

Structural Panel Bayesian VAR with Multivariate Time-varying Volatility to jointly deal with Structural Changes, Policy Regime Shifts, and Endogeneity Issues

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Abstract

This paper improves a standard Structural Panel Bayesian Vector Autoregression model in order to jointly deal with issues of endogeneity, because of omitted factors and unobserved heterogeneity, and volatility, because of policy regime shifts and structural changes. Bayesian methods are used to select the best model solution for examining if international spillovers come from multivariate volatility, time variation, or contemporaneous relationship. An empirical application among Central-Eastern and Western Europe economies is conducted to describe the performance of the methodology, with particular emphasis on the Great recession and post-crisis periods. Findings from evidence-based forecasting are also addressed to evaluate the impact of an ongoing pandemic crisis on the global economy.

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1 Introduction

Vector Autoregressions (VARs) are widely used when studying macroeconomic–financial linkages to detect interdependencies and co-movements among multiple economic time-series. In the simplest form, error terms in the VAR models are assumed to have constant variances. While convenient, assuming time-invariant coefficients and variances, it turns out to be highly restrictive in capturing the evolution and thus the dynamics of multiple economic time-series. When time-varying series are introduced in a VAR to highlight the evolving relationship between multiple economic-financial variables, its state space structure need to be modeled and then used in the empirical analysis to estimate unobserved time variations and volatility. Since allowing time-varying coefficients and/or volatility introduces too many parameters than data points, the literature have proposed random processes to time variations or volatility to deal with the curse of dimensionality (see, for instance, Koop and Korobilis (2013)). The randomness in these time-varying parameters fits sufficiently well with Bayesian methods because there is no strict distinction between fixed (true) parameters and random samples.

In this context, multicountry Bayesian VAR (BVAR) models have given a new impulse to the literature to evaluate macroeconomic-financial linkages, to test specification hypotheses, and to conduct policy exercises (see, e.g., Ciccarelli et al. (2018), Canova and Ciccarelli (2009), Canova et al. (2007, 2012), and Koop (1996)). Nevertheless, although estimation of time-varying structures is feasible with a large homogeneous cross-section, heterogeneous dynamics due to an unexpected shock combined with not directly observed or measured factors make it difficult to exploit cross-sectional information to estimate time-series variations in multicountry setups. More precisely, these empirical models tend to be non-structural and constrained because of time-invariant or exogenous factors in the system. Thus, when formulating policies or forecasting, it is not possible to identify – for example – the reasons underlying different cross-country reactions given an unexpected shock, the causality between real and financial variables, how additional transmission channels allow shocks to spill over, and how economic and institutional implications matter in driving shock transmission.

My approach and empirical application aim to contribute to this debate. More precisely, they build on Pacifico (2019b), who developed a structural version of the BVAR – labeled as Structural Panel Bayesian VAR (SPBVAR) – in order to deal with model misspecification and unobserved heterogeneity problems when jointly modeling and quantifying multicountry data using the information contained in a large set of endogenous and economic–financial variables¹. The advantage of this approach is that it is easier to match endogenous variables to additional time-variant factors. However, the framework is valid if and only if prior specifications are satisfied and a fully hierarchical structure is provided. The latter focuses on a state-space factorization structure where the factors driving the coefficients of the SPBVAR are restricted to evolve over

¹See, for further studies, Pacifico (2019a), Pacifico (2020a), and Curcio et al. (2020).

time as random walks so as to: *(i)* reduce the number of parameters; *(ii)* allow for the evaluation of permanent shifts; *(iii)* investigate any type of coefficient factors via their interactions; and *(iv)* replace volatility changes by coefficient changes. This latter turns out to be highly restrictive to evaluate multiple time-varying change-points (or structural breaks) when studying macroeconomic and financial time-series. For example, international business cycle dynamics, policy interactions, and interdependencies and co-movements among different sectors and countries have changed substantially during the recent global crisis and successive consolidation periods. In addition, the increasing volatility and uncertainty in financial markets have confirmed the close volatility linkage between economic-financial data and thus the need to investigate shifts in either coefficients or volatility when describing these changes in a time-varying multicountry framework (see, e.g., Primiceri (2005), Canova and Gambetti (2009), Clark (2009), Cogley et al. (2010), and Sims and Zha (2006)).

The methodological implementation described in this paper consists of overtaking these limits and thus jointly deal with multiple structural breaks, policy regime shifts, and policy interactions among countries and sectors. The model suggested in this paper takes the name of multicountry SPBVAR with Multivariate Time-varying Volatility (SPBVAR-MTV). The two main differences with respect to a standard SPBVAR lie in an additional component to investigate fiscal and monetary policy implications and interactions, and in the variance-covariance matrix allowed to be time-variant. The latter is an useful way of modeling time-varying conditional second moments to provide an alternative to the stochastic volatility specification; therefore, in this context, volatility changes are not more replaced by coefficient changes. The computational costs involved in using that specification are moderate since the high dimensionality is avoided via Bayesian inference and Monte Carlo Markov Chain (MCMC) implementations. For instance, Kalman-Filter technique is used to get appropriate posterior distributions for time-varying coefficients and Metropolis-Hastings algorithm is used to draw posteriors for log-volatilities evolving over time. A Structural Normal Linear Regression (SNLR) model is obtained via Bayesian methods to work with smaller systems in which all the regressors are endogenous, observable, directly measured, and time-varying linear combinations of the right-hand variables of the SPBVAR-MTV model. I also account for three more indices² in order to quantify international spillover effects and thus evaluate their size (or intensity in terms of volatility) and dynamics (or spreading) among countries and sectors over time. *(i)* The Bilateral Net Spillover Effect (BNSE) is used to account for cross-unit interdependencies, feedback effects from the impulse variables, and temporary or persistent long-run effects of a potential shock (or excess spillover effects). *(ii)* The Systemic Contribution (SC) index is used to evaluate sequential features associated with systemic events. And *(iii)* the Total Contagion Index (TCI) is addressed to investigate contagion measures in real economy and financial markets when dealing with both issues of endogeneity (because of omitted variables and unobserved heterogeneity) and volatility (because of policy regime shifts and structural changes).

In this paper, the SPBVAR-MTV model incorporates the econometric literature on standard Time-Varying

²The first two indices have been constructed as Pacifico (2019b).

Parameter Vector Autoregressions (TVP-VARs) with stochastic volatility, become a benchmark model for analysing and forecasting the evolving inter-relationships between multiple macroeconomic variables (see, e.g., Koop et al. (2009), Koop and Korobilis (2013), Liu and Morley (2014), D’Agostino et al. (2013), and Clark and Ravazzolo (2015)). Despite the empirical success of these flexible time-varying models, they show a relevant limit about their potential and feasible over-parameterization. More precisely, on the methodological side, the literature makes out two popular Bayesian methods for TVP-VARs with stochastic volatility: Marginal Likelihood (ML), evaluating how likely the observed data are occurred within the system, and Deviance Information Criterion (DIC), trading off between model fit and model complexity. As regards ML estimates, they are usually obtained by using the harmonic mean³ of a conditional likelihood⁴ that tends to have a substantial bias selecting the wrong model (see, for instance, Chan and Grant (2015) and Frühwirth-Schnatter and Wagner (2008)). Concerning DIC procedure, the MCMC integration based on the conditional likelihood tends to associate higher probability to the most complex models (overfitting⁵).

The methodology proposed in this paper overtakes these limits by using analytically integrations for integrating out the time-varying volatilities. More precisely, integrated likelihood evaluation is achieved by integrating out the time-varying parameters analytically (e.g., Kalman-Filter technique), whereas the log-volatilities are integrated out numerically via importance sampling. The latter consists of two steps: (*i*) the Metropolis-Hastings algorithm is used to draw posteriors for time-varying log-volatilities from the proposal density distributio, and then (*ii*) the Newton-Raphson (N-R) method⁶ algorithm is involved to find the maximum of the (log) conditional density. In this way, the computational costs are further reduced focusing on band and sparse matrix algorithms instead of the conventional Kalman filter.

An empirical application is developed by accounting for the Central, Eastern, and Western European (CEWE) countries in order to include a large pool of advanced and emerging economies, with particular emphasis to the most recent recession and successive post-crisis periods. The United States (US) are included in the analysis to assess international spillover effects and possible contagion measures among financial markets. In this study, I focus on the latest two alternative monetary policy regimes that have been in place since the 1990 (see, for instance, Kallianiotis (2019)): (1) the Inflation Stabilization Era (ISE) from 1994 to 2008 and (2) the Zero Interest Rate Era (ZIRE) from 2008 to 2015. I also consider two more additional periods: (1) 2006q1 – 2009q4 to investigate possible commonality between financial markets and real economy during the Great recession and (2) 2010q1 – 2018q4 to evaluate fiscal implications and policy perspectives during post-crisis consolidation. Moreover, since most of countries joined in with Euro Area (EA), one is also able to investigate how policy regime shifts and endogeneity issues matter when studying macroeconomic–financial

³See, for instance, Gelfand and Dey (1994).

⁴It would correspond to the conditional density of the data given the log-volatilities, but marginal of the time-varying parameters.

⁵Overfitting and thus overestimation of effect sizes refers to a common problem in Bayesian Model Averaging since more complex models will always provide a somewhat better fit to the data than simpler models, where the ‘complexity’ stands (for example) for the number of unknown parameters. See, for instance, Pacifico (2020b).

⁶See, for instance, Tjalling (1995).

linkages. The analysis focuses on five main questions. First, I investigate how different economic–institutional characteristics affect the transmission of fiscal and monetary policy shocks among countries and sectors. Second, I investigate how policy interactions affect the benefits of consolidation among countries. Third, I evaluate how endogeneity and volatility issues affect inter-sector and inter-country linkages in panel setups. Fourth, I evaluate the role of policy regime shifts and their interactions when structural changes and contagion effects matter.

The remainder of this paper is organized as follows. Section 2 introduces the econometric model and the estimation procedure. Section 3 describes the dynamic analysis focusing on prior assumptions strategy, posterior distributions, and MCMC implementations. Section 4 presents the data and the empirical analysis. Section 5 addresses a counterfactual assessment on macroeconomic–financial linkages in multicountry dynamic setups by investigating in depth how structural changes and policy regime shifts affect the spreading and the evolution of international spillover effects, with particular attention on triggering events and policy recommendations for decision makers. The final section contains some concluding remarks.

2 Econometric Model

According to Pacifico (2019b), I extend and improve the standard version of the multicountry SPBVAR model in order to jointly account for time-varying parameters and multivariate volatility evolving over time.

Thus, the SPBVAR-MTV model developed in this study includes two additional components: (i) a set of lagged endogenous variables in order to assess different policy regimes and their interactions and (ii) time-varying log-volatilities to capture further evolving inter-relationships between multiple economic–financial data. The model has the form:

$$Y_{i,t}^m = \mu_t + \sum_{\lambda=1}^l \left[A_{it,j}^m(L) Y_{i,t-\lambda}^m + B_{it,j}^q(L) W_{i,t-\lambda}^q + \ddot{B}_{it,j}^{\tilde{q}}(L) \ddot{W}_{i,t-\lambda}^{\tilde{q}} + C_{it,j}^\xi(L) Z_{i,t-\lambda}^\xi \right] + \varepsilon_{it} \quad (1)$$

where the subscripts $i, j = 1, 2, \dots, N$ are country indices, $t = 1, 2, \dots, T$ denotes time, L stands for the lag operator, μ_t is an $NM \cdot 1$ vector of time-varying intercepts stacked for i , $A_{it,j}$ is an $NM \cdot NM$ matrix of coefficients for each pair of countries (i, j) for a given m , $Y_{i,t-\lambda}$ is an $NM \cdot 1$ vector of lagged variables of interest for each i for a given m , $B_{it,j}$ is an $NQ \cdot NQ$ matrix of coefficients for each pair of countries (i, j) for a given q , $W_{i,t-\lambda}$ is an $NQ \cdot 1$ vector including a set of lagged directly observed variables for each i for a given q , $\ddot{B}_{it,j}$ is an $N\tilde{Q} \cdot N\tilde{Q}$ matrix of coefficients for each pair of countries (i, j) for a given \tilde{q} , $\ddot{W}_{i,t-\lambda}$ is an $N\tilde{Q} \cdot 1$ vector including a set of additional lagged endogenous variables for each i for a given \tilde{q} , $C_{it,j}$ is an $N\xi \cdot N\xi$ matrix of coefficients for each pair of countries (i, j) for a given ξ , $Z_{i,t-\lambda}$ is an $N\xi \cdot 1$ vector including a set of lagged proxy⁷ variables for each i for a given ξ , and $\varepsilon_{it} \sim i.i.d.N(0, \Sigma_t)$ is an $NM \cdot 1$ vector of

⁷A proxy variable is an easily measurable variable used in place of a variable that cannot be directly measured.

disturbance terms. The subscripts $\lambda = 1, 2, \dots, l$ are lags for each of the $m = 1, \dots, M$ endogenous variables, $q = 1, \dots, Q$ (directly) observed variables to account for additional transmission channels, $\tilde{q} = 1, \dots, \tilde{Q}$ (directly) additional observed variables to account for policy shifts and interactions, and $\xi = 1, \dots, \Xi$ proxy variables to account for economic-institutional implications and macroeconomic imbalances among countries and sectors. Here, all variables in the system are endogenous and time-varying.

