

# DOCTORAL THESIS

## Heterogeneity, Commonality, and Interdependence in the Euro Area: Size and Dynamics of Fiscal Spillover Effects in Macroeconomic-Financial Linkages

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### Abstract

The paper develops empirical measures to estimate the strength and dynamic of fiscal spillover effects in the Euro Area. It moves for estimating a Bayesian VAR model of real and financial variables in order to examine in depth economic policy coordination and policy making, with a strong attention on the current financial crisis. Spillovers are estimated recursively with weakly-exogenous common factors. The aim of the project accounts for interdependencies across countries within the Euro Area and derives impulse response functions and conditional forecasts with the output of an Monte Carlo Markov Chain routine. However, the paper attempts to estimate the systemic contribution and cross-country transmission of unexpected shocks on the productivity in the Eurozone between June 1995 and March 2014. Overall, the positive impact on outputs in the financial dimension indicates the importance of coordinated fiscal actions among euro area members. Shocks overflow in a heterogeneous way across countries. Moreover, financial variables show higher amplification of spillover effects which can be seen as a result of increased interdependence between variables. Finally, the analysis is consistent and robust with the more recent literature on business cycles, which recognizes the importance of both group-specific and global factors in evaluating cross-country spillovers and responses to an unexpected shocks.

Keywords: Fiscal Policy, Cross-country Spillovers, Impulse Response Functions, Conditional Forecasts, Panel Bayesian VAR.

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# 1 Introduction

In the debate on global imbalances, the euro area countries did not receive much attention so far. While the current account has been close to balance over the past decades at the aggregate level, divergences between individual member states have increased since the introduction of the common currency. Disparities across the member states are striking, for example persistent current account deficits of Portugal, Ireland, Greece, and Spain (PIGS) are accompanied by huge surpluses in Germany, Austria, and Netherlands.

Since the Euro started<sup>1</sup>, the most research findings focused on debt dynamics, current account balances, contagion effects, and fiscal rules. However, there are many channels and factors through which macroeconomic and financial linkages can be analyzed. For instance, they can arise by a deterioration of financial conditions affecting the economy through a negative wealth effect on consumption and investment decisions. Additionally, business cycles, demographic developments, and fiscal policy are important determinants of empirical realisations of inward growth spillover effects. Furthermore, the European integration process certainly made stronger interdependencies across countries to a fiscal shock. On the other hand, in the meantime tight institutional and economic interdependencies may have made euro area countries more alike, the recent recession has shown that there may still be a substantial degree of heterogeneity, with some common behavior, in real and financial linkages across countries. In addition, those linkages may have changed over time because of economic/institutional implications. Up to now, research on these issues have still not been analysed in the necessary depth. The aim of the project is to try to fill this gap, accounting for break-time effects, time-varying variables, and macroeconomic-financial linkages.

This paper addresses the topic of commonality and heterogeneity across countries and over time within the euro area. It analyzes sign, dimension, and transmission of fiscal spillover effects across countries, with a particular emphasis on the recent recession and fiscal consolidations. Finally, it quantifies the prominent role of transmission channels and economic/institutional implications in driving height difference and spreading of shocks and *cause-effect* relationships. The project accounts for a Bayesian Vector Auto-Regressive (BVAR) model where real, financial, and selected *latent* factors<sup>2</sup> are jointly modelled for a total of 12 countries of the eurozone for the period from

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<sup>1</sup>The euro area consists of those European Union (EU) Member States which have adopted the euro as their single currency. The euro area were introduced on 1 January 1999, as stage III of economic and monetary union began, in 11 countries and expanded through a series of enlargements to 18 countries, so far.

<sup>2</sup>Latent or hidden factors are variables that are not directly observed but are rather inferred from other variables that are observed and, hence, directly measured.

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1999 to 2014. The project runs out evaluating a Seemingly Unrelated Model (SUR) in order to analyze the evolution (and, hence, strength and dynamic), commonality, and heterogeneity of fiscal spillover effects in macroeconomic-financial linkages. The selected eurozone countries are: Italy (IT), Spain (ES), France (FR), Belgium (BE), Netherlands (NL), Austria (AT), Germany (DE), Finland (FI), Luxembourg (LU), Ireland (IE), Portugal (PT), and Greece (GR). The first 11 countries are the founding euro-area Member States. On 1 January 2001, Greece joined the Euro Area (EA).

The specification of the econometric model is the same for all countries considered. Bayesian methods and Maximum Likelihood Estimates (MLE) are used to reduce the dimensionality of the model, put structure on the time variations, and simultaneously evaluate omitted variable bias and issues of endogeneity. In the case of fully hierarchical priors, a MCMC method (or alternatives) can be employed to calculate posterior distributions. To be more precise, MCMC methods are used to model for bayesian inference and numerical integration, to compute impulse responses and conditional forecasting experiments to unexpected perturbations in the innovations of either the VAR or the factors. A BVAR model is used for the following reasons. First, it provides a flexible coefficient factorization that renders estimation easy. Second, the econometric approach makes model selection and inference tractable measuring the evolution of heterogeneity and spillovers in an unified framework. Third, possible commonalities can be analyzed jointly for all variables and countries. The evidence would confirm the need to allow for cross-country and cross-variable interdependencies when studying real and financial linkages. However, country-specific factors remain very important explaining the presence of a heterogeneous pattern across members and of co-movements in economic activity. The specification model used in this study is consistent with the recent literature which recognizes the importance to separate common shocks from propagation of country-specific shocks through different channels.

The paper is structured as follows. Section 2 describes the empirical model and further specification. Section 3 discusses related literature. Section 4 explains the structure of multicountry VAR model. Section 5 accurately describes model estimation, prior assumptions, and posterior distributions with MCMC methods and Bayesian statistics. Section 6 describes dynamic analyses of the model. Section 7 illustrates the data. Section 8 provides estimates of fiscal spillover effects, systemic contributions, and contagion index in real and financial dimension before crisis period. Section 9 examines in depth common and country-specific factors during the recent crisis. Section 10 discusses the role of commonalities and heterogeneity across countries over time. Section 11

contains concluding remarks.

## 2 Econometric and Specification Model

The paper estimates the following BVAR model:

$$Y_{it} = A_{it,j}(L)Y_{i,t-1} + B_{it,j}(L)W_{i,t-1} + C_{it,j}(L)Z_{i,t-1} + \varepsilon_{it} \quad (1)$$

where  $C_{it,j}$  is a  $\xi \cdot 1$  vector that include common and idiosyncratic factors<sup>3</sup>. Here, there are  $p_3$  lags for each of the NM endogenous variables. Thus,  $p = p_1 = p_2 = p_3 = 1$ .

The models hold three important features. (i) The coefficients of the specification are allowed to vary over time. (ii) Dynamic relationship are allowed to be country-specific. In this way, heterogeneity biases are minimized. (iii) Cross-unit lagged interdependencies exist whenever the matrix  $A_t(L) = [A_{1t}(L), A_{2t}(L), \dots, A_{Nt}(L)]'$  is not block diagonal for some L. To be more precise, stacking the elements of  $A_{it,j}$  over i, a matrix that is not block diagonal for at least one j can be obtained. Thus, dynamic feedback across countries is possible.

This feature adds flexibility to the specification but it is costly. In fact, the number of coefficients is increased by factor N ( $k = NM \cdot p_1 + q \cdot p_2 + \xi \cdot p_3$  in equation 1). However, in 1 the dynamic relationships are allowed to be unit specific and the coefficients could vary over time. Following the framework in Canova and Ciccarelli (2009), the model 1 can be re-written in a simultaneous-equation form in order to avoid the matter of dimensionality. Let  $\delta_{it}^m$  be  $k \cdot 1$  vectors, with  $\delta_{it} = (\delta_{it}^1, \delta_{it}^2, \dots, \delta_{it}^M)'$ , which contains, stacked, the M rows of the matrices  $A_{it,j}$  and  $B_{it,j}$ , a  $NMk \cdot 1$  vector  $\delta_t = (\delta'_{1t}, \delta'_{2t}, \dots, \delta'_{Nt})'$  can be defined. The specification of models assumes the form:

$$Y_t = X_t \cdot \delta_t + E_t \quad (2)$$

where, accounting for equation 1,  $X_t = I_{NG} \otimes X'_t$ , with  $X_t = (Y'_{t-1}, W'_t, W'_{t-1})'$ ,  $Y_t$  and  $E_t$  are  $NM \cdot 1$  vectors containing the endogenous variables and the random disturbances of the model.

<sup>3</sup>The variables are unemployment rate, final consumption, and unit labour costs for all selected period. During financial crisis and fiscal consolidations, other private and public factors are also added, such as private consumptions and international investment positions.

Here,  $Y_{it}$  is expressed in terms of  $X_t$ . The crucial aspect of equation 2 is that there is no subscript  $i$  since variables of all countries in the system are stacked in  $X_t$ . However, in equation 1, the vector  $X_t = (Y'_{t-1}, W'_t, W'_{t-1}, Z'_t, Z'_{t-1})'$  contains endogenous and exogenous variables of the system.

Now, since  $\delta_t$  varies in different time periods for each country-variable pair, whenever  $\delta_{it}$  is unrestricted, it is impossible to estimate it. Moreover, its sheer dimension ( $k=NMp$  parameters in each equation) could prevent any meaningful unconstrained estimation. There are more coefficients than data points. To solve it, a flexible structure where  $\delta_t$  is factored can be assumed:

$$\delta_t = \Xi \cdot \theta_t + u_t \quad u_t \sim N(0, \Sigma \otimes V) \quad (3)$$

where  $\Xi$  is a matrix of coefficients,  $\dim(\theta_t) \ll \dim(\delta_t)$ , and  $u_t$  captures unmodelled and idiosyncratic variations present in  $\delta_t$ .

The selection of the type of factors is often a matter of choice, that is typically dictated by the needs of the investigation. In a cross-country study of business cycle transmissions, for example, common and country-specific factors are probably sufficient although, when constructing indicators of GDP, one way want to specify, at least, a common, a country-specific, and a variable-specific factor. In equation 3, all factors are permitted to be time-varying and, hence, time invariant structures can be obtained via restrictions on their law of motion, as explained below.

Empirical evidence are discussed in Section 10.1 in order to estimate economic and structural implications in driving the transmission of a shock in macroeconomic-financial linkages. Running equations 2 and 3 for equation 1, the SUR model is:

$$\Xi \cdot \theta_t = \Xi_1 \cdot \theta_{1t} + \Xi_2 \cdot \theta_{2t} + \Xi_3 \cdot \theta_{3t} + \Xi_4 \cdot \theta_{4t} + \Xi_5 \cdot \theta_{5t} + \Xi_6 \cdot \theta_{6t} \quad (4)$$

where  $\Xi_1$  and  $\Xi_2$  are matrices of dimensions  $NMk \cdot N$ , with  $k = NM \cdot p_1 + q \cdot p_2$ .  $\theta_{1t}$  and  $\theta_{2t}$  are mutually orthogonal  $NM \cdot 1$  factors capturing, respectively, the country specific components (12x1 vectors) for real and financial dimension accounting for the *weights* component.

They account, respectively, for real and financial variables plus variable-specific factors, so that  $\Xi_{1it} = \sum_{m=1}^3 \cdot \sum_j y_{tim-p-j}$  and  $\Xi_{2it} = \sum_{m=4}^6 \cdot \sum_j y_{tim-p-j}$ ,  $i = 1, \dots, 12$ ,  $p = p_1 = p_2 = 1$ .

$\Xi_3$  and  $\Xi_4$  are matrices of dimensions  $NMk \cdot N$ , with  $k = NM \cdot p_1 + q \cdot p_2 + \xi \cdot p_3$ .  $\theta_{3t}$  and  $\theta_{4t}$

are mutually orthogonal  $NM \cdot 1$  factors capturing, respectively, the country specific components (12x1 vectors) for real and financial dimension accounting for the *weights* and *imbalances* components. They account, respectively, for real and financial variables plus variable-specific factors and exogenous variables, so that  $\Xi_{3it} = \sum_{m=1}^3 \cdot \sum_j y_{tim_p-j}$  and  $\Xi_{4it} = \sum_{m=4}^6 \cdot \sum_j y_{tim_p-j}$ ,  $i = 1, \dots, 12$ ,  $p = p_1 = p_2 = p_3 = 1$ .

$\Xi_5$  is a matrix of dimension  $NMk \cdot M_1$ .  $\theta_{5t}$  is mutually orthogonal  $NM_1 \cdot 1$  factor. It corresponds to the variable-type component (4x1 vector) for four groups of variables: *real* + trade flows; *fin* + capital flows; *real* + trade flows + *imbalances*; and *fin* + capital flows + *imbalances*, where  $M_1 \leq M$  denotes the number of variable groups.  $\Xi_{5it} = \sum_{m=1}^6 \cdot \sum_j y_{tim_1p-j}$ ,  $p = p_1 = p_2 = p_3 = 1$ .

$\Xi_6$  is a matrix of dimensions  $NMk \cdot 1$ ,  $\theta_{6t}$  is mutually orthogonal  $N \cdot 1$  factor capturing movements in the common component to all countries and variables (2x1 vector): *real* + *fin* + *weights* and *real* + *fin* + *weights* + *imbalances*.  $\Xi_{6it} = \sum_{m=1}^6 \cdot \sum_j y_{tim_p-j}$ ,  $p = p_1 = p_2 = p_3 = 1$ .

The idea behind the first four effects is to provide a possible reason about different reactions or co-movements across countries to a common shock (e.g., fiscal consolidations to improve the economic growth against increasing local and economic imbalances). The fifth and sixth effect would highlight the importance of economic and structural factors and transmission channels in driving the spreading of a shock when studying macroeconomic-financial linkages.

Hence,  $\theta_t = (\theta'_{1t}, \theta'_{2t}, \theta'_{3t}, \theta'_{4t}, \theta'_{5t}, \theta'_{6t})'$  is a  $NMk \cdot 1$  vector and the estimated model is:

$$Y_t = \chi_{1t}\theta_{1t} + \chi_{2t}\theta_{2t} + \chi_{3t}\theta_{3t} + \chi_{4t}\theta_{4t} + \chi_{5t}\theta_{5t} + \chi_{6t}\theta_{6t} + \eta_t \quad \text{with} \quad \theta_t = \theta_{t-1} + v_t \quad (5)$$

where  $\chi_t \equiv X_t \cdot \Xi$ , with  $\Xi = [\Xi_1, \Xi_2, \Xi_3, \Xi_4, \Xi_5, \Xi_6]$ .

Factoring  $\delta_t$  as in equation 3 reduce the problem of estimating  $NMk$  coefficients into the one of estimating for example,  $NM + M_1 + 1$  factors characterizing their dynamics. Moreover, this finding is able to transform the overparametrized multicountry VAR into a parsimonious Seemingly Unrelated Regression (SUR) model. By equations 3 and 2, it can be written as:

$$Y_t = \chi_t\theta_t + \eta_t \quad (6)$$

By construction,  $\chi_{it}$  are linear combinations of right-hand side variables of the multicountry VAR

and correlated among each other. The correlation decreases as  $M$  or  $N$  or  $p = \max[p_1, p_2, p_3]$  increase, and comovements are emphasized across lagged variables.

