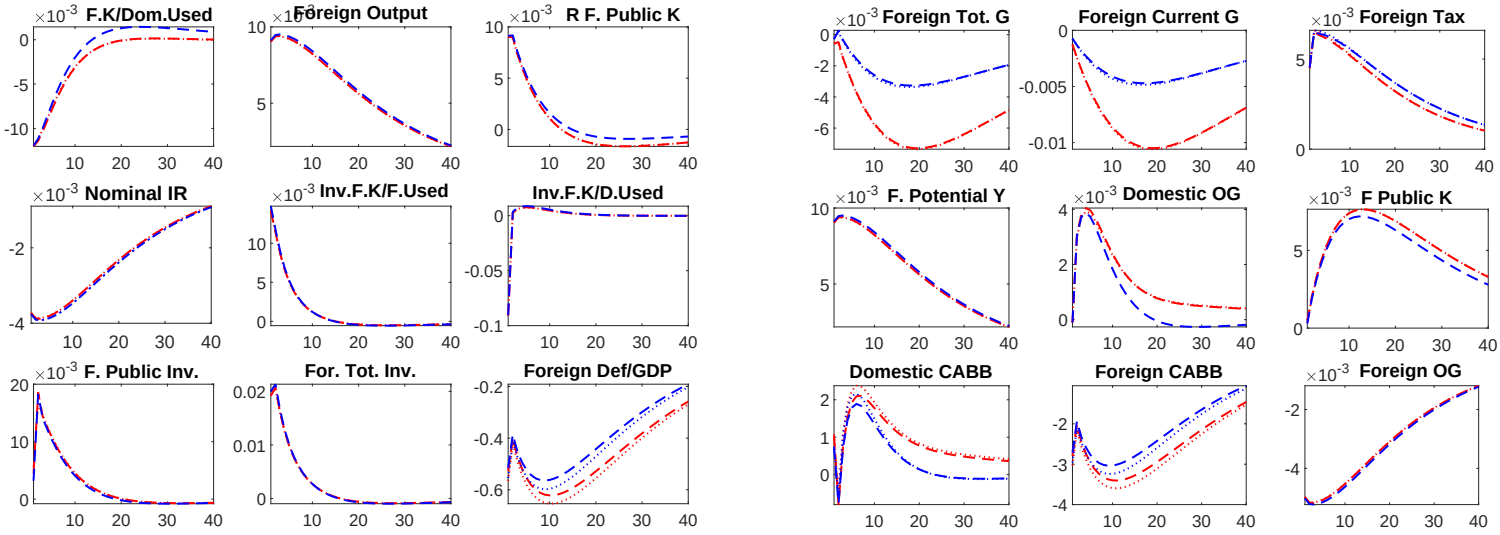

Fig.2.6: *System responses to a 1% innovation in foreign productivity. The red and blue lines represent respectively the fiscal policy scenarios of ouput and debt stabilization. The dotted line shows the benchmark scenarios (I-III) and the dash line the golden rule scenarios (II-IV).*

Finally, also in this case, a reduction of the foreign Output Gap for values of 0.05 in terms of log-deviation and of the levels of structural deficits is confirmed, in line with the economic theory and reference literature (Gali (2006)), with the same differentiation dynamics noted in scenarios II-IV with respect to the presence of fiscal policies of output and debt stabilization.

Domestic Government Spending Shock

Finally, the responses of the model economy to a domestic and foreign current expenditure shock are shown below. In the presence of public expenditure shocks, it is important to underline the dynamics of some systemic variables with respect to their behaviour in the different policy scenarios as well as with respect to the four different regulatory scenarios. There are important qualitative and quantitative differences in the II-IV scenarios. In fact, a negative response to impact and high levels of persistence of values around 0.005 and 0.01 in terms of log-deviation of private household consumption is immediately noticeable. Such is the phenomenon of "Crowding-Out" of private consumption with respect to government expenditure. What happens? Basically, government spending crowds out private consumption which sees a reduction as a result of the current expenditure shock. At the same time, as specified by Straub and Tchakarov (2007) "such an increase in government spending and the corresponding expected increase in future taxes generate a negative wealth effect, inducing households to reduce consumption and increase labour supply". These effects are more pronounced in the benchmark scenarios (I-III) while they are less pronounced in the golden rule scenarios. This phenomenon is in line with the results obtained in the similar studies by Zeynologlu (2018), where a similar phenomenon occurs. It is also important to underline that, in the presence of a demand shock, this effect is more mitigated by a debt-stabilising fiscal policy than by an output-stabilising one, where crowding-out is more evident in both scenarios I-III and II-IV. In line with theory, an increase in domestic public debt and a reduction in foreign public debt are confirmed, with significant persistence of the shock led by a general increase in the deficit, which also in this case presents the same dynamics of the supply shock with respect to scenarios II-IV, although with differentiations with respect to the application of fiscal policy to stabilise output and debt. In fact, in the presence of the latter, our expectations are confirmed, since there is a smaller rise in domestic public debt as well as in the deficit/GDP ratio, while abroad this dynamic is supported by a policy of output stabilization, especially in the medium-long term. The dynamics and positive (negative) responses to the impact of domestic (foreign) bond returns and the related risk premiums were also confirmed.

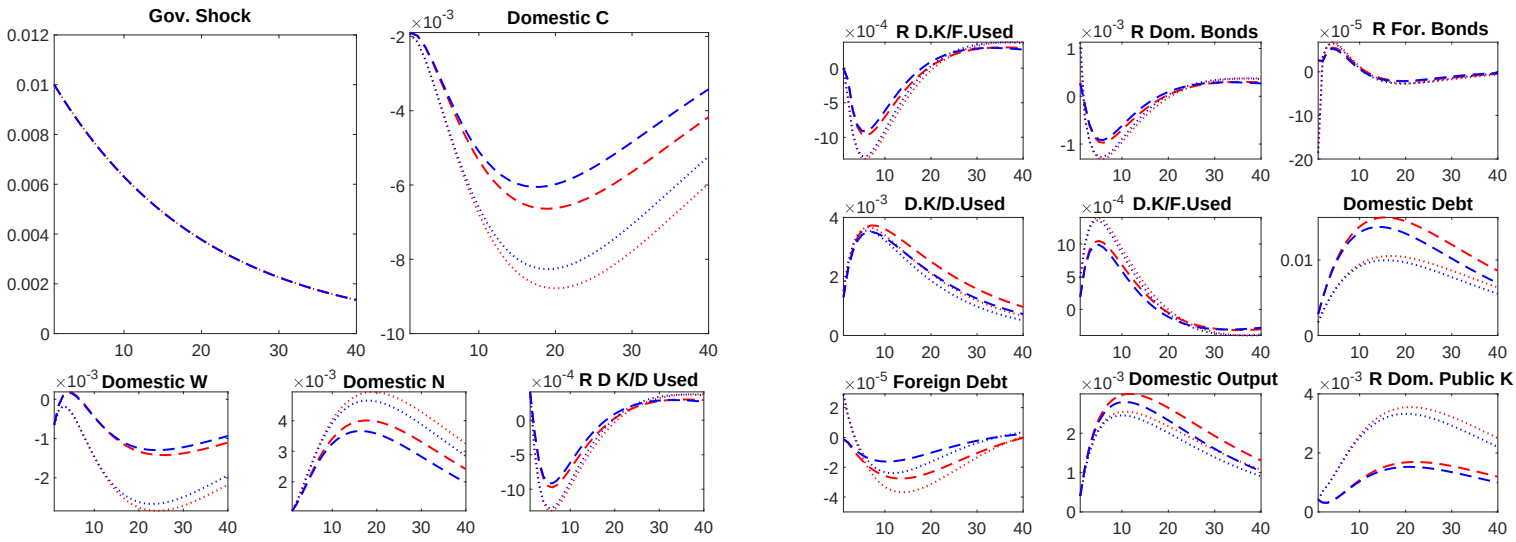

Fig.2.7: *System responses to a 1% innovation in domestic government spending. The red and blue lines represent the fiscal policy scenarios of ouput stabilization (OSFP) and debt stabilization (DSFP) respectively. The dotted line shows the benchmark scenarios (I-III) and the dash line the golden rule scenarios (II-IV).*

It is obviously confirmed (see among others Ferrero (2009), Straub and Tchakarov (2007) and QUEST III ModelRatto et al. (2008)) an increase in domestic GDP around 0.02 in terms of log-deviations and of its potential counterpart given the growing component of Current Expenditure, which also in this case presents the same dynamics noted in the presence of the different fiscal policies and greater in the cases of the unbundling of public investments. In this case, however, it is important to focus on the persistence of such dynamics in models characterised by nominal rigidities. It is indeed confirmed in this case the persistence of a positive fiscal multiplier of such an increase in GDP, although in presence of "crowding-out" on consumption but never sufficient to lead to the reduction of the same output as in the previous studies by Gali and Monacelli (2005). Therefore, the increases in public capital government expenditure and public current government expenditure are confirmed for values respectively of 0.02 and 0.05 in terms of log-deviation as well as an increase, as previously analysed, of domestic tax revenues in line with the results of Zeynologlu (2018) analysed in the previous review.

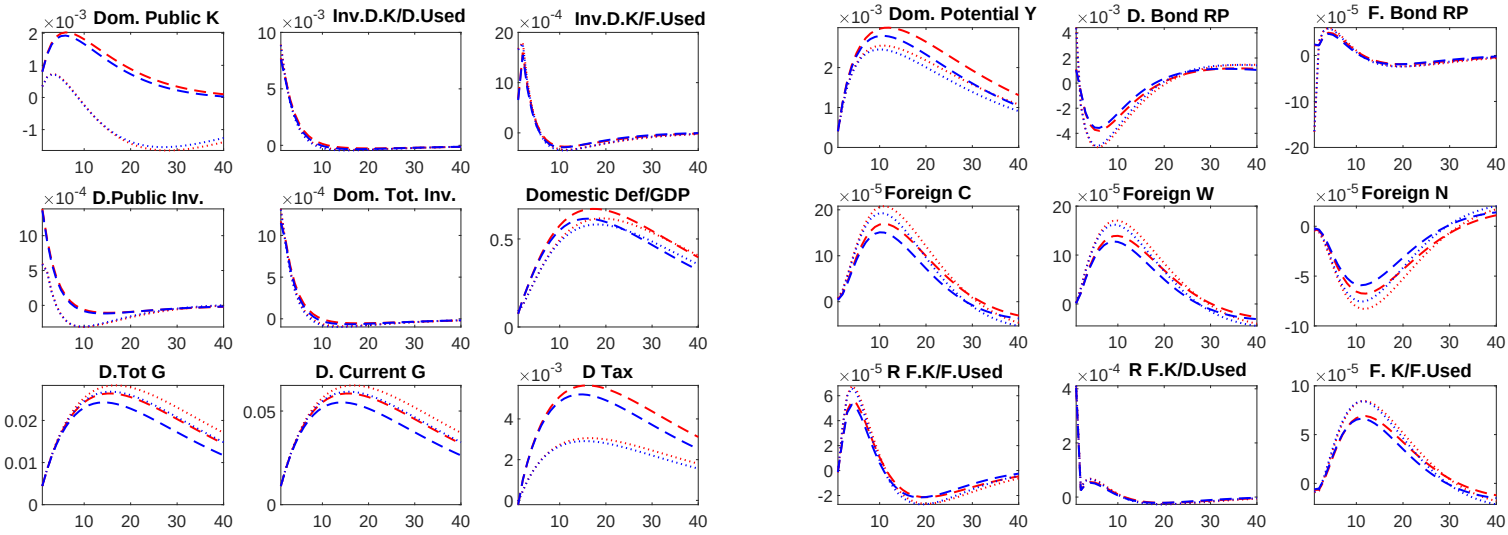

Fig.2.8: *System responses to a 1% innovation in domestic government spending. The red and blue lines represent the fiscal policy scenarios of ouput stabilization (OSFP) and debt stabilization (DSFP) respectively. The dotted line shows the benchmark scenarios (I-III) and the dash line the golden rule scenarios (II-IV).*

Of particular interest in this research are the results that can be analysed with respect to public investment. In this case, in the benchamark scenarios with respect to a shock of domestic government expenditure, the empirical evidence of Straub and Tchakarov (2007) noted in figures 2.5 and 2.6 is confirmed with respect to the SGP constraints and with the consequent growth dynamics of the D/GDP ratios and of the public debt. However, it is very important to underline that these dynamics are different in the investment separation phase, allowing a higher growth in the range of 0.02 and 0.01 in terms of log-deviation of public capital and public investment with significant persistence levels until steady-state is reached again. This result is probably due to the government's ability to increase public investment while allowing public capital to grow more in the production function without being bound by European fiscal rule. This phenomenon then has repercussions on the systemic variables, probably giving rise to the effects highlighted above.

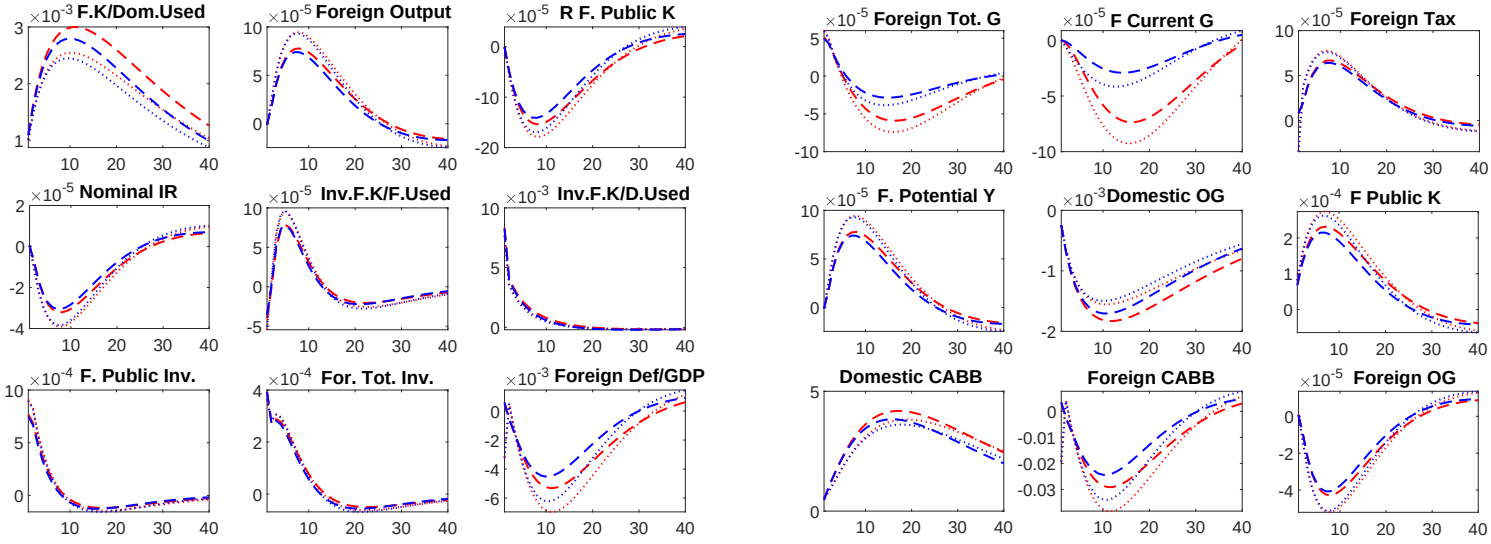

Fig.2.9: *System responses to a 1% innovation in domestic government spending. The red and blue lines represent the fiscal policy scenarios of ouput stabilization (OSFP) and debt stabilization (DSFP) respectively. The dotted line shows the benchmark scenarios (I-III) and the dash line the golden rule scenarios (II-IV).*

The remaining answers are in line with the results obtained in the literature in models characterised by nominal rigidities and distortionary taxation (see among others Straub and Tchakarov (2007) and Ferrero (2009)). Finally, there is no particular differentiation of the analysed demand shock from the previously analysed results in the presence of supply shocks, except for the application of different fiscal policies in scenarios II-IV with respect to the dynamics of the structural deficit in the second phase. In these cases, in fact, the greater stabilization dynamics of a fiscal policy of debt stabilization in the II-IV scenarios are confirmed with respect to the output counterpart in the I-III benchmark scenarios, where the increase effect reaches higher levels.

Foreign Government Spending Shock

Finally, it is possible to note what are the dynamics that occur at an aggregate level EA to a shock of current government expenditure. Well, also in this case we

can note the increase in current expenditure due to the shock to which the same dynamics present in the transmission of the shock at the international level are immediately evident. An increase in government expenditure at the EU level has positive effects on the domestic country with a general increase in consumption and a reduction in the returns of domestic bonds and an increase in foreign ones, with the associated dynamics of increase and reduction of the respective risk premiums in line with the analysis carried out at the European level by the main reference modeling (see Vitek (2014), (2016)).

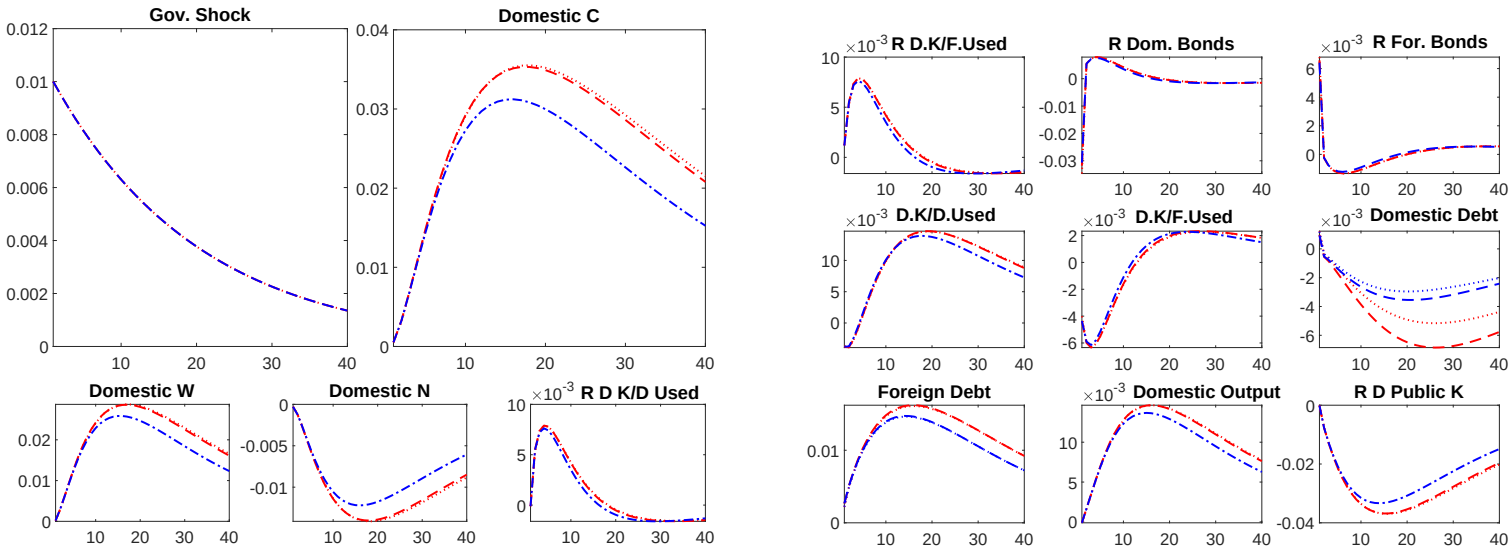

Fig.2.10: *System responses to a 1% innovation in foreign government spending. The red and blue lines represent the fiscal policy scenarios of ouput stabilization (OSFP) and debt stabilization (DSFP) respectively. The dotted line shows the benchmark scenarios (I-III) and the dash line the golden rule scenarios (II-IV).*

In addition, economic theory confirms a general increase in foreign debt with a reduction in domestic debt. Even in these dynamics, this result is in line with the dynamics of the II-IV scenario with respect to the benchmark ones, leading to a lower reduction at the external level in the presence of debt stabilization policies rather than of output and especially in the presence of the application of a separation of public investments. Symmetrically, the same dynamics can be observed at European level, where the current expenditure shock is mitigated in its growth

effects on debt by a policy of greater debt stabilization.