The two main differences with respect to a standard SPBVAR lie in the additional component $\ddot{B}_{it,j}\ddot{W}_{i,t-\lambda}$ and – particularly – in the variance-covariance matrix of the vector of innovations (ε_{it}). More precisely, $\Sigma_t = \text{diag}\left(\exp(h_{1t}), \exp(h_{2t}), \dots, \exp(h_{Nt})\right)$, where $h_t = (h_{1t}, h_{2t}, \dots, h_{Nt})'$ denotes the time-varying log-volatilities according to the following random walk:

$$h_t = h_{t-1} + v_t \quad \text{where} \quad v_t \sim N(0, \Sigma_h) \quad (2)$$

where $\Sigma_h = \text{diag}(\sigma_{h,1t}^2, \sigma_{h,2t}^2, \dots, \sigma_{h,Nt}^2)$ is a block diagonal covariance matrix and h_0 denotes the initial conditions to be estimated. The random-walk assumption in (2) is very common in the time-varying VAR literature, having the advantage of focusing on permanent shifts and reducing the number of parameters in the estimation procedure. The variance in v_t is allowed to be time-variant and it is an useful way of modeling time-varying conditional second moments to provide an alternative to the stochastic volatility specification. The main usefulness is that volatility changes are not more replaced by coefficient changes and the computational costs – involved in using that specification – are moderate since the high dimensionality can be avoided via Bayesian inference and MCMC integrations.

In equation (1), the dynamic relationships are allowed to be unit-specific and all the (potential) structural changes are allowed to vary over time. In addition, whenever the matrices $A_{it,j}(L)$, $B_{it,j}(L)$, $\ddot{B}_{it,j}(L)$, and $C_{it,j}(L)$ differ⁸ for some L , cross-unit lagged interdependencies matter, and then dynamic feedback and interactions among countries and variables are possible. Thus, the framework of the model (1) makes it able to connect the empirical results to the existing literature and contemporaneous developments when quantifying international business cycles, evaluating policy interactions, and performing conditional forecasting. Nevertheless, even if this feature adds flexibility to the specification, it is very costly. In fact, the number of coefficients is increased by $N(M + Q + \tilde{Q} + \Xi)l$ factors.

Let $k = N[M + Q + \tilde{Q} + \Xi]l$ be the number of all matrix coefficients in each equation of the SPBVAR-MTV model for each pair of countries (i, j) , a $1 \cdot k$ vector $X_t = (I, Y'_{i,t-1}, Y'_{i,t-2}, \dots, Y'_{i,t-l}, W'_{i,t-1}, W'_{i,t-2}, \dots, W'_{i,t-l}, \ddot{W}'_{i,t-1}, \ddot{W}'_{i,t-2}, \dots, \ddot{W}'_{i,t-l}, Z'_{i,t-1}, Z'_{i,t-2}, \dots, Z'_{i,t-l})'$ can be defined containing all lagged (endogenous) variables in the system for each i . Then, I define an $NMk \cdot 1$ vector $\gamma_{it,j}^k = \text{vec}(g_{it,j}^k)$ containing all columns, stacked into a vector⁹, of the matrices $A_t(L)$, $B_t(L)$, $\ddot{B}_t(L)$, and $C_t(L)$ for each pair of countries (i, j) for a given k , with $g_{it,j}^k = (\mu_t, A'_{it,j}, A'_{it,j}, \dots, A'_{it,j}, B'_{it,j}, B'_{it,j}, \dots, B'_{it,j}, \ddot{B}'_{it,j}, \ddot{B}'_{it,j}, \dots, \ddot{B}'_{it,j}, C'_{it,j}, C'_{it,j}, \dots, C'_{it,j})'$,

⁸See, for instance, Pacifico (2019b).

⁹The vec operator transforms a matrix into a vector by stacking the columns of the matrix, one underneath the other.

and $\gamma_t = (\gamma'_{1t}, \gamma'_{2t}, \dots, \gamma'_{Nt})'$ denoting the time-varying coefficient vectors, stacked for i , for each country–variable pair. With these specifications, I can express the model (1) in a simultaneous-equation form:

$$Y_t = \tilde{X}_t \gamma_t + E_t \quad (3)$$

where $Y_t = (Y'_{1t}, \dots, Y'_{Nt})'$ and $E_t = (\varepsilon'_{1t}, \dots, \varepsilon'_{Nt})'$ are $NM \cdot 1$ vector containing the observable variables of interest and the random disturbances of the model for each i for a given m , respectively, and $\tilde{X}_t = (I_{NM} \otimes X_t)$ contains all the lagged time-varying variables within the system stacked in X_t .

Now, because the coefficient vectors in γ_t vary in different time periods for each country–variable pair and there are more coefficients than data, it is impossible to eliminate γ_t . Thus, to avoid the curse of dimensionality, I adapt the framework in Pacifico (2019b) and assumes γ_t to have the following factor structure:

$$\gamma_t = \sum_{f=1}^F G_f \cdot \beta_{ft} + u_t \quad \text{with} \quad u_t \sim N(0, \Sigma_u) \quad (4)$$

where $F \ll NMk$ and $\dim(\beta_{ft}) \ll \dim(\gamma_t)$ by construction, $G_f = [G_1, G_2, \dots, G_F]$ are $NMk \cdot \varkappa^f$ matrices obtained by multiplying the matrix coefficients $(g^k_{it,j})$, stacked in the vector γ_t , by conformable matrices D_f with elements equal to zero and one, with \varkappa^f being a numerical index that depends on the typology of the factorization, u_t is an $NMk \cdot 1$ vector of unmodeled variations present in γ_t , and $E(u_t u'_t) = \Sigma_u = \Sigma_e \otimes V$, with Σ_e denoting the covariance matrix of the vector E_t that includes time-varying log-volatilities and $V = (\sigma^2 I_k)$ as in Kadiyala and Karlsson (1997). In this framework, unobserved heterogeneity and functional forms of misspecification are absorbed in the $\varkappa^f \cdot 1$ time-varying coefficient vectors β_{ft} . They are observable smooth linear functions of the lagged variables and thus can be easily estimated with a gain in efficiency and accuracy.

The idea is to shrink γ_t to a much smaller dimensional vector β_t , with $\beta_t = (\beta'_{1t}, \beta'_{2t}, \dots, \beta'_{Ft})'$, containing all the regression coefficients stacked into a vector. In this way, further investigations (e.g., policy regime shifts and interactions, international business cycles, and economic–institutional linkages) can be performed. Finally, the factorization of γ_t becomes exact as long as σ^2 converges to zero.

In equation (4), all factors are permitted to be time-varying, and thus time-variant structures can be obtained via implementations of MCMC algorithms. Moreover, time variations in the variance of shocks u_t to the factors β_{ft} are also allowed so that Y_t can capture (potential) structural changes among countries and variables. Running equations (3) and (4) for equation (1), the factorization is:

$$\sum_{f=1}^F G_f \cdot \beta_{ft} = G_1 \cdot \beta_{1t} + G_2 \cdot \beta_{2t} + \dots + G_F \cdot \beta_{Ft} \quad (5)$$

Given the factorization in equation (5), the reduced-form SPBVAR-MTV model in equation (3) can be

transformed into a Structural Normal Linear Regression model with an error covariance matrix of an Inverse–Wishart (IW) distribution¹⁰. By equations (3) and (4), the SNLR model can be written as

$$Y_t = \tilde{X}_t \left(\sum_{f=1}^F G_f \beta_{ft} + u_t \right) + E_t \equiv \chi_{ft} \beta_{ft} + \eta_t \quad (6)$$

where $\chi_{ft} \equiv \tilde{X}_t G_f$ is an $NM \cdot \varkappa^f$ matrix that stacks all coefficients and their possible interactions in the SPBVAR-MTV model in (1), with $\chi_t = \text{diag}(\chi'_{1t}, \chi'_{2t}, \dots, \chi'_{Ft})$, and $\eta_t \equiv \tilde{X}_t u_t + E_t \sim N(0, \sigma_t \cdot \Sigma_u)$ has a particular heteroskedastic covariance matrix that needs to be accounted for, with $\sigma_t = (I_N + \Sigma_h \otimes \tilde{X}'_t \tilde{X}_t)$.

To complete the specification, I suppose the following state-space structure for the time-varying regression coefficients:

$$\beta_t = \beta_{t-1} + \tilde{v}_t \quad \text{with} \quad \tilde{v}_t \sim N(0, P_t) \quad (7)$$

where $\beta_t = (\beta_{1t}, \beta_{2t}, \dots)'$, $P_t = \text{diag}(\bar{P}_{1t}, \bar{P}_{2t}, \dots, \bar{P}_{Ft})$ is a block diagonal matrix, and $\bar{P}_{ft} = (p_{ft} \cdot I_k)$, where p_{ft} controls the tightness (stringent conditions) of the factorization (f) of the time-varying coefficient parameters (β_t) in order to make them estimable. Here, some considerations on the innovations are in order: (i) the errors E_t , u_t , and v_t are mutually independent; (ii) the error terms η_t and \tilde{v}_t are allowed to be correlated between them; and (iii) v_t and η_t are correlated between them by construction.

Finally, if the factorization in equation (5) is exact, $\sigma^2 \rightarrow 0$ and one has to act on three competing models:

1. Model I (M_I): A benchmark model with no change-points, denoting the '**General Case**'.

Here, $\eta_t \sim N(0, \dot{\Sigma})$ and depends on the only disturbances contained in u_t , with

$\dot{\Sigma} = \text{diag}(\Sigma'_e, \Sigma'_e, \dots, \Sigma'_e)$. The M_I would correspond to the standard SPBVAR, with $h_0 = 0$, $h_t = h_{t-1}$, and $\Sigma_t = \Sigma$.

2. Model II (M_{II}): A benchmark model with change-points in the only log-volatilities, denoting the '**Special Case**'.

Here, $\eta_t \sim N(0, \tilde{\Sigma})$ and depends on the only disturbances contained in E_t , with $\tilde{\Sigma} = \sigma_t$. The M_{II} refers to the case of structural breaks because of (potential) unmodeled dynamics¹¹ in γ_t , with $h_0 \neq 0$, h_t evolving over time, $\Sigma_t \neq \Sigma$, and uncorrelatedness between the innovations η_t and \tilde{v}_t .

3. Model III (M_{III}): A benchmark model with change-points in either time-varying parameters or log-volatilities, denoting the '**Full Case**'.

¹⁰See, for instance, Pacifico (2019b), concerning the conformation of the SNLR model and the exact form of the β_{ft} 's and the G_f 's.

¹¹They denote both unobserved heterogeneity and misspecification problems.

Here, $\eta_t \sim N(0, \ddot{\Sigma})$ and depends on the disturbances contained in v_t and u_t , with $\ddot{\Sigma} = \sigma_t \cdot \text{diag}(\Sigma'_e, \Sigma'_e, \dots, \Sigma'_e)$.

The M_{III} refers to the case of structural breaks because of both unmodeled dynamics and policy regime shifts, with $h_0 \neq 0$, $\Sigma_t \neq \Sigma$, and h_t evolving over time.

Finally, once the (conditional) marginal likelihood¹² is obtained for any model, the exact and final solution can be obtained via MCMC integrations, corresponding to the *best*¹³ model solution with higher log Bayes factor (lBF):

$$lBF_{k,k^*} = \log\left(\frac{L(Y_T|M_k)}{L(Y_T|M_{k^*})}\right) \quad (8)$$

where M_k denotes all the possible model solutions accounting for the 'General Case' (M_I) with no change-points and M_{k^*} refers to all the possible model solutions according to the 'Special Case' (M_{II}) or the 'Full Case' (M_{III}). The higher lBF refers to the final solution having higher Posterior Model Probabilities (PMPs)¹⁴ according to a generalised version of the Kass and Raftery (1995)'s scale of evidence:

$$\left\{ \begin{array}{ll} 0 < lB_{\xi,l} < 2 & \text{no evidence for submodel } M_{\xi} \\ 2 < lB_{\xi,l} < 6 & \text{moderate evidence for submodel } M_{\xi} \\ 6 < lB_{\xi,l} < 10 & \text{strong evidence for submodel } M_{\xi} \\ lB_{\xi,l} > 10 & \text{very strong evidence for submodel } M_{\xi} \end{array} \right. \quad (9)$$

The methodology does not include studies focused on Markov-Switching dynamics to model covariance matrices of country-specific Markov chains¹⁵, since this paper aims to extend and improve recent works assessed when studying macroeconomic–financial linkages in multicountry dynamic panel setups. Nevertheless, to solve potential overfitting problems and cross-unit unobserved heterogeneity, Markov-switching models follow similar hierarchical prior specification strategy proposed in those works, where their empirical results met with positive feedback in the empirical analysis.