The vector of endogenous variables depends on a small number of observable indices,  $\chi_{it}$ , and the factors  $\theta_{it}$  load on the indices. They are vector time-varying loadings to be estimated. In fact, they are smooth linear functions of the lagged endogenous variables. Thus, in equation 5,  $\chi_{1t}\theta_{1t}$ ,  $\chi_{2t}\theta_{2t}$ ,  $\chi_{3t}\theta_{3t}$ ,  $\chi_{4t}\theta_{4t}$  are observable country specific indicators for  $Y_t$ ,  $\chi_{5t}\theta_{5t}$  is observable variable specific indicator for  $Y_t$ , and  $\chi_{6t}\theta_{6t}$  is observable common indicator for  $Y_t$

### 3 Discussion and Relationship with the literature

Macroeconomic coordination of fiscal policies is a recurring theme both in the policy discussions and in the economic literature. Recent developments have given a new impulse to the study of this issue using multicountry VAR models and panel VAR for applied macroeconomic analysis. For example, Benassy-Quere (2006) estimate the impact of fiscal spillovers from Germany to the remaining G7 countries using an augmented SVAR model. Blanchard and Perotti (2002), Perotti (2005), and Canova and Pappa (2002) estimate fiscal shocks in the U.S. economy and selected G7 economies using an augmented country-specific VAR model. Beetsma et al. (2005) consider fiscal spillover effects through trade proceeding in two steps. First, they obtain estimates on the effects of a fiscal shock on output using an European panel VAR. Second, they impose homogeneity restrictions and plug the panel VAR estimates into a trade-gravity type of model. Pesaran (2003) and Pesaran and Shin (1998) develop a multi-country Global Vector Autoregression (GVAR) approach in which estimate the spillover effects of a domestic budget balance shock on the members of the EA by combining all country-specific VAR models in one multi-country model and treating all variables as endogenous. In a recent paper, Canova et al. (2012) investigate heterogeneity and spillovers in macro-financial linkages across developed economies adopting a panel Bayesian VAR model.

By the same token, empirical measures for the strenght and dynamic of spillover effects have attracted widespread attention by academia, policy makers and market participants, specially during the recent financial crisis. After the collapse of Lehman Brothers in autumn 2008, the fear of contagion has been one of the most prominent issues on the agenda both for financial research and policy making. A variey of approaches and methods on how to measure contagion has been proposed. Allen and Gale (2000) define contagion as a consequence of excess spillover. More

generally, he summarises five main criteria to identify contagion. First, an idiosyncratic negative shock that affects a financial institution and spreads to other parts of the financial system. Second, the interdependencies between assets are different than in tranquil times. Third, the excess dependencies cannot be explained by common shock. Fourth, events associated with extreme left tail returns. Fifth, interdependencies evolve sequentially. Constancio (2012) extends the identification of contagion in the abnormal speed and strength of potential spillovers as a consequence of a trigger-event (e.g., financial instability).

One advantage of the flexible coefficient factorization in equation 3 is that the overparametrization of the original multicountry TVC-VAR is dramatically reduced. In fact, in the resulting SUR model, estimation and specification searches are constrained only by the dimensionality of  $\theta_t$  ( $\delta_t$  is integrated out). A second advantage is that, given the MA nature of many  $chi_{it}$ , the regressors of equation 6 capture low-frequency comovements present in the lags of the VAR. Since the model averages out not only cross section, but also time-series noise, reliable and stable estimates of  $\theta_t$  can potentially be obtained, making the framework useful for a variety of medium-term policy analyses exercises. A third advantage is that the SUR model in equation 6 has some economic content. For example, if  $\theta_{1t}$  and  $\theta_{2t}$  capture information that is common to all lags and the coefficients of the VAR,  $\chi_{1t}\theta_{1t}$  and  $\chi_{2t}\theta_{2t}$  are indicators for  $Y_t$  based on common information. By the same token, indicators containing other types of information can also be easily constructed. Finally, since  $\chi_{it}$  are predetermined, leading versions of these indicators can be obtained projecting  $\theta_t$  on the information available at  $t - \tau$ , whit  $\tau = 1, 2, \dots$

Some commentators have argued that the equal and exogenous weights that equation 3 imposes on the regressors of 6 are restrictive and suggested the possibility to estimate the  $\Xi$ 's. The structure of this framework is no more restrictive than the one used in related literature. Clearly, the equal weighting scheme is appropriate if all variables are measured in the same units (e.g., growth rates) and their variability is comparable; otherwise, preliminary transformations need to be used or the vector of  $\Xi_i$  appropriately scaled. For example, if the variability of the variables of country 1 is considerably larger than the variability of the variables of country 2, then one could specify  $Xi_1 = (\sigma_1^{-1}, \dots, \sigma_1^{-1}, \sigma_2^{-1}, \dots, \sigma_2^{-1}, \dots)$  where  $\sigma_1$  and  $\sigma_2$  measure the average standard deviation of the variables in country 1 and 2. The idea of estimating the  $\Xi$ 's is not considered in this framework; the weights are a priori determined by the flexible factorization used. This latter is feasible if one directly starts from equation 6, treats  $\Xi_i$  as unknown, and employs the factor models techniques described in Section 5. Further, this approach has two types of advantages over single-country or



two-country VARs. (i) If the information is weak or the sample short, cross sectional information may help to get better estimates and smaller standard errors. (ii) If the momentum that shocks induce across countries is the result of lagged interdependencies, the model described in equation 6 will be able to capture it.

In addition, the SUR model has also some similarities with the models used by Pesaran (2003) and Pesaran and Shin (1998) to model global interdependencies, even though the starting point, the underlying specification, and the estimation technique differ. In fact, in these papers, the baseline specification is a traditional micro-panel structure with unobservable common components in the error term, instead of a VAR; no time variations are allowed in the coefficients and no lagged interdependencies are present;  $N$  is assumed to be large. In this setup, it is possible to obtain a consistent estimate of the common unobservable component by arithmetically averaging the dependent and the independent variables of the unit-specific regressions. Therefore, the estimated specification looks like a set of unrelated single-country VARs, where common factors are proxied by averages of the variables across countries. The specification model described in equation 6 shares the idea of using arithmetic averages as regressors. It can be interpreted as an F-factor generalization of these authors' approach, where each factor spans a difference space; instead, the model described in equation 6 allows for lagged interdependencies in the error term and for time-varying loading. Finally, this specification does not need a large  $N$  to work.

The recent recession has shown two main matters. First, there are institutional and economic interdependencies across countries, specially between Eurozone countries having relinquished independent monetary and exchange rate policies. Second, there may still be a substantial degree of heterogeneity with some common behaviours in macroeconomic-financial linkages across countries and that those linkages may have changed over time. There exists a variety amount of empirical work on spillovers effects from fiscal policy shock. A possible reason for this is that, while the theoretical literature suggests a variety of possible channels through which fiscal policy may cause cross-border spillovers, empirically, it has proved difficult to find significant spillover effects. Nevertheless, there are some exceptions. In a study that is closest to the current one, Canova et al. (2012) confirm the need to allow for cross-country and cross-variable interdependencies when studying real and financial linkages. Moreover, country-specific factors remain very important, which explains the presence of a heterogeneous pattern in macroeconomic-financial linkages. They extend recent empirical work that assesses the macro-economic effects of impulses in the economy by using time-varying multicountry VAR models to study interdependence and

time variation simultaneously across a panel of countries.

According to the above-mentioned literature, the aim of the project is to understand common and heterogeneous patterns between financial and real variables, with a strong attention on the recent recession. This study presents a method to estimate the strength and dynamic of fiscal spillover effects in the EA using a Bayesian VAR approach to study cross-unit interdependencies, unit-specific dynamics, group- and variable- specific effects, and time variations in the coefficients. The framework of analysis is Bayesian in order to reduce the dimensionality of the model and put structure on the time variations. Posterior of impulse response functions and conditional forecasts are obtained with the output of an MCMC simulations. This study contributes to the literature for the effects of fiscal shocks on the economies in the EA. Bilateral trade and capital for real and financial variables are respectively computed in order to account for cross-country linkages. The paper finds that spillovers of fiscal coordination in the real dimension are no more larger than financial dimension. Moreover, cross-border spillovers have exacerbated the negative effects of consolidations, with a substantial degree of heterogeneity in real dimension and a deeper interdependence in financial dimension. From a policy perspective, optimal policy coordination in the EA would have required a differentiation of consolidation efforts depending on the fiscal space to minimise the negative spillovers.

## 4 Multicountry VAR Setup and Related literature

To illustrate the structure of the matrices  $\Xi'$  s and of  $X_{it}$  suppose there are  $M=2$  variables for each of  $n=2$  countries and that the BVAR has  $p=1$  lags and no intercept:

$$\begin{bmatrix} y_t^1 \\ x_t^1 \\ y_t^2 \\ x_t^2 \end{bmatrix} = \begin{bmatrix} b_{1,1,t}^{1,y} & b_{2,1,t}^{1,y} & b_{1,2,t}^{1,y} & b_{2,2,t}^{1,y} \\ b_{1,1,t}^{1,x} & b_{2,1,t}^{1,x} & b_{1,2,t}^{1,x} & b_{2,2,t}^{1,x} \\ b_{1,1,t}^{2,y} & b_{2,1,t}^{2,y} & b_{1,2,t}^{2,y} & b_{2,2,t}^{2,y} \\ b_{1,1,t}^{2,x} & b_{2,1,t}^{2,x} & b_{1,2,t}^{2,x} & b_{2,2,t}^{2,x} \end{bmatrix} \cdot \begin{bmatrix} y_{t-1}^1 \\ x_{t-1}^1 \\ y_{t-1}^2 \\ x_{t-1}^2 \end{bmatrix} + \eta_t \quad (7)$$

Here,  $\delta_t = [b_{1,1,t}^{1,y}, b_{1,1,t}^{1,x}, b_{1,1,t}^{2,y}, b_{1,1,t}^{2,x}, b_{2,1,t}^{1,y}, b_{2,1,t}^{1,x}, b_{2,1,t}^{2,y}, b_{2,1,t}^{2,x}, b_{1,2,t}^{1,y}, b_{1,2,t}^{1,x}, b_{1,2,t}^{2,y}, b_{1,2,t}^{2,x}, b_{2,2,t}^{1,y}, b_{2,2,t}^{1,x}, b_{2,2,t}^{2,y}, b_{2,2,t}^{2,x}]'$  is a  $(16 \cdot 1)$  vector containing the time varying coefficients of the model. Note that the typical element of  $\delta_t$ ,  $b_{i,s,t}^{j,l}$ , is indexed by the country  $i$ , the variable  $j$ , the variable in an equation  $l$  (in-

dependent of the country) and the country in an equation  $s$  (independent of the variable). Given the factorization described in equation 3, the VAR() can be rewritten as:

$$\begin{bmatrix} y_t^1 \\ x_t^1 \\ y_t^2 \\ x_t^2 \end{bmatrix} = \begin{bmatrix} \chi_{1,t} \\ \chi_{1,t} \\ \chi_{1,t} \\ \chi_{1,t} \end{bmatrix} \theta_{1t} + \begin{bmatrix} \chi_{2,t}^1 & 0 \\ \chi_{2,t}^1 & 0 \\ 0 & \chi_{2,t}^1 \\ 0 & \chi_{2,t}^1 \end{bmatrix} \theta_{2t} + \begin{bmatrix} \chi_{3,t}^1 & 0 \\ \chi_{3,t}^1 & 0 \\ 0 & \chi_{3,t}^1 \\ 0 & \chi_{3,t}^1 \end{bmatrix} \theta_{3t} + \eta_t \quad (8)$$

where  $\chi_{1t} = y_{t-1}^1 + x_{t-1}^1 + y_{t-1}^2 + x_{t-1}^2$ ,  $\chi_{2,t}^1 = y_{t-1}^1 + x_{t-1}^1$ ,  $\chi_{2,t}^2 = y_{t-1}^2 + x_{t-1}^2$ ,  $\chi_{3,t}^1 = y_{t-1}^1 + y_{t-1}^2$ ,  $\chi_{3,t}^2 = x_{t-1}^1 + x_{t-1}^2$ . In the empirical application, all variables are measured in standardized and demeaned growth rates and therefore this type of averaging will indeed be appropriate. Note that if  $\theta_{1t}$  is large relative to  $\theta_{2t}$ ,  $y_t^1$  and  $x_t^1$  comove with  $y_t^2$  and  $x_t^2$ . On the other hand, if  $\theta_{1t}$  is zero,  $y_t^1$  and  $x_t^1$  may drift apart from  $y_t^2$  and  $x_t^2$ . In the general case when  $p > 1$ , lags could be weighted using a decay factor in the same spirit as Doan *et al.* (1984).

The regressors in equation 6 are combinations of lags of the right hand side variables of the VAR, while  $\theta_{it}$  play the role of time varying loadings. Using averages as regressors is common in the signal extraction literature (see e.g., Sargent, 1989) and in the factor model literature (see e.g., Forni and Reichlin (1998)). However, there are three several important differences between regressors in equation 6 and standard factor models. (i) The indices are used weighting equally the information in all variables, while in factor models the weights generally depend on the variability of the components. (ii) The indices dynamically span lagged interdependencies across units and variables, while in standard factor models they statistically span the space of the variables of the system. (iii) The indices are directly observable, while in factor models they are estimated. However, these indices are correlated by construction since the factorization is applied on the coefficient vector rather than on the variables. Finally, this averaging approach creates moving average terms of order  $p$  in the regressors of equation 6, even when  $y_{it}$  are serially independent. Hence, contrary to what occurs in factor models, the indicators implicitly filter out from the right hand side variables of the VAR high frequency variability.

Exploiting SUR model, the regressors emphasize the low frequencies movements in the variables of the VAR. This finding is important in forecasting in the medium run and in detecting turning points of GDP growth (Ciccarelli and Rebucci (2007)). The SUR model in Section 2 has some similarities with the Global VAR (GVAR) model (see e.g., Pesaran *et al.*, 2005), even though the

starting point. Nevertheless, the underlying specification and the estimation technique differ. To be more precise, in the GVAR models the estimated specification looks like a set of unrelated single country VARs where common factors are proxied by averages of the variables across countries. The empirical approach would share the idea of using arithmetic averages as regressors and can be interpreted as an F-factor generalization of these author's approach, where each factor spans a difference space allowing for lagged interdependencies in the error term and for time-varying loading.

## 5 Model Estimation

As stated in Section 2, Bayesian VAR model is a feasible solution to the overfitting problem. To be more precise, there are three statistical regularities of time-series data. (i) Trending behaviour. (ii) More recent values contain more information than past values. (iii) Past values of the variable contain more information than past values of other variables. Here, regularities are transformed in prior assumptions. Bayesian estimation requires the specification of these prior assumptions.