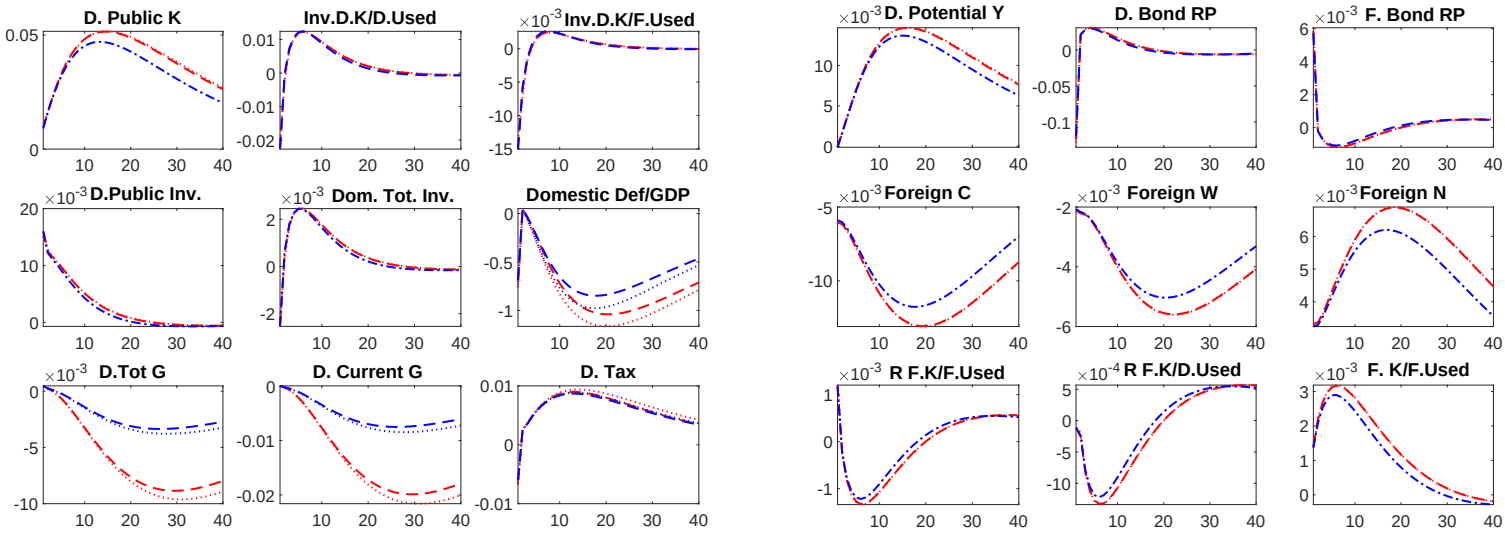

Fig.2.11: *System responses to a 1% innovation in foreign government spending. The red and blue lines represent the fiscal policy scenarios of ouput stabilization (OSFP) and debt stabilization (DSFP) respectively. The dotted line shows the benchmark scenarios (I-III) and the dash line the golden rule scenarios (II-IV).*

A general increase in the foreign deficit/GDP ratio and a reduction in the domestic deficit/GDP ratio have also been confirmed. These positive effects are an element of originality and result in this sense, once again emphasising the dynamics of international investment with respect to foreign capital and its growth in the domestic production function. The result is a general increase in domestic output and related investment, with the same differentiation as in scenarios II-IV and the different applications of the relevant fiscal policies. In line with the results analysed and with the literature (Ferrero (2009), Ratto et al. (2008) and Straub and Tchakarov (2007)) we note also at EA level the consequent crowding-out effect on private consumption and the same dynamics analysed above with respect to labour and wages due to the increase of the current component of public expenditure.

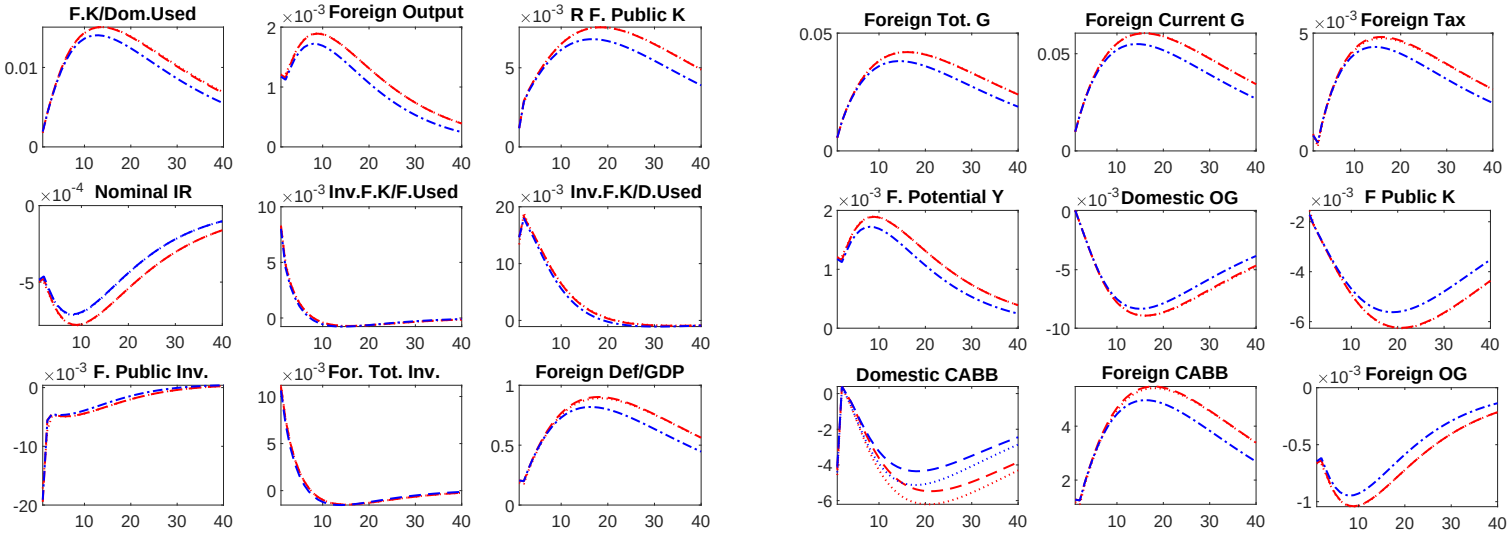

Fig.2.12: *System responses to a 1% innovation in foreign government spending. The red and blue lines represent the fiscal policy scenarios of ouput stabilization (OSFP) and debt stabilization (DSFP) respectively. The dotted line shows the benchmark scenarios (I-III) and the dash line the golden rule scenarios (II-IV).*

Finally, the growing elements of foreign GDP and its potential counterpart are analysed, confirming a general increase in current and capital expenditure. An interesting result to underline, however, are the dynamics related to public capital and public investments. Well, in this case, the same dynamics highlighted in the case of the domestic country in the scenarios of unbundling of public investment cannot be observed. In fact, the application of a golden rule does not seem to have the same dynamics of increase of the same investments with respect to the benchmark scenarios, although similar results can be found with respect to the dynamics of the same fiscal policies. In this sense this analysis seems to confirm the empirical evidence at the European level of Straub and Tchakarov (2007) and Zeynologlu (2018).

2.5 References

1. Albonico, A., Calés, L., Cardani, R., Croitorov, O., Ferroni, F., Giovannini, M., Vogel, L. "Comparing post-crisis dynamics across Euro Area countries with the Global Multi-country model" Economic Modelling, 2019

2. Basu P., Wada K., "Unconventional Monetary Policy and the Bond Market in Japan: A New-Keynesian Perspective", No. 18-E-12. Institute for Monetary and Economic Studies, Bank of Japan, 2018 3. Batini, N., Cantelmo, A., Melina, G., Villa, S., "How loose, how tight? A measure of monetary and fiscal stance for the euro area", Bank of Italy, Temi di Discussione (Working Paper) No, 1295, 2020

4. Calvo G., "Staggered prices in a utility-maximizing framework", Journal of Monetary Economics, 12, issue 3, p. 383-398, 1983

5. Cassou S., Lansing K., "Optimal fiscal policy, public capital, and the productivity slowdown," Journal of Economic Dynamics and Control, Elsevier, vol. 22(6), pages 911-935, June, 1998

6. Elmgren, P. G. V., "The effects of debt stabilising fiscal rules in a macroeconomic (DSGE) model", Hanken School of Economics, Helsinki, 2018

7. Ferrero, A., "Fiscal and monetary rules for a currency union", Journal of international Economics, 77(1), 1-10, 2009

8. Furceri, D, Mourougane, A., "The effects of fiscal policy on output: A DSGE analysis", OECD, 2010

9. Galí J., "Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework", Princeton University Press, 2008

10. Gali, J., Monacelli, T., "Optimal monetary and fiscal policy in a currency union", Journal of international economics, 76(1), 116-132., 2008

11. Guo J., Lansing K., "Tax structure and welfare in a model of optimal fiscal policy," Economic Review, Federal Reserve Bank of Cleveland, issue Q I, pages 11-23, 2007

12. Leeper E. M., Walker B., Yang S., "Government investment and fiscal stimulus," Journal of Monetary Economics, Elsevier, vol. 57(8), pages 1000-1012, November, 2010

13. Leeper E. M., Walker, T. B., Yang, S. C. S., "Fiscal foresight and information flows". Econometrica, 81(3), 1115-1145, 2013

14. Leeper E., Campbell L., “Understanding Inflation as a Joint Monetary-Fiscal Phenomenon”, ch. Chapter 30, p. 2305-2415, 2016

15. Muscatelli V., Tirelli P., “Analyzing the Interaction of Monetary and Fiscal Policy: Does Fiscal Policy Play a Valuable Role in Stabilisation?”, CESifo Economic Studies, CESifo, vol. 51(4), pages 549-585, 2005

16. Rudebusch, G. D., Swanson E. T., ”The bond premium in a DSGE model with long-run real and nominal risks”, American Economic Journal: Macroeconomics, 2012

17. Schmitt-Grohé, S., Uribe, M., “Optimal fiscal and monetary policy under sticky prices”. Journal of economic Theory, 114(2), 198-230, 2004

18. Smets F., Wouters R., “Shocks and frictions in US business cycles: A Bayesian DSGE approach”, American economic review, 97(3), 586-606, 2007

19. Tchakarov I., Straub R., , ”Assessing the Impact of a Change in the Composition of Public Spending: A DSGE Approach”, No 2007/168, IMF Working Papers, International Monetary Fund, 2007

20. Torres J. L., ”Introduction to Dynamic Macroeconomic General Equilibrium Models,” Vernon Press Titles in Economics, Vernon Art and Science Inc, edition 2, number 54, July, 2016

21. Vieira, P., Machado C., Ribeiro A., ”Optimal fiscal simple rules for small and large countries of a monetary union”, EcoMod2016 [9685], 2016

Basu P., Wada K., “Unconventional Monetary Policy and the Bond Market in Japan: A New-Keynesian Perspective” No. 18-E-12. Institute for Monetary and Economic Studies, Bank of Japan, 2018

Chapter 3

Estimation and Bayesian Dynamics in a Monetary-union Bayesian-DSGE

3.1 Introduction

Once we have introduced in the first chapter the evolutions and dynamics of DSGE modelling with respect to the analysis of economic policies in the context of the European Monetary Union since the introduction of the various fiscal rules in the regulatory framework and how they have been answered by modelling through their theoretical implementation as in the second chapter of this thesis, it is necessary to provide the necessary premises for the continuation of this analysis in its empirical version.

In fact, if the first RBC models, later developed into more complex DSGE and DSGE-NK models, as analysed above, allowed a more detailed analysis of macroeconomic dynamics, it is also true that an important part of policy analysis derives from the possibility of forecasting and empirically verifying these models in order to compare the results obtained with the data own by the researcher's. In fact, in the early 1960s, empirical analysis of macroeconomic dynamics began to follow what were the first implementations on structural statistical autoregressive models (SVAR). However, the possibility presented by DSGE models, both through their theoretical structure and through their parametric invariance to economic policy choices, soon allowed to overcome Lucas' Critique (Lucas (1976)), with respect to the possibility of conducting macroeconomic analysis on models that could be precisely micro-founded in their structure and not merely statistical, and in order to

be able to analyse the mechanisms of response and transmission of macroeconomic shocks on individual agents and with respect to their individual behaviour and provide a correct economic interpretation. (Justiniano et al. (2017))

The SVAR models, based on a set of totally endogenous variables and therefore based on a more empirical than theoretical consistency, have gradually been replaced by a more solid theoretical apparatus.

The communication between the two research strands has however led over time to a form of complementarity, allowing both methodologies to provide useful answers to both approaches (on the one hand the DSGE with a more theoretical structuring useful to limit or expand the empirical models, the other, on the part of the SVAR useful for a correct specification of parameters within the theoretical models themselves). In their first versions, obviously more stylised, therefore, a first form of communication between VAR and DSGE models was slowly sought in order to better guide economic policy choices based on the responses of these systems. (although as specified by Sims (1980) still too little informative with respect, on the one hand, to the complexity of the economy and, on the other hand, to the structural interpretation of the same IRFs). Examples include the models of Rotemberg and Woodford (1997), Gali (1999) and Christiano et al. (2005).

Over time, however, the various restrictions placed on the possible communication between the two approaches, such as those related to the number of shocks in the empirical models with respect to the theoretical models (in SVAR models the number of shocks must be equal to the number of endogenous variables) as well as those related to the possible long-run forecasts and the necessary re-evaluation of the forecasting capacity of the empirical models with respect to its own lagged variables, prompted economists to move towards new methodologies in order to improve communication between the two approaches on the one hand and to maintain theoretical coherence on the other, especially for stochastic dynamic models.

As a result of these needs, the first models therefore sought to implement new empirical connection techniques for the theoretical models themselves. Examples were the models of Del Negro and Schorfheide (2004) by augmenting their data set with data generated by the theoretical model itself through the introduction of Bayesian priors (the priors, as we shall see below, drive the empirical parameters towards

the parameters implied by the DSGE model, with weights that are determined by the proportion of simulated data compared to real data used in the estimation Assenmacher (2017)) and then arrive at the pioneering study of Smets and Wouters (2007) with an implementation of Bayesian estimation for the US economy and the EA territory. The development of Bayesian techniques for DSGE estimation has thus been an essential ingredient in transforming these models into credible tools for data description and forecasting thus allowing a stronger validation of the volatility analysis of macroeconomic variables to different exogenous shocks (Justiniano et al. (2017)). This is why, in the following research, the theoretical model presented in the second chapter, with its innovative implementations with respect to fiscal and financial scenarios in the European Monetary Union, will be validated through the implementation of a Bayesian estimation with respect to European empirical data in order to analyse its dynamics also in the light of the present empirical evidence.

3.1.1 The Bayesian Estimation Methodology

With regard to the methodology used in the following chapter, we will make explicit reference to the empirical analyses carried out using Bayesian-DSGE models. The theoretical structure of the model allows us to obtain important answers regarding the volatility of the main macroeconomic variables with respect to the shocks to which the economy is subjected. Nevertheless, an empirical verification of the model and of the responses obtained from the calibration of the structural parameters is necessary to validate its effectiveness. In these terms, in the economic literature on macro-structural models, starting with the techniques introduced by the work of Smets and Wouters (2007), the technique of Bayesian estimation of model parameters in the light of the data input into the model has been consolidated over time.
Although the theoretical structure of the behavioural equations remains the same, the estimation system allows, as we shall see, to introduce and analyse, through specific econometric methods, forms of convergence, both as regards the individual deep parameters of the model, and as regards the model as a whole.
The choice of Bayesian estimation for stochastic dynamics models has many advantages over other estimation methods in the economic literature. In fact, it is based on the plausibility of the model itself by considering the ”priors” distributions of our

structural parameters (i.e. calibrated in the theoretical model) as weights to be used for the estimation of their a posteriori distributions. In this way the Bayesian estimation process avoids reaching extreme points for the a posteriori estimates where the likelihood would reach anomalous high levels. This is in fact a problem linked to the stylised nature and miss-specification of theoretical models and in this sense there is a risk of having a likelihood tending towards maximum zones in parameter space, giving rise to problems such as the so-called "Dilemma of absurd parameter estimation" (Pfifer (2013))[1]. In this way, using priors as weights in the estimation process, we have the possibility of a better identification of the parameters, which is facilitated by the inclusion of exogenous shocks in the estimation process, which act as observation errors of the equations of the theoretical model. (Bizzotto and Castelnuovo (2011)).

The advantages of Bayesian estimation are undoubtedly those relating to the possibility of comparing the goodness of fit of the theoretical model with the data included within it, thus allowing comparisons with alternative models on the basis of the maximum likelihood values obtained.

The operation of Bayesian estimation is therefore based on Bayes' estimation rule, from which a prior distribution is first defined with a probability density function such as: $p(\theta_A|\mathbf{A})$ where $\mathbf{A}$ is the theoretical model considered and θ_A denotes the parameters of the theoretical model $\mathbf{A}$ and the function $p(x)$ describes the probability density function (PDF).

From here we decrypt the likelihood function, which thus indicates the probability density of the observed data as a function of the model itself and its structural parameters (Canova (2012), De Luca and Castelnuovo (2011)), such that:

$$L(\theta_A|Y_t, \mathbf{A}) \equiv p(Y_t|\theta_A, \mathbf{A})$$

with $p(Y_t|\theta_A, \mathbf{A}) = p(y_0|\theta_A, \mathbf{A})\Pi_{t=1}^{T} p(y_t|Y_{t-1}, \theta_A, \mathbf{A})$ and substituting in the likelihood function we get $L(\theta_A|Y_t, \mathbf{A}) = p(y_0|\theta_A, \mathbf{A})\Pi_{t=1}^{T} p(y_t|Y_{t-1}, \theta_A, \mathbf{A})$ which is our prior probability function combined with the likelihood function with respect to the observed data. Knowing also that according to Bayes' theorem it is possible

[1] ML estimates could be indeed often in contrast with information provided from outside of the model. For this reason using of Bayesian techniques (as implemented in Bayesian-DSGE models with their numerical alghorithms) that augment the likelihood with prior informations makes the posterior distributions more well-behaved (Pfifer (2013))

to obtain a parameter density once the data have been supplied to the theoretical model according to which: $p(\theta|Y_t) = \frac{p(\theta;Y_t)}{p(Y_t)}$ and that $p(Y_t|\theta) = \frac{p(\theta;Y_t)}{p(\theta)}$ and therefore: $p(\theta;Y_t) = p(\theta|Y_t)p(\theta)$ it is possible finally to obtain our posterior function by combining the density functions of the priors with the likelihood function L, obtaining the canonical form of Bayesian estimation: $p(\theta_A|Y_t,\mathbf{A}) = \frac{p(Y_t|\theta_A,A)p(\theta_A|\mathbf{A}\ )}{p(Y_t|\mathbf{A})}$ where the normalization constant $p(Y_t|\mathbf{A}) = \int_{\omega_A} p(\theta_A;Y_t|\mathbf{A})d\theta_A$ is the marginal density of the vector of stochastic variables Y indipendent from θ. Finally substituting we obtain:

$$p(\theta_A|Y_t,\mathbf{A}) = \frac{p(Y_t|\theta_A,\mathbf{A})p(\theta_A|\mathbf{A})}{\int_{\omega_A} p(\theta_A;Y_t|\mathbf{A})d\theta_A}$$

Where: $p(\theta_A|Y_t,\mathbf{A})$ is the a posteriori estimate of the model conditional on the given data, $p(\theta_A)$ is the priori distribution of the model, θ is the vector of structural parameters, Y is the stochastic variable with its own distribution and finally where $p(Y_t|\theta_A,\mathbf{A})p(\theta_A|\mathbf{A})$ as numerator of the a posteriori density is the posterior Kernel function useful for estimating all the moments of the posterior distributions themselves. (i.e., the non-normalised posterior density (given a constant or equal marginal density for each parameter)) such that: $p(Y_t|\theta_A,\mathbf{A})p(\theta_A|\mathbf{A}) \equiv K(\theta_A|Y_t,\mathbf{A})$.
In this way, the Bayesian estimation method allows the information provided by the prior calibrations to be linked to the input data in the system getting an update of the information, which therefore allows the statistical confidence level of the newly obtained estimates to be continuously updated in order to construct the posterior distributions of the model.