3 Dynamic Analysis

Before specifying prior assumptions and posterior distributions, I recall the state-space structure of the reparameterized SPBVAR-MTV model in (6):

¹²See Section 3.2 for further detail.

¹³Here, *best* stands for the model solution (or combination of predictors affecting the outcomes) with better prediction accuracy (see, for instance, Pacifico (2020b)).

¹⁴The PMPs, in Bayesian statistics, are computed by updating the prior probabilities through Bayes' theorem. See, for instance, Pacifico (2020b) concerning PMPs' computations in time-varying high dimensional data.

¹⁵See, for instance, Krolzig (1997, 2000) and Sims and Zha (2006).

$$Y_t = (\tilde{X}_t \cdot G)\beta_t + \eta_t \quad \left(\text{'Measurement Equation'} \right) \quad (10)$$

$$\beta_t = \beta_{t-1} + \tilde{v}_t \quad \left(\text{'State-Transition Equation'} \right) \quad (11)$$

3.1 Hierarchical Prior Setups and Assumptions

Supposing exact factorization in (5), in order to complete the model, I need to define prior moments on $(\Sigma_e^{-1}, \Sigma_h^{-1}, p_{f0}, \beta_0)$. Thus, collecting them in a vector ϕ_0 , with $\phi_0 = (\Sigma_e^{-1}, \Sigma_h^{-1}, p_{f0}, \beta_0)$ being prior densities, the conditional likelihood function can be derived from the sampling density $p(Y|\phi_0)$ by using a mixture hierarchical distribution. In other words, (i) a Normal distribution for factors β and log-volatilities h ; (ii) a Wishart distribution for Σ_e^{-1} ; and (iii) an Inverse-Gamma distribution for Σ_h and p_f , where $p_f = \text{vec}(P_t)$. That is,

$$\beta|\Sigma_e^{-1}, \Sigma_h^{-1}, Y \sim N\left(\hat{\beta}, \ddot{\Sigma}^{-1} \otimes (\chi' \chi)^{-1}\right) \quad (12)$$

$$\Sigma_e^{-1}|Y \sim W\left(S_p^{-1}, T - k - 1\right) \quad (13)$$

$$h|\Sigma_h^{-1}, Y \sim N\left(\alpha_h, V_h\right) \quad (14)$$

$$\Sigma_h^{-1}|Y \sim G\left\{\frac{\omega_h}{2}, \frac{S_h}{2}\right\} \quad (15)$$

$$p_f^{-1}|Y \sim G\left\{\frac{\omega_p}{2}, \frac{S_p}{2}\right\} \quad (16)$$

with $\ddot{\Sigma}$ depending on the benchmark model: (i) $\ddot{\Sigma} = \dot{\Sigma} = \Sigma_e$ in Model I; (ii) $\ddot{\Sigma} = \tilde{\Sigma}$ in Model II; and (iii) $\ddot{\Sigma} = \tilde{\Sigma}$ in Model III.

Here, α_h , V_h , ω_h , and ω_p are hyperparameters, $S_p = (Y_t - \hat{\beta}\chi_t)'(Y_t - \hat{\beta}\chi_t)$ is the sum of the squared errors, with $\hat{\beta} = (\Sigma_t \chi_t' \chi_t)^{-1} \cdot (\Sigma_t \chi_t' Y_t)$ referring to the OLS estimate of β , S_h denotes the least squares estimate of σ_h based on the (saturated) model, with $\sigma_h = \text{vec}(\Sigma_h)$. Finally, the equation (14) corresponds to the proposal distribution obtained by MCMC integration (such as Metropolis-Hastings algorithm¹⁶).

Since the above-mentioned hierarchical prior specification strategy is affected by common or subjective beliefs because of the marginal effect of economic-financial variables, the Independent Normal-Wishart Prior is used in this analysis so as to assume that tentative beliefs for $\phi_0 = (\Sigma_e^{-1}, \Sigma_h^{-1}, p_{f0}, \beta_0)$ are derived from

¹⁶See, for instance, Levine and Casella (2014).

separate considerations.

Given the state-space structure in equations (10) and (11), MCMC methods and implementations (such as Gibbs sampling, Kalman Filter algorithm, and Metropolis-Hastings algorithm) can be computed numerically and joint distributions characterised analytically. The first step is to suppose that data run from ($t = 0$) to ($t = T$) in order to obtain a training sample $(-\tau, 0)$ and then to estimate the features of the priors. When such a sample is unavailable, it is just sufficient to modify the expressions for the prior moments in equations (12)–(16) as:

$$\mathbf{p}\left(\Sigma_e^{-1}, \Sigma_h^{-1}, p_{f0}, h_0, \beta_0\right) = \mathbf{p}\left(\Sigma_e^{-1}\right) \cdot \prod_f \mathbf{p}\left(h_0\right) \cdot \prod_f \mathbf{p}\left(p_{f0}\right) \cdot \mathbf{p}\left(h_0\right) \cdot \mathbf{p}\left(\beta_0\right) \quad (17)$$

where

$$\mathbf{p}\left(\Sigma_e\right) = iW\left(z_1, \beta_1\right) \quad (18)$$

$$\mathbf{p}\left(\Sigma_h\right) = IG\left(\frac{\bar{\omega}_0}{2}, \frac{\bar{S}_0}{2}\right) \quad (19)$$

$$\mathbf{p}\left(p_{f0}\right) = IG\left(\frac{\bar{\omega}_0}{2}, \frac{\tilde{S}_0}{2}\right) \quad (20)$$

$$\mathbf{p}\left(h_0\right) = N\left(\alpha_0, V_0\right) \quad (21)$$

$$\mathbf{p}\left(\beta_0|\mathfrak{F}_{-1}\right) = N\left(\bar{\beta}_0, R_0\right) \quad (22)$$

Here, $N()$ stands for a Normal distribution, $iW()$ denotes an Inverse-Wishart distribution, $IG()$ indicates an Inverse-Gamma distribution, and \mathfrak{F}_{-1} refers to the information available at time -1 . The prior for β_0 in (22) and the law of motion for the factors imply that:

$$\mathbf{p}\left(\beta_t|\mathfrak{F}_{-1}\right) = N\left(\bar{\beta}_{t-1|t-1}, R_{t-1|t-1} + P_t\right) \quad (23)$$

where $\bar{\beta}_{t-1|t-1}$ and $R_{t-1|t-1}$ denote mean and variance-covariance matrix of the conditional distribution of $\bar{\beta}_{t|t}$, respectively.

All the hyperparameters are known. More precisely, collecting them in a vector δ , where $\delta = (z_1, \beta_1, \bar{\omega}_0, \bar{S}_0, \tilde{S}_0, \alpha_0, V_0, \bar{\beta}_0, R_0)$, they are treated as fixed and are either obtained from the data to tune the prior to the specific applications (such as $z_1, \bar{\omega}_0, \alpha_0$, and $\bar{\beta}_0$) or selected a priori to produce relatively loose priors (such as $\beta_1, \bar{S}_0, \tilde{S}_0, V_0$, and R_0). In this context, the only fully Bayesian approach that leads to analytical results requires

the use of a natural conjugate prior. According to equations (12), (13), and (15), the natural conjugate prior has the form¹⁷:

$$\beta_t | \Sigma_e^{-1}, \Sigma_h^{-1}, Y^T \sim N(\bar{\beta}_{t|t}, R_{t|t} + P_t) \quad \text{or} \quad \mathbf{p}(\beta_t | \Sigma_e^{-1}, \Sigma_h^{-1}, Y^T) = N(\bar{\beta}_{t|t}, R_{t|t} + P_t) \quad (24)$$

$$\Sigma_e | Y^T \sim iW(z_1, \beta_1) \quad \text{or} \quad \mathbf{p}(\Sigma_e | Y^T, \beta) = iW(z_1, \beta_1) \quad (25)$$

$$\Sigma_h | Y^T \sim IG\left(\frac{\omega_\alpha}{2}, \frac{S_V}{2}\right) \quad \text{or} \quad \mathbf{p}(\Sigma_h | Y^T, \beta) = IG\left(\frac{\omega_\alpha}{2}, \frac{S_V}{2}\right) \quad (26)$$

where $\bar{\beta}_{t|t}$ and $R_{t|t}$ are hyperparameters collected in the vector δ , and ω_α and S_V are parameters to be estimated.

If $P_t = 0$, allowing for time-variant factors and volatilities, draws of p_{ft} and σ_{hi}^2 can be taken from Normal-Inverse-Gamma distributions.

According to the natural conjugate prior (24), β_t depends on Σ_e and Σ_h . Thus, β_t , Σ_e , and Σ_h are not independent of one another. To allow different equations in the VAR to have different explanatory variables, previous specifications have to be modified. More precisely, given the SNLR model in (10), general priors that do not involve the restrictions inherent in the natural conjugate prior are the Independent Normal-Wishart (INW) and the Independent Inverted Gamma (IIG) distributions. The latter has different scale and shape parameters with respect to p_{ft} and is obtained by maximum likelihood estimates. Thus, the natural conjugate priors (24)–(26) can be re-written as:

$$\mathbf{p}(\beta_t, \Sigma_e^{-1}, \Sigma_h^{-1} | Y^T) = \mathbf{p}(\beta_t | Y^T) \cdot \mathbf{p}(\Sigma_e^{-1} | Y^T) \cdot \mathbf{p}(\Sigma_h^{-1} | Y^T) \quad (27)$$

where

$$\beta_t | Y^T \sim N(\bar{\beta}_{t|t}, R_{t|t}) \quad \text{or} \quad \mathbf{p}(\beta_t | Y^T) = N(\bar{\beta}_{t|t}, R_{t|t}) \quad (28)$$

$$\Sigma_e | Y^T \sim iW(z_1, \beta_1) \quad \text{or} \quad \mathbf{p}(\Sigma_e | Y^T, \beta) = iW(z_1, \beta_1) \quad (29)$$

$$\Sigma_h | Y^T \sim IG\left(\frac{\hat{\omega}_\alpha}{2}, \frac{\hat{S}_V}{2}\right) \quad \text{or} \quad \mathbf{p}(\Sigma_h | Y^T, \beta) = IG\left(\frac{\hat{\omega}_\alpha}{2}, \frac{\hat{S}_V}{2}\right) \quad (30)$$

Here, the hyperparameters $\hat{\omega}_\alpha$ and \hat{S}_V denotes scale and shape parameters, respectively, collected in the

¹⁷Analytical integration for integrating out the time-varying log-volatilities are explained in depth in Section 3.2.

vector δ . Moreover, the prior (28), with $P_t = 0$, allows for the prior covariance matrix $R_{t|t}$ to be anything the researcher chooses, rather than the restrictive $(\Sigma_e|Y^T \otimes R_{t|t} + \Sigma_h)$ form of the natural conjugate prior.

3.2 Posterior Distributions and MCMC Implementations

3.2.1 Conditional Likelihood and Kalman Filter Technique for Time-varying Parameters

The posterior distributions for $\phi = (\Sigma_e^{-1}, \Sigma_h^{-1}, p_{ft}, h_t, \{\beta_t\}_{t=1}^T)$ are calculated by combining the prior with the (conditional) likelihood for the initial conditions of the data. The resulting function is then proportional to

$$L(Y^T|\phi) \propto \left(\ddot{\Sigma}\right)^{-\frac{T}{2}} \cdot \exp\left\{-\frac{1}{2}\left[\Sigma_t\left(Y_t - (\tilde{X}_t G)\beta_t\right)'\right] \cdot \ddot{\Sigma}^{-1} \cdot \left[\Sigma_t\left(Y_t - (\tilde{X}_t G)\beta_t\right)\right]\right\} \quad (31)$$

where $Y^T = (Y_1, \dots, Y_T)$ denotes the data and $\phi = (\Sigma_e^{-1}, \Sigma_h^{-1}, p_{ft}, h_t, \{\beta_t\}_{t=1}^T)$ refers to the unknowns whose joint distribution needs to be found, with ϕ_{-k} standing the vector ϕ , excluding the parameter k .