### 5.1 Prior Information

In hierarchical models, many problems involve multiple parameters which can be regarded as related in some way by the structure of the problem. A joint probability model for those parameters should reflect their mutual dependence. Typically, the dependence can be summarized by viewing these parameters as a sample from a common population distribution. Hence, the problem can be modelled hierarchically, with observable outcomes ( $Y_i$ ) modeled conditionally on certain parameters ( $\theta_i$ ), which themselves are assigned a distribution in terms of further (possibly common) parameters, hyperparameters ( $\alpha$ ). This hierarchical thinking may help solve the trade-off between inaccurate fit and overfitting, and also plays an important role in developing computational strategies.

Given equation described in equation 6, the prior  $p(\theta)$  typically depends on hyperparameters. Collecting the latter in a vector  $\alpha$ , it leads that:

$$p(\theta|Y, \alpha) = \frac{p(\theta, \alpha, Y)}{p(Y|\alpha)} = \frac{p(Y|\theta, \alpha)p(\theta|\alpha)}{p(Y|\alpha)} \quad (9)$$

If  $\eta$  is unknown, the second stage prior distribution (hyperprior),  $p(\theta)$ , is:

$$p(\theta) = \int p(\theta, \alpha) d\alpha = \int p(\theta|\alpha)p(\alpha) d\alpha \quad (10)$$

The posterior will be:

$$p(\theta, \alpha|Y) = p(\theta|\alpha, Y)p(\alpha|Y) \propto p(Y|\theta, \alpha) \cdot p(\theta|\alpha)p(\alpha) \quad (11)$$

Then,

$$p(\theta|Y) = \int p(\theta, \alpha|Y) d\eta = \int p(\theta|\alpha, Y)p(\alpha|Y) d\alpha \quad (12)$$

Furthermore, equation 6 can be alternatively written in the following manner, accounting for indices<sup>4</sup>:

$$Y_{ij} = \chi_{ij}\theta_i + \eta_{ij} \quad (13)$$

where,  $i=1,2, \dots, n$  and  $j=1,2, \dots, J$ . Stacking, this latter become:

$$Y_j = \chi_j\theta + \eta_j \quad (14)$$

where,  $Y_j$  is a  $(n \cdot 1)$  vector,  $\chi_j$  is a  $(n \cdot k)$  matrix,  $\theta$  is a  $(k \cdot 1)$  vector, and  $\eta_j$  is a  $(n \cdot 1)$  vector, with  $k = \sum_{i=1}^n k_i$ . Stacking further:

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<sup>4</sup>Remember that  $Y_{ij}$  represents the effect on the productivity given the impulse response of the variable  $i$  for a shock in the variable  $j$ .

$$Y = \chi\theta + \eta \quad (15)$$

If  $\eta_j \sim N(0, \Sigma)$ , then  $\eta \sim N(0, \Omega)$ , where  $\Omega = (\Sigma \otimes I)$ .

The non-zero covariances imply that equation 13 is related and individual regressions are tied into a system of equations that can be analyzed together. Variances can also differ across  $j$ , while  $\eta_i$  are independent across  $i$ . Generally speaking, equation 15 is a linear regression model, where:

$Y_j = (Y_{1j}, Y_{2j}, \dots, Y_{nj})'$  ;  $\theta = (\theta_1, \theta_2, \dots, \theta_n)'$  ;  $\eta_j = (\eta_{1j}, \eta_{2j}, \dots, \eta_{nj})'$  ;  $Y = (Y_1, Y_2, \dots, Y_J)'$  ;  $\chi = (\chi_1, \chi_2, \dots, \chi_J)'$  ;  $\eta = (\eta_1, \eta_2, \dots, \eta_J)'$ .  $\chi_j$  and  $\Omega$  are matrix having the following form:

$$\chi_j = \begin{pmatrix} \chi'_{1j} & 0 & \dots & 0 \\ 0 & \chi'_{2j} & 0 & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \dots & 0 & \chi'_{nj} \end{pmatrix} \quad \Omega = \begin{pmatrix} \Sigma & 0 & \dots & 0 \\ 0 & \Sigma & 0 & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \dots & 0 & \Sigma \end{pmatrix}$$

.

The specification allows the disturbance to productivity across fiscal shocks on real and financial dimension for a particular country to be correlated, but it assumes zero correlation across countries. Finally, equation 15 has the following state-space structure:

$$Y_t = (X_t \cdot \Xi_t)\theta_t + \eta_t \quad \eta_t = X_t u_t + E_t \quad (16)$$

$$\theta_t = (I - C)\bar{\theta} + C\theta_{t-1} + v_t \quad v_t \sim N(0, B) \quad (17)$$

$$\bar{\theta} = P\mu + \varepsilon \quad \varepsilon \sim N(0, \Psi) \quad (18)$$

where,  $E_t \sim N(0, \Omega)$  and  $v_t \sim N(0, \Sigma \otimes V)$ . Moreover,  $\bar{\theta}$  is the unconditional mean of  $\theta_t$ ;  $P$ ,  $C$  are known matrices;  $\eta_t$  and  $\varepsilon$  are mutually independent and independent of  $E_t$  and  $u_t$ ; and  $B$  is a

block diagonal matrix, with  $B = \text{diag}(\bar{B}_1, \dots, \bar{B}_F)$ . Let  $\Omega = \Sigma \otimes I$  and  $V = \sigma^2 \cdot I_k$ , where  $V$  is a  $(k \cdot k)$  matrix and  $\Omega$  is a  $(NM \cdot NM)$  matrix. Here,  $\sigma^2$  is known and  $B_f = b_f \cdot I$ , where  $f = 1, \dots, F$  and  $b_f$  controls the tightness of factor  $i$  in the coefficients.

The intuition behind this specification is simple. The factors obey the stochastic restrictions implied by equation 17 permitting time variations. In the model 16, it is assumed a general AR() structure. Since the matrix  $C$  is arbitrary, many posterior are allowed in the specification. Although,  $C$  is treated as fixed, hence it is possible to make it function of a small set of hyperparameters whose posterior can be jointly obtained with one of the other parameters. This approach is not followed here since that a choice joins the computational costs and that a near random walk specification for  $\theta_t$  is for all purposes satisfactory. Moreover, the spherical assumption on  $V$  reflects the fact that factors are measured in common units. The block diagonality of  $B$  is needed to guarantee the orthogonality of the factors, which is preserved a-posterior and, hence, their identifiability. The assumption of  $\Omega = \Sigma \otimes I$  is standard (see e.g., Kadiyala and Karlsson, 1997). In this way, prior assumptions can be specified and, hence, Bayesian computations are feasible. Further, the factors  $\theta_{it}$  drive the coefficients vector  $\delta_t$ . The idea is to shrink  $\delta_t$  into  $\theta_{it}$  obtaining a much smaller dimensional vector.  $\Xi_j$  are matrices with elements equal to zero or one. Finally, independence among the errors is standard. To be more precise,  $E_t$ ,  $u_t$ ,  $\eta_t$ , and  $\varepsilon$  are assumed to have normal distribution, but it easy to allow for fat tails if non-normal observations are presumed to be present.

For example, let  $(u_t | X_t \Xi) \sim N(0, (X_t \Xi)(\Omega \otimes V))$ , where  $X_t^{-1} \sim \chi^2(\nu, 1)$ . This latter holds since, unconditionally,  $u_t \sim t_\varepsilon(0, \Omega \otimes V)$ . Nevertheless, by construction, the forecast errors of specified SUR model already display fat tail distributions, even when all disturbances are normal. Hence, this extension will not be considered here. Further complication allowing, for example, for skewness in the errors or for time variations in the variance of shocks to the factors are easy to introduce (see e.g., Canova, 1993, and Fernandez and Steel, 1998). All of these additions go in the directions of capturing non-normal patterns in  $Y_t$ , if this is needed.

Numerous specifications are nested in the model 16. For example, a factor is time invariant when  $B_{it} = 0$  and the appropriate elements of  $C$  are set to zero. No exchangeability obtains when  $\Psi$  is large; whereas, exact pooling obtains when  $\Psi = 0$  and the factorization becomes exact when  $\sigma^2 = 0$ .

## 5.2 Bayes Factor and Model Selection

According to the factorization in equation 3, the type of factors  $\delta_t$  depends on the nature of the problem. Nevertheless, one may be interested in having a method to statistically determine the number of indices needed to capture the heterogeneities present across time, units, and variables in the multicountry VAR, or to verify general hypotheses on the type of indices to be included. In order to discriminate across models with different indices, the (conditional) marginal likelihood for a  $h$  generic index can be defined as:

$$L(Y^T|M_h) = \int \mathcal{F}(Y^T|\phi_h, M_h)p(\phi_h|M_h)d\phi_h \quad (19)$$

where,  $Y_T$  denotes the data,  $P(\phi_h|M_h)$  is the prior density for  $\phi$  in model  $M_h$ , and  $\mathcal{F}(Y_T|\phi_h, M_h)$  is density of the data under the parameterization produced by  $M_h$ . Equation 19 is conceptually simple, but can be evaluated analytically only in few elementary cases. More often, it is intractable and must be computed by numerical methods, using the output of the MCMC sampler, as suggested by Raftery et al. (1997), Chib and Jeliazkov (2001), and Chib (1995).

Given, the complexity of the model, these numerical computation are not entirely straightforward. As an alternative, one can rely on asymptotic (normal) approximations to 19, for example Laplace's method, which takes a second-order expansion of 19 around the model (or the Schwarz criterion) which expands 19 around the maximum-likelihood estimator. Since in hierarchical models, asymptotic normality might not be a sensible approximation. Thus, a good idea is to compute alternative measures of marginal likelihood before taking decisions about the size of  $h$ . Once the marginal likelihood is obtained for any model  $h$ , the Bayes factor is:

$$B_{hh} = \frac{L(Y_T|M_h)}{L(Y_T|M'_h)} \quad (20)$$

It can be used to decide whether  $M_h$  or  $M'_h$  fits the data better. Since marginal likelihoods can be decomposed into the product of one-step ahead predictive record. Moreover, since the ML implicitly discounts the performance of models with a larger number of indices, the equation 21 directly trades off the predictive record with the dimensionality of the model. By equation 19,



it is also possible to conduct useful specification searches. For example, it is possible to examine whether the factorization in equation 3 is exact, letting  $\phi_h$  unrestricted and  $\phi'_h = (\dots, \sigma^2 = 0, \dots)$ ; or whether there are time variations in  $\theta_t$ , letting  $\phi_h$  be unrestricted and  $\phi'_h = (\dots, b_f = 0, \dots)$  for some  $f$ . Finally, support for the presence of interdependencies is obtained by comparing the marginal likelihoods of the unrestricted model and that of a vector of country-specific time-varying VARs.

Instead of examining hypotheses on the structure of the model, one may want to incorporate model uncertainty directly into posterior estimates. Let  $M_1$  be the model with one index,  $M_h$  the model with  $h$  indices, with  $h=2, \dots, H$ , and that the Bayes factor  $B_{h1}$  for each  $M_h$  is computed. The posterior probability of model  $h$  is:

$$p(M_h|Y_T) = \frac{\beta_{it} B_{h1}}{\sum_{h=2}^H \beta_h B_{h1}} \quad (21)$$

where,  $\beta_h$  are the prior odds for  $M_h$ , and model uncertainty can be accounted for weighting  $G(\phi_h)$  by  $p(M_h|Y^T)$ , with  $G()$  denotes the Gamma distribution.

Given the SUR model in equation 16, prior densities are assumed for  $\phi_0 = (\Omega^{-1}, \mathcal{B}, \theta_0)$ , the factorization is exact (for example,  $\delta_t = \Xi_t \theta_t$ ),  $C = I$ ,  $\Psi = 0$ , so that hierarchical prior with exchangeability are allowed, and  $b_f$  controls the tightness of each factor in the coefficients (e.g.,  $X_t \Xi$ ). In this way, the equation 17 becomes:

$$\theta_t = \theta_{t-1} + v_t \quad v_t \sim N(0, \mathcal{B}) \quad (22)$$

The random-walk assumption is very common in the time-varying VAR literature and has the advantage of focusing on permanent shifts and reducing the number of parameters in the estimation procedure<sup>5</sup>.

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<sup>5</sup>See e.g., Primiceri (2005) for a discussion on alternative specifications.

### 5.3 Prior Assumptions

Let  $\phi_0 = (\Phi_{-1}, b_f, \theta_0)$  to be the prior densities, three tentative beliefs (assumptions) can be defined accounting for the model described in equation 15. (i) Conditional Normality:  $p(\eta|\phi_0) = N(0, \Phi)$ . This is a hierarchical prior for  $\eta$ . (ii) Conditional Independence:  $p(\eta|\phi_0) = p(\eta|\phi_0)p(\chi|\phi_0)$ . (iii) Exogeneity:  $p(\chi|\phi_0) = p(\chi)$ . With these assumptions, the likelihood function is:

$$p(Y, \chi|\phi_0) = p(Y|\phi_0)p(\chi) \propto p(Y|\phi_0) \propto (\Omega)^{-\frac{n}{2}} \cdot \exp\left[-\frac{1}{2}(Y - \chi\theta)'(\Omega)^{-1}(Y - \chi\theta)\right] \quad (23)$$

Running the likelihood function, the estimated model in 16 is:

$$Y = \chi\hat{\theta} + \eta \quad (24)$$

By equation 24, the likelihood in equation 23 can be developed as:

$$\begin{aligned} (\Omega)^{-\frac{n}{2}} \exp\left[-\frac{1}{2}(Y - \chi\hat{\theta})'(\Omega)^{-1}(Y - \chi\hat{\theta})\right] &= (\Omega)^{-\frac{n}{2}} \exp\left\{-\frac{1}{2}[(\chi\theta + \eta) - \chi\hat{\theta}]'\Omega^{-1} \cdot \right. \\ &\quad \left. \cdot [(\chi\theta + \eta) - \chi\hat{\theta}]\right\} \end{aligned} \quad (25)$$

By completing the square, equation 26 can be re-written for convenience:

$$\begin{aligned} &(\Omega)^{-\frac{n}{2}} \exp\left[-\frac{1}{2}\chi'(\theta - \hat{\theta})' + \eta'\Omega^{-1}\eta + \chi(\theta - \hat{\theta})\right] \\ &= (\Omega)^{-\frac{v}{2}}(\Omega)^{-\frac{k}{2}} \exp\left\{-\frac{1}{2}[(\theta - \hat{\theta})'\chi'\Omega^{-1}\chi(\theta - \hat{\theta}) + vS]\right\} \\ &= (\Omega)^{-\frac{k}{2}} \exp\left\{-\frac{1}{2}[(\theta - \hat{\theta})'\chi'(\Omega)^{-1}\chi(\theta - \hat{\theta})]\right\} (\Omega)^{-\frac{v}{2}} \exp\left\{-\frac{vS}{2}(\Omega)^{-1}\right\} \end{aligned} \quad (26)$$

where,  $\hat{\theta} = (\chi'\chi)^{-1}\chi'Y$ ,  $S = (\frac{1}{v})(Y - \chi\hat{\theta})'(Y - \chi\hat{\theta})$ , and  $v = n - k$ . In the following manner and

perspective, it is easy to notice that:  $(\Omega)^{-\frac{k}{2}} \exp\{-\frac{1}{2}[(\theta - \hat{\theta})' \chi' (\Omega)^{-1} \chi (\theta - \hat{\theta})]\}$  is the kernel of a  $N(\hat{\theta}, \Omega(\chi' \chi)^{-1})$  and  $(\Omega)^{-\frac{v}{2}} \exp\{-\frac{vS}{2}(\Omega)^{-1}\}$  is the kernel of an  $IG(\frac{v-2}{2}, \frac{vS}{2})$ .