3.2 Bayesian Estimation Procedure: Metropolis-Hastings algorithm, data and estimated model

Specifically, once the theoretical model is structured, and thus with the distinction between exogenous variables, endogenous and structural parameters, the solution of the system is estimated. The procedure starts from the knowledge of the prior distributions of the parameters and is carried out by finding, as seen, first the mode of the posteriors through the maximisation of the logarithmic posterior of Kernel. Specifically, the Bayesian estimation function is non-Gaussian with respect to θ but to a function of θ as in Bayes' relation seen before. Then, the posterior distribution of the structural parameters is found. To do so, the simulation considers

the Metropolis-Hastings sampling method (following a *Monte-Carlo-Markov Chain (MCMC)* algorithm) preferable in terms of reliability, efficiency and consistency with respect to alternative methods of indirect inference and maximum likelihood as specified in Canova and Sala (2009), Canova (2012), An and Schorfheide (2007) and Fernandez-Villaverde (2009) in Bizzotto and Castelnuovo (2011).

The M-H sampling algorithm allows to simulate posterior distribution through the so-called "sampling rejection" whereby a sequence of samples in the form of Markov chains is first generated from an unknown distribution and then, under the assumption of an asymptotically normal a posteriori parameter distribution, a Gaussian approximation is constructed around the posterior mode using an asymptotic covariance matrix scale of the prior proposed distribution.[2]

The MH algorithm allows in this sense to estimate the posterior distributions efficiently nearby the mode (An and Schorfheide (2007)).

The algorithm then follows the following procedure: It chooses a stochastic initial point of estimate θ^0 as the posterior mode as the most probable point of the distribution itself via the *"M-H Random Walk"* procedure by following a very large number of simulations and subsequently eliminating a first half of them in order to allow convergence to the ergodic distribution of the posterior through a detachment and independence of the algorithm itself from its starting point.

It computes a "jump" distribution in the form: $\theta' = \theta' + \epsilon$ with $\epsilon = N(0, c\Sigma)$ where Σ is the inverse of the Hessian computed from the posterior mode and where c denotes a constant inversely proportional to the magnitude of the acceptance rate r. It then calculates an acceptance rate, of the chosen point, r given by $r = \frac{p(\theta'|Y_t)}{p(\theta^{t-1}|Y_t)}$. It accepts or rejects the proposal θ' if $r > K$, where K is the realisation point of a uniform distribution.

This acceptance rule allows to visit the entire domain of the posterior distribution without running the danger of considering the posterior distribution as exact for a given parameter only by finding a local maximum. At the same time, the algorithm makes it possible to take small steps backwards in order to find a global maximum for the distribution itself. The algorithm does this by relying on the variance of its

[2] A Markov Chain is a particular stochastic process where known the present state of the process it presents the "Markov Property" being dependent only by its immediately past state. Specifically, the process looks like: $P(\theta^{n+1} \in A|\theta^n = x, \theta^{n-1} \in A_{n-1}, ..., \theta^0 A_0 = P(\theta^{n+1} \in A|\theta^n = x) \quad \forall A_0, ..., A_{n-1}, A \in S)$ where S is the set of states of the process.

Leap Distribution itself and in particular on the factor scale. When this is too small the acceptance rate will be too high. In this way the Markov Chain generated on the candidate parameters will follow a "slow shuffling", i.e. taking the distribution over a longer period to converge to the posterior distribution until the chain stops at the local maximum found. (Canova (2012), Bizzotto and Castelnuovo (2011)) Conversely, when the scaling factor is too high, the acceptance rate r will become too low and the Markov Chain will spend much more time in the tails of the posterior distribution.

3.2.1 Data and estimated model

The empirical estimation of the previously presented model follows a state-space representation of the same model. Specifically, we will have the vector of the model's structural parameters (ξ), endogenous variables (Z_t), exogenous shocks (ε), innovations (η_t) and observed variables (Y_t), provided through the processed data. Specifically, the state-space representation will be as follows:

$$\begin{bmatrix} Z_t \\ \varepsilon_t \end{bmatrix} = \mathbf{A}(\xi) \begin{bmatrix} Z_{t-1} \\ \varepsilon_{t-1} \end{bmatrix} + \mathbf{B}(\xi)\eta_t, \qquad Y_t = \mathbf{C}(\xi) \begin{bmatrix} Z_t \\ \varepsilon_t \end{bmatrix}$$

It consists of two different sets of systems. The first equation indicates the structural or transition equation of the model and expresses the dynamics of the theoretical state of the model that binds the endogenous variables to the exogenous ones through a first-order Autoregressive process ($AR_{(1)}$). The matrix $\mathbf{A}$ present in the equality is for this reason called Transition Matrix and the matrix $\mathbf{B}$ transformation matrix. It transforms the state of the observations. In the second group we find instead the so-called Measurement or Observation Equation (with the observed variables of the system) which allows to link the structural and transformation equations (i.e. the unobservable state of the system) to the provided data. It also performs the task of error selection with the matrix $\mathbf{C}$ containing deterministic elements. Once the state-space representation of the system has been provided, the estimation algorithm moves on to the previously introduced Kalman filter which allows the calculation of the verisimilitude function as seen above. Specifically, given the likelihood of the model $L([Y]_{t=1}^{T}|\xi)$ we proceed through a recursive procedure conditional on the

observed variables provided to the system up to the maximum available time. In fact, the filter, through two sets of equations (the so-called Prevision and Update equations) calculates the moments of the state vector of the transformation equation (the vector of innovations) conditioned to the vector of the observed variables of the system. In fact, given the distribution of the vector of innovations conditional on the observed variables in $AR_{(1)}$ process, the recursive procedure will thus allow the estimation procedure to be updated with each new observation provided. The likelihood L is thus multiplied by the prior distributions provided for the chosen structural parameters of the model in order to obtain the final posterior one as specified above through the application of Bayes' Rule. In order to allow the model parameters to be estimated and the estimates to converge, the observed variables are then provided. The selection made in the following work takes into consideration the recorded levels of log-GDP per head at current Italian and European prices in the period 1969-2021 and that of the level of log-Consumption per head at current prices also in the period 1969-2021. The number of years and data considered must necessarily be high enough in this sense to allow for an update of the data with each new simulation, as seen above.(Canova (2012)) Specifically, however, the data obtained in this way must undergo further processing, and in this sense require a little methodological study. In fact, the representation of the system in its log-linearised version (at the first order of approximation) requires that the data be processed in such a way as to permit uniformity with the theoretical model itself. (Pfifer (2013)). In the specific, as specified by the reference literature, for log-linear system at the first order the historical series so obtained must be treated through the application of the Hodrik-Prescott Filter and in the specific of the One-Side-HP filter that allows to obtain the "demeaned" trend with an average equal to zero with respect to the same steady state (as well as the variables expressed in log-deviation in the model approximated at the first order). The application of the filter, therefore, follows a computational procedure according to which once the time series are obtained in level, they are placed in logarithmic version and then see the application of the One-Side-H-P filter on the same data in log-levels per-head. It is particularly effective in eliminating the cyclical components and short-run fluctuations of the data obtained (detrending) by focusing more on long-run fluctuations. This is also one of the rea-

sons for the choice of sufficiently long time series to allow a correct application of this statistical filter.

3.3 Results of the estimation process

3.3.1 Prior-Posterior Distributions, Univariate and Multivariate Convergence Analysis

In merit the procedure of bayesian esteem so like exposed in the previous paragraph, it concurs to obtain several outputs. In such sense all the procedure of simulation comes carried out on the software DYNARE (Adjemian, Bastani, Juillard, Mihoubi, Perendia, Ratto and Villemot (2011)) on the software MATLAB. In such sense the computation concurs to obtain various results between which, like analyzed in the continuation, the irfs relative the Bayesian dynamics. The first result to be analyzed, in order to highlight problems in the estimation procedure and at the same time verify the correctness of the estimation procedure carried out is based on the analysis of convergence of the single estimated parameters of the model as well as the model as a whole. The first step to be taken is that of the assignment of the values and of the probability density (PDF) a priori assigned to the parameters of the system. The assignment of probability distributions follows the domain of the corresponding parameter. In the case of the estimated parameters of the model, their relative domain and the choice of the assigned priors, they are mainly based on observations and, in line with the main reference literature on model estimation of European and Italian economies, their choice mainly follows the reference literature and specifically the studies carried out in Smets and Wouters (2003, 2007), Straub and Tchakarov (2007) and Ferrero (2009). Once the densities of the individual parameters have been entered, the Metropolis-Hastings algorithm (as analyzed above) is initialized in order to simulate the a posteriori distributions. As seen, this algorithm is based on the MCMC (Monte Carlo Markov Chain) methodology (see Canova (2007)). In order to ensure a certain robustness of the results obtained and the goodness of the estimate it is therefore necessary to provide a number sufficiently high compared to the size of the Markov chains generated by the M-H algorithm. In this sense a dimension of 100'000 iterations is chosen. In this procedure is then maximized, in a first step the a posteriori density function as a combination of the a

priori information of the structural parameters with the likelihood of the same data and then obtain from the Metropolis-Hastings algorithm the likelihood of a posteriori distributions for each parameter (univariate) and for the model as a whole (multivariate). Of the 100'000 iterations it is necessary to see, after about the first half, then approximately after the first 50'000 iterations, if the priors converge then to the ergodic distribution of the posteriors. The posteriors themselves are then composed, as we will be able to analyze graphically in the following with the intervals present in the Bayesian irfs, by the same posterior mean correlated with the intervals of credibility 'Highest Posterior Density' (HPD) at 90%. We then evaluate the correctness of the estimates through the outputs provided by the procedure. The graphs reported in the following figures are therefore those relative to the convergences called "MCMC univariate Diagnostic" in order to evaluate the convergence between prior and posterior of the single parameters as well as the "Multivariate Diagnostic" useful to evaluate the convergence of the model as a whole according to the three measures provided by the diagnostics of Brooks and Gelman (1998) and specifically 'interval' according to the interval generated around the mean with HPD, as specified, at 90%, 'm2' related to the variance and 'm3' related to the third moments. To this end in the univariate analysis we analyze the variance measures internally and between the generated Markov chains. Two different lines can be analyzed, one blue and one red. The blue lines test the variance between the chains generated by the Metropolis-Hastings algorithm while the red lines test the variance within those chains. A robust and meaningful result is obtained when they are both relatively constant beyond their first half of acceptance (50'000 iterations in our case given the 100'000 set) and in convergence to the ergodic ditribution. On the axis of the abscissas is therefore reported the number of iterations of the algorithm M-H as well as on the ordinates the moments of the same parameter whose first value corresponds to the value of beginning iteration of the algorithm M-H. To evaluate the correctness we analyze the stabilization effects starting from the second half of the chains and their convergence (the blue and red lines must converge) as proposed by the diagnostics of Brooks and Gelman (1998). In the following graphs it is therefore possible to analyze how our variances converge for all the parameters considered maintaining a certain constancy just starting from the second half of the

iterations considered (100'000). In the case of non-convergence we would arrive at a verisimilitude not sufficiently informative by increasing the number of iterations of the M-H algorithm or by choosing different prior distributions (more informative than those considered). The convergence thus obtained on the diagnostics of Brooks and Gelman (1998) allows us therefore to affirm in our simulated scenarios that both the number of iterations chosen is sufficiently high and that our prior distributions are sufficiently informative allowing however the data to provide the right additional information on the model. As it regards the graphs brought back (Fig.3.1-Fig.3.8) relative the multivariate diagnostics it is possible also in this case to notice a stabilization of the curves after the first 50'000 iterations considered for all the scenarios of simulation considered allowing us therefore to affirm like also in the case of the model in its complexity is the number of iterations chosen is sufficiently elevated is like our prior distributions are behaved in sufficiently informative way allowing however to the data to supply the just additional information on the model. Finally, the last graphs obtained allow us to note the dynamics of the smoothed structural shocks of the system obtained through the previously described application of the Kalman filter. They allow us to obtain a reconstruction of the values of the unobserved shocks on the sample, using all the information contained in the observation sample. It is computed via the 'Kalman Smoother'. In these terms the Bayesian estimation procedure starts from the assumption that the structural shocks have zero mean. In these terms the estimation procedure of structural shocks allows us to state a correct estimate if they do not systematically deviate from the zero value. In such cases it would come in fact to introduce a problem in the estimated model regarding the calibrated values, regarding some constant missing or regarding a lacked correspondence between the meaning of the variable in your model and in our data. We can therefore affirm also in this case one their correspondence in the around of the zero for all the scenarios considered in our simulation. Finally, the last table describes the values of the structural parameters estimated mainly by following the methodologies used in the reference literature, both behavioral and relative to capital depreciation rates (in our case, both private and public), to the elasticities of substitution of the productive factors in the production function, given the structure of the model between labor, domestic capital, foreign capital and public capital,

to the autoregressive processes ($AR_{(1)}$) and finally to the deviations of the relative structural shocks. We then present the obtained values of the posterior estimated with the Bayesian estimation procedure with respect to the respective priors always for each simulation and estimation scenario considered in our iterations.

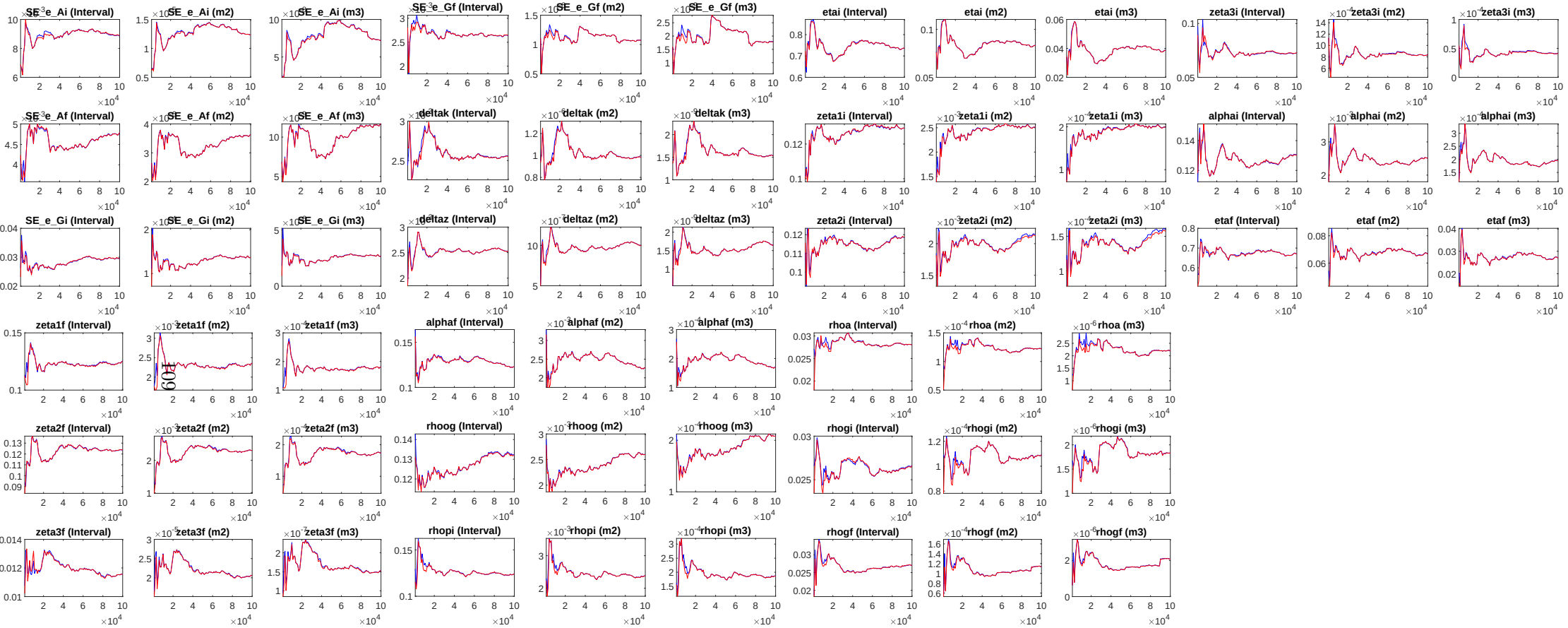

Fig.3.1: *Univariate Convergence Analysis generated by the Metropolis-Hastings algorithm*

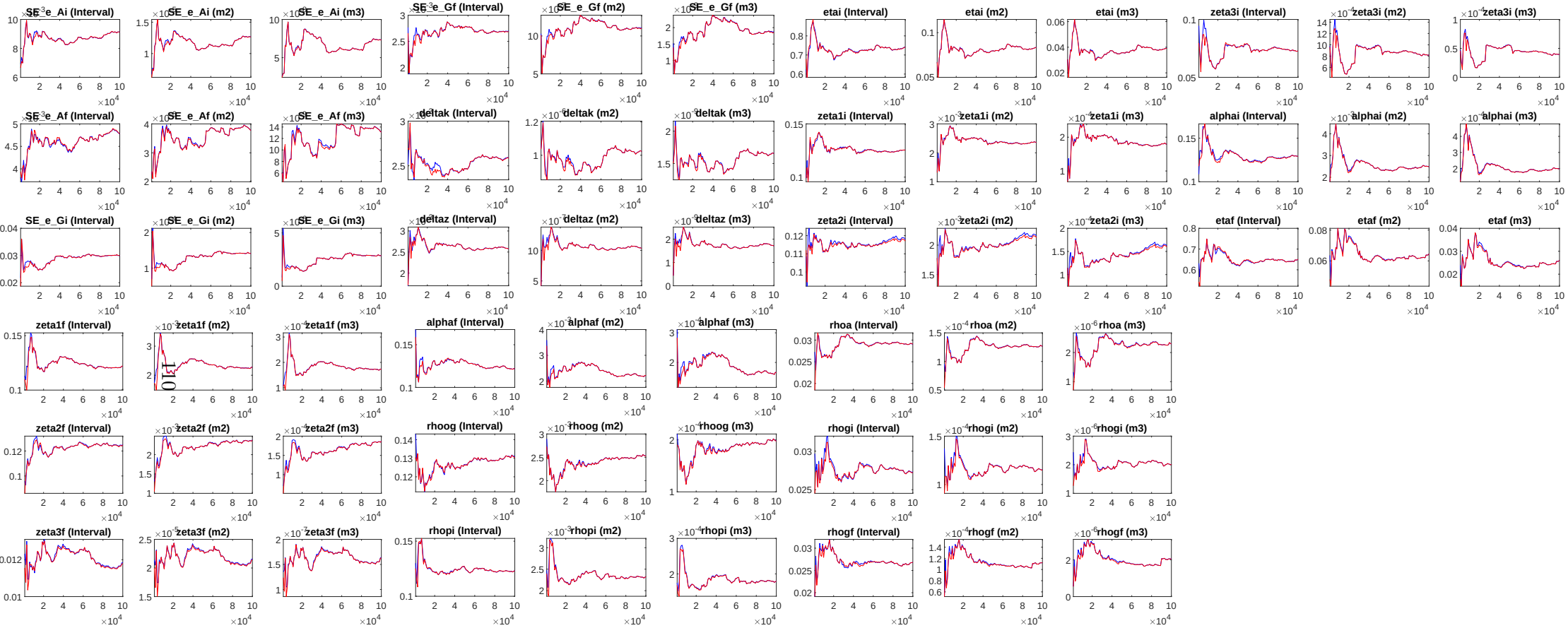

Fig.3.2: *Univariate Convergence Analysis generated by the Metropolis-Hastings algorithm*