Despite the dramatic parameter reduction obtained with equation (10), the analytical computation of posterior distributions ($\phi|Y^T$) is unfeasible. Thus, a variant of the Gibbs sampler approach – Kalman-Filter technique – can be used through MCMC integrations. More precisely, for the conditional posterior distribution of $(\beta_1, \dots, \beta_T|Y^T, \phi_{-\beta_t})$, it gives the following forward recursions for posterior means and the covariance matrix, respectively:

$$\bar{\beta}_{t|t} = \bar{\beta}_{t-1|t-1} + \left[R_{t|t-1}(\tilde{X}_t G)F_{t|t-1}^{-1}\right] \left[Y_t - (\tilde{X}_t G)' \bar{\beta}_{t-1|t-1}\right] \quad (32)$$

$$R_{t|t} = \left[I_k - \left(R_{t|t-1} \cdot (\tilde{X}_t G) \cdot F_{t|t-1}^{-1} \cdot (\tilde{X}_t G)'\right)\right] \cdot (R_{t|t-1}) \quad (33)$$

where

$$F_{t|t-1} = \left[(\tilde{X}_t G)' \cdot R_{t|t-1} \cdot (\tilde{X}_t G)\right] + \Sigma_e \quad (34)$$

$$R_{t|t-1} = R_{t-1|t-1} + \Sigma_h \quad (35)$$

Starting from $\beta_{T|T}$ and $R_{T|T}$, the marginal distributions of β_t can be computed by averaging over draws in the nuisance dimensions, and the Kalman filter backward can be run to characterise posterior distributions for ϕ :

$$\beta_t|\beta_{t-1}, Y^T, \phi_{-\beta_t} \sim N(\bar{\beta}_{t|t+1}, R_{t|t+1}) \quad \text{or} \quad \mathbf{p}(\beta_t|\beta_{t-1}, Y^T, \phi_{-\beta_t}) = N(\bar{\beta}_{t|t+1}, R_{t|t+1}) \quad (36)$$

where

$$\bar{\beta}_{t|t+1} = \tilde{R}_{t|t+1} \left[\left(R_{t|t+1}^{-1} \cdot \bar{\beta}_{t|t} \right) + \left(\sum_{t=1}^T (\tilde{X}_t G)' \cdot \Sigma_e^{-1} \cdot (\tilde{X}_t G) \hat{\beta} \right) \right] \quad (37)$$

$$R_{t|t+1} = \left[I_k - \left(R_{t|t} \cdot R_{t+1|t}^{-1} \right) \right] \cdot (R_{t|t}) \quad (38)$$

with

$$\tilde{R}_{t|t+1} = \left[\left(R_{t|t+1}^{-1} + \Sigma_h \right) + \left(\sum_{t=1}^T (\tilde{X}_t G)' \cdot \Sigma_e^{-1} \cdot (\tilde{X}_t G) \right) \right]^{-1} \quad (39)$$

$$\hat{\beta} = \left[(\tilde{X}_t G)' \cdot \Sigma_e^{-1} \cdot (\tilde{X}_t G) \right]^{-1} \cdot \left[(\tilde{X}_t G)' \cdot \Sigma_e^{-1} \cdot Y_t \right] \quad (40)$$

The equations (38) and (40) refer to the variance-covariance matrix of the conditional distribution of $\bar{\beta}_{t|t+1}$ and the Generalized Least Square (GLS) estimator, respectively. By rearranging the terms, equation (37) can be rewritten as

$$\bar{\beta}_{t|t+1} = \tilde{R}_{t|t+1} \cdot \left[\left(R_{t|t+1}^{-1} \bar{\beta}_{t|t} \right) + \left(\sum_{t=1}^T (\tilde{X}_t G)' \cdot \Sigma_e^{-1} \cdot Y_t \right) \right] \quad (41)$$

where $\bar{\beta}_{t|t+1}$ and $R_{t|t+1}$ denote the smoothed one-period-ahead forecasts of β_t and of the variance-covariance matrix of the forecast error, respectively.

The above output of the Kalman filter is used to generate a random trajectory for $\{\beta_t\}$ by using the backward recursion starting with a draw of $\{\beta_t\}$ from $N(\bar{\beta}_{T|T}, R_{T|T})$ ¹⁸. Given (36), the other posterior distributions can be defined as:

$$\Sigma_e | Y^T, \phi_{-\Sigma_e} \sim iW(\hat{z}_1, \hat{\beta}_1) \quad \text{or} \quad \mathbf{p}(\Sigma_e | Y^T, \phi_{-\Sigma_e}) = iW(\hat{z}_1, \hat{\beta}_1) \quad (42)$$

$$\Sigma_h | Y^T, \phi_{-\Sigma_h} \sim IG(\bar{\omega}_\alpha, \bar{S}_V) \quad \text{or} \quad \mathbf{p}(\Sigma_h | Y^T, \phi_{-\Sigma_h}) = IG(\bar{\omega}_\alpha, \bar{S}_V) \quad (43)$$

$$h_t | Y^T, \phi_{-h_t} \sim N(\tilde{a}_{h,\bar{m}}, \tilde{V}_{h,\bar{m}}) \quad \text{or} \quad \mathbf{p}(h_t | Y^T, \phi_{-h_t}) = N(\tilde{a}_{h,\bar{m}}, \tilde{V}_{h,\bar{m}}) \quad (44)$$

$$p_{ft} | Y^T, \phi_{-p_{ft}} \sim IG\left(\frac{\bar{\omega}_p}{2}, \frac{\tilde{S}_p}{2}\right) \quad \text{or} \quad \mathbf{p}(p_{ft} | Y^T, \phi_{-p_{ft}}) = IG\left(\frac{\bar{\omega}_p}{2}, \frac{\tilde{S}_p}{2}\right) \quad (45)$$

¹⁸See, for instance, Carter and Kohn (1994).

Here, some considerations are in order.

In equation (42), $\hat{z}_1 = z_1 + T$ and $\hat{\beta}_1 = \beta_1 + \Sigma_t u_t' u_t$, with z_1 and β_1 denoting the arbitrary degree of freedom and the arbitrary scale parameter, respectively. In this analysis¹⁹, $z_1 \cong N(M + M_v + M_c)$ and $\beta_1 \cong 1.0$.

In equation (43), $\bar{\omega}_\alpha = \bar{\omega}_0 + \hat{\omega}_\alpha$ and $\bar{S}_V = \bar{S}_0 + \hat{S}_V$, with $\bar{\omega}_0$ and \bar{S}_0 denoting the arbitrary degree of freedom (sufficiently small) and the arbitrary scale parameter, respectively, $\hat{\omega}_\alpha = \left(\sum_{t=1}^T \log(\tau_t) / t \right) + \log\left(\sum_{t=1}^T (1/\tau_t) \right) - \log(t)$ and $\hat{S}_V = (t \cdot \hat{\omega}_\alpha) / \left(\sum_{t=1}^T (1/\tau_t) \right)$ referring to the Maximum Likelihood Estimates (MLEs). In this analysis, $\tau_t = \{\tau_1, \dots, \tau_T\}$ is the random sample from the data $\{0, T\}$, $\bar{\omega}_0 \cong 0.1 \cdot \exp(M + M_v + M_c)$, $\bar{S}_0 \cong 0.01$, and \hat{S}_V is obtained by numerically computing $\hat{\omega}_\alpha$.

In equation (44), $\tilde{a}_{h,\bar{m}} = \alpha_0 \cdot \bar{\omega}$ is obtained by Metropolis-Hastings algorithm²⁰ and $\tilde{V}_{h,\bar{m}} = V_0 + \hat{\Sigma}_{h^*}$ is computed by MCMC-based EM algorithm²¹, with α_0 and V_0 denoting the arbitrary degree of freedom and the arbitrary scale parameter, respectively, and $\hat{\Sigma}_{h^*} = (\hat{\Sigma}_{1T}, \dots, \hat{\Sigma}_{nT})$ referring to the estimated covariance matrix for each i in a regime \bar{m} given t . In this analysis, $\tilde{a}_{h,\bar{m}}$ is constructed to be close to zero, with $\bar{\omega} \cong 0.1 \cdot \exp(NM)$, and h^* is an arbitrary vector given \bar{m} regimes according to the state-transition equations (2) and (11).

In equation (45), $\bar{\omega}_p = \bar{\omega}_0 + k$ and $\tilde{S}_p = \tilde{S}_0 + \Sigma_t (\beta_t^f - \beta_{t-1}^f)^{-1} \cdot (\beta_t^f - \beta_{t-1}^f)$, with \tilde{S}_0 and β_t^f denoting the arbitrary scale parameter and the f^{th} subvector of β_t , respectively. In this analysis, $\tilde{S}_0 \cong 0.1$, f refers to the factors described in equation (5), and k denotes the number of all matrix coefficients in each equation of the SPBVAR-MTV model in (1).

Finally, the last two hyperparameters to be defined in the vector δ are $\bar{\beta}_0 = \hat{\beta}_0$, with $\hat{\beta}_0$ denoting the Ordinary Least Squares (OLS) estimates of equation (10), and $R_0 = I_k$.

3.2.2 Metropolis-Hastings Algorithm for h_{it}

Suppose \bar{m} regimes, with $\bar{m} = 0, 1, \dots, s$, and use Metropolis-Hastings algorithm to draw posteriors for h_{it} from the proposal density distribution $\delta^*(h_{it})$ with probability $\alpha_{\bar{m}}$ equals:

$$\alpha_{\bar{m}} = \frac{\mathbf{p}\left(h_{it}^{\bar{m}} | h_{it-1}^{\bar{m}}, h_{it+1}^{\bar{m}-1}, Y^T, \{\beta_t\}, \Sigma_h\right) \cdot \delta^*(h_{it}^{\bar{m}-1})}{\mathbf{p}\left(h_{it}^{\bar{m}-1} | h_{it-1}^{\bar{m}}, h_{it+1}^{\bar{m}-1}, Y^T, \{\beta_t\}, \Sigma_h\right) \cdot \delta^*(h_{it}^{\bar{m}})} \quad (46)$$

According to the SNLR described in equation (10), let β_t^* denote the time-varying coefficient vectors when $\bar{m} \neq 0$ (some forms of break occur), the probability function takes the form:

$$\mathbf{p}\left(\beta_t | Y^T\right) \cdot \delta^*(\beta_t^* | \beta_t) \cdot \alpha(\beta_t^*, \beta_t) = \mathbf{p}\left(\beta_t^* | Y^T\right) \cdot \delta^*(\beta_t | \beta_t^*) \quad (47)$$

¹⁹See Section 4 for the form of the M_v 's and the M_c 's.

²⁰See Section 3.2.2.

²¹See Section 3.3.

where

$$\alpha(\beta_t^*, \beta_t) = \min \left[\frac{\mathbf{p}(\beta_t^* | Y^T) \cdot \delta^*(\beta_t | \beta_t^*)}{\mathbf{p}(\beta_t | Y^T) \cdot \delta^*(\beta_t^* | \beta_t)}, 1 \right] \cong \alpha_{\bar{m}} \quad (48)$$

In (48), $\alpha(\beta_t^*, \beta_t)$ is the probability to accept or reject a draw²². In addition, since the posterior distribution corresponds – by construction – to a multivariate normal distribution, the Optimal Acceptance Rates (OARs)²³ are:

$$\begin{cases} \bar{m} = 1 & \text{with } OAR = 44\% \\ 1 < \bar{m} \leq 5 & \text{with } OAR = 28\% \\ \bar{m} > 5 & \text{with } OAR = 23.4\% \\ & \text{(large dimension)} \end{cases} \quad (49)$$

3.3 Analytical Integration for Integrating out the Time-varying Volatilities

Given the proposal density distribution $\delta^*(\beta_t^*, \beta_t)$ in Section 3.2.2, with probability $\alpha(\beta_t^*, \beta_t)$ in (48), one needs to integrate out h_{it} using importance sampling. More precisely, in this study, I approximate the log conditional marginal density $\log[\mathbf{p}(h|Y, P_t, \Sigma_h, \beta_0, h_0)]$ by using a Gaussian density, which is then used as the importance sampling density. Thus, the Expectation-Maximization (EM) algorithm is used to find the maximum of the log conditional marginal density and consists of two steps: the Expectation step (E-step) and the Maximization step (M-step)²⁴.