A hierarchical prior for  $\eta$  has been already specified. Thus, in order to complete the model, a prior moments on  $(\chi_0, \Omega^{-1}, b_f)$  need to be defined. It is just viewed that the likelihood function can be derived from the sampling density  $p(Y|\phi_0)$ , thus it is considered as a function of the parameters. To be more precise, it can be shown to be of a form that breaks into three parts. (i) A distribution for factors  $\theta$  given  $\Omega$ . (ii) A distribution where  $\Omega^{-1}$  has a Wishart distribution. (iii) A distribution for  $b_f$ , where  $b_f = \text{vec}(\mathcal{B})$  has a Inverse Gamma distribution. That is:

$$\theta|\Omega, Y \sim N(\hat{\theta}, \Omega \otimes (\chi' \chi)^{-1}) \quad (27)$$

$$\Omega^{-1}|Y \sim W(S^{-1}, T - K - M - 1) \quad (28)$$

$$b_f|Y \sim IG(\frac{\bar{\omega}}{2}, \frac{vS}{2}) \quad (29)$$

Furthermore, such prior assumptions will generally be influenced, for example, by common or subjective beliefs about marginal effects of economic variables. Hence, Independent Normal Wishart Prior is used in this analysis, since it assumes that tentative beliefs on  $(\theta_0, \Omega^{-1}, b_f)$  derive from separate considerations.

## 5.4 Inference

Rearranging equations 16 and 22, the SUR model described in equation 15 can be easily re-written as:

$$Y_t = (X_t \cdot \Xi_t) \theta_t + \eta_t \quad \eta_t = X_t u_t + E_t \quad (30)$$

$$\theta_t = \theta_{t-1} + v_t \quad v_t \sim N(0, \mathcal{B}) \quad (31)$$

In order to conduct inference, letting  $\theta_t = \theta \forall t$ , the estimation is easy since it only requires regressing each element of  $Y_t$  on appropriate averages, adjusting estimates of the standard errors for the presence of heteroschedasticity. With a prior for  $\theta$ , posterior estimates would be straightforward to construct. However, when the  $\theta_t$ 's are time-varying, with  $\mathcal{B}_{it} \neq 0$ , MCMC methods can be employed to construct their exact posterior distributions.

Let data run from  $(-\tau, T)$ , where  $(-\tau, 0)$  is a training sample used to estimate features of the prior. When such a sample is unavailable, it is sufficient to modify the expressions for the prior moments in equations 27, 28, and 29 as:

$$p(\Omega^{-1}, b_f, \theta_0) = p(\Omega^{-1})\Pi_f p(b_f)p(\theta_0) \quad (32)$$

where,  $p(\Omega^{-1}) = W(\theta_1, z_1)$ ,  $p(b_f) = IG(\frac{\bar{\omega}_0}{2}, \frac{S_0}{2})$ , and  $p(\theta_0|\mathcal{F}_{-1}) = N(\bar{\theta}_0, \bar{R}_0)$ . Here,  $N()$  stands for Normal,  $W()$  for Wishart, and  $IG()$  for Inverse Gamma distributions; while  $\mathcal{F}_{-1}$  denotes the information available at time  $-1$ . The prior for  $\theta_0$  and the law of motion for the factors imply that  $p(\theta_t|\mathcal{F}_{-1}) = N(\bar{\theta}_{t-1|t-1}, \bar{R}_{t-1|t-1} + \mathcal{B}_t)$ , where  $\bar{\theta}_{t-1|t-1}$  and  $\bar{R}_{t-1|t-1}$  are, respectively, the mean and the (variance)-covariance matrix of the conditional distribution of  $\bar{\theta}_{t-1|t-1}$ . The hyperparameters are all known<sup>6</sup>. To be more precise, collecting them in a vector  $\alpha$ , where  $\alpha = (z_1, Q_1, \bar{\omega}_0, S_0, \bar{\theta}_0, \bar{R}_0)$ , they are treated as fixed and are either obtained from the data (this is the case for  $\bar{\theta}_0$  and  $Q_1$ ) to tune the prior to the specific applications or selected a-priori to produce relatively loose priors (this is the case for  $z_1, \bar{\omega}_0, S_0, \bar{R}_0$ ). The value used are:  $z_1 = N \cdot M + 7$ ,  $Q_1 = \hat{Q}_1$ ,  $\bar{\omega}_0 = 10^7$ ,  $S_0 = 1.0$ ,  $\bar{\theta}_0 = \hat{\theta}_0$ , and  $\bar{R}_0 = I_f$ . Here,  $\hat{Q}_1$  is a block diagonal matrix, where  $\hat{\theta}_1 = \text{diag}(Q_{11}, \dots, Q_{1N})$  and  $Q_{1i}$  is the estimated covariance matrix of the time invariant version for each country VAR, and  $\hat{\theta}_0$  is obtained with the OLS on a time invariant version of equation 15.

The posterior distributions for  $\phi = (\Omega^{-1}, b_f, \{\theta_t\}_{t=1}^T)$  are calculated combining the prior with the (conditional) likelihood on initial conditions of the data, which is proportional to:

$$L(Y^T|\phi) \propto (\Omega)^{-\frac{T}{2}} \exp\left\{-\frac{1}{2}[\Sigma_t(Y_t - (X_t\Xi)\theta_t)'\Omega^{-1}\Sigma_t(Y_t - (X_t\Xi)\theta_t)]\right\} \quad (33)$$

where,  $Y^T = (Y_1, \dots, Y_T)$  denotes the data,  $\phi = (\Omega^{-1}, b_f, \{\theta_t\})$  the unknowns whose joint distri-

<sup>6</sup>For instance, prior hyper-parameters are own computations.

bution needs to be found, with  $\phi_{-k}$  standing the vector  $\phi$  excluding the parameter  $k$ .

## 5.5 Posterior Distributions and MCMC Methods

Given a prior  $p(\phi)$ , according to the Bayes rule, the conditional posterior  $p(\phi|Y^T)$  is proportional to:

$$p(\phi|Y^T) = \frac{p(\phi)L(Y^T|\phi)}{p(Y^T)} \propto p(\phi)L(Y^T|\phi) \quad (34)$$

Given  $p(\phi|Y^T)$ , the posterior distribution for the elements of  $\phi$  can be obtained by integrating out nuisance parameters from  $p(\phi|Y^T)$ . Once these distributions are found, location and dispersion measures can be obtained for  $\phi$  or for any interesting continuous function of these parameters. Despite the dramatic parameter reduction obtained with equation 15, analytical computation of posterior distributions  $p(\phi|Y^T)$  is unfeasible. However, through Monte Carlo techniques, a variant of the Gibbs sampler approach can be used in this framework it only requires knowledge of the conditional posterior distribution of  $\phi$ . Thus, the posterior distributions for  $\phi$  are:

$$\theta_t|Y^T, \phi_{-\theta_t} \sim N(\tilde{\theta}_{t|T}, \tilde{R}_{t|T}) \quad \text{with} \quad t \leq T \quad (35)$$

$$\Omega|Y^T, \phi_{-\Omega} \sim iW(\hat{z}_1, \hat{\theta}_1) \quad (36)$$

$$b_f|Y^T, \phi_{-b_f} \sim IG\left(\frac{\bar{\omega}_f}{2}, \frac{\bar{S}}{2}\right) \quad (37)$$

where,  $\tilde{\theta}_{t|T} = \tilde{R}_{t|T}(\bar{R}_0^{-1}\bar{\theta}_0 + \Sigma_t(X_t\Xi)'\Omega^{-1}Y_t)$  and  $\tilde{R}_{t|T} = (\bar{R}_0^{-1} + (X_t\Xi)'\Omega^{-1}(X_t\Xi))^{-1}$ , with  $\tilde{\theta}_{t|T}$  and  $\tilde{R}_{t|T}$  denoting the smoothed one-period-ahead forecasts of  $\theta_t$  and of the variance-covariance matrix of the forecast error, respectively;  $\hat{z}_1 = z_1 + T$  and  $\hat{\theta}_1 = [\theta_1 + (Y_1 - (X_1\Xi)\theta_1)'\Omega^{-1}(Y_1 - (X_1\Xi)\theta_1)]^{-1}$ ;  $\bar{\omega}_f = K + \bar{\omega}_0$  and  $\bar{S} = S_0 + \Sigma_t(\theta_t^f - \theta_{t-1}^f)'(\theta_t^f - \theta_{t-1}^f)$ , with  $\theta_t^f$  denoting the  $f^{th}$  subvector of  $\theta_t$ ,  $K = NM$ , and  $f$  the factors described in equation 4.

The conditional posterior of  $(\theta_1, \dots, \theta_T|Y^T, \phi_{-\theta_t})$  can be obtained with a run of the Kalman filter

and of a simulation smoother as in Greenberg and Chib (1995). To be more precise, the Kalman (1960, 1963) filter technique is adopted to estimate linear regression models with time-varying coefficients. This class of models consists of two equations. (i) The transition equation, describing the evolution of the state variables. (ii) The measurement equation, describing how the observed data are generated from the state variables. This approach is extremely useful for investigating the issue of parameters constancy, because it is an updating method producing estimates for each time period based on the observations available up to the current period. It is important to realise that recursive OLS estimation is not a suitable technique to use here. Recursive estimation is essentially a test of structural stability. For example, given  $H_0$  be the null hypothesis that the parameters are constant and  $H_1$  be that alternative that the parameters are estimated through recursive estimation. But as the underlying assumption of OLS is always that the parameters are constant, recursive estimation does not provide a consistent estimate of a time-varying parameters. In particular, given  $\theta_{0|0}$  and  $R_{0|0}$ , the Kalman filter gives the recursions<sup>7</sup>:

$$\theta_{t|t} = \tilde{\theta}_{t-1|t-1} + [\tilde{R}_{t|t-1}(X_t\Xi)F_{t|t-1}^{-1}][Y_t - (X_t\Xi)\theta_{t-1|t-1}] \quad (38)$$

$$R_{t|t} = [I - \tilde{R}_{t|t-1}(X_t\Xi)'F_{t|t-1}^{-1}(X_t\Xi)](\tilde{R}_{t-1|t-1} + \mathcal{B}) \quad (39)$$

$$F_{t|t-1} = (X_t\Xi)' \tilde{R}_{t|t-1}(X_t\Xi) + \Omega_t \quad (40)$$

Hence, in order to obtain a sample  $\{\theta_t\}$  from the joint posterior distribution  $(\theta_1, \dots, \theta_T | Y^T, \phi_{-\theta_t})$ , the output of the Kalman filter is used to simulate  $\theta_T$  from  $N(\theta_{T|T}, R_{T|T})$ ,  $\theta_{T-1}$  from  $N(\theta_{T-1|T-1}, R_{T-1|T-1})$ , and  $\theta_1$  from  $N(\theta_1, R_1)$ , where:

$$\theta_t = \theta_{t|t} + R_{t|t} \cdot R_{t+1|t}^{-1} \cdot (\theta_{t+1} - \theta_{t|t}) \quad (41)$$

$$R_t = R_{t|t} - R_{t|t} \cdot R_{t+1|t}^{-1} \cdot R_{t|t} \quad (42)$$

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<sup>7</sup>For instance, see the dynamic analysis described in Section 6.

The recursion can be started choosing  $R_{0|0}$  to be diagonal with elements equal to small values, whereas  $\theta_{0|0}$  can be estimated in the training sample or initialized using a constant coefficient version of the model.

Under regularity conditions (see e.g., Geweke (2001)), cycling through the conditional distributions in equation 35 in the limit it produces draws from the joint posterior of interest. In fact, convergence only requires the algorithm to be able to visit all partitions of the parameter-space in a finite number of iterations. Thus, the marginal distributions of  $\theta_t$  can be computed averaging over draws in the nuisance dimensions and the posterior distributions of indicators can be obtained. A credible 95% interval for every indicator described in equation 4 is obtained ordering the draws of  $\chi_{it}\theta_{it}$  for each  $t$  and taking the 5th and the 95th percentile of the distribution.

However, the regressors of the SUR model in equation 30 are correlated, but the presence of correlation, even of extreme form, does not create problems in identifying the loading as long as the priors are proper (see e.g., Ciccarelli and Rebucci (2007)). In addition, the choice of making  $E_t$  and  $u_t$  correlated allows conjugation between the prior and the likelihood, avoids identification issues, and greatly simplifies the computation of the posterior. This latter is also used in Minnesota prior (see e.g., Doan *et al.*, 1984). Hence, as stated in Section 5.3, the forecast error  $\eta = Y_t - (X_t\Xi)\theta_t$  has the form  $(\eta|\sigma^2) \sim N(0, \sigma_t\Omega)$ . Therefore, unconditionally,  $\eta_t$  has a multivariate  $t$ -distribution centered at  $\theta$ , with a scale matrix proportional to  $\Omega$  and  $\nu_n$  degree of freedom. Thus, the innovations of the model described in equation 30 are endogenously allowed to have fat tails. Finally, since the fit improves when  $\sigma^2 \rightarrow 0$ , the model in equation 30 presents an exact factorization of  $\delta_t$ .

In order to compute conditional heteroschedasticity in  $Y_t$ , Sargent and Cogley (2005) specify  $\Omega$  to be a function of a set of stochastic volatility processes. The above discussion shows that a similar result can be equivalently obtained with a simpler set of assumptions. The selection model in equation 30 appeals on another count. To be more precise, since shocks to the model may alter its dynamics, by construction, it has built-in an endogenous adaptive scheme that allows coefficients to adjust when breaks in the relationship occur. Posterior distributions for any continuous function  $G(\phi)$  can be obtained using the output of the MCMC algorithm and the ergodic theorem. For example,  $E[G(\phi)] = \int G(\phi)p(\phi|Y_T)d\phi$  can be approximated using  $\frac{1}{L}[\sum_{l=\bar{L}+1}^{\bar{L}+L} G(\phi^l)]$ , where the first  $\bar{L}$  observations represent a burn-out sample discarded in the calculation. Predictive distributions for future  $Y_{it}$ 's can be estimated using the recursive nature of the model and the conditional structure of the posterior. Let  $Y^{t+\tau} = (Y_{t+1}, \dots, Y_{t+\tau})$ , consider the conditional density of  $Y^{t+\tau}$ ,

given the data up to  $t$ , and a function  $G(Y^{t+\tau})$ , then:

$$F[G(Y^{t+\tau})|Y_t] = \int F[G(Y^{t+\tau})|Y_t, \phi] p(\phi|Y_t) d\phi \quad (43)$$

Here, forecasts for  $Y^{t+\tau}$  can be obtained drawing  $\phi^{(l)}$  from the posterior distribution and simulating the vector  $Y^{t+\tau}$  from the density  $F(Y^{t+\tau}|Y_t, \phi^{(l)})$ . Turning point distributions can also be constructed by appropriately choosing  $G$ . Impulse responses and conditional forecasts can be obtained with the same approach as detailed in Section 6.