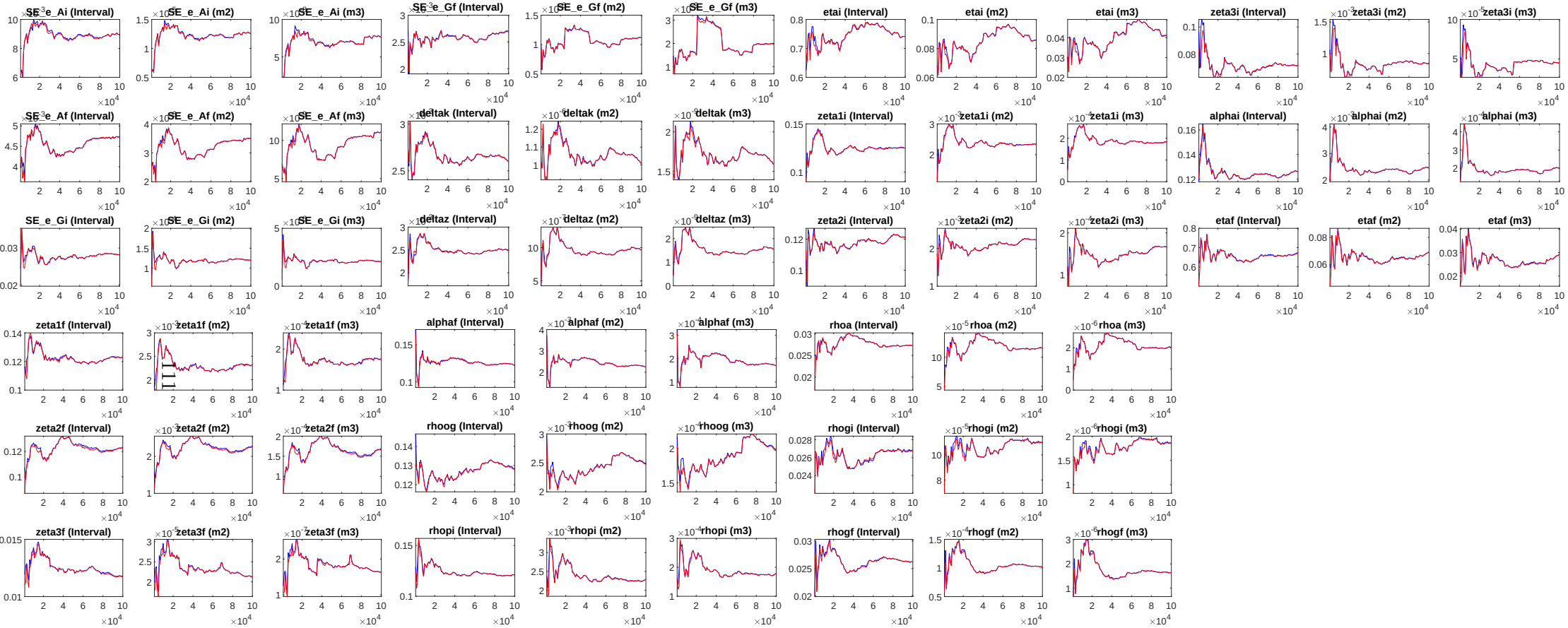

Fig.3.3: *Univariate Convergence Analysis generated by the Metropolis-Hastings algorithm*

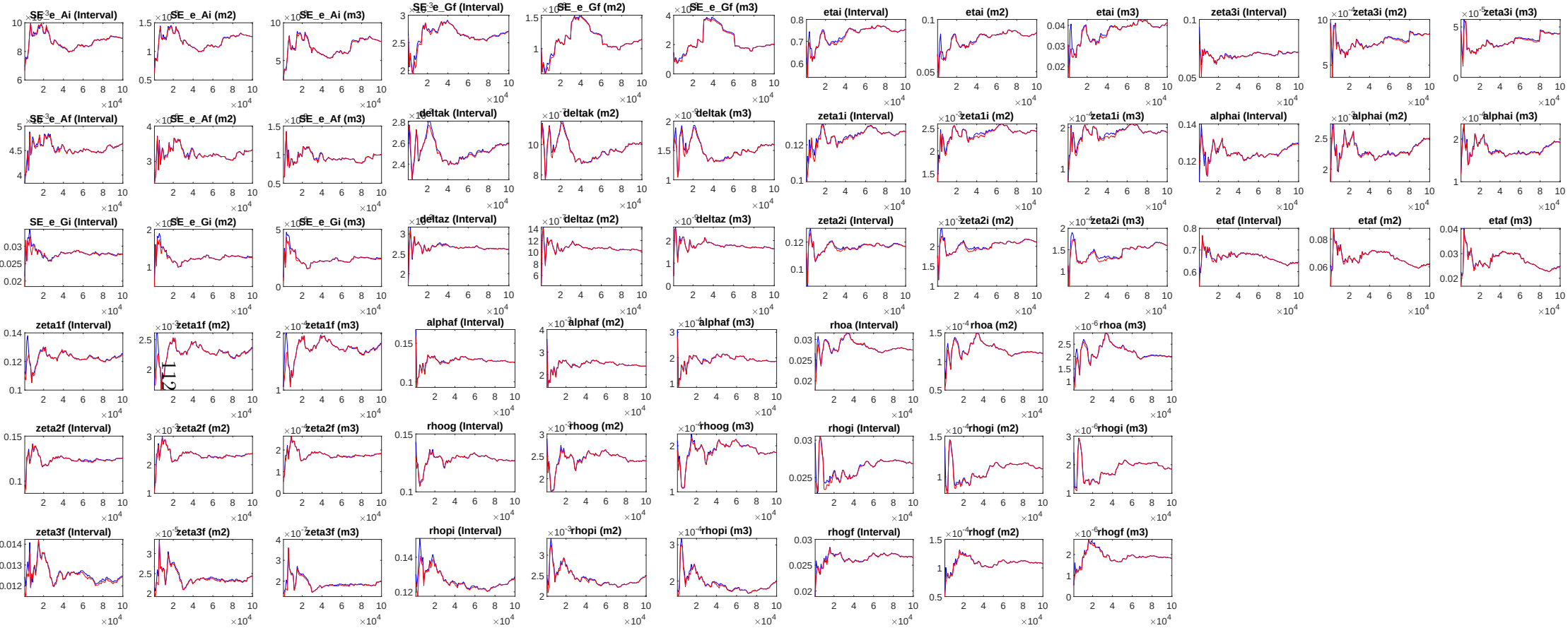

Fig.3.4: *Univariate Convergence Analysis generated by the Metropolis-Hastings algorithm*

Multivariate Convergence: OSFP

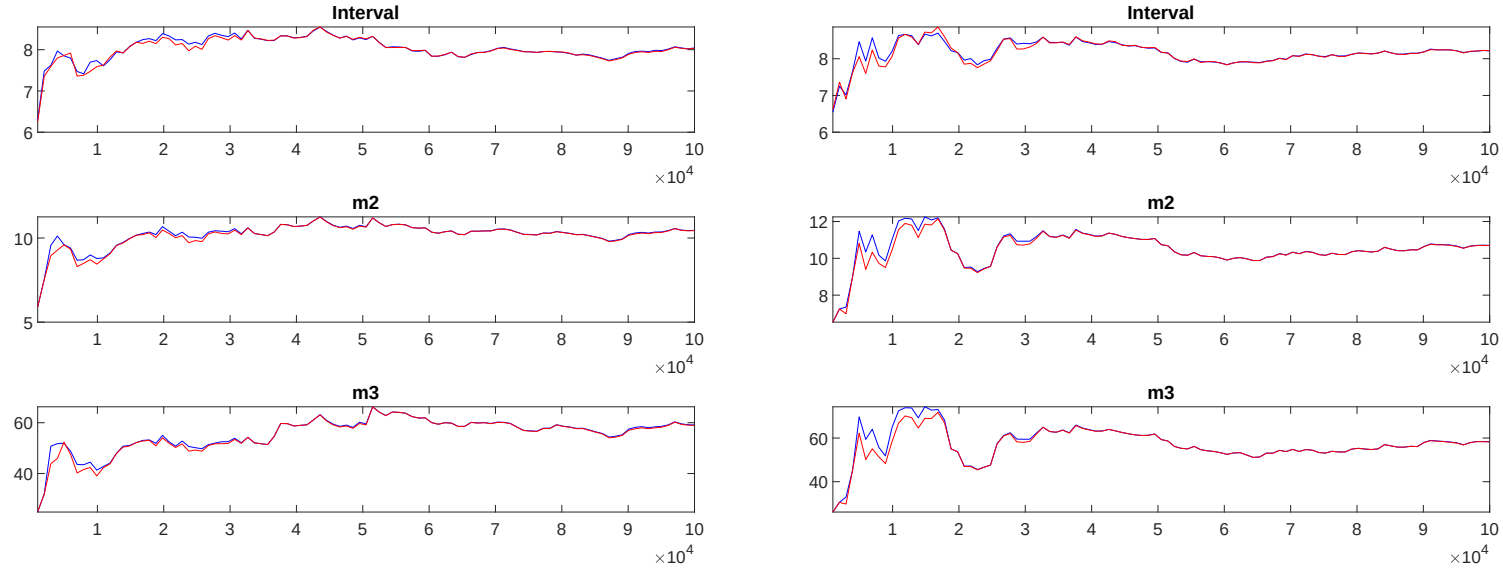

Fig.3.5: *Multivariate Convergence Analysis generated by the Metropolis-Hastings algorithm*

Multivariate Convergence: DSFP

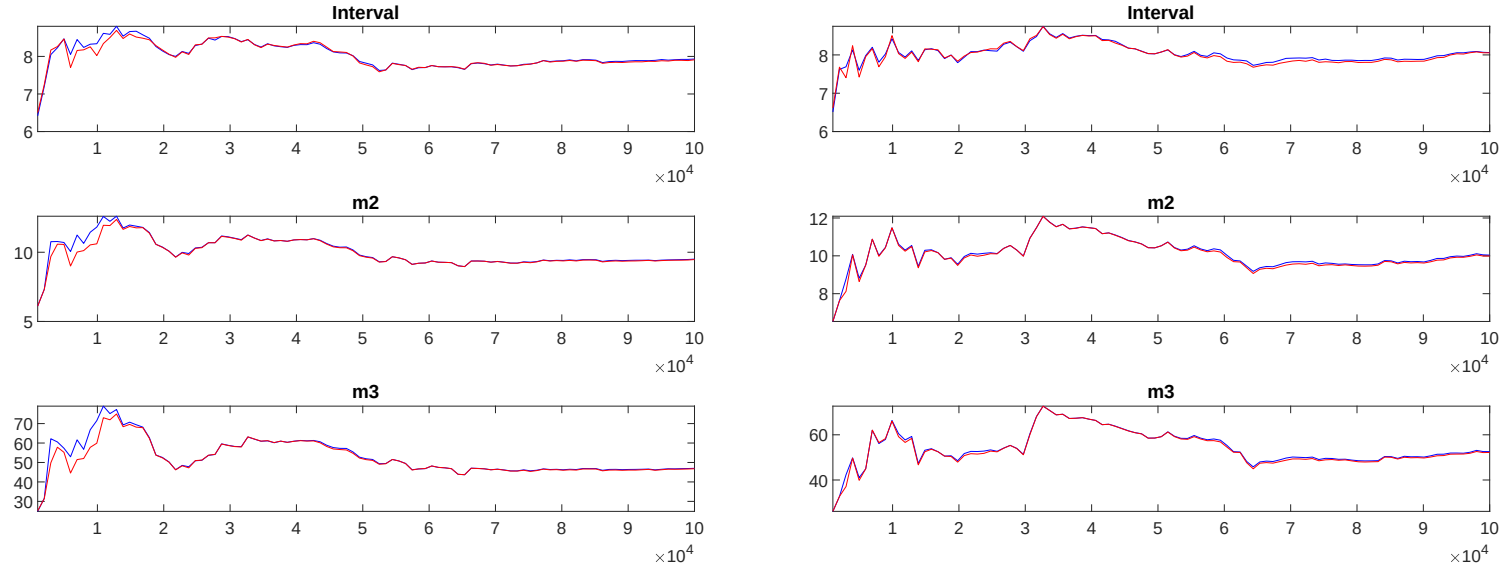

Fig.3.6: *Multivariate Convergence Analysis generated by the Metropolis-Hastings algorithm*

Smoothed Shocks: OSFP

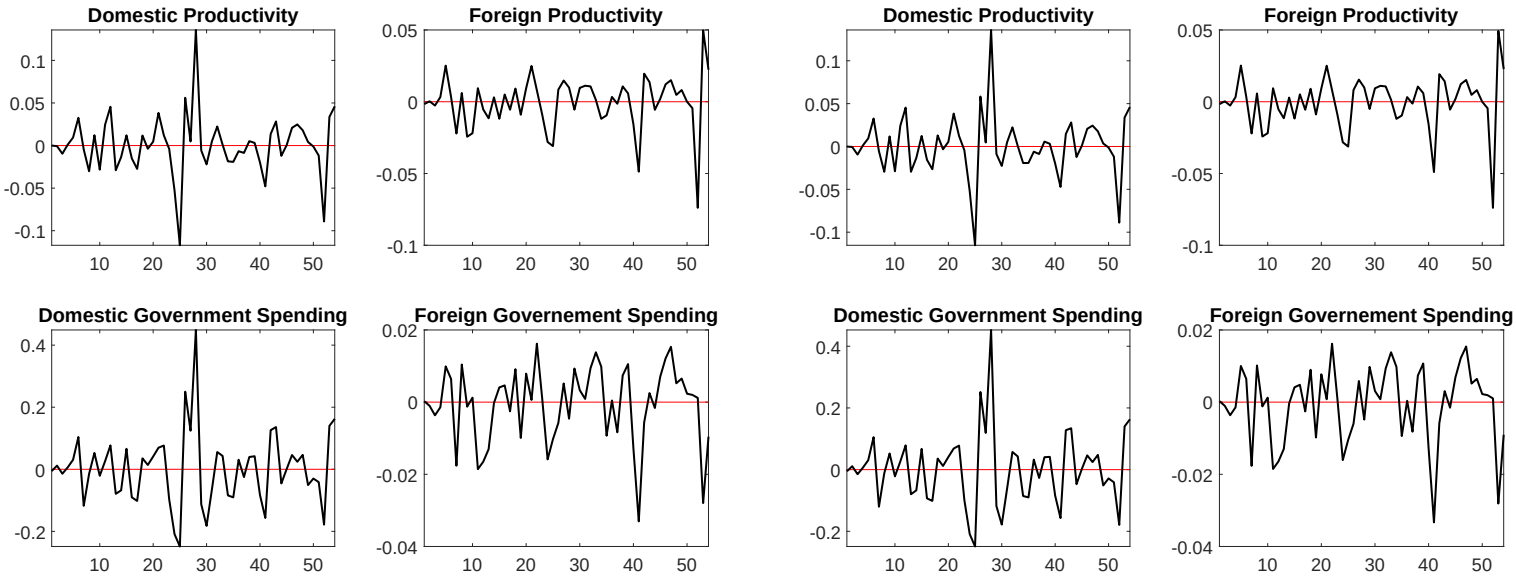

Fig.3.7: *Smoothed Shocks generated by the Metropolis-Hastings algorithm and Kalman smoother*

Smoothed Shocks: DSFP

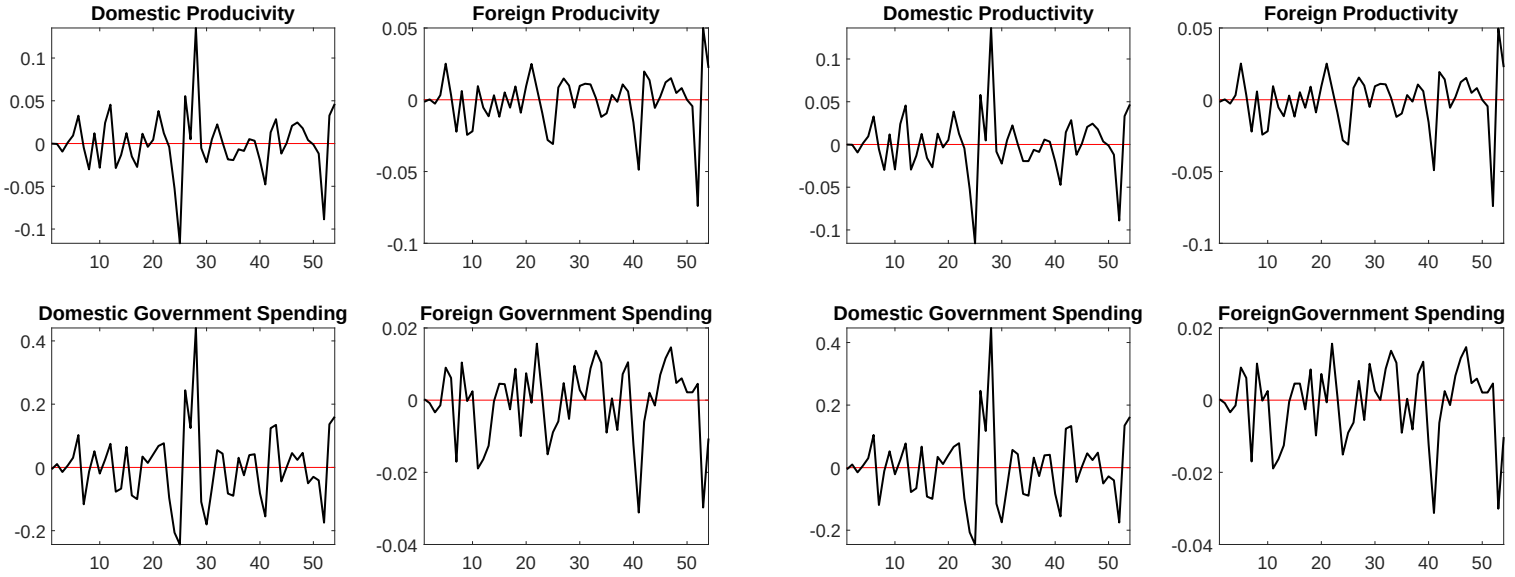

Fig.3.8: *Smoothed Shocks generated by the Metropolis-Hastings algorithm and Kalman smoother*