3.3.1 Expectation Step (E-step)

The E-step is implemented by computing the following conditional expectation:

$$\Psi(h|h^*) = \mathbb{E}_{\beta|h^*} \left[\log \left(\mathbf{p}(h, \beta | Y, P_t, \Sigma_h, \beta_0, h_0) \right) \right] \quad (50)$$

where the expectation is taken with respect to $\mathbf{p}(\beta|Y, h^*, P_t, \Sigma_h, \beta_0, h_0)$ for an arbitrary vector h^* as Kroese and Chan (2014). More precisely, given the SNLR described in (10) and let \tilde{P}_β denote the first difference matrix, the state-transition equation (11) can be rewritten as:

$$\tilde{P}_\beta \cdot \beta = \bar{\alpha}_\beta + \tilde{\eta} \quad \text{with} \quad \tilde{\eta} \sim N(0, \Upsilon_\beta) \quad (51)$$

²²See, for instance, Jacquier et al. (1994).

²³See, for instance, Roberts and Rosenthal (2001).

²⁴See, for instance, McLachlan and Krishnan (1997) and Steele (1996).

where $\bar{\alpha}_\beta = (\beta'_0, 0, \dots, 0)'$, $\Upsilon_\beta = I_T \otimes P_t$, and \tilde{P}_β is a lower triangular matrix of dimension k . Thus, the conditional marginal density for β would be distributed as a Normal according to standard linear regression results:

$$\mathbf{p}(\beta|Y, h^*, P_t, \Sigma_h, \beta_0, h_0) \sim N(\tilde{\beta}, \Phi_\beta^{-1}) \quad (52)$$

where

$$\tilde{\beta} = \Phi_\beta^{-1} \cdot \varphi_\beta \quad (53)$$

$$\Phi_\beta = \left(\tilde{P}'_\beta \cdot \Upsilon_\beta^{-1} \cdot \tilde{P}_\beta \right) + \left[(\tilde{X}_t G)' \cdot \tilde{\Sigma}^{-1} \cdot (\tilde{X}_t G) \right] \quad (54)$$

$$\varphi_\beta = \left(\tilde{P}'_\beta \cdot \Upsilon_\beta^{-1} \cdot \tilde{P}_\beta \right) \bar{\beta}_0 + \left[(\tilde{X}_t G)' \cdot \tilde{\Sigma}^{-1} \cdot Y \right] \quad (55)$$

Here, the precision sampler of Chan and Jeliazkov (2009) can be used to sample from $N(\tilde{\beta}, \Phi_\beta^{-1})$ efficiently. In other words, the mean vector $\tilde{\beta}$ and the precision matrix Φ_β are computed using h^* .

The expectation in (50) can then be written in terms of an explicit expression:

$$\begin{aligned} \Psi(h|h^*) &= -\frac{1}{2} \left\{ (h - \alpha_0)' \cdot \left[\Pi'_h \cdot (I_T \otimes \Sigma_h^{-1}) \cdot \Pi_h \right] \cdot (h - \alpha_0) \right\} + \\ &\quad - \frac{1}{2} (\mathbf{1}'_{nT} \cdot h) - \frac{1}{2} \text{tr} \left\{ \text{diag}(e^{-h}) \cdot \left[\left((\tilde{X}_t G) \cdot \Phi_\beta^{-1} \cdot (\tilde{X}_t G)' \right) + \right. \right. \\ &\quad \left. \left. + \left(Y - (\tilde{X}_t G) \tilde{\beta} \right) \left(Y - (\tilde{X}_t G) \tilde{\beta} \right)' \right] \right\} + c \end{aligned} \quad (56)$$

where $\text{tr}()$ is the trace operator, c is a constant independent of h , $\mathbf{1}'_{nT}$ is a vector of ones, Π_h is a lower triangular matrix of dimension n , $\alpha_0 \cong \Pi_h^{-1} \cdot \bar{\alpha}_0$, with $\bar{\alpha}_0 = (h'_0, 0, \dots, 0)$, and $\varepsilon_i = \left(Y - (\tilde{X}_t G) \tilde{\beta} \right)$ is the error term.

3.3.2 Maximization Step (M-step)

The M-step consists of maximizing the function $\Psi(h|h^*)$ with respect to h by using the Newton-Raphson method²⁵. Thus, the gradient (g_Ψ) and the Hessian (\mathcal{H}_Ψ) are, respectively:

²⁵See, for instance, Kroese et al. (2011).

$$g_{\Psi} = - \left[\Pi'_h \left(I_T \otimes \Sigma_h^{-1} \right) \Pi_h \cdot (h - \alpha_0) \right] - \frac{1}{2} (\mathbf{1}_{nT} - e^{-h} \odot \bar{\theta}) \quad (57)$$

$$\mathcal{H}_{\Psi} = - \left[\Pi'_h \left(I_T \otimes \Sigma_h^{-1} \right) \Pi_h \right] - \frac{1}{2} (e^{-h} \odot \bar{\theta}) \quad (58)$$

where ' \odot ' refers to the entry-wise product and $\bar{\theta} = (s_{1T}^2 + \hat{\varepsilon}_{1T}^2, \dots, s_{nT}^2 + \hat{\varepsilon}_{nT}^2)'$, with s_{iT}^2 denoting the $i - th$ diagonal element of $\left[(\tilde{X}_t G) \cdot \Phi_{\beta}^{-1} \cdot (\tilde{X}_t G)' \right] \cong V_0$ and $\hat{\varepsilon}_{iT}^2$ denoting the $i - th$ element of $\left[Y - (\tilde{X}_t G) \tilde{\beta} \right] \cong \hat{\Sigma}_{h^*}$.

Here, \mathcal{H}_{Ψ} is negative definite for all h , ensuring fast convergence of the N-R method, and Φ_{β} , since it is a band matrix, guarantees that its Cholesky factor L_{β} can be obtained without further effort. More precisely, given $\Phi_{\beta} = L_{\beta} \cdot L'_{\beta}$, with L_{β} denoting a lower triangular matrix, $\mathcal{G} = L'_{\beta} \cdot (\tilde{X}_t G)$ can be obtained by solving the linear system $L_{\beta} \mathcal{G} = (\tilde{X}_t G)$ for \mathcal{G} . Thus, the diagonal elements of $\left[(\tilde{X}_t G) \cdot \Phi_{\beta}^{-1} \cdot (\tilde{X}_t G)' \right]$ will be the row sums of the squares of \mathcal{G} .

Finally, the MCMC Expectation-Maximization (MCMC-EM) algorithm can be summarised in this way:

- A. **E-step:** Compute Φ_{β} , $\tilde{\beta}$, and $\bar{\theta}$ given the current value $h_{it}^{\nu-1}$, with ν denoting the $\nu - th$ iteration.
- B. **M-step:** Maximise $\Psi(h|h^{\nu-1})$ with respect to h by the N-R method. That is,

$$h^{\nu} = \underset{h}{\operatorname{argmax}} \quad \Psi(h|h^{\nu-1})$$

- C. Compute g_{Ψ} and \mathcal{H}_{Ψ} from Ψ_{β} , $\tilde{\beta}$, and $\bar{\theta}$ obtained in (A), and set $h = h^{(\bar{m}-1, \nu-1)}$.

- D. Update $h^{(\bar{m}, \nu-1)} = h^{(\bar{m}-1, \nu-1)} - \left(\mathcal{H}_{\Psi}^{-1} \cdot g_{\Psi} \right)$.

- E. Repeat steps (A)-(D) until some convergence criterion is met at the OARs in (49). Thus, terminate the iteration and set $h^{\nu} = h^{(\bar{m}, \nu-1)}$, denoting that a certain change-point among time-varying coefficient vectors and log-volatilities has been assessed correctly.

4 Data Description and Empirical Model

The SPBVAR-MTV model in (1) contains 17 country-specific models, including the United States, 8 Central-Eastern Europe (CEE) economies²⁶ and 8 Western Europe (WE) economies²⁷. The CEE and WE countries – except for *SL* – also refer to European emerging and advanced economies, respectively. Moreover, all the European countries are Eurozone members, with the exception of *CZ*, *HU*, and *PO*, and thus inter-sector and inter-country linkages can be investigated in depth.

²⁶Czech Republic (*CZ*), Hungary (*HU*), Estonia (*EE*), Latvia (*LV*), Lithuania (*LT*), Poland (*PO*), Slovak Republic (*SK*), and Slovenia (*SL*).

²⁷Austria (*AT*), Belgium (*BE*), France (*FR*), Germany (*DE*), Ireland (*IE*), Italy (*IT*), Portugal (*PT*), and Spain (*ES*).

The dataset contains the following collection of variables. (i) Six endogenous variables are involved to describe real economy ($real_{it,j}$) and financial markets ($fin_{it,j}$): three real variables to capture real business cycles (general government spending, gross fixed capital formation, GDP growth rate) and three financial variables to highlight the situation in the lending markets (bank leverage, flow of credit into economy, inflation rate). (ii) Bilateral flows of trade ($rweights_{it,j}$) and financial transactions ($fweights_{it,j}$) are used to deal with endogeneity issues when studying international spillover effects among countries and variables. (iii) Three policy variables ($policy_{it,j}$) are used to investigate monetary and fiscal policy implications and interactions among countries and sectors (international interest policy rate, general government debt, and current account balance). (iv) Five (directly) observed variables are used as *proxy* variables ($structures_{it,j}$) to evaluate economic–institutional implications in driving the evolution of international spillovers and transmission of shocks over time among countries and variables: three indicators to deal with internal imbalances (financial consumption expenditure, private sector consumption, change in unemployment rate); one indicator to capture competitiveness developments and catching-up effects (nominal labour cost); and one indicator to monitor the probabilities of transitions between expansion/recession phases in business cycles and potential macroeconomic imbalances (house price indices). (v) The real GDP per capita in logarithmic form (*productivity*) is used to evaluate the size and the spreading of international spillover effects over time among countries and sectors given an unexpected shock. The $weights_{it,j}$ ²⁸, the $policy_{it,j}$, and the $structures_{it,j}$ components are treated endogenously and used to jointly deal with endogeneity issues, structural changes, and policy regime shifts.

The series are expressed in standard deviations with respect to the same quarter of the previous year (q_t/q_{t-1}), and seasonally and calendar adjusted. All variables are used in year-on-year growth rates and all data comes from OECD data source.

The estimation sample covers the period from December 1994 to December 2018. It amounts, without restrictions, to 26,384 regression parameters. More precisely, each equation of the time-varying SPBVAR-MTV in (1) has $k = [17(6+2+3+5)] \cdot 1 = 272$ coefficients, and there are 97 equations in the system. Given the structural conformation of the model and a sufficiently large number of quarters describing economic–institutional and policy implications, it is able to capture: (i) endogeneity issues because of unobserved heterogeneity and misspecified dynamics across the sample; (ii) interdependency, commonality, and homogeneity because of potential international macroeconomic-financial linkages among countries and sectors; and (iii) relevant monetary and fiscal policy interactions and contagion measures.

Given the factor structure in (4), I assume that the coefficient vector γ_t depends on ten factors. Thus,

²⁸The $weights_{it,j}$ component corresponds to the sum of $rweights_{it,j}$ and $fweights_{it,j}$.

$$\begin{aligned}
G_f \beta_{ft} &= G_1 \beta_{1t} + G_2 \beta_{2t} + G_3 \beta_{3t} + G_4 \beta_{4t} + G_5 \beta_{5t} + G_6 \beta_{6t} + G_7 \beta_{7t} + G_8 \beta_{8t} + \\
&+ G_9 \beta_{9t} + G_{10} \beta_{10t} + u_t
\end{aligned} \tag{59}$$

where, stacking for t , $\beta_f = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9, \beta_{10})'$ contains all time-varying coefficient vectors to be estimated. Given the factorization in (59), the SNLR model in (10) can be written as:

$$Y_t = \tilde{X}_t \left(\sum_{f=1}^{10} G_f \beta_{ft} + u_t \right) + E_t \equiv \sum_{f=1}^{10} \chi_{ft} \beta_{ft} + \eta_t \quad \text{with} \quad \tilde{X}_t = \left(I_{NM} \otimes X_t \right) \tag{60}$$

According to diagnostic tests (Table 1), the marginal (conditional) likelihood estimation confirms the exact γ_t 's factorization in (59) and the estimates are asymptotically consistent given the absence of serial correlations across the residuals. Thus, the specified factors in (60) can be made clearer and estimated in terms of posterior means.

The indicators $\chi_{1t} \beta_{1t}$ and $\chi_{2t} \beta_{2t}$ are $NM \cdot 1$ vectors of observable country-specific indicators for Y_t , and account for the only $real_{it,j}$ and $fin_{it,j}$ components, respectively, in order to evaluate international spillover effects and transmission of shocks among countries in real economy and financial markets.