## 5.6 Variance Component Model

Considering the model with the following state-space structure:

$$Y_{it} = \gamma_{it} + T_t \quad (1 - \rho_t L)T_t = e_t \quad (44)$$

$$\gamma_{it} = \gamma_i + \vartheta_{it} \quad (1 - w_i L)\vartheta_{it} = (X\Xi)_{it} \quad (45)$$

$$\gamma_i = \gamma_0 + \epsilon \quad (46)$$

where,  $e_t$  is i.i.d. across  $t$ ,  $\vartheta_{it}$  is i.i.d. across  $t$ , and  $Y_{it}$  is a  $NM \cdot 1$  vector for each  $i=1,2,\dots, N$ . This model has the following VAR representation<sup>8</sup>:

$$Y_t = \gamma_{0t}^* + A_t Y_{t-1} + B_t W_{t-1} + \eta_t = \gamma_{0t}^* + \delta_t (X_t \Xi) + \eta_t \quad (47)$$

where,  $X_t = (Y'_{t-1}, W'_t, W'_{t-1}, C'_t, C'_{t-1})'$ ,  $Y_t$  is a  $NM \cdot 1$  vector each  $t$ ,  $\gamma_{0t}^* = \text{diag}\{(1 - 1\omega\epsilon a_i)\}(1 - \rho_t)\gamma_0$ ,  $\eta_{it} = (1 - \omega_i L)e_t + (1 - \omega_i L)(1 - \rho_t L)\epsilon + (1 - \rho_t L)(X\Xi)_{it}$ ,  $A_{it} = \rho_t + \omega_i$ , and  $B_{it} = \rho_t + \omega_i$ . Therefore, an error component model generates a particular error structure in the VAR. Note that

<sup>8</sup>In this case, the multicountry selected VAR model corresponds to equation 1.



$\gamma_{0t}^*$  are time trends common to all the  $M$  variables for unit  $i$ . Thus, according to equation (2.6),  $\delta_t = [\text{vec}(A_t), \text{vec}(B_t), \text{vec}(C_t)]$  is factorized as:

$$\delta_{timp} = \Xi_1\theta_{1t} + \Xi_2\theta_{2t} + \Xi_3\theta_{3t} + \Xi_4\theta_{4t} + \Xi_5\theta_{5t} + u_{timp}^\delta \quad (48)$$

where,  $\theta_{1t}$ ,  $\theta_{2t}$ ,  $\theta_{3t}$ , and  $\theta_{4t}$  are  $NM_1 \cdot 1$  vectors of country-specific factors common to all lags  $p$ ;  $\theta_{5t}$  is  $M_1 \cdot 1$  vector of group-variable specific common to all variables  $m$  and lags  $p$ .  $\Xi_1$ ,  $\Xi_2$ ,  $\Xi_3$ , and  $\Xi_4$  are matrices of dimensions  $NMk \cdot 1$ ;  $\Xi_5$  is matrix of dimension  $NMk \cdot M_1$ . Therefore,  $\gamma_{0t}^*$  is assumed to be:

$$\gamma_{0t}^* = \Xi_6\theta_{6t} + u_{pit}^\gamma \quad (49)$$

where,  $\theta_{6t}$  is  $NM \cdot 1$  vector, and  $\Xi_6$  is matrix of dimension  $NMk \cdot 1$ . Equations 48 and 49 represent a version of the model described in equation 5. Here, the number of parameters to be estimated is  $NM + NM_1 + M_1$ , which is still relatively large. To further reduce the dimensionality of the parameter vector one could make  $\theta_{6t}$  time- or unit- independent and exploit averages in the remaining dimensions to construct the appropriate regressors. Disregarding how  $\gamma_{0t}^*$  is parametrized, the SUR model is:

$$(Y_t - \gamma_{0t}^*) = \theta_{1NM_1}\chi_{1t} + \theta_{2NM_1}\chi_{2t} + \theta_{3NM_1}\chi_{3t} + \theta_{4NM_1}\chi_{4t} + \theta_{5M_1}\chi_{5t} + \eta_t \quad (50)$$

where,  $\chi_{1t}$ ,  $\chi_{2t}$ ,  $\chi_{3t}$ , and  $\chi_{4t}$  are country-specific indices,  $\chi_{5t}$  is a group-variable specific factors,  $\chi_{6t} = (Y_t - \gamma_{0t}^*)$  denotes common and idiosyncratic components across all countries and variables, and  $\eta_t$  is composite error whose variance depends on group-specific factors, on a common index, on variable-specific effects, on the lags  $p$ , and on a time- or unit- independence index. Hence, the reparametrization maintains the original error component structure, but somewhat reduces the dimensionality of the parameters space.

## 6 Dynamic Analyses

Dynamic Analysis is non-standard in the SUR model as described in equation 30, because of the specification of the error term and the time variations potentially present in the coefficients. Hence, in the following sections, impulse responses and conditional forecasts are described providing statistics useful for academics and policymakers.

### 6.1 Impulse Responses

Impulse responses are generally computed as the difference between two realizations of  $y_{t+\tau}$ , with  $\tau = 1, 2, \dots$ , which are identical up to time  $t$ . Thus, between  $t+1$  and  $t+\tau$ , one can assume two time impulses in the  $j$ th component of  $e_{t+\tau}$ . (i) One that occurs only at time  $t+1$ . (ii) The other that no shocks take place at all dates between  $t+1$  and  $t+\tau$ . In a model with time-varying coefficients, the approach is inadequate since it overlooks that between  $t+1$  and  $t+\tau$ , structural coefficients may also change. Therefore, impulse responses are obtained as the difference between two conditional expectations of  $y_{t+\tau}$ . In both cases, they are conditioned on the history of the data  $Y_t$  and of the factors  $\theta^t$ , the parameters of the law of motion of the coefficients, and all future shocks. However, impulse responses are conditioned on a random draw for the current shocks, whereas in the other the current shocks is set to its unconditional value. Hence, they are worked out on future shocks instead of integrating them out because, computationally, such a choice gives more stable responses, even though this makes standard error bands larger than in the case where future shocks are integrated out. Given the equation 30, one has two potential types of impulses. (i) One to the variables of the system. (ii) One to the factors.

Here, the reparametrized SUR is:

$$y_t = \chi_t \theta_t + (E_t + X_t u_t) \quad \text{with} \quad \theta_t = \theta_{t-1} + v_t \quad (51)$$

where  $\theta_t = [\theta'_{1t}, \theta'_{2t}, \dots, \theta'_{Ft}]'$ ,  $\chi_t = [\chi_{1t}, \dots, \chi_{Ft}]$ ,  $\chi_{it} = \Xi X_t$ ,  $X_t = [Y_{t-1}, W_t, W_{t-1}]$ . Let  $\mathcal{U} = [(E_t + X_t u_t)', v_t']'$  be the vector of reduced-form shocks and  $Z_t = [H_t^{-1}(E_t + X_t u_t)', H_t^{-1}v_t']'$  be the vector of structural shocks where  $E_t = H_t v_t$ ,  $H_t H_t' = \Omega$  so that  $\text{var}(v_t) = \sigma^2 I_k$  and  $H_t = J \cdot K_t$  where  $K_t K_t' = I$  and  $J$  is a matrix that orthogonalizes the VAR shocks.

Here, a Choleski system is obtained setting  $K_t = I, \forall t$ , and choosing  $J$  to be lower triangular whereas more structural identification schemes are obtained letting  $J$  be an arbitrary square root matrix and  $K_t$  a matrix implementing certain theoretical restrictions. The identification matrix  $K_t$  is allowed to be time-varying since, when recursive estimations are used, estimates of  $\Omega$  depends on  $t$ .

Let  $\mathbb{Z}_t = (\Omega, \sigma^2, B_t, \Phi)$ , let  $\bar{Z}_{j,t}$  be a particular realization of  $Z_{j,t}$  and  $Z_{-j,t}$  indicate the structural shocks, excluding the one in the  $j$ th component. Let  $F_t^1 = \{Y^{t-1}, \theta^t, \mathbb{Z}_t, H_t, Z_{j,t}\}$ , with  $Z_{j,t} = \{\bar{Z}_{j,t}, Z_{-j,t} \mathcal{U}_{t+1}^{t+\tau}\}$ , and  $F_t^2 = \{Y^{t-1}, \theta^t, \mathbb{Z}_t, H_t, Z_{j,t}\}$ , with  $Z_{j,t} = \{EZ_{j,t}, Z_{-j,t} \mathcal{U}_{t+1}^{t+\tau}\}$  be two conditioning sets. Thus, responses to a shock at  $t$  in the  $j$ th component of  $Z_t$  are obtained as:

$$IR(t, t + \tau) = E(Y_{t+\tau}|F_t^1) - E(Y_{t+\tau}|F_t^2) \quad t = 1, 2, \dots \quad (52)$$

In order to see what definition equation 52 involves, rewrite the original VAR model ?? in a companion form<sup>9</sup>:

$$Y_{t+\tau} = A_{t+\tau} Y_{t+\tau-1} + B_{t+\tau} W_{t+\tau-1} + E_{t+\tau} \quad (53)$$

and let

$$\delta_{t+\tau} = \Xi[\theta_{t+\tau-1} + v_{t+\tau}] + u_{t+\tau} \quad (54)$$

where  $\delta_{t+\tau} = [vec(A_{1t+\tau}), vec(B_{t+\tau})]$  and  $A_{1t+\tau}$  is the first row of  $A_{t+\tau}$ . Taking  $Y^{t-1} = (Y_{t-1}, Y_{t-2}, \dots, W_{t-1})$ ,  $A^t = (A_t, A_{t-1}, \dots)$ ,  $B^t = (B_t, B_{t-1}, \dots)$ , and  $H_{t+\tau} = H_t$  for  $\forall \tau$  as given.

Solving backward, equations 53 and 54 can be rewritten as:

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<sup>9</sup>The same computations are done for the model 1 accounting for exogenous variables  $Z_t$ .

$$\begin{aligned}
Y_{t+\tau} &= \left( \prod_{k=0}^{\tau} A_{t+\tau-k} \right) Y_{t-1} + B_{t+\tau} W_{t+\tau-1} + \sum_{h=1}^{\tau} \left( \prod_{k=0}^{h-1} A_{t+\tau-k} \right) B_{t-\tau-h} W_{t+\tau-h-1} + \\
&+ H_{t-\tau} \eta_{t+\tau} + \sum_{h=1}^{\tau} \left( \prod_{k=0}^{h-1} A_{t+\tau-k} \right) H_{t+\tau-h} \eta_{t+\tau-h} \quad (55)
\end{aligned}$$

and as

$$\delta_{t+\tau} = \Xi \theta_{t-1} + \Xi \sum_{k=0}^{\tau} v_{t+\tau-k} + u_{t+\tau} \quad (56)$$

Consider first the case of a  $(m+1)$ -period impulse in the  $j$ th component of  $v$ . For example:  $v_{j,t+k} = \bar{v}_{j,t+k}$ ;  $v_{-j,t+k}$ ,  $k = 0, 1, 2, \dots, m$  and  $v_{t+m'}$ , with  $\forall m' > m$ , are restricted. Then,

$$\begin{aligned}
IR_{t,t+\tau} &= E_t[Y_{t+\tau} | Y_{t-1}, A^t, B^t, \mathbb{Z}_t, H_t, \{\bar{\eta}_{jt+m}\}_{k=0}^m, \{\eta_{-jt+k}\}_{k=0}^m, \{\eta_{t+k}\}_{k=m+1}^{\tau}] - \\
&- E_t[Y_{t+\tau} | Y^{t-1}, A^t, B^t, \mathbb{Z}_t, H_t, \{\eta_{t+k}\}_{k=0}^{\tau}] \\
&= E_t \left[ \left( \prod_{k=0}^{\tau-1} \right)^j H_t^j (\bar{\eta}_{jt} - E\eta_{jt}) + \left( \prod_{k=0}^{\tau-2} A_{t+\tau-k} \right)^j \cdot H_{t+1}^j (e\bar{a}_{jt+1} - E\eta_{jt+1}) + \dots \right. \\
&\quad \left. \dots + \left( \prod_{k=0}^{\tau-m-1} A_{t+\tau-k} \right)^j \cdot H_{t+m}^j (e\bar{a}_{jt+m} - E\eta_{jt+m}) \right] \quad (57)
\end{aligned}$$

where the superscript  $j$  refers to the  $j$ th column of the matrix. It is easy to see that, when  $A_t = A$  and  $B_t = B$ ,  $\forall t$ , equation 57 reduces to standard impulse responses and, when  $E_t$  and  $v_t$  are correlated (that is both the sign and the size of the shocks matter a shock in  $v_t$ ), may induce changes in  $A_t$  or  $B_t$ .

Given 52, responses in the SUR model can be computed as follows:

1. Choosing  $t$ ,  $\tau$ , and  $J_t$ . Draw  $\Omega^l = H_t^l (H_t^l)', (\sigma^2)'$  from their posterior distribution and  $u_t^l$

from  $N(0, (\sigma^2)^2 I \otimes H_t^l (H_t^l)')$ . Computing  $y_t^l = \chi_t \theta_t + H_t \bar{\eta}_t + X_t u_t^l$ .

2. Drawing  $\Omega = H_{t+1}^l (H_{t+1}^l)', (\sigma^2)^l, \mathcal{B}_{t+1}^l, \phi^l$ . Drawing  $\eta_{t+1}^l$  from their posterior distribution. Using the law of motion of the factors to compute  $\theta_{t+1}^l, l = 1, 2, \dots, L$ , and the definition of  $\Xi$  to compute  $\chi_{t+1}$ . Drawing  $u_{t+1}^l$  from  $N(0, (\sigma^2)^l I \otimes H_{t+1}^l (H_{t+1}^l)')$  and computing  $y_{t+1}^l = \chi_{t+1} \theta_{t+1} + H_{t+1} \bar{\eta}_{t+1} + X_{t+1} u_{t+1}^l, l = 1, 2, \dots, L$ .
3. Repeating Step 2 and computing  $\theta_{t+k}^l, \theta_{t+k}^l, k = 1, 2, \dots, \tau$ .
4. Repeating Steps 1 - 3 by setting  $\eta_{t+K} = E(\eta_{t+1}), k = 1, 2, \dots, m$  and using the draws for the shocks in 1 - 3.

Responses to structural shocks to the law of motion of the factors can be computed in the same way. An impulse in  $v_t = \bar{v}$  lasting  $(m+1)$  periods implies from equation 56 that:

$$E(\bar{\delta}_{t+\tau} - \delta_{t+\tau}) = \Xi \sum_{k=0}^m H_{t+k} (\bar{\eta}_{t+\tau-k} - E\eta_{t+\tau} - k) \quad (58)$$

and

$$\begin{aligned} IR_{t,t+\tau} &= E_t \left[ \prod_{k=0}^{\tau} (\bar{A}_{t+1,\tau-k} - A_{t+\tau-k}) Y_{t+1} + \sum_{h=1}^{\tau} \prod_{k=0}^{h-1} (\bar{A}_{t+1,\tau-k} - A_{t+\tau-k}) \cdot B_{t+\tau-h-1} + \right. \\ &\quad \left. + \sum_{h=1}^{\tau} \prod_{k=0}^{h-1} (\bar{A}_{t+1,\tau-k} - A_{t+\tau-k}) H_{t+\tau-h} \eta_{t+\tau-h} \right] \quad (59) \end{aligned}$$

## 6.2 Conditional Forecasts

There are two types of conditional forecasts one can compute in this framework. Those involving displacement of the exogenous variables  $W_t$  from their unconditional path, and those involving a particular path for a subset of the endogenous variables. Both types of conditional forecasts can be constructed using the output of the Gibbs sampler routine.