Par.	PDF	Prior	Post(I)	Interval 90%(I)	Post(II)	Interval 90%(II)	Post(III)	Interval 90%(III)	Post(IV)	Interval 90%(IV)
OSFP										
δ_k	β	0.1000/0.0500	0.1336	[0.0858-0.1823]	0.1338	[0.0837-0.1821]	0.1336	[0.0858-0.1823]	0.1338	[0.0837-0.1821]
δ_z	β	0.1000/0.0500	0.0965	[0.0225-0.1686]	0.0977	[0.0211-0.1688]	0.0965	[0.0225-0.1686]	0.0977	[0.0211-0.1688]
η_i	N	2.0000/0.3000	2.2590	[1.8166-2.7441]	2.2797	[1.8193-2.7494]	2.2590	[1.8166-2.7441]	2.2797	[1.8193-2.7494]
ζ_1^i	β	0.3850/0.0500	0.4324	[0.3507-0.5127]	0.4355	[0.3526-0.5160]	0.4324	[0.3507-0.5127]	0.4355	[0.3526-0.5160]
ζ_2^i	β	0.2230/0.0500	0.2217	[0.1444-0.2959]	0.2199	[0.1418-0.2937]	0.2217	[0.1444-0.2959]	0.2199	[0.1418-0.2937]
ζ_3^i	β	0.0900/0.0500	0.0518	[0.0078-0.0939]	0.0496	[0.0079-0.0901]	0.0518	[0.0078-0.0939]	0.0496	[0.0079-0.0901]
α_i	β	0.7500/0.0500	0.7505	[0.6711-0.8340]	0.7484	[0.6710-0.8310]	0.7505	[0.6711-0.8340]	0.7484	[0.6710-0.8310]
ρ_i^g	β	0.9500/0.0500	0.9457	[0.9280-0.9639]	0.9456	[0.9280-0.9625]	0.9457	[0.9280-0.9639]	0.9456	[0.9280-0.9625]
η_f	N	2.0000/0.3750	2.9399	[2.4880-3.3406]	2.9332	[2.4867-3.3651]	2.9399	[2.4880-3.3406]	2.9332	[2.4867-3.3651]
ζ_1^f	β	0.3620/0.0500	0.3602	[0.2764-0.4361]	0.3583	[0.2751-0.4392]	0.3602	[0.2764-0.4361]	0.3583	[0.2751-0.4392]
ζ_2^f	β	0.3500/0.0500	0.3797	[0.2958-0.4645]	0.3792	[0.2995-0.4605]	0.3797	[0.2958-0.4645]	0.3792	[0.2995-0.4605]
ζ_3^f	β	0.0200/0.0050	0.0206	[0.0129-0.0285]	0.0205	[0.0120-0.0283]	0.0206	[0.0129-0.0285]	0.0205	[0.0120-0.0283]
α_f	β	0.7500/0.0500	0.7474	[0.6688-0.8260]	0.7498	[0.6701-0.8274]	0.7474	[0.6688-0.8260]	0.7498	[0.6701-0.8274]
ρ_f^g	β	0.9500/0.0500	0.9483	[0.9313-0.9648]	0.9476	[0.9307-0.9643]	0.9483	[0.9313-0.9648]	0.9476	[0.9307-0.9643]
ρ_a	β	0.9500/0.0100	0.9464	[0.9301-0.9638]	0.9468	[0.9302-0.9634]	0.9464	[0.9301-0.9638]	0.9468	[0.9302-0.9634]
$\epsilon_{i,t}^A$	Inv.γ	0.0100/2.0000	0.0348	[0.0290-0.0402]	0.0350	[0.0292-0.0406]	0.0348	[0.0290-0.0402]	0.0350	[0.0292-0.0406]
$\epsilon_{i,t}^G$	Inv.γ	0.0100/2.0000	0.1117	[0.0938-0.1298]	0.1123	[0.0936-0.1292]	0.1117	[0.0938-0.1298]	0.1123	[0.0936-0.1292]
$\epsilon_{f,t}^A$	Inv.γ	0.0100/2.0000	0.0189	[0.0159-0.0219]	0.0189	[0.0159-0.0218]	0.0189	[0.0159-0.0219]	0.0189	[0.0159-0.0218]
$\epsilon_{f,t}^G$	Inv.γ	0.0100/2.0000	0.0082	[0.0066-0.0097]	0.0083	[0.0066-0.0099]	0.0082	[0.0066-0.0097]	0.0083	[0.0066-0.0099]
DSFP										
δ_k	β	0.1000/0.0500	0.1110	[0.0641-0.1563]	0.1123	[0.0653-0.1592]	0.1110	[0.0641-0.1563]	0.1123	[0.0653-0.1592]
δ_z	β	0.1000/0.0500	0.0968	[0.0233-0.1690]	0.0953	[0.0213-0.1651]	0.0968	[0.0233-0.1690]	0.0953	[0.0213-0.1651]
η_i	N	2.0000/0.3000	2.2758	[1.7917-2.7696]	2.2779	[1.8042-2.7469]	2.2758	[1.7917-2.7696]	2.2779	[1.8042-2.7469]
ζ_1^i	β	0.3850/0.0500	0.4332	[0.3495-0.5169]	0.4308	[0.3520-0.5128]	0.4332	[0.3495-0.5169]	0.4308	[0.3520-0.5128]
ζ_2^i	β	0.2230/0.0500	0.2248	[0.1473-0.2995]	0.2155	[0.1418-0.2886]	0.2248	[0.1473-0.2995]	0.2155	[0.1418-0.2886]
ζ_3^i	β	0.0900/0.0500	0.0523	[0.0080-0.0959]	0.0508	[0.0082-0.0917]	0.0523	[0.0080-0.0959]	0.0508	[0.0082-0.0917]
α_i	β	0.7500/0.0500	0.7501	[0.6705-0.8375]	0.7507	[0.6666-0.8299]	0.7501	[0.6705-0.8375]	0.7507	[0.6666-0.8299]
ρ_i^g	β	0.9500/0.0500	0.9460	[0.9290-0.9634]	0.9459	[0.9294-0.9632]	0.9460	[0.9290-0.9634]	0.9459	[0.9294-0.9632]
η_f	N	2.0000/0.3750	2.9570	[2.5322-3.4016]	2.9528	[2.5171-3.3941]	2.9570	[2.5322-3.4016]	2.9528	[2.5171-3.3941]
ζ_1^f	β	0.3620/0.0500	0.3578	[0.2772-0.4389]	0.3565	[0.2791-0.4340]	0.3578	[0.2772-0.4389]	0.3565	[0.2791-0.4340]
ζ_2^f	β	0.3500/0.0500	0.3824	[0.3023-0.4658]	0.3859	[0.3030-0.4696]	0.3824	[0.3023-0.4658]	0.3859	[0.3030-0.4696]
ζ_3^f	β	0.0200/0.0050	0.0203	[0.0122-0.0281]	0.0208	[0.0127-0.0288]	0.0203	[0.0122-0.0281]	0.0208	[0.0127-0.0288]
α_f	β	0.7500/0.0500	0.7509	[0.6718-0.8324]	0.7477	[0.6697-0.8291]	0.7509	[0.6718-0.8324]	0.7477	[0.6697-0.8291]
ρ_f^g	β	0.9500/0.0500	0.9477	[0.9313-0.9654]	0.9479	[0.9314-0.9652]	0.9477	[0.9313-0.9654]	0.9479	[0.9314-0.9652]
ρ_a	β	0.9500/0.0100	0.9469	[0.9307-0.9647]	0.9475	[0.9314-0.9634]	0.9469	[0.9307-0.9647]	0.9475	[0.9314-0.9634]
$\epsilon_{i,t}^A$	Inv.γ	0.0100/2.0000	0.0351	[0.0289-0.0409]	0.0350	[0.0291-0.0407]	0.0351	[0.0289-0.0409]	0.0350	[0.0291-0.0407]
$\epsilon_{i,t}^G$	Inv.γ	0.0100/2.0000	0.1105	[0.0929-0.1276]	0.1112	[0.0927-0.1299]	0.1105	[0.0929-0.1276]	0.1112	[0.0927-0.1299]
$\epsilon_{f,t}^A$	Inv.γ	0.0100/2.0000	0.0188	[0.0160-0.0218]	0.0190	[0.0159-0.0220]	0.0188	[0.0160-0.0218]	0.0190	[0.0159-0.0220]
$\epsilon_{f,t}^G$	Inv.γ	0.0100/2.0000	0.0087	[0.0070-0.0103]	0.0088	[0.0071-0.0105]	0.0087	[0.0070-0.0103]	0.0088	[0.0071-0.0105]

Tab.3.1: *Prior-Posterior Distribution generated by Bayesian Estimation*

3.3.2 Bayesian Dynamics and IRFs Analysis

Domestic Productivity Shock

In what follows, the results of the Bayesian estimatation of the model's variables in the four different fiscal scenarios are then presented, along with comparisons of the application of the different response coefficients of the fiscal policies considered (OSFP-DSFP). The following results follow in part the qualitative dynamics noted in the calibrated model of Chapter II although there are noteworthy quantitative differences due to the data entry and empirical estimation of the system. At a first impact of the domestic productivity shock, in fact, results are noted that are in line with the same reference literature regarding models estimated in simulation scenarios characterized by nominal rigidities as analyzed previously. In fact, this confirms an increase in supply with an associated increase in domestic consumption and a general increase in the respective remuneration of production factors for the values recorded. These results reflect the dynamics of further empirical estimation models on the EA and Italy (see among others Smets and Wouters (2007), Straub and Tchakarov (2007) and Ratto et al. (2008)). The dynamics relating to the decoupling of public investment from European budget constraints is also of particular interest. In fact, the dynamics makes it possible to note how there is a general change in some of the macroeconomic variables when the golden rule is applied with respect to the benchamark scenarios, especially in the presence (as analyzed below) of demand shocks. With regard to the domestic supply shock, consumption responds to the increase in productivity in a positive manner leading to their growth around values this time of 0.06 in terms of log-deviation around the fifth period with high levels of persistence compared to the calibrated levels of 0.02. Similarly, it is possible to note and confirm a greater positive response of the same in the scenarios of application of the OSFP compared to the DSFP.

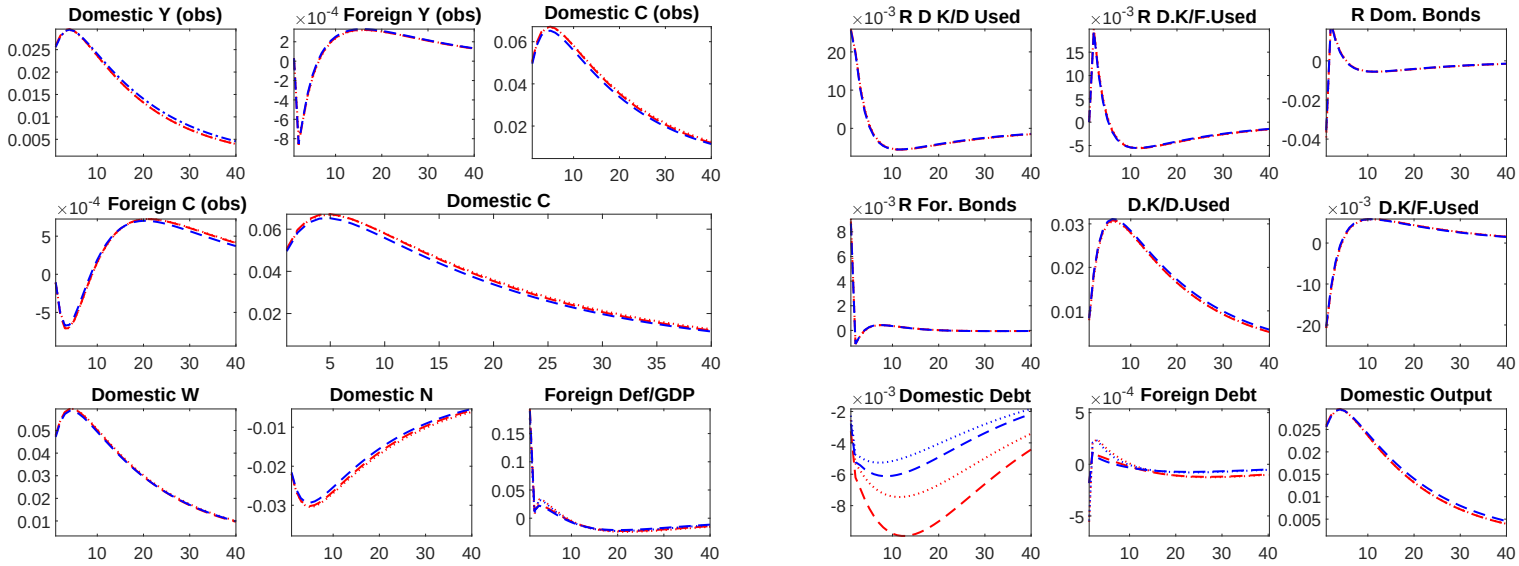

Fig.3.9: *Bayesian dynamics of the system at a domestic productivity innovation. The red and blue lines represent respectively the fiscal policy scenarios of stabilization of ouput and debt. On the dotted line it is possible to see the benchmark scenarios (I-III) while on the dash line the golden rule application scenarios (II-IV).*

The same dynamics apply to the increase in wage levels and a general reduction in working hours, although in this case less reduced in scenarios II-IV. This phenomenon again reflects what has been analyzed in the literature and empirical evidence (see among others Gali and Rabanal (2005) and Ratto et al. (2008)) and Gali (1999) (see Chapter II). We note as analyzed an increase in the respective returns to domestic capital used both domestically and abroad by log-deviation values of 0.2 and 0.15 relative to the calibrated levels of 0.06 and 0.005. At the same time, it is possible to note increases in the quantities of the same capital used by firms in the production function, although no differences are found between the different scenarios. However, it is important to underline a new result which is more visible than the increased return recorded by public capital, in the area of 0.02 values in terms of log-deviation, in scenarios II-IV. Greater public investment, therefore, free to grow from European constraints, has a slight impact on this dynamic. A reduction in domestic debt is always confirmed due to the supply shock, greater in scenarios II-IV and of greater magnitude in cases of output stabilization policies, as well as

symmetrically an almost instantaneous increase in foreign debt always presumably acquired from the domestic country in strong growth with the same dynamics noted in the calibration phase with respect to the OSFP and DSFP although with log-deviation levels recorded of -0.006 and -0.008 around the tenth period. This process always leads to responses in line with the results reached in the literature Badarau et al. (2014) and with the empirical evidence of further Bayesian estimation models (Vitek (2014, 2017) and Christoffel et al. (2011)) with respect to both domestic and foreign bond returns, with the associated risk premiums, which always respond to domestic (foreign) impact in a negative (positive) manner without however any particular differentiation between scenarios. A rapid increase in domestic output is then confirmed, this time for maximum values of 0.030 in terms of log-deviation for all periods considered compared to the values of 0.01 evaluated in the calibrated model.

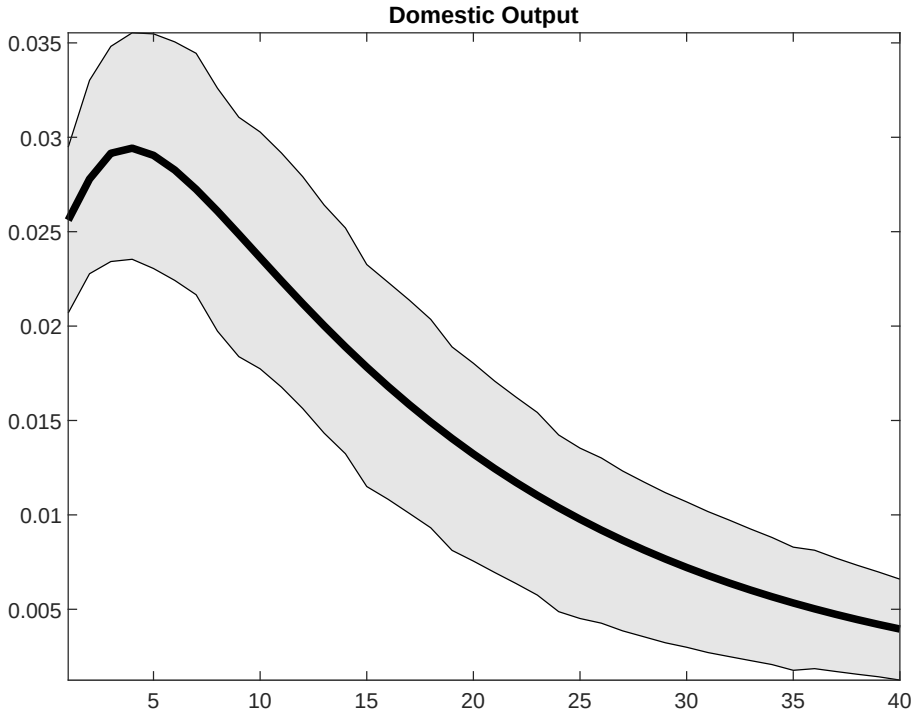

Fig.3.10: *Bayesian dynamics of the output growth to a domestic productivity shock.*

In confirmation of what has been seen, in fact, there is a general increase in public capital used by domestic companies, which sees a greater increase of 0.02 in

terms of log-deviation, as well as a confirmation of the increase in public investment (0.015) and therefore a general increase in total domestic investment, driven by the greater quantity of capital used by firms. Also in confirmation with the calibrated model and in line with the results achieved in the literature with respect to economic theory, the same dynamics of reduction of the Deficit/GDP ratio is noted, greater in the II-IV scenarios as can be seen from the respective inflection points, due to the rapid growth of the domestic economy. In this sense, the empirical estimate makes it possible to note, in fact, how output stabilization (OSFP) in the presence of supply shocks leads to a greater reduction in this ratio in the fifteenth period compared to the equivalent baseline scenario relative to the greater response coefficients in debt stabilization (DSFP). It is always confirmed the response of Domestic Capital purchased externally: well, even the empirical estimate of the model shows how the shock of domestic supply, combined with the presence of nominal rigidities on prices, leads to have a response to negative impact on the stocks of this capital always due, in all probability, just that excess supply due to the increase of TFP but that does not find levels of conjunction in the foreign demand of the same good seeing a lowering as confirmed, however, by the negative response to impact for values of 0.06 in terms of log-deviation of Investments in Domestic Capital used internationally and the symmetrical increase (always 0.06) of domestic investments in domestic capital used domestically. With regard to these variables in their empirical estimate, the differentiation of the scenarios does not entail any particular differences between them other than a very small increase in domestic investment in domestic capital used in the area of values slightly above 0.01 in terms of log-deviation.