The indicators $\chi_{3t} \beta_{3t}$ and $\chi_{4t} \beta_{4t}$ are $NM \cdot 1$ vectors of observable country-specific effects for Y_t , and account for one additional component: (1) $real_{it,j}$ with $policy_{it,j}$ and (2) $fin_{it,j}$ with $policy_{it,j}$. They are able to investigate monetary and fiscal policy implications and interactions among countries in the real and the financial dimensions, respectively.

The indicators $\chi_{5t} \beta_{5t}$ and $\chi_{6t} \beta_{6t}$ are $NM \cdot 1$ vectors of observable country-specific effects for Y_t , and account for two components further: (1) $real_{it,j}$ with $rweights_{it,j}$ and $policy_{it,j}$, and (2) $fin_{it,j}$ with $fweights_{it,j}$ and $policy_{it,j}$. They are able to jointly evaluate how international transmission channels and policy issues affect the size and spreading of spillover effects given an unexpected shock among countries in real economy and financial markets, respectively.

The indicators $\chi_{7t} \beta_{7t}$ and $\chi_{8t} \beta_{8t}$ are $NM \cdot 1$ vectors of observable country-specific effects for Y_t , and account for one component further: (1) $real_{it,j}$ with $rweights_{it,j}$, $policy_{it,j}$, and $structures_{it,j}$, and (2) $fin_{it,j}$ with $fweights_{it,j}$, $policy_{it,j}$, and $structures_{it,j}$. They are able to assess the role that international transmission channels, macroeconomic-institutional implications, potential macroeconomic imbalances, and policy implications play in allowing shocks to spill over among countries in real and financial sectors, respectively.

The indicator $\chi_{9t} \beta_{9t}$ is a $NM \cdot M_v$ vector of observable cross-country variable-specific effects for Y_t , where $M_v = (M_{v1}, M_{v2}, M_{v3}, M_{v4})$ denotes the number of variable groups: (i) $M_{v1} = real_{it,j}$ and $policy_{it,j}$; (ii) $M_{v2} = fin_{it,j}$ and $policy_{it,j}$; (iii) $M_{v3} = real_{it,j}$, $rweights_{it,j}$, $policy_{it,j}$, and $structures_{it,j}$; and (iv) $M_{v4} =$

$fin_{it,j}$, $fweights_{it,j}$, $policy_{it,j}$, and $structures_{it,j}$. The variable-specific factor is able to investigate endogeneity issues, policy regime shifts, and multivariate structural breaks among variables in real economy and financial markets.

Finally, the indicator $\chi_{10t}\beta_{10t}$ is a $NM \cdot M_c$ vector of observable common effects for Y_t , where $M_c = (M_{c1}, M_{c2})$ denotes the number of common groups: (i) $M_{c1} = real_{it,j}$, $fin_{it,j}$, and $policy_{it,j}$ and (ii) $M_{c2} = real_{it,j}$, $fin_{it,j}$, $weights_{it,j}$, $policy_{it,j}$, and $structures_{it,j}$. The common factor is able to assess idiosyncratic spillover effects due to different reactions or co-movements among countries and variables for a given common unexpected shock in the real and financial dimensions. This latter (M_{c2}) is also used to investigate and then quantify contagion measures during triggering events and policy regime shifts.

Table 1: Diagnostic Tests

Test	Test Statistics	Degrees of freedom	p-value
LGB_π	16573	1649	0.00
P_π	837.3	1261	0.30
MLE_f	67.44	10	0.00

Here, LGB_π stands for a Multivariate Ljung-Box Test of the series, with lags $\pi = 30$; P_π refers to the Portmanteau (Asymptotic) Test on the residuals, with lags $\pi = 30$; MLE_f is the Marginal (Conditional) Likelihood Estimation Test obtained through the Schwartz approximation, with $f = 10$.

Dynamic analyses have been conducted via accurate MCMC implementations. The total number of draws was $5000 + 1000 = 6000$, which corresponds to the sum of the final number of draws to discard and draws to save, respectively. A total of 1000 draws has been used to conduct posterior inference at each t . The outcomes absorb the conditional forecasts computed for a time frame of 9 quarters (2 years and a quarter) in order to also address potential findings concerning the impact of an ongoing pandemic crisis on the global economy. The natural conjugate prior refers to four subsamples: (i) 1994q4–2008q3 and (ii) 2008q4–2015q4 in order to evaluate how monetary policy regimes affect the dynamics of the GDP growth; and (iii) 2006q1–2009q4 and (iv) 2010q1–2018q4 in order to highlight the impact of the most recent financial crisis and fiscal consolidation when investigating international spillover effects.

According to the log Bayes Factor in (8) and the exact factorization in (59), most of the time-varying estimated coefficient vectors ($\hat{\beta}_{ft}$) embrace the 'Full Case' (M_{III}), where structural changes and policy regime shifts hold in either time-varying parameters or log-volatilities (see Table 2). It accounts for: two of the country-specific factors ($\chi_{7t}\hat{\beta}_{7t}$ and $\chi_{8t}\hat{\beta}_{8t}$); the cross-country variable-specific factor ($\chi_{9t}\hat{\beta}_{9t}$) belonging to the variable groups M_{v3} and M_{v4} ; and the common factor ($\chi_{10t}\hat{\beta}_{10t}$) belonging to the common group M_{c2} . All remaining empirical results embrace the 'Special Case' (M_{II}), except for two factors concerning the 'General Case' (M_I). They correspond to the first two country-specific indicators ($\chi_{1t}\hat{\beta}_{1t}$ and $\chi_{2t}\hat{\beta}_{2t}$). These findings highlight the performance and then the potential of the SPBVAR-MTV model pointing out that: (i) change-points and policy regime shifts need to be taken into account when dealing with macroeconomic–financial linkages in multicountry dynamic panel setups; (ii) multiple structural changes in time-varying

log-volatilities occur when evaluating international transmission channels and policy implications among countries and sectors in both the real and the financial dimensions; and (iii) change-points and policy regime shifts in either time-varying coefficients or log-volatilities occur when accounting for economic–institutional implications to investigate unobserved heterogeneity and misspecified dynamics among country- and variable-specific factors and common features.

Table 2: Empirical Results on the Benchmark Model

Time-varying Factors	'General Case' (M_I)	'Special Case' (M_{II})	'Full Case' (M_{III})
$\chi_{1t}\hat{\beta}_{1t}$, $\chi_{2t}\hat{\beta}_{2t}$	IBF >10	$2 \leq lBF \leq 6$	$0 \leq lBF \leq 2$
$\chi_{3t}\hat{\beta}_{3t}$, $\chi_{4t}\hat{\beta}_{4t}$	$6 \leq lBF \leq 10$	IBF >10	$2 \leq lBF \leq 6$
$\chi_{5t}\hat{\beta}_{5t}$, $\chi_{6t}\hat{\beta}_{6t}$	$0 \leq lBF \leq 2$	IBF >10	$6 \leq lBF \leq 10$
$\chi_{7t}\hat{\beta}_{7t}$, $\chi_{8t}\hat{\beta}_{8t}$	$0 \leq lBF \leq 2$	$6 \leq lBF \leq 10$	IBF >10
$\chi_{9,1t}\hat{\beta}_{9,1t}$, $\chi_{9,2t}\hat{\beta}_{9,2t}$	$6 \leq lBF \leq 10$	IBF >10	$2 \leq lBF \leq 6$
$\chi_{9,3t}\hat{\beta}_{9,3t}$, $\chi_{9,4t}\hat{\beta}_{9,4t}$	$0 \leq lBF \leq 2$	$6 \leq lBF \leq 10$	IBF >10
$\chi_{10,1t}\hat{\beta}_{10,1t}$	$6 \leq lBF \leq 10$	IBF >10	$2 \leq lBF \leq 6$
$\chi_{10,2t}\hat{\beta}_{10,2t}$	$0 \leq lBF \leq 2$	$6 \leq lBF \leq 10$	IBF >10

The first column denotes the time-varying factors and the other three columns refer to all three (potential) best benchmark models. The *best* model solution is highlighted in bold and corresponds to the highest log Bayes Factor with respect to the generalised version of the Kass and Raftery (1995)'s scale of evidence as (49).

5 Macroeconomic-financial Linkages with Structural Changes and Policy Regime Shifts: a Counterfactual Assessment

The aim of this empirical analysis is to improve the existing literature on macroeconomic–financial linkages in multicountry dynamic panel setups when dealing with either endogeneity or volatility issues. Thus, the SPBVAR-MTV model is appropriate to be used for investigating in depth how structural changes and policy regime shifts affect the intensity and the evolution (or dynamics) of international spillover effects among countries and sectors.

More precisely, intra-CEWE dynamics are assessed in four related contexts. (i) Firstly, international spillovers and policy issues are evaluated in an international and broader European setting (Section 5.1). (ii) Then, the empirical results are reevaluated accounting for additional time-varying factors to address endogeneity issues because of unobserved heterogeneity and misspecified dynamics (Section 5.2). (iii) A deepened investigation is further conducted accounting for multivariate change-points and policy regime shifts during different phases of financial cycles, where many emerging market economies have experienced a large surge of capital inflow following the notably expansionary monetary policies of major advanced countries, and fiscal consolidation adjustments, playing a central role in the disinflation process (Section 5.3). (iv) Finally, policy implications and suggestions for decision makers are addressed according to all the aforementioned findings (Section 5.4).

5.1 International Spillovers and Policy Issues among CEWE Economies

In Figure 1, where I consider the first two country-specific indicators (χ_{1t}, χ_{2t}) for the overall sampled time-series, all the CEE economies tend to be net receivers (or inward spillovers) in the real dimension and thus would be affected by the conditional impulse responses received from the European advanced countries (net senders). Overall, the size of the spillover effects is larger in the financial dimension because of highly strong cross-country interdependencies. These results find confirmation in previous related works such as Pacifico (2019a,b), Pacifico (2020a), and Curcio et al. (2020).

However, contrary to them, the findings highlight that a consistent cross-country heterogeneity across the spillovers' dynamics would matter more in financial markets (Figure 1b), while a persistent degree of homogeneity and larger co-movements among countries tend to occur in the real dimension despite stronger inter-country linkages in the financial one (Figure 1a). The results confirm the presence of potential functional form of misspecifications that need to be investigated thoroughly when studying macroeconomic-financial linkages.

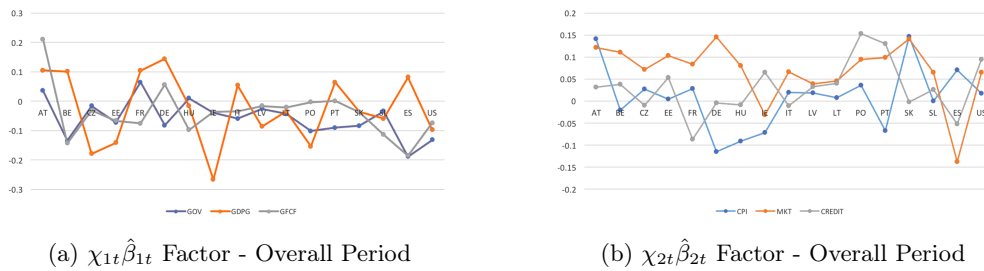


Figure 1: Systemic Contributions of the *productivity* given a 1% shock to real and financial dimensions are drawn as standard deviations of the variables in the system and in year-on-year growth rates. They account for $\chi_{1t}\hat{\beta}_{1t}$ (plot a) and $\chi_{2t}\hat{\beta}_{2t}$ (plot b) cross-country indicators, referring to the overall sampled time-series, where $\hat{\beta}_{1t}$ and $\hat{\beta}_{2t}$ are posterior means.

In Figure 2, I account for the two country-specific indicators dealing with policy issues and their interactions (χ_{3t}, χ_{4t}). In contrast to the previous results, most CEE economies tend to be net senders (outward international spillovers) in their real dimension (Figure 2a). Cross-country heterogeneity follows to be consistent and stronger in real economy and even more in financial markets (Figure 2b). In addition, larger commonality and homogeneity matter across the spreading and the intensity of spillover effects. The findings confirm the importance to account for either endogeneity and volatility issues.

From a global perspective, the same dynamic behaviour is observed in the transmission of US financial shocks, with outward spillover effects. The results are consistent and robust with the more recent literature on multicountry dynamic panel setups. More precisely, they confirm that US seem to be an important driver in allowing unexpected shocks to spill over and thus affecting European financial markets, mainly regarding CEE economies with inward spillovers. Then, intra-country shocks directly affect a country's own output growth in the real economy because of consistent cross-country interdependencies²⁹.

²⁹See, for instance, Pacifico (2020a) and Curcio et al. (2020).