Consider first displacing the exogenous variables from their expected future path for  $m+1$  periods. Calling the new path  $\bar{W}_{t+k}, k = 0, 1, \dots, m$ . Then, the response of  $Y_{t+\tau}$  is:

$$\begin{aligned}
IR_{t,t+\tau} = & E_t \left[ \left( \prod_{k=0}^{\tau-2} A_{t+\tau-k} \right) B_{t+1} (\bar{W}_{jt} - W_{jt}) + \left( \prod_{k=0}^{\tau-3} A_{t+\tau-k} \right) B_{t+2} (\bar{W}_{jt+1} - W_{jt+1}) + \dots \right. \\
& \left. \dots + \left( \prod_{k=0}^{\tau-2-m} A_{t+\tau-k} \right) B_{t+m+1} (\bar{W}_{jt+m} - W_{jt+m}) \right] \quad (60)
\end{aligned}$$

Thus, to compute conditional forecasts of this type in the SUR model, one need to:

1. Choosing  $t$ ,  $\tau$ , and a path  $\{\bar{W}_{t+k}\}_{k=0}^m$ . Drawing  $\Omega^l$ ,  $(\sigma^2)^l$  from their posterior, drawing  $E_t^l + X_t u_t^l$  and computing  $y_t^l$ .
2. Drawing  $(B_t)^l$ ,  $\Psi^l$  from their posterior distribution; drawing  $v_{t+1}^l$  and using the law of motion of the factors to draw  $\theta_{t+1}^l$ ,  $1, 2, \dots, L$  and the definition of  $\Xi$  to compute  $\chi_{t+1}$ . Then,  $E_{t+1}^l + X_{t+1} u_{t+1}^l$  are drawn to compute  $y_{t+1}^l = \chi_{t+1} \theta_{t+1}^l + (E_{t+1}^l + X_{t+1} u_{t+1}^l)$ ,  $l = 1, 2, \dots, L$ .
3. Repeating Step 2 in order to compute  $\theta_{t+k}^l$ ,  $y_{t+k}^l$ ,  $k = 1, 2, \dots, \tau$ .
4. Repeating Steps 1 - 3. In this way, it sets  $W_{t+k} = E(W_{t+k})$ ,  $k = 0, 1, \dots, m$ , using the draws for the shocks in 1 - 3.

Finally, considering the case in which the future path of a subset of  $Y_t$ 's is fixed. For example, in a system with output growth, inflation, and the nominal rate, one would like to work out on a given path for the future interest rate.

Hence, partitioning  $Y_t = A_t Y_{t-1} + B W_{t-1} + E_t$  in two blocks, let  $Y_{2t+k} = \bar{Y}_{2t+k}$  be the fixed variables and  $Y_{1t+k}$  those allowed to adjust, the Impulse Responses are:

$$\begin{aligned}
IR_{t,t+\tau} = & E \left[ H_t^1 \left( \prod_{k=0}^{t-1} A_{t+\tau-k} \right)^1 (\bar{\eta}_{2t} - \eta_{2t}) + H_{t+1}^1 \left( \prod_{k=0}^{t-2} A_{t+\tau-k} \right)^1 (\bar{\eta}_{2t+1} - \eta_{2t+1}) + \dots \right. \\
& \left. \dots + H_{t+m}^1 \left( \prod_{k=0}^{t-1-m} A_{t+\tau-k} \right)^1 (\bar{\eta}_{2t+m} - \eta_{2t+m}) \right] \quad (61)
\end{aligned}$$

where  $\bar{\eta}_{2t+k} = \bar{Y}_{2t+k} - A_{21,t+k} Y_{1t-k-1} - A_{22,t+k} Y_{2t-k-1} - B_{2t+k} W_{t+k-1}$  and the super-script 1 refers

to the first row of the matrix. Hence, to compute this type of conditional forecasts one need to:

1. Partitioning  $y_t = (y_{1t}, y_{2t})$ , choosing  $t$  and a path  $\{y_{2t+k}\}_{k=0}^T$ . Using the model to solve for the  $\bar{\eta}_{2t}$  that gives  $y_{2t} = \bar{y}_{2t}$ , backing out the implied  $y_{1t}^l$  once draws for  $E_{1t}^l$ , and computing  $u_t^l$  from their posterior distribution. Thus,  $upsilon_{t+1}^l$  can be drawn using the law of motion of the factors to obtain  $\theta_{t+1}^l$ , with  $l = 1, 2, \dots, L$ , and the definition of  $\Xi$  to compute  $\chi_{t+1}$ .
2. Using the model to solve for  $\bar{\eta}_{2t}$  that gives  $y_{2t+1} = \bar{y}_{2t+1}$ , backing out the implied  $y_{1t+1}^l$  once draws for  $E_{1t+1}^l$ , and computing  $u_{t+1}^l$  as above. Hence, once can draw  $v_{t+2}^l$ , using the law of motion of the factors to compute  $\theta_{t+2}^l$ , with  $l = 1, 2, \dots, L$ , and the definition of  $\Xi$  to compute  $\chi_{t+2}$ .
3. Repeating Step 2 and computing  $\theta_{t+k}^l, y_{t+k}^l, k = 2, 3, \dots$
4. Repeating Steps 1 - 3, once can set  $\eta_{t+k}^l = E(\eta_{t+k}), \forall k$  using the draws for the shocks in 1 - 3.

In Step 2 of all algorithms, it has implicitly assumed that selecting a path for the shocks does not alter neither the law of motion of the factors nor the beliefs about the true structural shocks. If this were not the case, an intermediate Step, where a run of the Kalman filter updates the information about the factors, needs to be used (for instance, see Section 5.5).

### 6.3 Recursive Unconditional Forecasts

Given the information at time  $t$ , unconditional forecasting exercises only require the computation of the predictive distribution of future observations. In some cases, recursive unconditional forecasts are needed, in which case the predictive density of future observations has to be constructed for every  $t = \bar{t}, \dots, T$  once recursive estimates of  $p(\phi_h|Y^T)$  are computed<sup>10</sup>. These recursive distributions are straightforward to obtain (e.g., a MCMC routine need to be run for every  $t$ ) and, although computationally demanding, they are feasible on available machines.

<sup>10</sup>See for instance Section 5.2 according to the Bayes factor.

## 7 Data Description

The last 15 years have observed an increased globalization of world economies. The model is estimated for 12 economies of the Euro Area: Italy, Spain, France, Belgium, Netherlands, Austria, Germany, Finland, Luxembourg, Ireland, Greece, Portugal. The sample period is 1998q4 - 2014q2. This span of data includes a large number of quarters before and after the financial crisis. Hence, the model is able to capture not only possible time variation around business cycle phases, but also time variation caused by possible structural changes (see e.g., Koop (1996)).

For each of the EA12 countries, the real variables (*real*) included are general government spending, real GDP growth rate, and gross fixed capital formation in order to capture business cycles and main spillover channels in real dimension. To be more precise, the general government spending (*gov*) denotes all financial accounts in percentage of GDP. Real GDP growth rate (*gdpg*) is computed respect to the same quarter of the previous year ( $q/q-4$ ). Gross fixed capital formation (*cap*), also known as Investments, consists of resident producers' acquisitions, less disposals, of fixed assets plus certain additions to the value of non-produced assets. These assets acquired are intended for use in processes of production. GFCF includes acquisition less disposals of, e.g. buildings, structures, machinery and equipment, mineral exploration, computer software, literary or artistic originals and major improvements to land such as the clearance of forests.

The financial variables (*fin*) included are interest rate, general government debt, and general government deficit, which are most suitable to capture business cycles and spillover channels in financial dimension. To be more precise, the interest rate (*int*) denotes EMU convergence criterion series relates to interest rates for long-term government bonds denominated in national currencies. Selection guidelines require data to be based on central government bond yields on the secondary market, gross of tax, with a residual maturity of around 10 years. The General government debt (*debt*) corresponds to quarterly non-financial accounts for the general government sector which are conceptually consistent with the corresponding annual data compiled on a national accounts basis. The general government sector comprises central government, state government, local government, and social security funds and is observed in percentage of GDP. General government surplus/deficit (*curr*), also known as current account balance, is defined in the Maastricht Treaty as general government net borrowing/lending according to the European System of Accounts and observed in percentage of GDP.

The (directly) observable variable to measure the effects from fiscal shocks in real and financial



components, respectively, is the productivity as proxy for economic growth (*prod*). It is defined as  $prod_{it} = \ln\left(\frac{Y_{it}}{Y_{it-1}}\right)$ , by considering the computations of Sala-i Martin (1996).

Bilateral flows of trade and bilateral flows of capital are computed to capture interactions between real and financial variables across countries, respectively [hereafter, they will call *weights*]. To be more precise, the trade flows denote exports and imports by Member States of the Euro Area at the current prices and weighted for the GDP. The capital flows denote financial transactions computed on the total economy in million units of national currency and weighted for the GDP. The values are expressed at the net on the total transactions.

There are five indicators which describe macroeconomic imbalances and, hence, economic/structural implications (called *imbalances*). The project considers one indicator monitoring external positions, one indicator capturing competitiveness developments and *catching-up* factors, three indicators reflecting internal imbalances. The specification model described in equation 6 is able to observe interdependence and time-varying effects across countries and over time. For the all selected period, net investment position, nominal unit labour cost, general government consumption, private sector consumption, unemployment rate.

To be more precise, international investment positions (*inv*) are observed in million euro and weighted for the GDP of eurozone countries.

Unit labour costs (*lab*) measure the average cost of labour per unit of output and are calculated as the ratio of total labour costs to real output. In broad terms, unit labour costs show how much output an economy receives relative to wages, or labour cost per unit of output. Generally, it represents a direct link between productivity and the cost of labour used in generating output. A rise in an economy's unit labour costs represents an increased reward for labour's contribution to output. However, a rise in labour costs higher than the rise in labour productivity may be a threat to an economy's cost competitiveness, if other costs are not adjusted in compensation.

The variable (*cons*) denotes the final consumption aggregates at the current prices and weighted for the GDP.

The variable (*priv*) consists of expenditure incurred by resident institutional units on goods or services that are used for the direct satisfaction of individual needs or wants or the collective needs of members of the community. The consumption expenditure may take place on the domestic territory or abroad. Private final consumption expenditure includes households' and Non Profit Institutions Serving Households' (NPISH) final consumption expenditure. NPISH are separate legal entities serving households and account, for example, for trade unions, professional

societies, political parties, sports clubs and so on.

Finally, unemployment rate (*unem*) denotes the growth rates of the unemployment by sex and age groups in percentage of GDP.

Running the BVAR model in equation 1, the above mentioned variables are able to evaluate bilateral spillover effects and systemic contribution between real and financial sectors<sup>11</sup>. Running the SUR model in equation 6, they can estimate possible co-movements, heterogeneity, and dispersion indices. Empirical evidence are heavily discussed in Section 10, where *catching-up* and competitiveness factors and *cause-effect* relationship due to economic/institutional implications can be explained<sup>12</sup>.

All endogenous and exogenous variables in the system are expressed in quarters. Since the analysis does not consider all Euro Area Countries, adjusted weights by the EA12 GDP are used to compute posterior of IRFs (except for the interest rate which is taken on the basis of EMU convergence criterion bond yields). However, all variables in the system are observed in annual growth rates with respect to the same quarter of the previous year ( $q/q-4$ ). Mostly data come from Eurostat, OECD, and IMF databases.

In equation 1, all real and financial variables are treated as endogenous, while group-variable specific are treated as weakly-exogenous factors. To be more precise, trade flows *real* corresponds to bilateral net exports and capital flows *fin* are determined as bilateral net financial transactions. Thus, they correspond to weak-exogenous variables in order to capture co-movements, spreading of spillover effects, heterogeneity, and some economic/structural implications when studying macroeconomic-financial linkages. The variables are significant and potentially strongly correlated with real and financial dimension. Moreover, the variable specific *fin* is more significant than transmission channels in real dimension. This result has implications for the extent to which some economies can be considered engines of global or regional growth or alternatively can be considered transmitters of growth shocks that originate elsewhere.

The advantage of this approach is that it is easier to model the endogenous variables conditional on the exogenous variables if these show some kind of irregular behaviour, which would be difficult to model within a VAR framework. It is very tempting to draw inference from the conditional or partial model rather than modelling the exogenous variables less carefully or not at all. Thus, one would work with smaller systems in terms of the parameters to be estimated with a gain in efficiency. The above-mentioned approach is valid if and only if the assumption of

<sup>11</sup>Bilateral net spillover effects and systemic contribution are discussed in Section 8.

<sup>12</sup>In Section 8.1, preliminary tests are carried out in order to highlight the presence of those relationships.

weak exogeneity is satisfied (see e.g., Engle and Granger (1987) and Barassi and Hall (2002)). To be more precise, the basis for this discussion is provided by the analysis of joint and conditional densities and sequential factorisation.

For example, let  $D_x(Y_t, W_t | X_{t-1}, \delta)$  be the sequential density at time  $t$  of the random vector  $X_t = (Y_t : W_t)'$  conditional on  $X_{t-1} = (X_0, X_1, \dots, X_{t-1})$ , where  $\delta = (\delta_1, \dots, \delta_n)' \in \Delta$  which is compact subset of  $\mathfrak{R}^n$ . Generally speaking,  $W_t$  is endogenous in the framework of the joint density function, but if  $W_t$  is weakly exogenous it is possible to factorise the joint density such that knowledge of how the process  $W_t$  is determined is not necessary in order to investigate the properties of the process  $Y_t$ . Thus, allowing for the existence of many one-to-one transformations from the original  $n$  parameters  $\delta = (\delta_1, \dots, \delta_n)' \in \Delta$  to any new set of parameters  $\alpha \in \dot{A}$ , with  $\alpha = (\alpha_1, \alpha_2)'$ , the factorization of the joint density function is:  $D_x(Y_t, W_t | X_{t-1}, \delta) = D_{y|w}(Y_t | W_t, X_{t-1}, \alpha_1) D_w(W_t | X_{t-1}, \alpha_2)$ . This latter involves a subset  $\phi$  of the parameters  $\delta$ , where  $\phi$  is a vector of parameters of interest.

However, the first requirement for a variable  $W_t$  to be regarded as weakly exogenous for a set of parameters of interest  $\phi$  is that the marginal process for  $W_t$  should add no useful information about  $\phi$ , that is one must be able to learn about  $\phi$  from  $\alpha_1$  alone. The second condition one needs to justify taking  $W_t$  as given is that  $\alpha_1$  should not depend on  $\alpha_2$ . If this were the case one could learn indirectly about  $\phi$  from  $\alpha_2$ . Thus,  $W_t$  would be weakly-exogenous for  $\phi$  if and only if:  $\phi$  is function of  $\alpha_1$  and does not depend on  $\alpha_2$ ;  $\alpha_1$  and  $\alpha_2$  are variation-free.