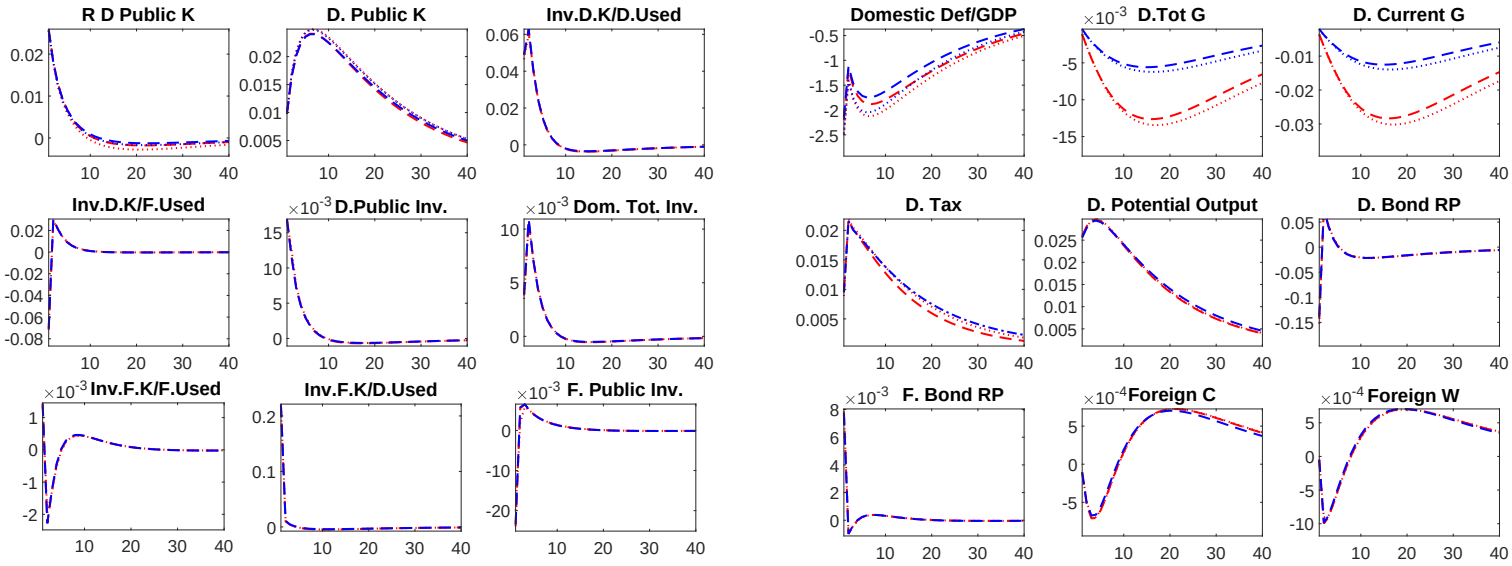

Fig.3.11: *Bayesian dynamics of the system at a domestic productivity innovation. The red and blue lines represent respectively the fiscal policy scenarios of stabilization of ouput and debt. On the dotted line it is possible to see the benchmark scenarios (I-III) while on the dash line the golden rule application scenarios (II-IV).*

Therefore, the countercyclical nature of public expenditure (greater in the case of output stabilization policies due to the rapid growth of domestic GDP recorded and a reduction in the respective debt), both capital and current account, is confirmed, which sees a reduction driven by the rapid growth of the domestic economy but less in the following estimate in scenarios II-IV (probably driven by the greater public spending allowed by the same public investments), as well as the procyclical nature of taxation which sees a rapid increase slightly greater in the scenarios of golden rule as well as in the cases of DSFP, probably precisely for the purpose of greater debt stabilization. In line with the increase in potential capacity of the economy, the data also confirm an increase in potential output. Bayesian dynamics also confirm how abroad the high level of domestic productivity has negative effects almost everywhere except for investments from the domestic country as well as the higher returns recorded for foreign capital used domestically (where this increase in Total Factor Productivity is recorded).

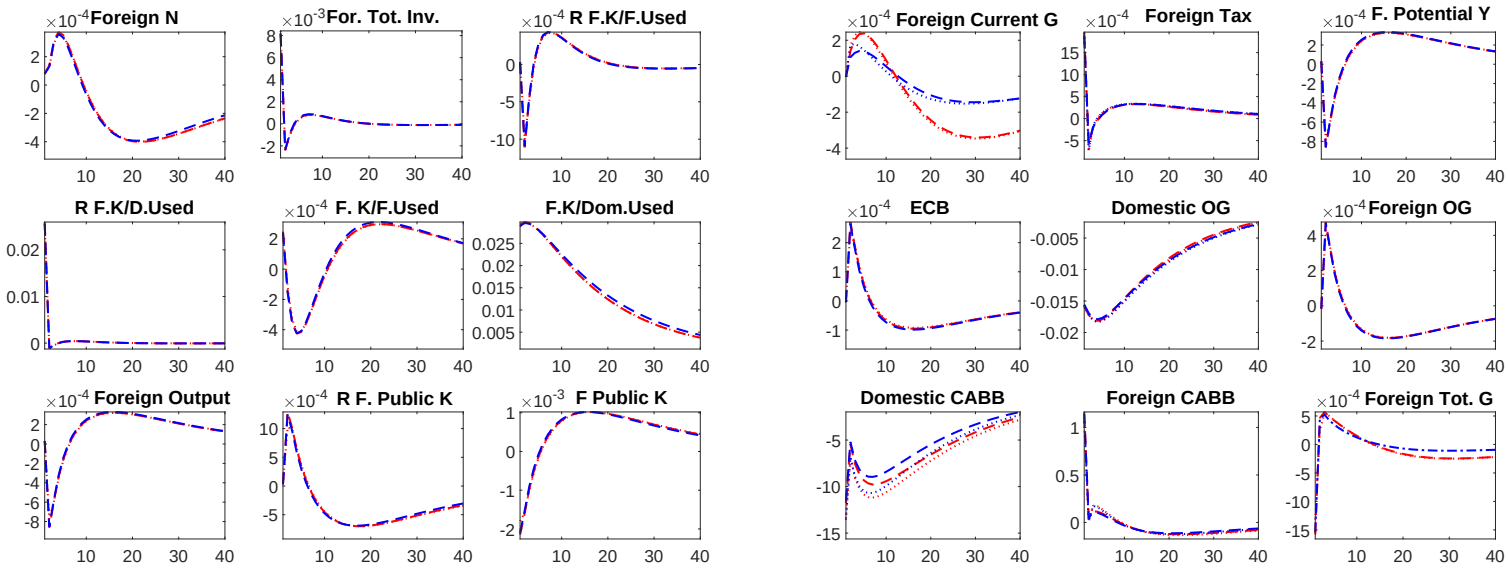

Fig.3.12: *Bayesian dynamics of the system at a domestic productivity innovation. The red and blue lines represent respectively the fiscal policy scenarios of stabilization of ouput and debt. On the dotted line it is possible to see the benchmark scenarios (I-III) while on the dash line the golden rule application scenarios (II-IV).*

Also confirmed are the dynamics of the countercyclical foreign response of public spending, both in current and capital account (although of lesser magnitude) in response to the short and medium term decline in foreign output, recorded as greater in cases of greater stabilization of the same output (given its own rapidly decreasing dynamics) rather than on debt levels as well as greater in the short term in scenarios of separation of investments (II-IV). In line with economic theory and with the literature previously analyzed on calibration models only (see always Gali (2006), in the presence of this supply shock there is also an increase in the response of the ratio of foreign deficit/GDP, slightly mitigated in the II-IV scenarios in foreign application of a DSFP as well as a reduction of the domestic Output Gap of values this time around 0.015 in terms of log-deviation slightly lower in the scenarios of separation of investments. Finally, it is interesting to note a bullish response by the European central bank, albeit with low levels of persistence aimed at restoring the equilibrium levels as well as on the negative (positive) responses of the domestic and foreign structural deficit also in line with the reduction (increase) of the deficit

itself and with the same dynamics recorded on the II-IV scenarios and the greater effect of fiscal policy stabilization of output recorded this time since the impact of the productivity shock compared to the benchmark scenario I-III of the debt stabilization policy. It is therefore possible to extract a new result from this positive analysis with respect to the dynamics of domestic CABB, which sees a greater reduction in cases where investments are unbundled. This phenomenon is probably explained by greater compensation on the part of, on the one hand, the dynamics of the deficit/GDP ratio and, on the other, the levels of reduction of the domestic OG. In such terms, it is possible to note how this dynamic abroad is always more offset in scenarios II-IV compared with the benchmarks, confirming the same dynamics noted in the presence of application of the Golden Rule. It is therefore possible to extrapolate an important result from this analysis. In the presence, in fact, of increases in domestic productivity, a policy at the domestic level of greater stabilization of output (with higher response coefficients to its own fluctuations) leads to levels of greater reduction in debt and domestic deficit/output ratios, as well as the recorded levels of structural deficit, if combined at the same time with a fiscal rule of separation of public investment capable of promoting greater economic growth (as in scenarios II-IV). These reduction (increase) effects can also be seen on foreign budget balances in relation to the application of a golden rule, both in relation to increases in deficit levels and in relation to adjustments with respect to structural deficit levels more in the short term if accompanied by a foreign policy of debt stabilization. The results analyzed with this empirical estimate allow us to confirm what was preliminarily analyzed in the calibrated model, although obviously, and in line with the objectives of Bayesian estimation, with different magnitudes recorded on the main macroeconomic variables.

Foreign Productivity Shock

Even in the presence of a foreign productivity shock, the same dynamics noted previously are empirically confirmed, and the same dynamics noted in the presence of the domestic shock are also confirmed, thus confirming the empirical studies previously carried out on the EA territory (Gali and Rabanal (2005) and Ratto et al (2008) and Gali (2008)) always with the given differences due to the different European parameterization and the weight of the given economies in the respective comparisons and Bayesian estimation. It is also interesting to dwell on how, at the aggregate level of the Union, qualitatively and quantitatively interesting effects can be found in the various scenarios of the response coefficients of the fiscal policy, even though the results confirm how, at the aggregate level, the presence of the golden rule does not affect some of the variables most affected at the domestic level by the same fiscal rule.

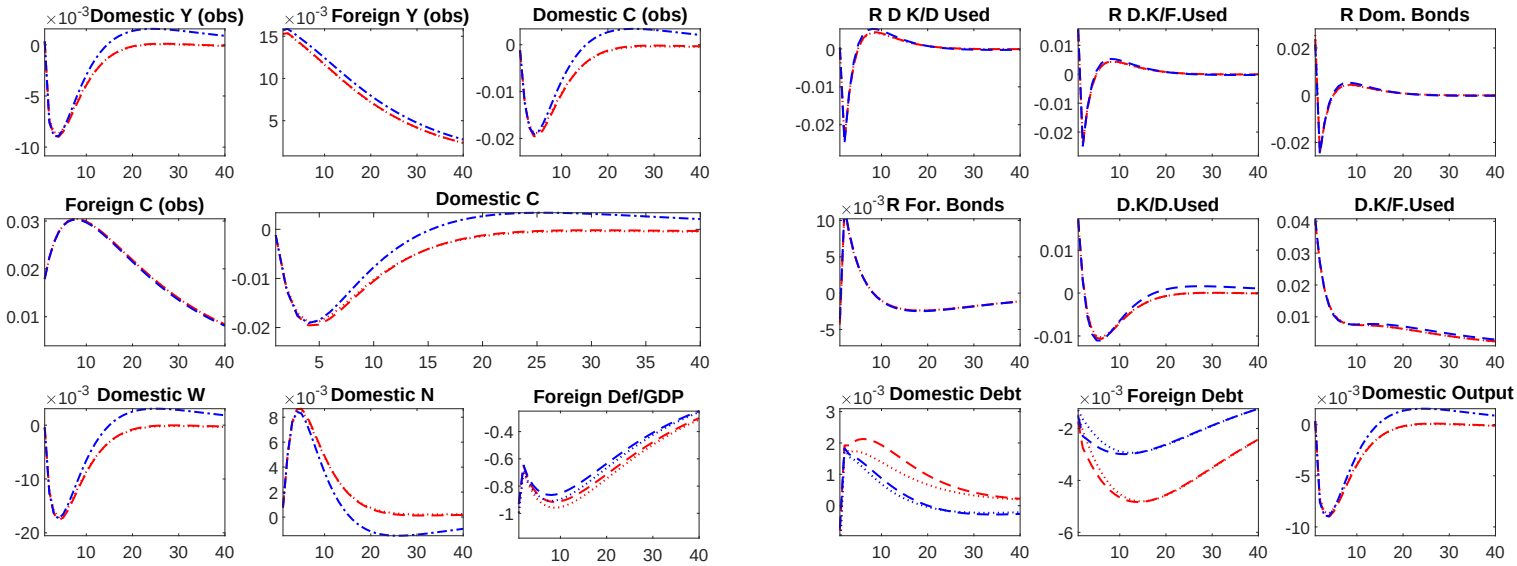

Fig.3.13: *Bayesian dynamics of the system at a foreign productivity innovation. The red and blue lines represent respectively the fiscal policy scenarios of stabilization of ouput and debt. On the dotted line it is possible to see the benchmark scenarios (I-III) while on the dash line the golden rule application scenarios (II-IV).*

We note in this sense, at first, an increase confirmed by the shock of foreign supply with a negative effect almost everywhere referred to domestic variables, as occurred in the previous simulation of the shock of domestic TFP, while there is a general increase in foreign consumption for values higher in the Bayesian dynamics to 0.03 in terms of log-deviation compared to the calibrated 0.02 always due to the greater supply generated by companies in the presence of increased productivity together with a greater increase in the presence of a domestic policy of greater response to fluctuations in output rather than debt. Similarly, an increase in foreign capital investment and an increase in international investment on the part of the EA is confirmed, as well as an immediate reduction (increase) in the levels of foreign public debt (domestic) which are greater, also in this case in the presence of output stabilization policies and application of the golden rule (II-IV) with respect to the baseline scenarios (I-III) respectively for almost double values in terms of log-deviation starting from the tenth period (respectively 0.02-0.03, 0.04-0.05)

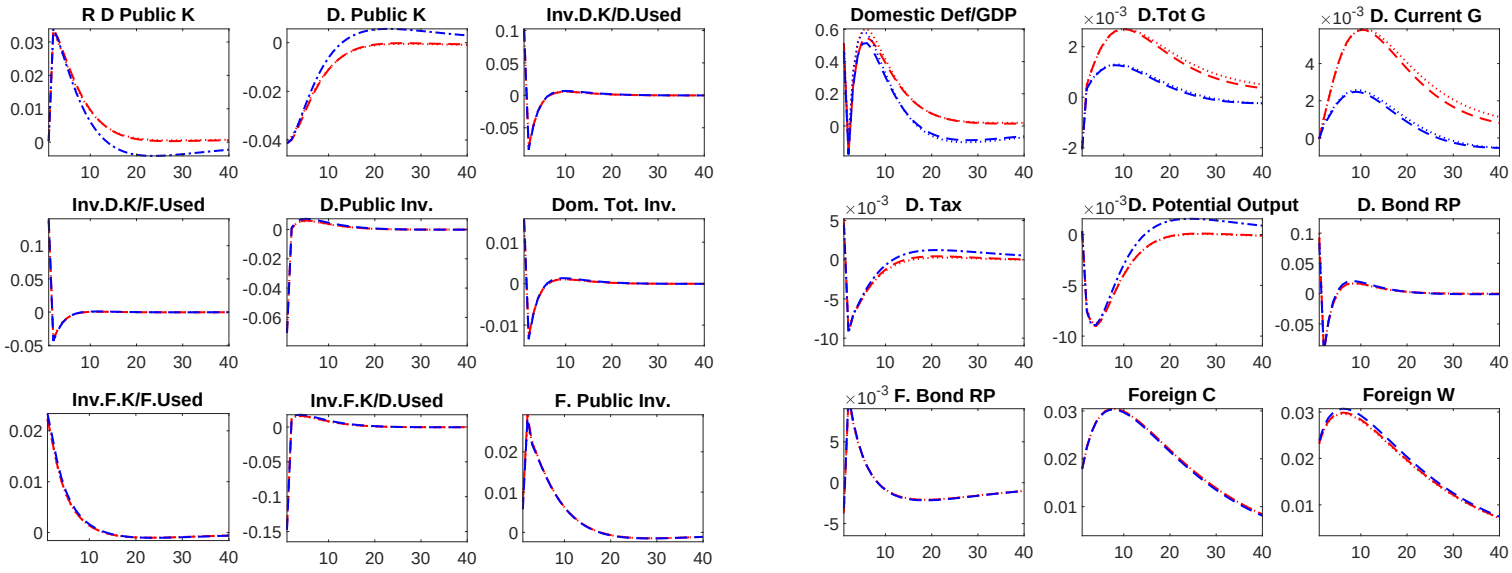

Fig.3.14: *Bayesian dynamics of the system at a foreign productivity innovation. The red and blue lines represent respectively the fiscal policy scenarios of stabilization of ouput and debt. On the dotted line it is possible to see the benchmark scenarios (I-III) while on the dash line the golden rule application scenarios (II-IV).*

Thus, the empirical dynamics on the EA territory (see Gali and Rabanal (2005)) are confirmed, given by the increase in the returns of all foreign productive factors, both from the wage point of view and with regard to the returns of foreign capital used by companies, both domestic and European, accompanied by the previously analyzed increase in their capacity utilization in the international production function. Also confirmed is the result of an increase in private and public investment with a general increase in public capital in the production function, once again more so in the OSFP scenarios (0.01 in terms of log-deviation) rather than DSFP (0.08) at the peak around the tenth period. This shock of supply and increased productivity also leads in this case to an increase in total output (also in this case higher in the Bayesian estimate than in the calibrated model) and potential output and to a reduction (increase) in the Deficit/GDP ratios which is greater also in this case in the presence of output stabilization policies at a European level and debt stabilization policies at a domestic level together with the results analyzed with respect to scenarios II-IV of the OSFP with respect to the baseline of greater response to foreign debt fluctuations.

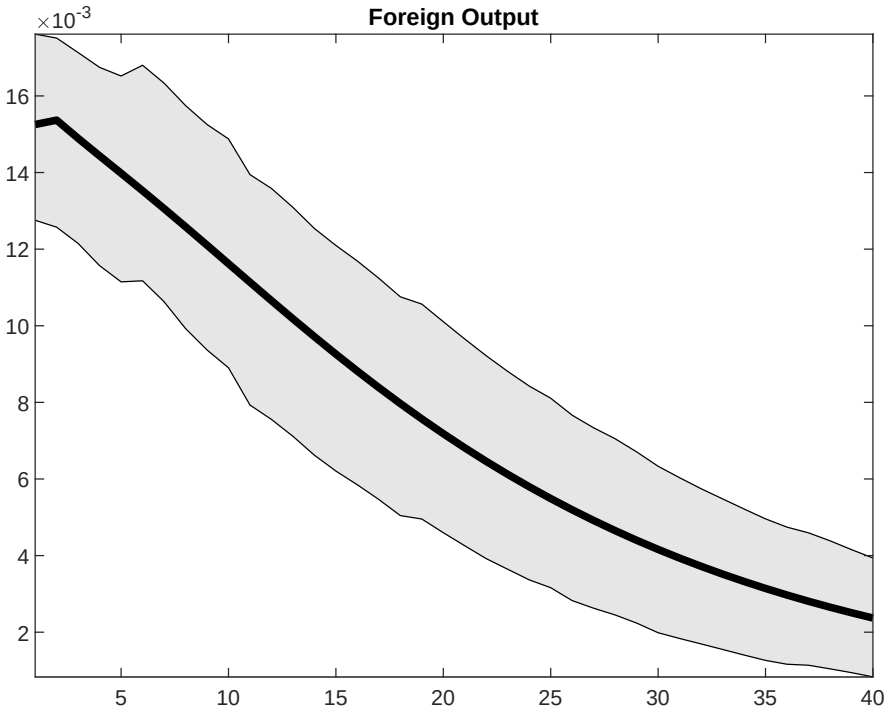

Fig.3.15: *Bayesian dynamics of the output growth to a foreign productivity shock.*

Therefore, the counter-cyclical dynamics of foreign capital and current public spending are confirmed, as well as a pro-cyclical increase in foreign tax revenues.