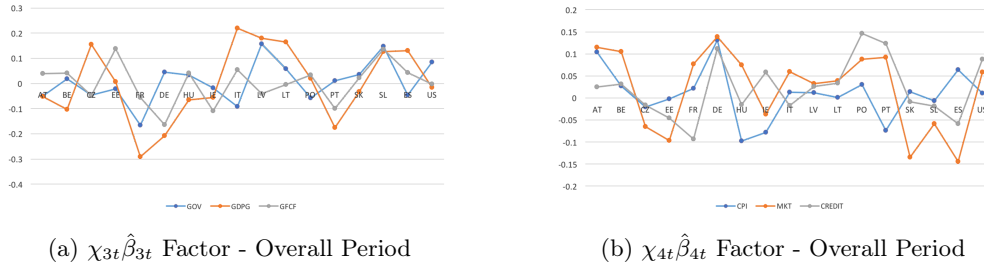


Figure 2: Systemic Contributions of the *productivity* given a 1% shock to real and financial dimensions are drawn as standard deviations of the variables in the system and in year-on-year growth rates. They account for $\chi_{3t}\hat{\beta}_{3t}$ (plot *a*) and $\chi_{4t}\hat{\beta}_{4t}$ (plot *b*) cross-country indicators, dealing with policy regime shifts and structural changes, where $\hat{\beta}_{3t}$ and $\hat{\beta}_{4t}$ are posterior means.

Established that structural changes and policy shifts affect macroeconomic-financial linkages among countries in an international and broader context, I consider the first two group-variable factors ($\chi_{9,1t}, \chi_{9,2t}$) in order to examine in depth how monetary policy regimes and fiscal implications drive international shocks among real and financial sectors (Figure 3). Here, the countries are grouped in three clusters: WE, CEE, and BLS³⁰.

During the ISE Regime, most countries tend to be net receivers and net senders in the real and the financial dimensions, respectively (Figure 3a). Moreover, larger homogeneity in the spreading and the intensity of international spillovers would matter more among CEE economies given an unexpected financial shock. From a policy perspective, since in that period (1994 – 2008) the only country joined in with EU was Slovenia, the results highlight that the transmission of shocks among sectors are mainly affected by highly strong cross-country interdependencies rather than policy implications (e.g., because of high persistent inflation among emerging and then CEE countries).

During ZIRE Regime, emerging economies become net senders in real economy with larger spillover effects than advanced economies (Figure 3b). In financial markets, international spillovers show higher intensity than real economy due to a deeper state of severe fiscal reforms. In addition, higher co-movements among sectors in both the real and the financial dimensions matter than the ISE period because of persistent financial measures to foster the output stabilization.

From a modeling perspective, inward spillovers during the ISE Regime highlight that CEE countries are less competitive than WE economies, requiring appropriate emergency programs in order to be up against triggering events. From a policy perspective, outward spillovers during the ZIRE Regime in real economy point out that more stringent fiscal constraints would need to support developing economies in absorbing the effects of unexpected financial shocks (misspecified dynamics).

³⁰It stands for the Baltic States (*EE, LV, and LT*).

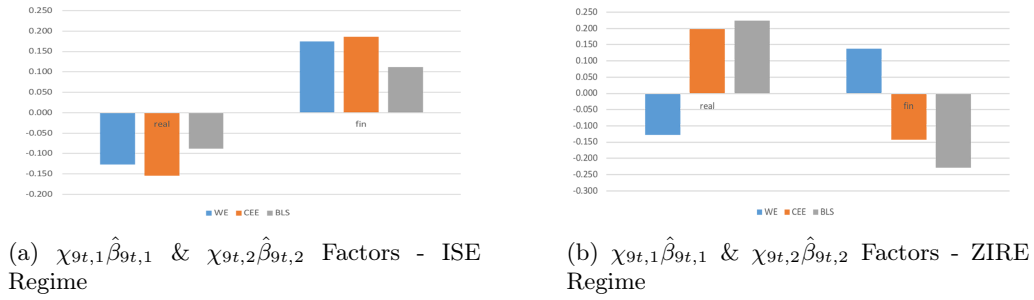


Figure 3: Systemic Contributions of the *productivity* given a 1% shock to real and financial dimensions are drawn as standard deviations of the variables in the system and in year-on-year growth rates. They account for the variable-specific indicators $\chi_{9t,1}\hat{\beta}_{9t,1}$ and $\chi_{9t,2}\hat{\beta}_{9t,2}$ during ISE (plot *a*) and ZIRE (plot *b*) regimes, dealing with policy issues and their interactions, where $\hat{\beta}_{9t,M_v}$'s are posterior means with $\hat{v} = v1, v2$.

5.2 Unobserved Heterogeneity and Misspecified Dynamics accounting for Additional Time-variant Factors

Accounting for additional time-variant factors in order to investigate in depth policy regime shifts and structural breaks along with endogeneity issues, relevant empirical results and policy perspectives are derived (Figure 4).

As regards $weights_{it,j}$ component (standing for omitted factors), most countries follow to show inward and outward spillovers in the real and the financial dimensions, respectively. Despite a consistent heterogeneity persists in their own output growth responses, larger co-movements matter accounting for additional shock transmission channels in real economy and even more in financial markets because of stronger cross-country financial linkages (Figures 4a and 4b). From a global perspective, outward spillovers in US confirm the importance about international spillover effects affecting European financial shocks (see, for instance, Pacifico (2020a) and Curcio et al. (2020)). From a modeling perspective, capital flows tend to matter more than trade flows in allowing shocks to spill over among countries (see, for instance, Pacifico (2019b) and Pacifico (2020a)). However, higher intra-CEWE heterogeneity in the financial dimension, in terms of spillovers' intensity and spreading, emphasises more consistent difference among financial markets due to tighter monetary policies.

Concerning $structures_{it,j}$ component (standing for unobserved heterogeneity), the intensity of spillover effects tends to increase confirming that economic–institutional linkages significantly affect countries' responses (Figures 4c and 4d). Cross-country commonality would be larger in real economy and thus if capital flows tend to matter more in driving shock transmission among financial markets, trade flows would matter more in affecting the spreading of spillover effects among countries. Moreover, output responses over time are larger in WE countries despite catching-up effects in CEE economies due to persistent and consistent cross-country heterogeneity. From a policy perspective, the results face a situation of trade-off. More precisely, if on one hand the adoption of sounder macroeconomic policies and economic–institutional changes – put in place to foster consolidated policy actions – have helped to bring inflation in emerging (and then CEE) economies back under control, on the other hand, in case of a noteworthy unexpected financial shock – without appropriate

coordinated structural reforms in trade, product, and labour markets – outward government benefits will be not able for supporting the process of international financial integration among countries and boosting the output to potential growth.

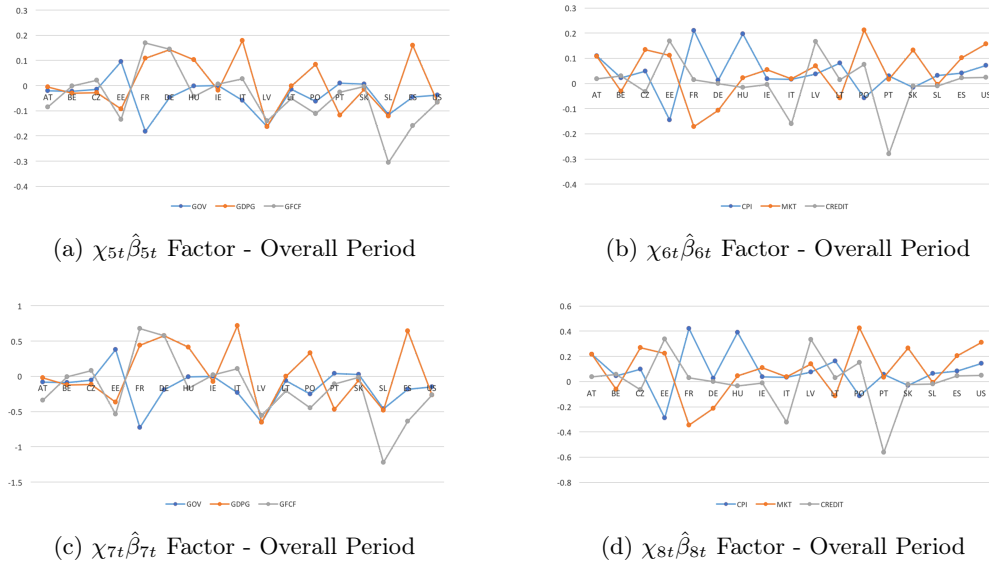


Figure 4: Systemic Contributions of the *productivity* given a 1% shock to real and financial dimensions are drawn as standard deviations of the variables in the system and in year-on-year growth rates. They account for $\chi_{5t}\hat{\beta}_{5t}$ (plot a) and $\chi_{6t}\hat{\beta}_{6t}$ (plot b) cross-country indicators, dealing with policy regime shifts and omitted factors, and $\chi_{7t}\hat{\beta}_{7t}$ (plot c) and $\chi_{8t}\hat{\beta}_{8t}$ (plot d) cross-country indicators, dealing with endogeneity and volatility issues, where $\hat{\beta}_5$, $\hat{\beta}_6$, $\hat{\beta}_7$, and $\hat{\beta}_8$ are posterior means.

Established that policy regime shifts and endogeneity issues (both omitted factors via additional transmission channels and unobserved heterogeneity via economic–institutional linkages) affect the spreading and the intensity of international spillovers, the same analysis is conducted by focusing on the last two cross-country variable-specific factors ($\chi_{9,3t}$, $\chi_{9,4t}$ in Figure 5).

In this context, some main considerations are in order. First, during ZIRE Regime, CEE and BLS countries from net senders become net receivers in the financial dimension. It highlights that, even if substantial structural reforms in terms of radical fiscal adjustments were able to absorb unexpected financial shocks (outward spillovers), consistent cross-country interdependencies among financial sectors – because of EA’s common monetary policy – brought about ‘pseudo-shock’ in the short term to catch up with the economic growth of the other euro participants³¹ (inward spillovers). Second, structural–institutional implications along with policy reforms affect the intensity (or volatility) of spillover effects in CEE and even more in BLS countries – because of larger current account deficits and lower real economic convergence – via international transmission channels, that allow in turn financial shocks to spill over. Third, persistent cross-country heterogeneity during monetary policy regimes emphasises that the fairly well synchronized business cycles among emerging and advanced economies might be unlikely, mainly on account of triggering events in the long run. Thus, the increasing need of consistent reforms of the international financial system to accelerate well-suited financial integration in developing countries. These findings are against existing studies that support similarity across

³¹They refer to the advanced and then WE countries.

business cycles in CEWE economies because of dealing with too short periods. For instance, they consider up to seven years or less, implying that only a single business cycle would be covered by the available data.

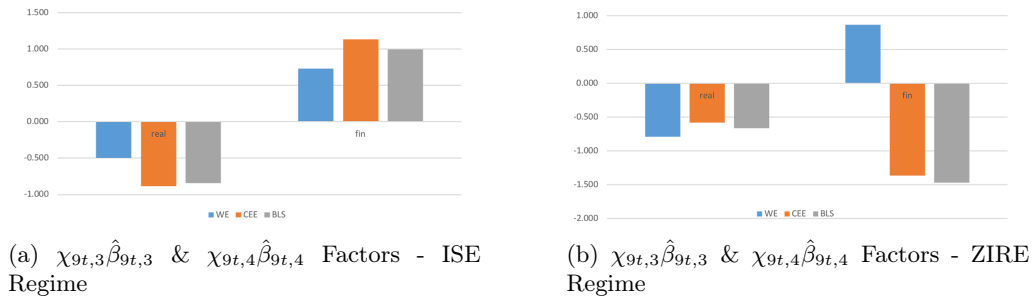


Figure 5: Systemic Contributions of the *productivity* given a 1% shock to real and financial dimensions are drawn as standard deviations of the variables in the system and in year-on-year growth rates. They account for the variable-specific indicators $\chi_{9t,3}\hat{\beta}_{9t,3}$ and $\chi_{9t,4}\hat{\beta}_{9t,4}$ during ISE (plot a) and ZIRE (plot b) regimes, dealing with endogeneity and volatility issues, where $\hat{\beta}_{9t,M_{\tilde{v}}}$'s are posterior means with $\tilde{v} = v3, v4$.