The exogenous variables observed in equation 1 and in the specification model 6 account for common and idiosyncratic component, which describes latent factors that both change over time and across countries affecting either real or financial dimension.

The analysis of cross-country growth spillovers and, more generally, multi-country estimations is generally hampered by dimensionality constraints. Four different VAR-based approaches have been suggested to tackle this issue: Bayesian VARs, factor model VARs, global VARs, and VARs based on regional groupings. All four techniques require additionally an approach for resolving the identification issue. The Bayesian VAR approach tackles the problem with the use of priors about the cross-country correlation patterns, which are subsequently updated with the data (see e.g., Banbura *et al.*, 2007, and Canova and Ciccarelli, 2006). Factor models, instead, collapse cross-country co-movements of several variables into common factors which are then allowed to affect the dynamics of the individual countries (see e.g., Bénassy-Quéré and Cimadomo, 2006). Global VARs reduce the individual countries' spillovers to their share in a weighted average for

the variable of interest, which then affects the individual countries' dynamics. The spillover in the global VAR has thus a direct interpretation, unlike the spillover in the factor VAR (see e.g., Bussière *et al.*, 2009, Galesi and Sgherri, 2009, and Dees *et al.*, 2007). A fourth approach focuses on a small set of countries or regions and then use the traditional structural VAR (SVAR) approach (see e.g., Bayoumi and Swiston, 2009, and Danninger, 2008). The degrees of freedom are preserved by reducing the number of regressors, for example by reducing either the number of countries involved or the number of variables considered, or a combination of both. Bayesian VARs and SVARs are more general than global VARs or factors VARs since they impose less structure on the inter-linkages. Compared to SVARs, Bayesian VARs require making more assumptions on the data generating process in return for more degrees of freedom, which makes the estimation feasible, if the number of regressors is high relative to the size of the available data sample. The SVAR approach proposed by Bayoumi and Swiston (2009) requires an extensive dataset, but has the advantage that it imposes no structure on the inter-linkages, and thus the coefficient estimates are purely data driven.

In this analysis, to study interdependence across countries, one may estimate a large VAR model that includes variables of all countries in the vector  $Y_{it}$ . In such model, all variables are treated as endogenous. However, for the large number of variables and of coefficients to be estimated and the relatively small number of observations, a large VAR may be intractable. A BVAR model offers an alternative approach by treating all variables as endogenous and bilateral flows of trade and capital as weakly exogenous. They highlight the possible existence of cross-country linkages. Due to the limited length of the time series, the model is estimated with only one lag of the endogenous variables ( $p_1 = 1$ ), a constant, one lag of the group-variables ( $p_2 = 1$ ), and one lag of the exogenous factors ( $p_3 = 1$ ).

Each equation of the VAR has  $k = [(12 \cdot 6) \cdot 1] + [(12 \cdot 2) \cdot 1] + [(12 \cdot 5 \cdot 1)] = 156$  coefficients and there are 72 equations in the system. Thus, without restrictions, there would be a total of  $156 \cdot 72 = 11232$  regression parameters. The total number of draws is  $5000 + 1000 = 6000$ , which corresponds to the sum of final number of draws to save and draws to discard, respectively. The study checked convergence recursively calculating the first two moments of the posterior of the parameters using 1000, 2000, 3000, 4000, 5000 draws and found that convergence is easily obtained with about 1000 draws. The analysis has been experimented with different combinations of runs and priors keeping the total number of iterations fixed. Thus, results would be robust to this choice. Finally, with this routine, unexpected shocks are computed in order to estimate

macroeconomic-financial linkages and fiscal spillover effects on the productivity in the Euro Area. For the specification model described in equation 6 are based on chains with 150000 draws. In particular, it corresponds to 3000 blocks of 50 draws and retained the last draw for each block. Finally, 2000 draws were used to conduct posterior inference at each  $t$ .

## 8 Fiscal spillover effects and shock transmission

In this Section, the aim of the analysis is to compute sign and dimension of fiscal spillovers in real and financial dimension accounting for cross-country and common factors and to answer the following questions: Is heterogeneity across countries the component mainly affecting output gap? What is the role of co-movements and interdependence across countries in driving spillovers between real and financial dimension? Are these components<sup>13</sup> larger in financial or real dimension? And why?

A central issue in economics and economic policy guidance is the effect of a change in fiscal policy on the domestic economy. Moreover, in an integrated world, domestic fiscal actions can also affect foreign economies. In the context of a currency union where the exchange rate between member countries is fixed, individual countries need some protection from shocks of uncoordinated fiscal policies (see e.g., fiscal agreements like the Stability and Growth Pact and successive measures adopted by EA).

The spillovers are defined as the transmission of an unexpected but identified shock from one variable to responding variables in the system. Aggregation of net spillover effects at each point in time yields then a contagion index. The article addresses two main spillover channels of an expansionary fiscal policy in one member country into the rest of the Eurozone. (i) Spillover effects through trade. A fiscal expansion stimulates domestic activities driving the exchange rate to appreciate and the domestic interest rate to increase. In a currency union, the exchange rate between members is fixed and the interest rate is ultimately determined at the union level, hence domestic money under circulation increases. (ii) Spillover effects through capital flows. The increase in the domestic interest rate attracts capital flows into domestic economy out of the rest of the union. Overall, economic theory provides reasoning to expect positive and negative spillover effects.

The BVAR model is able to determine how trends and changes in a particular variable in a

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<sup>13</sup>Heterogeneity, co-movements, and interdependence.

given country affect other countries, using generalized impulse response functions. Moreover, this analysis can assess whether negative financial shock in one country affects other countries and how affect real sector. In addition, interdependencies and transmission channels can be evaluated in this framework. Fiscal spillover effects are computed by generalized impulse response functions (GIRFs)<sup>14</sup> as the difference between a conditional and an unconditional projection of a variable for each country in a particular period. The analysis considers conditional impulse response functions (IRFs) for each variable in the system obtained over the same period conditionally on the actual path of another variable, that is an unexpected shock sent, for that period. The output deriving from the model absorbs conditional forecasts computed on the time frame of 10 quarters (2 years and half). The aim of this choose is to compute potential fiscal spillover effects, absorbing each single draw obtained from the posterior of regression coefficients. The prediction would reach December 2016 until the conclusion of actual *fiscal measure-path*.

In order to capture potential spillovers that could trigger financial contagion across the EA, a multicountry econometric framework is used to derive impulse responses from each variable to all other variables in the system. A matrix of potential spillover effects from each variable in the system has been constructed in order to define (individual) bilateral spillover effects. They describe the dynamics of impulse responses from a shock in real and financial variables within the Euro Area as weighted average of responses of each variables. Bilateral spillover effects can either be negative or positive. Here, two components can be defined, with  $N=1,2,\dots,12$ .

1. The average sum of the impulse responses to others defines (individual) bilateral OUT spillover effects:

$$SE_{OUT,y_i \rightarrow *} = \sum_{j=1}^N IR_{y_i \rightarrow y_j} \quad (62)$$

2. The average sum of the impulse responses from others defines (individual) bilateral IN spillover effects:

$$SE_{IN,* \rightarrow y_i} = \sum_{j=1}^N IR_{y_j \rightarrow y_i} \quad (63)$$

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<sup>14</sup>See e.g., Pesaran and Shin, 1998, for a definition of generalized impulse responses.

They account for time-varying impact in real and financial variables within the EA. They incorporate feedback effects from the impulse variables and temporary or persistent long-run effect of a potential shock. By the same token, bilateral net spillover effects is defined as the difference between the impulse responses sent and received from/to another variable. When the bilateral net spillover effect is positive, the variable (country) is a Net Sender of the system, and vice-versa. The following two equations are used in the framework:

$$SE_{NET,y_i \rightarrow y_j} = IR_{y_i \rightarrow y_j} - IR_{y_j \rightarrow y_i} \quad (64)$$

and

$$SE_{NET,y_j \rightarrow y_i} = IR_{y_j \rightarrow y_i} - IR_{y_i \rightarrow y_j} \quad (65)$$

They represent the amplification contribution of the impulse variable to the response variable and is able to capture sequential feature associated with systemic events.

$$SE_{NET,y_i \rightarrow y_j} + SE_{NET,y_j \rightarrow y_i} = 0 \quad (66)$$

Using  $SE_{NET,y_i \rightarrow y_j}$  for each variable, bilateral net spillover effects and its main components will be described and analyzed.

Finally, total bilateral net spillover effect can be computed by (62) and (63). It correspond to the sum of its bilateral net effects:

$$TSE_{NET,y_i} = \sum_{j=1}^N (IR_{y_i \rightarrow y_j} - IR_{y_j \rightarrow y_i}) \quad (67)$$

## 8.1 Summary statistics and Net Spillover matrices

Running the selected BVAR model,  $\beta$  posteriors drawn by bayesian computations are significant at the 95% of the confidence interval. The below Table shows the principal measures of fit in order to prove the robustness of the analysis.

The goodness of fit of the model is:

Table 1: Dependent Variable and Common Factors

1999 - 2014	Real Component	Financial Component
SSE	21.67	10.07
$R^2$	0.6073	0.6357
Adj. $R^2$	0.6047	0.6333
RMSE	0.2659	0.2557

In the Table are shown the principal measures of fit as the Sum of Squared Error ( $SSE$ ),  $R^2$  and Adj.  $R^2$ , and Root Mean Squared Error ( $RMSE$ )

Thus, mostly variability of the observable variable is explained by regression coefficients. It is closed to zero either for real or financial dimension. Moreover, financial sector shows a small improvement in measuring fitting objects. This results would highlights the bigger interdependencies across countries in financial dimension as after proved.

Nevertheless, financial components show the presence of greater omitted variables proving the importance to consider economic and institutional factors when studying real and financial linkages.

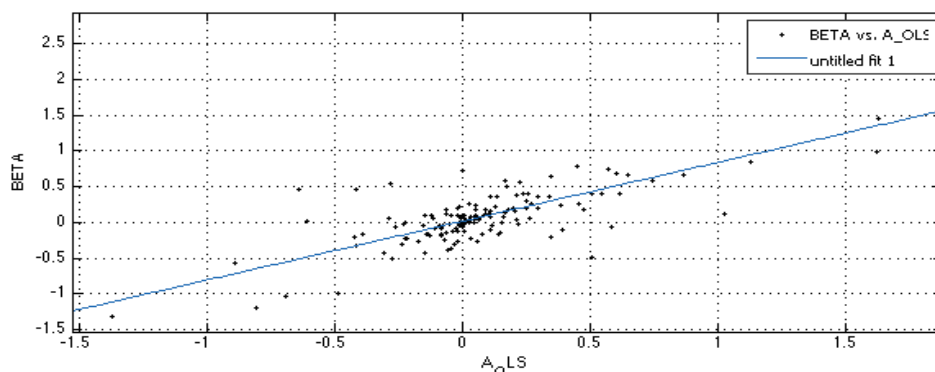


Figure 1: The Figure shows the robustness of the model drawing a scatter plot between the matrix of regression coefficients and the single draw from the posterior of  $\beta$  for real dimension accounting for weights (which corresponds to the variable  $real$ )

Using  $SE_{NET, y_i \rightarrow y_j}$  and focusing on the only  $respondents$  in Table 2, net spillover effects can be computed.



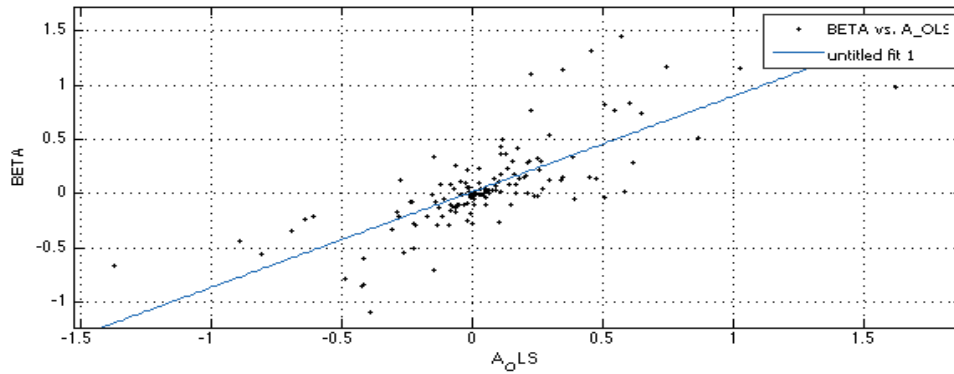


Figure 2: The Figure shows the robustness of the model drawing a scatter plot between the matrix of regression coefficients and the single draw from the posterior of  $\beta$  for financial dimension accounting for weights (which corresponds to the variable  $\beta_{in}$ )

The results would find partial but more significant spillovers to the same financial variables in mostly countries than to their real economy. Moreover, the transmission across countries through a variety of episodes seems to be stronger between trade exposures than between capital flows. A proof of this findings is heavily discussed in Section 8.2.

## Real Dimension

Accounting for each component in real dimension, the mean and standard deviation of estimated regression coefficients are shown for the before crisis period: 1999 - 2006. The Table 2 summarizes the impulse responses of productivity<sup>15</sup> to 1% shock to real variables:

Estimates for the sample 1999 - 2006 suggest a limited Germany's role in generating outward spillovers despite its large size. This result, in part, reflects Germany's own dependence on growth in the rest of the Eurozone<sup>16</sup>.

Accounting for the variable  $gov$ , Netherlands and Portugal also are net receiver of the system. Germany is particularly sensitive to shocks in the other two large EA countries (France and Spain) and in the other two smaller open economies (Finland and Belgium). France is much less sensitive to shocks in Germany than Germany is to shocks in France. Spain's growth is particularly affected by shocks in Italy and in other three smaller European countries (Belgium, Austria, and Netherlands). This result seems to hold for Italy's growth with lesser extent, but it is a net sender of unexpected shocks in Spain. Germany responds to a shocks in France more strongly than Italy. Overlooking responses of Luxembourg's growth, Germany responds to a growth shock in Eurozone countries more strongly than any of the other large EA countries and exhibits the

<sup>15</sup>Productivity is the current observable variable used in the model.