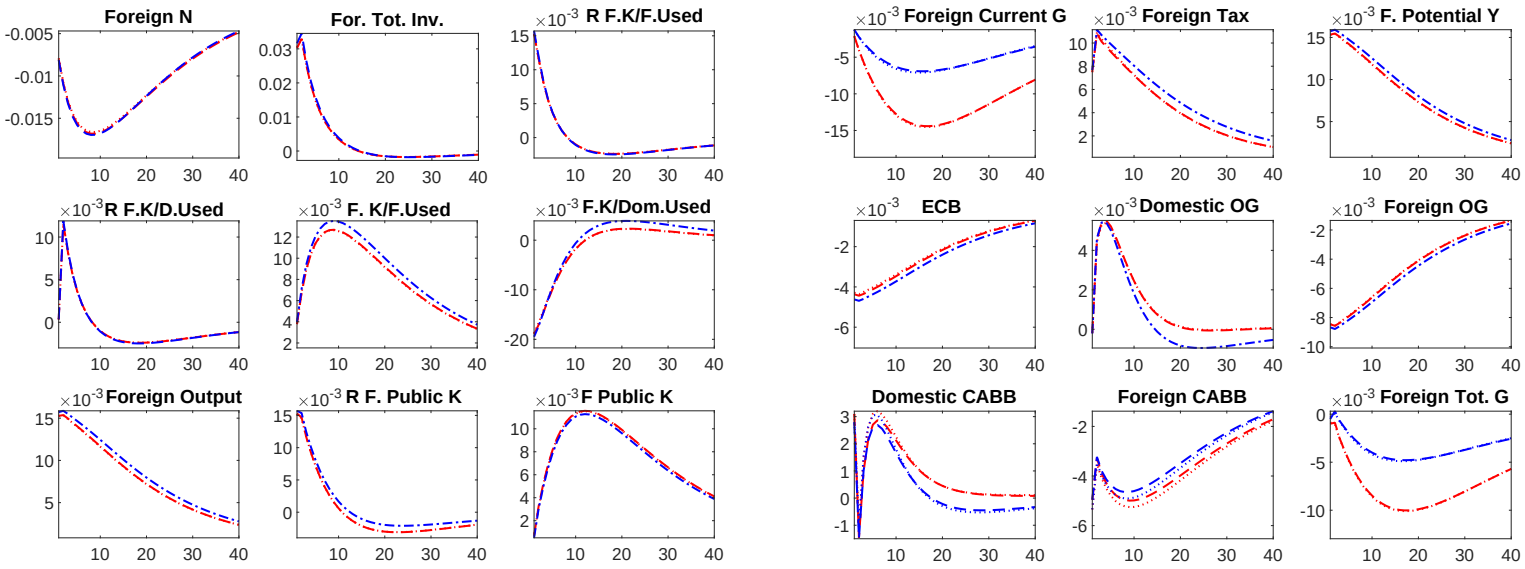

Fig.x: *Bayesian dynamics of the system at a foreign productivity innovation. The red and blue lines represent respectively the fiscal policy scenarios of stabilization of ouput and debt. On the dotted line it is possible to see the benchmark scenarios (I-III) while on the dash line the golden rule application scenarios (II-IV).*

Finally, also in this case, in accordance with economic theory and reference literature (Gali (2006)), there is a reduction in the foreign output gap estimated for values of 0.008 in terms of log-deviation and structural deficit levels with the same dynamics of differentiation noted in scenarios II-IV with respect to the presence of fiscal policy of stabilization of output and debt, but not with the same effects of compensation due to the dynamics of public investment and the foreign output gap in scenarios II-IV with respect to I-III noted and highlighted for the II-IV scenarios for the domestic country where they continue to persist, especially in the presence of a DSFP as well as in the application of the separation of investments in application of an OSFP, probably due to the different dynamics of the response of domestic public

spending in response to the countercyclical trend induced by the foreign productivity shock in the different fiscal policy scenarios, which at the same time leads to greater containment of the dynamics of debt growth and the deficit/GDP ratio.

Domestic Government Spending Shock

In the presence of domestic public spending shocks, the estimates made regarding the dynamics of the main variables analyzed previously are in line and confirmed. In fact, qualitatively and quantitatively important differences are confirmed in the II-IV scenarios with respect to the different parameterization of fiscal policy. In fact, it can be seen immediately that there is a negative response both to impact and with high levels of persistence of maximum estimated values in the baseline scenarios around 0.02 and 0.08 in terms of log-deviation of private domestic consumption. It is therefore empirically confirmed by our data the phenomenon of "Crowding-Out" of private consumption compared to government spending in line with the results noted in the literature and economic theory (see Albonico et al. (2019)) highlighted in an illustrative way in its dynamics for the benchmark scenarios in Fig.3.16 with its intervals at 90%.

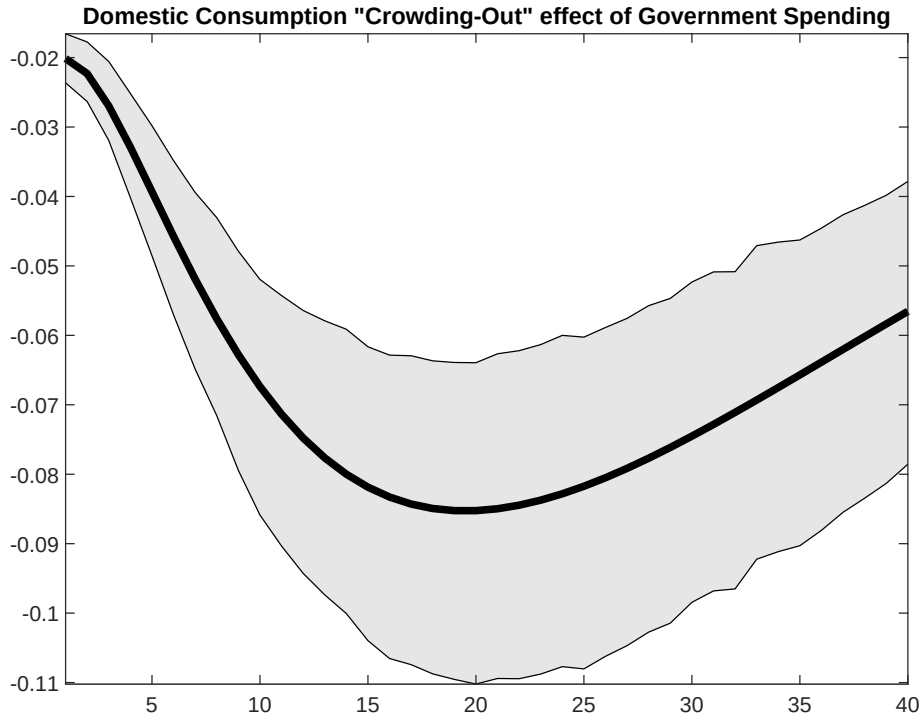

Fig.3.16: *Bayesian dynamics Crowding-Out effect on domestic private consumption following a domestic government spending shock.*

The procedure is always the same as previously analyzed. Public spending displaces private consumption which sees a reduction as a result of the demand shock registered as a result of greater public spending. In addition, the dynamics of Straub and Tchakarov (2007) are confirmed with respect to the wealth effect on labor driven by increased taxation. These effects are more noticeable in the benchmark scenarios (I-III) while, on the contrary, they are less reduced in the scenarios of application of the golden rule. This confirms empirically what was previously analyzed with regard to the mitigation effect of greater public investment with respect to the results obtained in the studies by Zeynologlu (2018). Moreover, in the presence of a demand shock, this effect is always more mitigated in the scenarios of application of a domestic fiscal policy of debt stabilization rather than of output where the "crowding-out" is more evident with respect to both scenarios I-III and II-IV probably due to the dynamics of greater containment of expenditure items following the objectives of greater countercyclicality of the debt itself. In line with economic theory, the increase in domestic public debt is always confirmed, with values of a maximum of 0.15, accompanied by a reduction in foreign debt with the persistence of a significant shock, in parallel with a general increase in the deficit/GDP ratio which sees the same dynamics of the supply shock confirmed by the estimates compared to scenarios II-IV, although always with differences compared to the experiments conducted in the presence of fiscal policy coefficients of stabilization of output and debt. In the presence of the latter, in fact, a lower rise in domestic public debt is always noted, driven by the dynamics of current and capital expenditure as well as the deficit/GDP ratio, while abroad this dynamic is always supported by a policy of greater stabilization of output, especially in the medium-long term. In these terms, it is important to underline the same dynamics noted in the presence of supply shocks, but which in this case takes hold in the twentieth period of greater restraint on deficit items moved by the separation of public investment in both OSFP and DSFP scenarios where the dynamics of public spending and greater tax revenues, accompanied by the growth recorded by output, probably compensate for this growth effect. In addition, the dynamics and positive (negative) responses to the impact of domestic (foreign) bond returns with the relative sovereign risk premiums remain

confirmed, although with momentary downturn points recorded for domestic bonds in scenarios II-IV around the eighth period, probably explained by the trend of the same returns, until a return to steady-state levels.

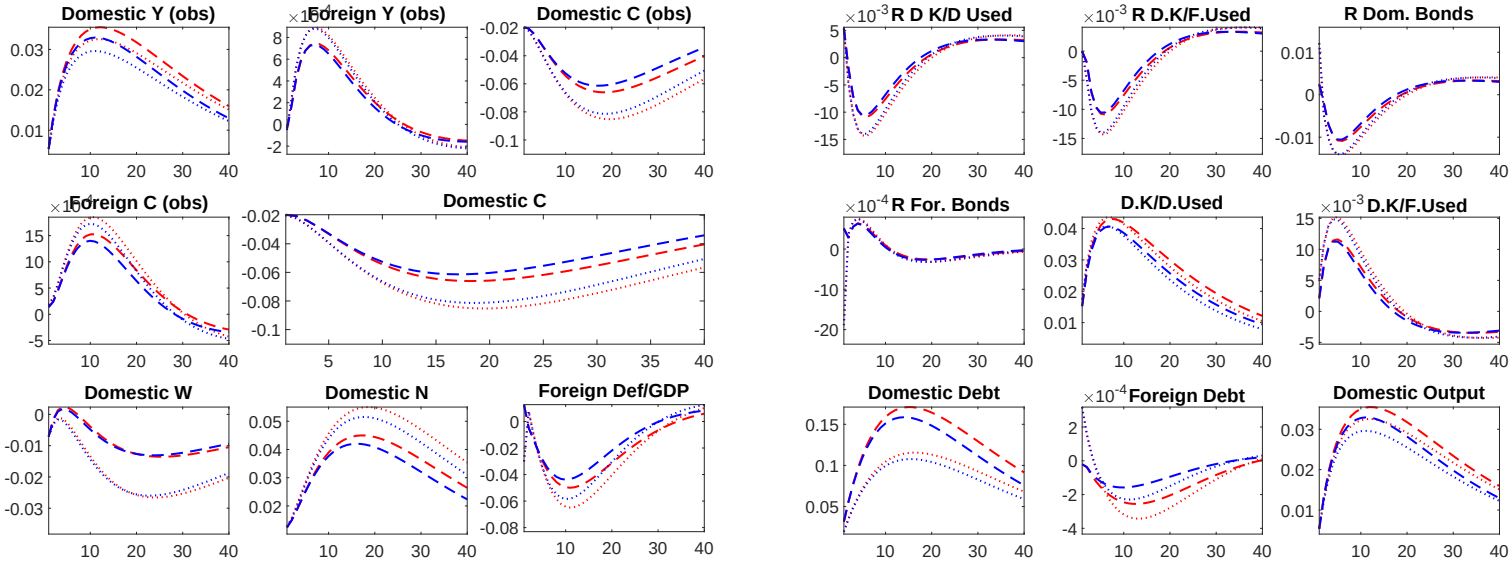

Fig.3.17: *Bayesian dynamics of the system at a domestic government expenditure innovation. The red and blue lines represent respectively the fiscal policy scenarios of stabilization of ouput and debt. In the dotted line it is possible to see the benchmark scenarios (I-III) while in the dash line the scenarios of application of the golden rule (II-IV).*

It is always confirmed as well as in the literature both on cases of calibrated models and estimation models (see among others Ferrero (2009), Straub and Tchakarov (2007) and Quest III of Ratto et al. (2008)) the increasing dynamics of domestic GDP in the region of 0.025 in terms of log-deviation in the benchmark scenarios, which this time records almost double values in terms of log-deviation (0.03-0.04) around the tenth period in the scenarios of separation of public investment and its potential counterpart, given the growing components of public expenditure (in response to the higher levels recorded in output and debt in addition to the relative dynamics analyzed in repercussion on the same private consumption and their displacement) and the propulsive effect of the higher levels recorded in the II-IV scenarios of the same public investment and public capital in fast and greater growth

in the production function. The remaining increases recorded in their growth dynamics with the theoretical results of Zeynologlu (2018) and with the empirical ones of Straub and Tchakarov (2007) noted in Figures 1.5 and 1.6 in eurozone analyzed above are thus confirmed.

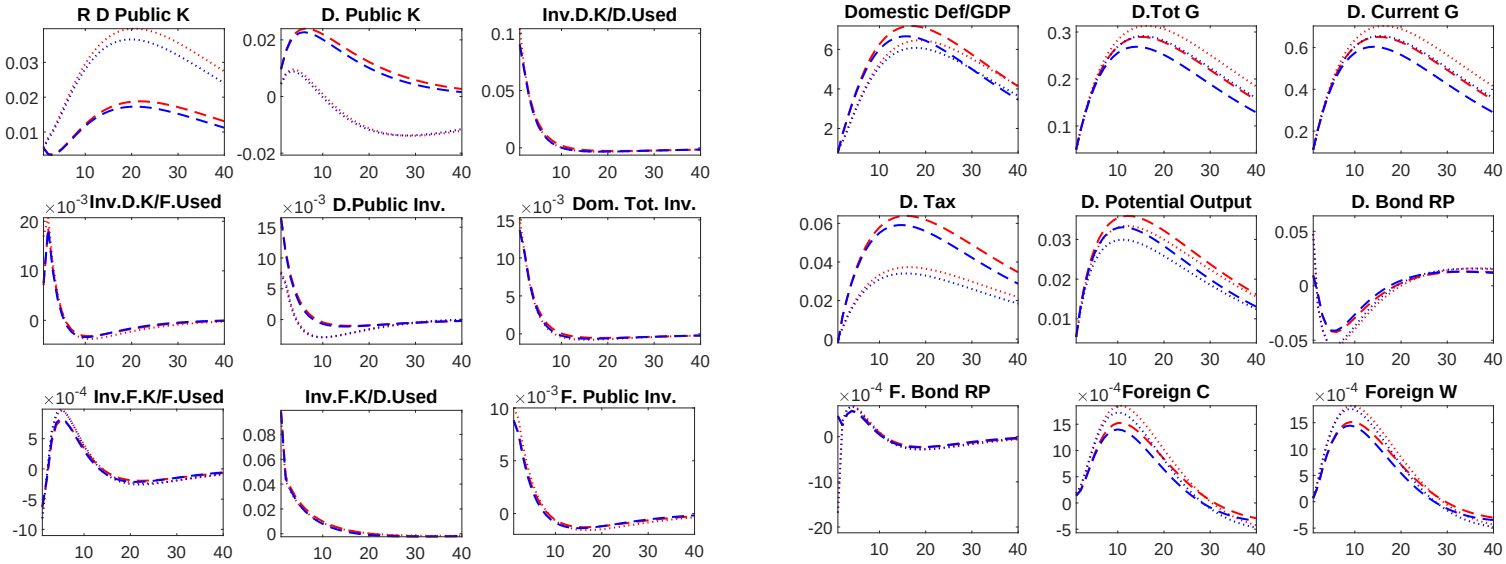

Fig.3.18: *Bayesian dynamics of the system at a domestic government expenditure innovation. The red and blue lines represent respectively the fiscal policy scenarios of stabilization of ouput and debt. In the dotted line it is possible to see the benchmark scenarios (I-III) while in the dash line the scenarios of application of the golden rule (II-IV).*

It is very important, however, to underline how these dynamics are always different in the phases of separation of investment, allowing for even greater growth (in estimates at 0.02 around the tenth period and 0.015 in terms of log-deviation) of public capital and public investment with significant levels of persistence until steady-state is once again reached. This result therefore confirms the possibility for the government to increase public investment and at the same time for public capital to grow more in the production function without being bound by European fiscal rules. This phenomenon then has repercussions on systemic variables.

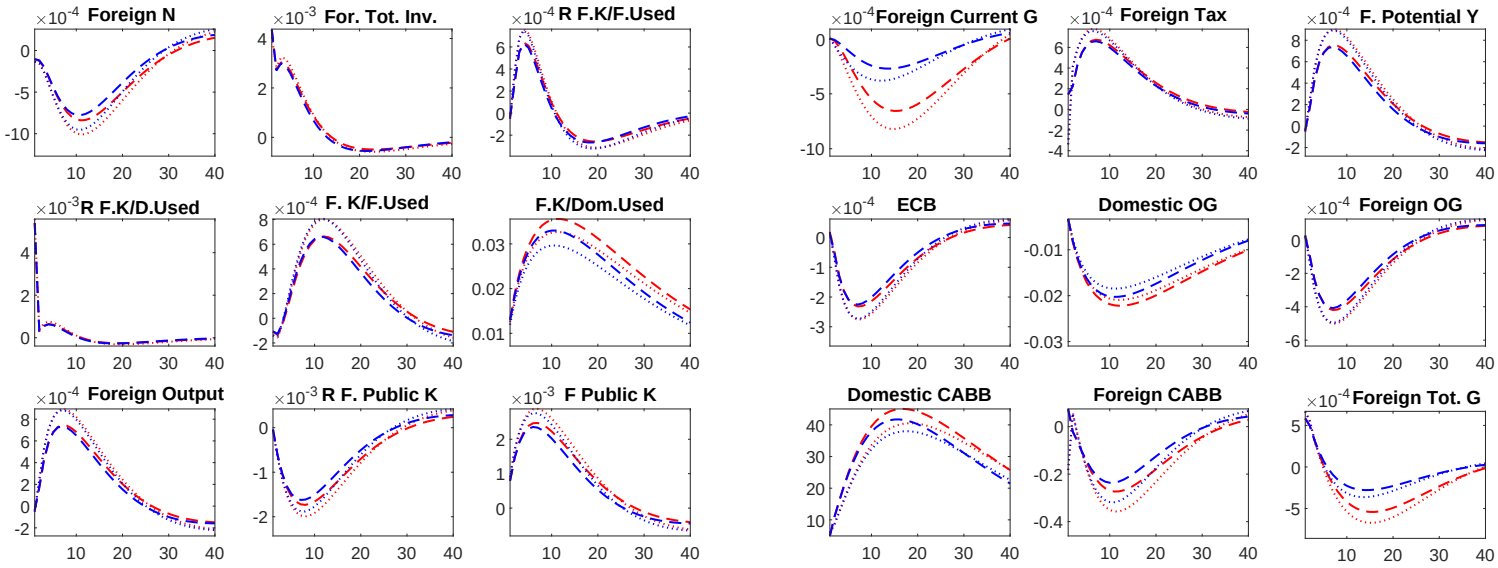

Fig.3.19: *Bayesian dynamics of the system at a domestic government expenditure innovation. The red and blue lines represent respectively the fiscal policy scenarios of stabilization of ouput and debt. In the dotted line it is possible to see the benchmark scenarios (I-III) while in the dash line the scenarios of application of the golden rule (II-IV).*

The remaining answers always confirm the results obtained in the literature in models characterized by nominal rigidity and distortionary taxation (see among others Straub and Tchakarov (2007) and Albonico et al. (2019)) with respect to the application of different fiscal policies in scenarios II-IV with respect to the dynamics of the structural deficit in its second phase. In these cases, in fact, the dynamics of greater stabilization of a fiscal policy of stabilization of the debt in the II-IV scenarios are confirmed with respect to the counterpart of the output in the I-III benchmark scenarios where the effect of increase reaches higher levels probably due to the compensation effects, recorded by resolving with respect to the CABB formula, with reference to the dynamics of the Output Gap and the same public investments.

Foreign Government Spending Shock

Finally, let us dwell on the dynamics estimated at the aggregate level of EA to a shock of current foreign government spending. The dynamics previously noted in the calibrated model are thus confirmed with respect to their volatility, but always with respect to different quantitative effects due to the Bayesian estimation of the model. In these terms, in fact, an increase in government spending at the Union level always has positive effects on the domestic country with a general increase in domestic consumption and with a reduction in domestic bond returns and an increase in foreign ones, with associated the same dynamics of increase and reduction of the respective risk premiums in line with the analyses carried out at the EA level by the main reference modeling (see Vitek (2014), (2016) and Christoffel et al. (2011)).