5.3 Policy Interactions, Common Features, and Contagion Measures among Countries and Sectors

In the aftermath of the Great Recession and an ongoing postcrisis consolidation, the intensity of spillover effects becomes larger in real and even more in financial dimension because of stronger inter-country linkages among financial markets behind stringent fiscal adjustments (Figures 6a and 6b). More precisely, the spreading and the size of spillover effects tend to be higher among CEE and even more BLS countries – because of extensive reforms – in real economy because of radical policy actions and among WE countries in financial markets because of stronger interdependencies. Despite a consistent homogeneity holds among CEWE economies, different countries' responses matter during financial crisis and even more fiscal consolidation periods due to coordinated but not fairly flexible fiscal actions, mainly among emerging economies suffering from lower competitiveness.

Finally, according to all the aforementioned findings, I compute the Total Contagion Index (TCI) on the only common indicator $\chi_{10t,2}\hat{\beta}_{10t,2}$, so as to investigate in depth commonality among sectors and countries in real economy and financial markets. To do it, the cumulative impulse responses are restricted in the interval $[0, 1]$ and the (individual) spillover effects are restricted in the interval $[-1, +1]$ so that the index will be bound between 0 and 100 (or between -100 and 0 if negative effects occur). Thus, the TCI is so obtained:

$$TCI_{y_i,j} = \frac{100}{N(N - \tilde{v})} \cdot \sum_{i=1}^N IR_{y_i \rightarrow y_j} \quad \text{with } i = j = 1, \dots, N \quad (61)$$

where, $IR_{y_i \rightarrow y_j}$ denotes individual (out) spillover effects and $N - \tilde{v}$ refers to the degrees of freedom depending on the needs of the investigation, with \tilde{v} accounting for the terms chosen in the factorization (59).

Here, some considerations are in order (Figure 6c). During crisis period, emerging and advanced economies

show inward and outward spillovers in the financial dimension, respectively. Contrary to postcrisis consolidation periods, where CEE and BLS countries become net senders and WE countries are net receivers. It highlights the presence of consistent policy interactions: an unexpected shock in financial markets (e.g., because of inflationary pressures, unsustainable credit boom, stiffening of banking supervision) affects the real economy through fiscal adjustments (e.g., public expenditure cuts, lowers increasing tax). Then, stringent economic–institutional linkages cause a ‘pseudo-shock’ among CEE and BLS economies because of larger fiscal adjustments – mainly in the last two decade – to catch up with the economic growth of the other advanced EA economies (from net receivers to net senders).

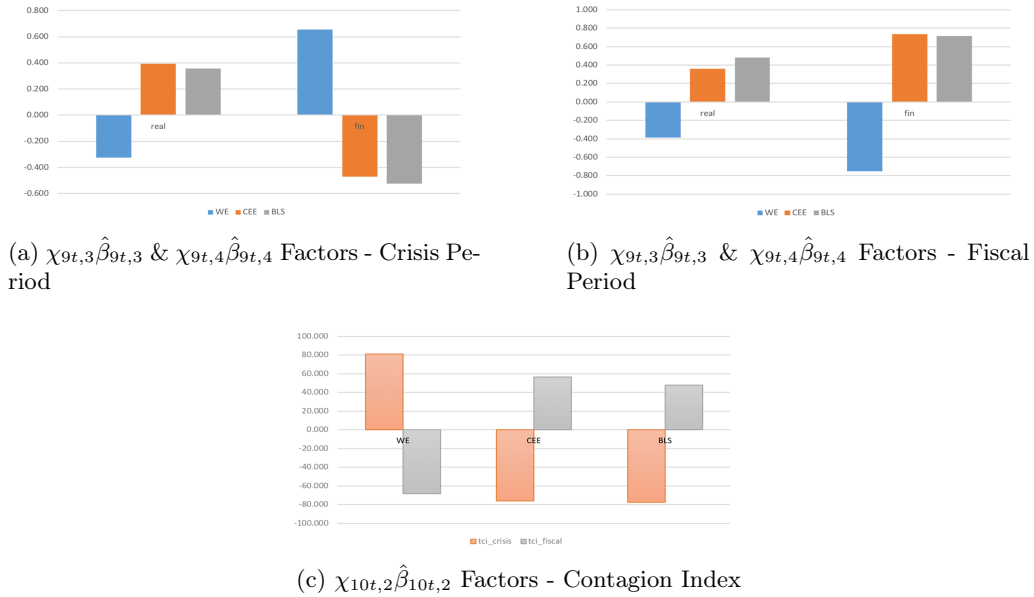


Figure 6: Systemic Contributions of the *productivity* given a 1% shock to real and financial dimensions are drawn as standard deviations of the variables in the system and in year-on-year growth rates. They account for the variable-specific indicators $\chi_{9t,3}\hat{\beta}_{9t,3}$ and $\chi_{9t,4}\hat{\beta}_{9t,4}$ during crisis (plot *a*) and post-crisis (plot *b*) periods, dealing with endogeneity and volatility issues, where $\hat{\beta}_{9t,M_i}$ ’s are posterior means with $\ddot{v} = v3, v4$.

5.4 Lessons and Matters for Future Policy Efforts

In summary, all the aforementioned results lead to four important chain-effect findings: (i) given an unexpected shock in financial markets, countries’ responses show higher heterogeneity among real sectors, pointing out non-homogeneous real economic convergence among countries (endogeneity issues); (ii) the related spillover effects show larger intensity among developing economies due to sever overheating periods, mainly during the recent financial crisis due to radical structural fiscal adjustments (policy-regime shifts); (iii) at the same time, higher intensity in the spreading of spillovers bring about larger volatilities among either sectors or countries over time (structural changes); and (iv) these volatility issues highlight an unlikely international business cycle synchronization among emerging economies and thus a solid but not properly achieved integration within EU, increasing the cost of participation in the European and Monetary Union (EMU).

Since developing countries tend to bear the brunt of triggering events due to their relatively low economic

weight (in terms of international trade exposures), 'quasi-flexible' policies should be conducted in order to ensure in a not-too-distant future: (i) the restoration of the confidence in financial systems, still recovering from the recent financial crisis; (ii) higher homogeneity across countries' responses in real economy given an unexpected financial shock so as to safeguard the inter-country real convergence; and (iii) stronger cross-correlations among CEWE economies when facing international shocks transmission.

In this context, 'quasi-flexible' policies stand for coordinated structural policy actions among foreign and domestic sectors along with more pointed fiscal adjustments according to country-specific requirements. Furthermore, the analysis highlights that, in case of a noteworthy unexpected shock in real economy, outward government benefits would be really beneficial for supporting the European integration and boosting the output to potential growth. Thus, the need of examining international spillovers accounting for both model misspecification problems and implied volatility changes.

In Figure 7, I display the generalized Entropy Index from 1994q4 to 2021q1. It corresponds to the Theil's Entropy, calculated by weighing the GDP with the population in terms of proportions with respect to the total, and can be used to measure the degree of divergence and economic inequality among countries. Here, forecasts from 2019q1 to 2021q1 correspond to conditional projections of each variable drawn in the SPBVAR-MTV in (1) and thus are able to point out the impact of an ongoing pandemic crisis on the global economy.

The coronavirus (or COVID-19) pandemic is a major global crisis negatively affecting sustainable development, economic growth, and stability and security across the globe. It constitutes an unprecedented challenge with very severe socio-economic consequences and highly strong deterioration of already existing humanitarian crises. In this study, the findings confirm the radical decrease of the economy in the last two quarters of the current year pursuant to the pandemic of coronavirus disease. However, a hint of the economic recovery shows up among countries in the next quarter. Thus, some considerations can be addressed. (i) First, coordinated and radical policy actions are necessary to deal with health emergency needs, support inter-country economic activity, and face the ground for the recovery. (ii) These adjustments should be implemented combining short, medium and long-term initiatives, but taking into account the dynamics of international spillovers and the cross-country economic-financial linkages so as to preserve confidence, stability, and financial integration (where highly strong heterogeneity and volatility matter). (iii) Moreover, even if several measures have already been taken at the national and EU levels, temporary and targeted discretionary fiscal stimulus have to keep on being adopted in a coordinated manner. More precisely, public resources and structural reforms in trade, product, and labour markets have to be directed to strengthen the healthcare sector and support affected economic-financial sectors. (iv) As regards monetary policy, closed resolute actions have to be taken by the European Central Bank to support liquidity and finance conditions to households and banks in order to preserve the smooth provision of credit to the economy. (v) Finally, to overcome the financing pressures faced by banks and households, all these policy adjustments need to be implemented by closely monitoring the evolution of the situation in each country and coordinating country-specific European and

national measures. However, if an increasing degree of divergence should overlook among countries, further and different actions, including legislative measures, will have to be taken – where appropriate – to mitigate the impact of Covid-19.

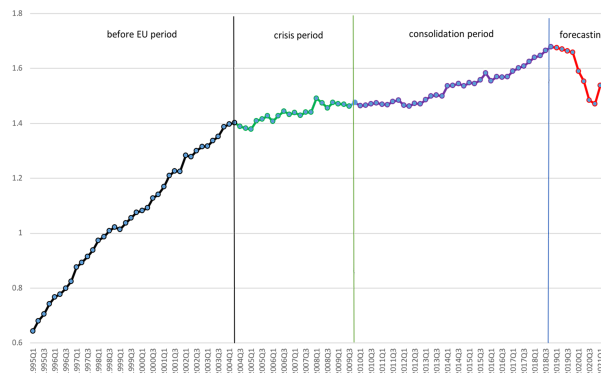


Figure 7: Generalized Entropy index according to the productivity growth from 1994q4 to 2021q1 is drawn. It corresponds to the Theil’s Entropy and is computed by weighing the GDP with the population in terms of proportions with respect to the total. The conditional projections of each variable drawn in the SPBVAR-MTV in (1) have been used to perform forecasting from 2019q1 to 2021q1.

6 Concluding Remarks

This paper provides new empirical insights in order to give a relevant contribution to the more recent literature on international macroeconomic-financial linkages when jointly modeling and quantifying multicountry data using the information contained in a large set of endogenous and economic-financial variables. A multicountry SPBVAR with Multivariate Time-varying Volatility is developed to jointly deal with issues of endogeneity, because of omitted factors and unobserved heterogeneity, and volatility, because of policy regime shifts and structural changes. The two main differences with respect to a standard SPBVAR lie in an additional component to investigate fiscal and monetary policy implications and interactions, and in the variance-covariance matrix allowed to be time-variant. The latter is an useful way of modeling time-varying conditional second moments to provide an alternative to the stochastic volatility specification; therefore, in this context, volatility changes are not more replaced by coefficient changes. The computational costs involved in using that specification are moderate since the high dimensionality is avoided via Bayesian inference and Monte Carlo Markov Chain (MCMC) implementations.

An empirical application is developed by accounting for the Central, Eastern, and Western European countries, with particular emphasis to the most recent recession and successive post-crisis periods. The United States are included in the analysis to assess international spillover effects and possible contagion measures among financial markets. In this study, I focus on the latest two alternative monetary policy regimes that have been in place since the 1990: (1) the Inflation Stabilization Era from 1994 to 2008 and (2) the Zero Interest Rate Era from 2008 to 2015. Two more additional periods are also considered: (1) 2006q1 – 2009q4 to investigate possible commonality between financial markets and real economy during

the Great recession and (2) 2010q1 – 2018q4 to evaluate fiscal implications and policy perspectives during post-crisis consolidation.

From a global perspective, the same dynamic behaviour is observed in the transmission of US financial shocks, with outward spillover effects. The findings are consistent and robust with the more recent literature on multicountry dynamic panel setups. More precisely, they confirm that US seem to be an important driver in allowing unexpected shocks to spill over and thus affecting European financial markets, mainly concerning CEE economies with inward spillovers. Then, intra-country shocks directly affect a country's own output growth in the real economy because of consistent cross-country interdependencies.

From a modeling perspective, the presence of highly strong intra-CEWE heterogeneity, in terms of intensity and spreading of spillover effects, emphasise more consistent difference among financial markets due to tighter monetary policies. In the aftermath of the Great Recession and an ongoing post-crisis consolidation, despite a consistent homogeneity holds among CEWE economies, different countries' responses tend to matter due to coordinated but not fairly flexible fiscal actions, mainly among emerging economies suffering from lower competitiveness. The findings confirm the need of examining international spillovers accounting for both misspecification problems and implied volatility changes.

From a policy perspective, the empirical results face a situation of trade-off. More precisely, if on one hand the adoption of sounder macroeconomic policies and economic–institutional changes – put in place to foster consolidated policy actions – have helped to bring inflation in emerging economies back under control, on the other hand, in case of a noteworthy unexpected financial shock – without appropriate coordinated structural reforms in trade, product, and labour markets – outward government benefits will be not able for supporting the process of international financial integration among countries and boosting the output to potential growth.

Compliance with Ethical Standards

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