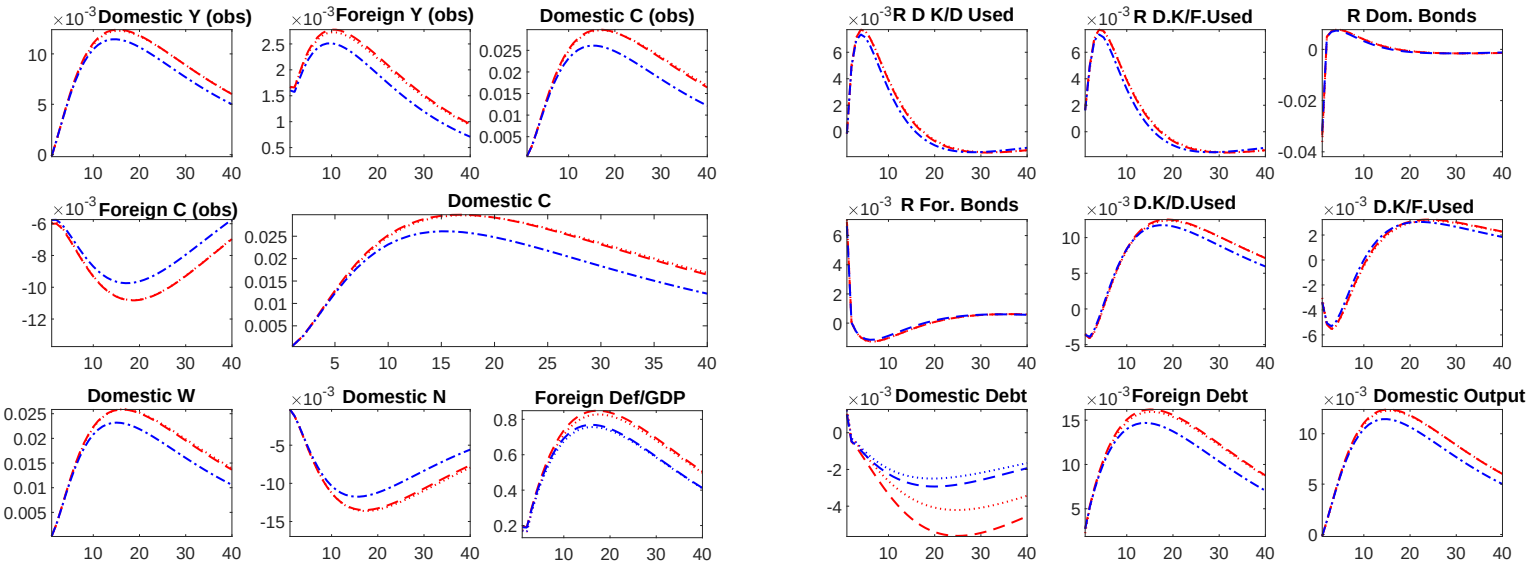

Fig.3.20: *Bayesian dynamics of the system at a foreign government expenditure innovation. The red and blue lines represent respectively the fiscal policy scenarios of stabilization of ouput and debt. In the dotted line it is possible to see the benchmark scenarios (I-III) while in the dash line the scenarios of application of the golden rule (II-IV).*

It is always confirmed, as per economic theory, a general increase in foreign debt with a reduction in domestic debt (for values estimated even higher than 0.004 in

terms of log-deviation). Also in these dynamics, this result is in line with the dynamics of the II-IV scenario compared to the benchmark ones, leading to a lower reduction at foreign level in the presence of debt stabilization policies rather than output and especially in the presence of the application of a separation of public investment, probably driven by the dynamics of the same expenditure, both capital and current. Symmetrically, the same dynamics can be seen at European level where the current expenditure shock is mitigated in its growth effects on debt by a policy of greater debt stabilization.

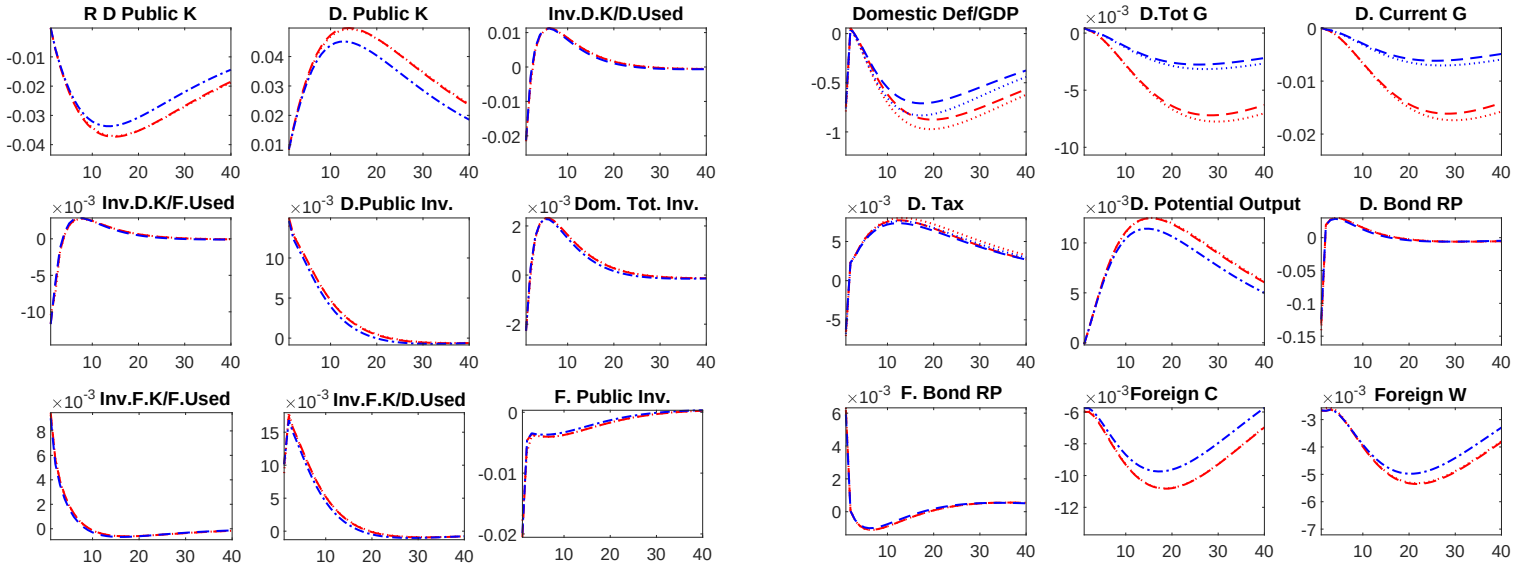

Fig.3.21: *Bayesian dynamics of the system at a foreign government expenditure innovation. The red and blue lines represent respectively the fiscal policy scenarios of stabilization of ouput and debt. In the dotted line it is possible to see the benchmark scenarios (I-III) while in the dash line the scenarios of application of the golden rule (II-IV).*

Also confirmed is a general increase in the foreign deficit/GDP ratio and a reduction in the domestic deficit/GDP ratio, respectively, for values around 0.5 and 0.8 in terms of log-deviation in the twentieth period, as noted. Once again these positive effects come to underline the dynamics of international investment with respect to foreign capital and with respect to its growth in the domestic production function. In this sense, the result is a general increase in domestic output with the

relative investments, always with the same differences due in scenarios II-IV and with the different applications of the relative fiscal policies. In line with the results analyzed and with the literature (Ferrero (2009), Ratto et al. (2008) and Straub and Tchakarov (2007)) it is noted also at EA level the consequent "crowding-out" effect on private consumption (detail for benchmark scenario in Fig.3.22), for values of 0.008 (always differentiated in the respective scenarios of different response of fiscal policy), and the same dynamics analyzed above with respect to labor and wages due to the increase in the current component of public spending.

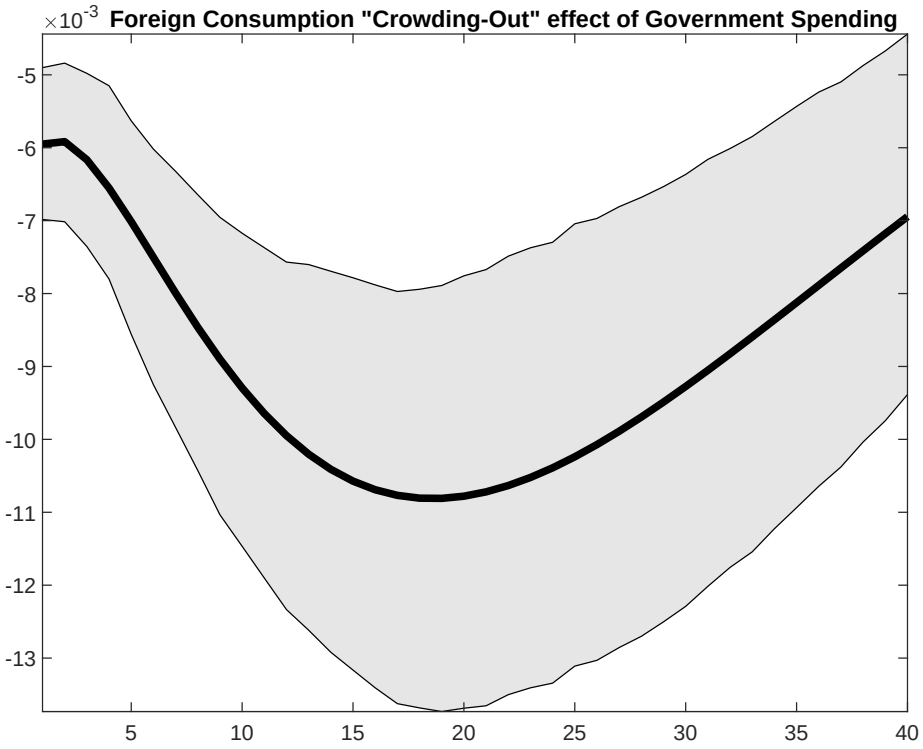

Fig.3.22: *Bayesian dynamics Crowding-Out effect on foreign private consumption following a foreign government spending shock.*

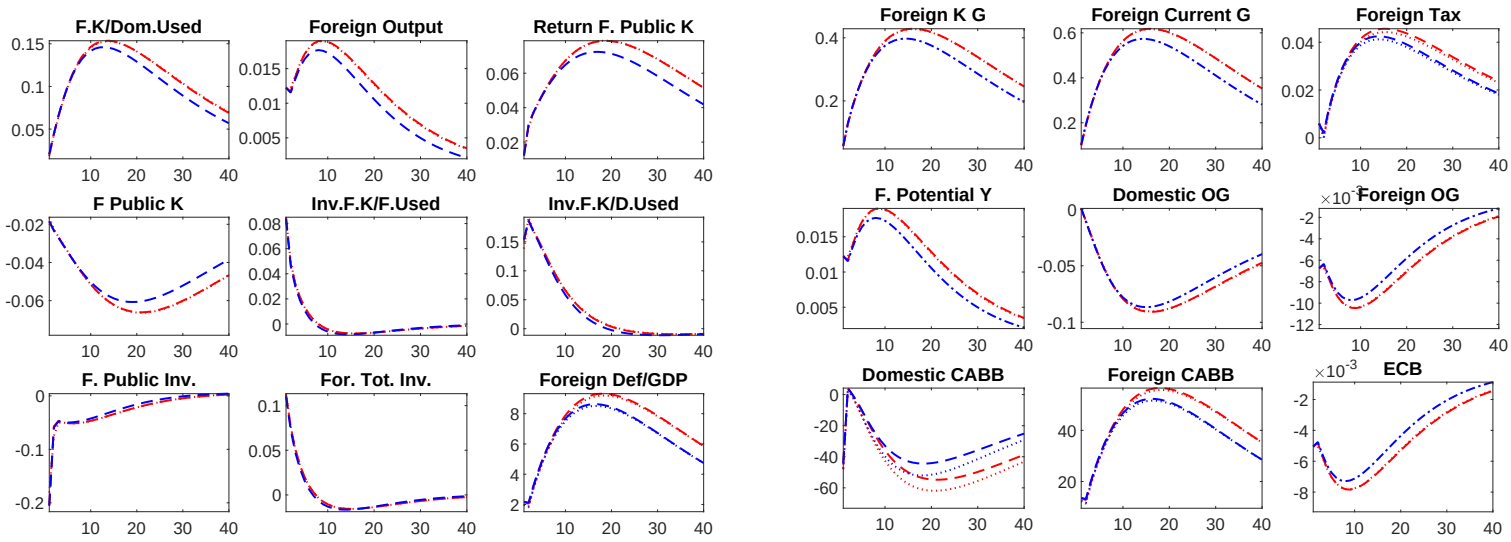

Fig.3.23: *Bayesian dynamics of the system at a foreign government expenditure innovation. The red and blue lines represent respectively the fiscal policy scenarios of stabilization of ouput and debt. In the dotted line it is possible to see the benchmark scenarios (I-III) while in the dash line the scenarios of application of the golden rule (II-IV).*

The Bayesian estimate also makes it possible to note new quantitatively important effects regarding the growing elements of foreign GDP for values estimated at 0.015 compared with the previous values of 0.002 given the growing component of public spending accompanied by a general increase, both if considered at current and capital account level. Again with reference to the dynamics of public capital and public investment, the same dynamics underlined in the case of the domestic country in the scenarios for the separation of public investment are not noted. The Bayesian estimate in this case shows how the application of a golden rule does not seem to have the same dynamics of increase in the same investments with respect to the benchmark scenarios, although similar results can be found in the dynamics of the same fiscal policies. In this sense, the estimation and the observed variables of the model seem to confirm the previous theoretical results and the empirical evidence at European level shown in Straub and Tchakarov (2007) and Zeynologlu (2018), also shown in the previous Figures 1.5 and 1.6, with respect to the absence of co-movements between European current expenditure and public investments probably

exactly because the european fiscal rules (SGP). Finally, the dynamics of increase (decrease) of the respective foreign (domestic) structural deficits are also evident.

3.4 References

1. An S., Schorfheide F., "Bayesian Analysis of DSGE Models," Econometric Reviews, Taylor and Francis Journals, vol. 26(2-4), pages 113-172, 2007

2. Assenmacher K., "Bridging the gap between structural VAR and DSGE models", DSGE Models in the Conduct of Policy: Use as intended, 2017

3. Bizzotto V., Castelnuovo E., "Identificazione delle Cause della Grande Moderazione Statunitense: Una Stima Bayesiana", Università degli Studi di Padova, 2011

4. Canova F., "Bayesian Methods for DSGE models", EUI and CEPR, 2012

5. Canova F., Sala, L., "Back to square one: Identification issues in DSGE models", Journal of Monetary Economics, 56(4), 431-449, 2009

6. Christiano L., Eichenbaum M., Evans, C., "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", Journal of Political Economy, 113(1), 1-45, 2005

7. De Luca D., Castelnuovo E., "Stima Di Modelli Di Persistenza Della Politica Monetaria Per L'area Dell'euro Con Tecniche Bayesiane", Università degli Studi di Padova, 2011

8. Del Negro M., Schorfheide F., "Priors from General Equilibrium Models for VARS", International Economic Review, 45: 643-673, 2004

9. Fernández-Villaverde J., "The Econometrics of DSGE Models," NBER Working Papers 14677, National Bureau of Economic Research, Inc., 2009

10. Galí, J., "Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations?" American Economic Review, 89 (1): 249-271, 1999

11. Justiniano A., Primiceri G., Tambalotti A., "Empirical DSGE Models: from the Great Moderation, to the Great Recession and beyond", DSGE Models in the Conduct of Policy: Use as intended, 2017

12. Lucas R., "Econometric policy evaluation: A critique," Carnegie-Rochester Conference Series on Public Policy, Elsevier, vol. 1(1), pages 19-46, 1976

13. Pfifer J., "A Guide to Specifying Observation Equations for the Estimation of DSGE Models", 2013

14. Rotemberg J., Woodford M., "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy", p. 297-361 in , NBER Macroeconomics

Annual, 1997

15. Sims, C., “Macroeconomics and Reality”, Econometrica, 48(1), 1-48., 1980

16. Smets F., Wouters R., “Shocks and frictions in US business cycles: A Bayesian DSGE approach”, American economic review, 97(3), 586-606, 2007

Conclusions and Further Developments

The above thesis concludes with a number of answers to the initial questions concerning both the theoretical soundness and alignment of the European Monetary Union as an Optimal Currency Area with the analysis conducted in the first chapter on its normative and modelling developments in the relevant literature, and the theoretical and empirical results of the analysis in the second and third chapters. In fact, the assumptions on fiscal differentiation and monetary centralisation lead to important resolved questions, especially in light of the current debates in the literature, on the possible interventions to be made with respect to the Eurozone countries and their national accounts in harmony with the rules imposed by the European treaties. Well, the results obtained allow, always bearing in mind, as in every scientific experiment of the reference "laboratory" expressed by the structure of the mathematical model exposed, that numerous interventions feared on possible positive effects of a possible application of a "golden rule" on public investments allow to achieve positive effects on some of the main macroeconomic aggregates. With regard to the positive effects and potential, as noted by the analysis carried out, it is possible to note immediately, in confirmation of some of the results achieved in the literature, how the application of a separation of public investment leads to an improvement, both in the presence of demand and supply shocks, of the volatility of both quantities such as private consumption, wages and output growth and of quantities such as Deficit/GDP and Cyclically-Adjusted Balances-Budget (CABB). In fact, they show promising results in terms of limiting crowding-out effects on private consumption conducted by an increase in the use of public capital within the production functions of firms, especially in the presence of demand shocks, leading to improvements in output growth and allowing for an improvement in tax rev-

enues. Positive effects would then appear in the area of the main public accounting items, where the application of the golden rule would allow a greater containment of the cyclically adjusted budget items starting from the medium term, probably due, and as analysed, to possible offsetting effects brought about by the evolution of the output gap itself conducted by the production dynamics created by not anymore limited public investments themselves. These results are then further differentiated in the presence of greater stabilisation on one side of output, in the presence of productivity shocks, and therefore on the side of the return on productive factors and of production itself, as well as greater stabilisation of debt in the presence of the same current expenditure shocks on the demand side. These greater offsetting effects would thus make it possible to bring the European public accounts in line with the Fiscal Rules imposed by the treaties themselves while at the same time maintaining compliance with the normative precepts of greater debt stabilisation by fiscal policy as analysed above and introduced and required by the Stability and Growth Pact (SGP). These scientific results allow us, therefore, not only to recognise how the separation of investment from the 3 per cent rule leads to positive results, but also how it has different results precisely through the inclusion of different stabilisation effects of fiscal policy. These results would thus be promising for a careful normative analysis aimed at analysing the effects on social welfare of such interventions, both on the normative side with respect to the treaties themselves, and on the same policies to be adopted in the presence of different types of shocks on the demand and supply side. In this sense it is possible to dwell on possible developments of this research and future improvements with respect to the reference model. Possible developments in this sense would be possible through an extension in the accumulation of capital through the inclusion of forms of adjustment costs as well as the possibility of differentiation, as specified in models in the literature, of different forms of public capital differentiated with respect to the respective levels of productivity. Specifically, it would be possible to include forms of public investment in infrastructural and non-infrastructural forms. Then it would be possible, on the demand side, to specify forms of persistence in consumption habits with possible shocks of preferences in order to analyse possible effects, from an empirical perspective and with more data as well, of public spending on the same crowding-out effect

in the presence of persistence in consumption habits. On the policy side, it would also be interesting to dwell on the developing forms of forward-guidance (FG) on fiscal policy as a new element in the literature, as well as a more complete structuring of financial frictions in the term structure of interest rates with respect to different risky and risk-free financial assets on financial markets. And finally, on the one hand, the inclusion of a more comprehensive financial and banking sector as well as a more in-depth examination of normative analysis with possible effects on social welfare. We leave such developments to the future of this research.