<sup>16</sup>IMF, 2011

Table 2: Responses of GDP to 1% shock to real variables

SPILLOVER MATRIX IN REAL DIMENSION						
	GOV		GDPG		CAP	
	1999-2006		1999-2006		1999-2006	
Country	Out	In	Out	In	Out	In
Italy	0.074	0.061	0.260	-0.008	0.147	0.045
Spain	0.037	0.026	0.502	0.006	0.086	0.056
France	0.721	0.011	0.292	-0.018	0.552	0.023
Belgium	0.136	0.142	-1.151	0.065	-1.288	0.034
Netherlands	-0.083	0.067	-0.024	-0.035	-0.161	-0.006
Austria	0.158	0.125	0.157	0.050	0.399	0.069
Germany	-0.176	0.130	-0.263	0.040	0.238	0.024
Finland	0.091	0.051	0.102	-0.031	-0.242	0.026
Luxembourg	0.007	0.169	0.339	-0.041	0.576	0.060
Ireland	0.143	0.004	0.198	0.221	0.062	-0.077
Greece	0.014	0.153	0.088	0.106	0.141	0.065
Portugal	-0.118	0.065	-0.074	0.072	-0.145	0.050
<b>Average</b>	<b>0.084</b>		<b>0.036</b>		<b>0.031</b>	

Outward and inward growth spillovers are shown for each country in real dimension for the period 1999 - 2006. They are computed running the selected BVAR model.

second largest response (after Belgium) to shocks in smaller european countries. This result seems to be consistent with Germany's large trade to the rest of EA. About GIPS countries (Greece, Ireland, Portugal, and Spain), they are sensitive to shocks in France (Portugal, Greece, and Ireland according to size). Greece and Portugal are sensitive to shocks in Spain and, for this latter, in Italy. On average, mostly countries are net receiver of shocks.

The Figure 3 draws inward growth spillovers to 1% shock to variable *gov*. Mostly countries show positive values, hence, they are net receiver of the system. In particular, Austria shows greater responses following by Finland, Germany, Greece, Belgium, and Spain. Ireland, Netherlands, and Italy tend to be net sender and, hence, affected from other economies with trade exposures. This result seems to comply with reforms following monetary union in 2001 - 2002 and the recovery processes in order to keep the stability of price strongly affected by real component in the majority of countries (commonalities).

In Table 2, growth's responses to shocks in the variable *gdpg* follow a similar path of the variable *gov*. Germany is a net receiver. According to size, Ireland, Greece, Belgium, Austria, and Portugal

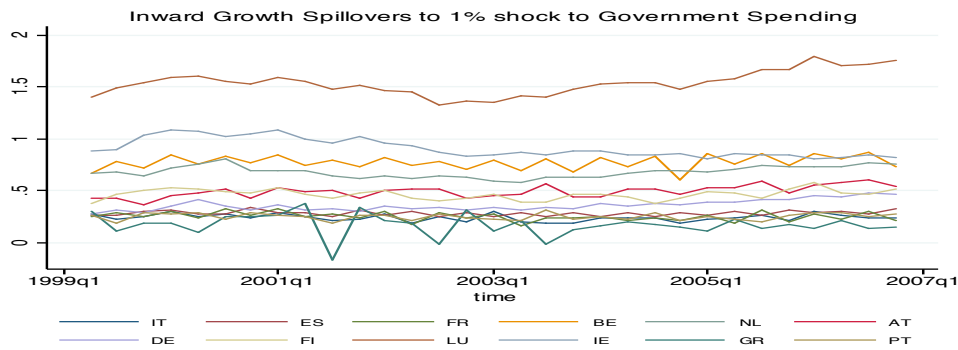


Figure 3: The Figure shows inward growth spillovers to a 1% shock to government spending for the period 1999 - 2007.

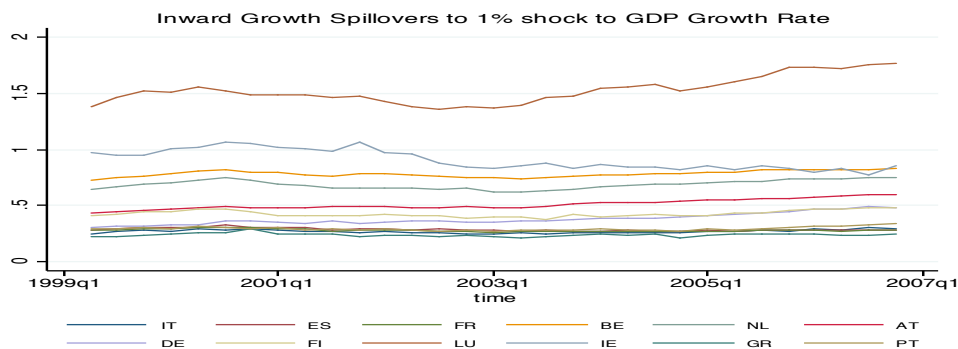


Figure 4: The Figure shows inward growth spillovers to a 1% shock to real GDP growth rate for the period 1999 - 2007.

are net receiver of the system.

Accounting for outward growth spillovers, Spain shows higher values with respect to other large countries (France and Italy) and smaller European economies (Austria and Finland). Italy's growth is sensitive to Germany but less than Spain. Growth in Ireland, Greece, and Portugal are sensitive to other EA countries.

The Figure 4 shows inward growth spillovers to 1% shock to variable *gdp*. Mostly countries show positive values, hence, they are net receiver of the system. As in the variable *gov*, Austria shows greater responses following by Finland, Germany, Greece, Belgium, and Spain. Ireland, Netherlands, and Italy tend to be net sender affecting other economies with trade exposures.

In Table 2, accounting for the variable *cap*, Germany's growth is a net sender proving large trade exposures with other European countries and, possibly, large dimension in international trade accounts. Belgium, Netherlands, Finland, and Portugal are net receiver and, hence, inward growth spillovers are sensitive to shocks sent by the rest of Europe. For example, Portugal's inward growths are less sensitive to Austria, Greece, Netherlands, and Belgium; like so Netherlands did not much affect by shocks to France, Finland, and Belgium.

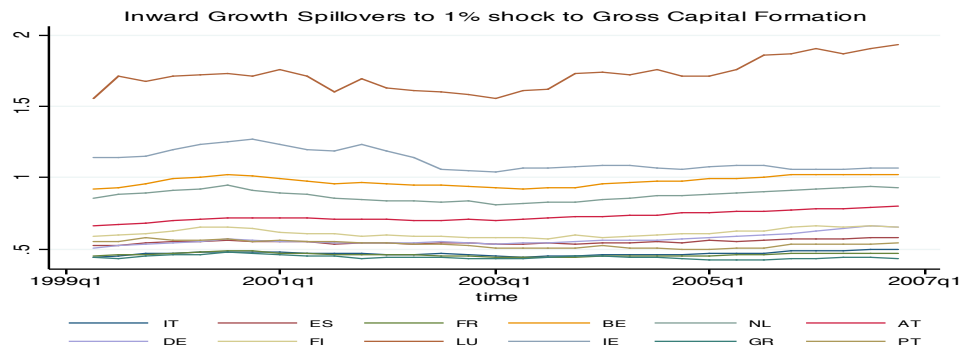


Figure 5: The Figure shows inward growth spillovers to a 1% shock to gross capital formation for the period 1999 - 2007.

In Figure 5, inward spillovers to 1% shock to variable *cap*, mostly countries show positive values, hence, they are net receiver of the system. As in the variable *gdpg*, Austria shows greater responses following by Finland, Germany, Greece, Belgium, and Spain. Ireland, and Italy tend to be net sender affecting other economies with trade exposures. The responses are bigger than *gov* and *gdpg* showing importance of trade transmission channels in driving spillover effects.

## Financial Dimension

Accounting for for each component in financial dimension, the mean and standard deviation of estimated regression coefficients are shown for the 'before crisis' period: 1999 - 2006. The Table 3 summarizes the impulse responses of productivity to 1% shock to financial variables:

Accounting for the variable *int*, inward growth spillovers are positive and, hence, Stated Members are net receiver of the system. It is evident since the variable *int* referred to the EMU convergence criterion chosen by policy. Given a shock in the variable, outward growth spillovers are sensitive except to some smaller open economies (as Netherlands and Belgium) which are particularly sensitive to growth shocks on other large Euro area countries (Italy, Spain, France, and Germany). Germany's growth is particularly sensitive to shock in Italy, Spain, and France and some smaller economies (Austria and Finland). France's growth is sensitive to a shock in Italy and Spain, but to a lower extent than Italy and Spain are to growth shocks in France.

The Figure 6 draws inward growth spillovers to 1% shock to variable *int*. The impulse responses are smoother with respect to real dimension. In fact, the variable is strictly correlated with fiscal actions and, hence, there have been coordinated recovery actions in the period 1998 - 2007 during the last EMU period (2001 - 2002). The inward growth spillovers was bigger during the period from 2000 to 2002. A significant increasing has been starting from 2006 with the worsening

Table 3: Responses of GDP to 1% shock to financial variables

SPILLOVER MATRIX IN FINANCIAL DIMENSION						
Country	INT		DEBT		CURR	
	1999-2006		1999-2006		1999-2006	
	Out	In	Out	In	Out	In
Italy	1.836	0.063	50.016	6.477	3.089	0.027
Spain	0.657	0.098	-0.174	2.489	0.083	0.066
France	1.828	0.092	0.145	4.904	0.555	0.177
Belgium	-1.979	0.144	0.245	13.594	-0.107	0.465
Netherlands	-2.109	0.108	0.032	4.177	-0.008	0.895
Austria	1.012	0.119	-0.143	12.190	0.079	0.338
Germany	0.263	0.120	0.000	4.571	-0.108	0.233
Finland	0.397	0.075	0.119	-2.410	0.194	0.915
Luxembourg	-3.077	0.194	-0.107	-0.393	-0.113	0.630
Ireland	0.162	0.144	-0.711	3.161	0.007	0.463
Greece	1.875	0.190	0.208	-2.234	-0.063	-0.343
Portugal	0.568	0.085	0.114	3.217	-0.021	-0.281
<b>Average</b>	<b>0.120</b>		<b>4.145</b>		<b>0.299</b>	

Outward and inward growth spillovers are shown for each country in financial dimension for the period 1999 - 2006. They are computed running the selected BVAR model.

in world economy. This result would prove the need to account for economic and institutional implications when studying real and financial linkages.

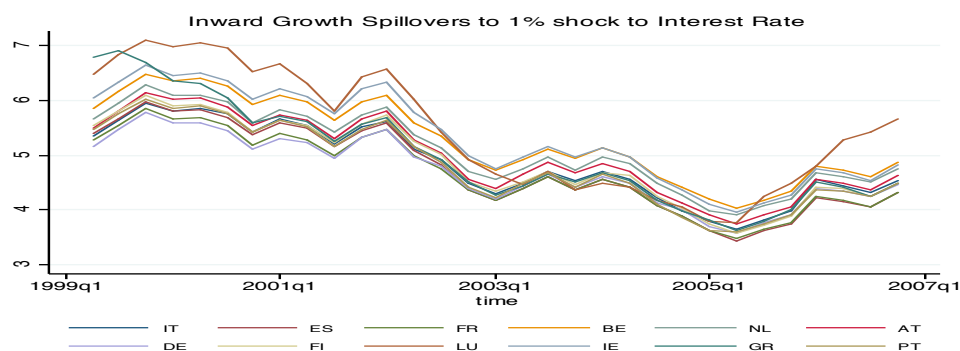


Figure 6: The Figure shows inward growth spillovers to a 1% shock to interest rate for the period 1999 - 2007.

In Table 3, mostly countries are net receiver of the system. Belgium, Austria, Italy, and France show higher inward growth spillovers than the rest of the EA12. An unexpected growth shock increases divergence and heterogeneity across countries due to the inflexibility of converge crite-

rions. In addition, the strong interdependence in a common-currency area affects real dimension and other correlated components.

Accounting for outward growth spillovers, mostly countries are net sender of the system; thus, an increasing in the variable *debt* negatively affects the countries' growth.

The Figure 7 shows inward growth spillovers to 1% shock to variable *debt*. Italy, Belgium, and Portugal shows higher inward spillover effects; in fact, the same countries have greater debt than others. There is heterogeneity across countries accentuated by divergence in public and private sectors.

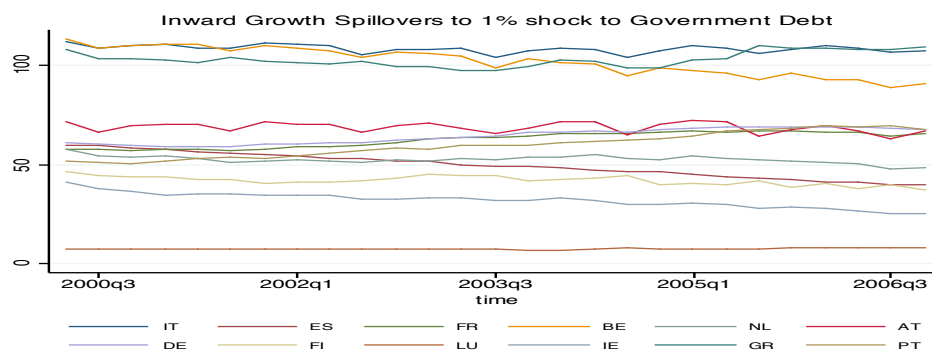


Figure 7: The Figure shows inward growth spillovers to a 1% shock to real GDP growth rate for the period 1999 - 2007.

In Table 3, accounting for the variable *curr*, mostly countries are net receiver of the system and, hence, potentially sensitive to growth shocks in other countries. Germany's growth is sensitive to Italy, Spain and other smaller economies (Austria and Finland). Spain's growth is much more sensitive to growth shocks in Germany than Germany is to growth shocks in Spain.

Accounting for outward growth spillovers, mostly countries are net sender of the system (e.g., Germany and other smaller open economies as Netherlands and Belgium) and, hence, affect growths in other countries because of large trade and capital exposures.

This findings would prove greater convergence in the variable *curr* with respect to others in either financial or real dimension because of GSP's monitoring.

In Figure 8, inward spillovers to 1% shock to variable *curr* are drawn. During 'before crisis' period, the inward growth spillovers develop in different way showing more trends over time. This result seems to prove the presence of latent factors increasing divergence across countries and worsening economic growth to an unexpected shock.

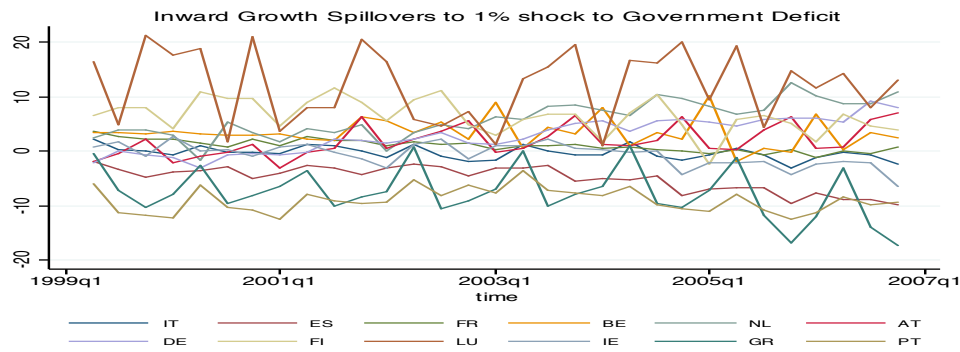


Figure 8: The Figure shows inward growth spillovers to a 1% shock to government surplus/deficit for the period 1999 - 2007.

## 8.2 Bilateral Spillover Effects for common and cross-country factors

Bilateral net spillover effects are computed in real and financial dimension for the period 1999 - 2006. They represent the amplification contribution of the first two lags of the impulse variable to the response variable in order to capture possible sequential features associated with systemic events.

### Real Dimension

According to real dimension, selected features of responses associated with unexpected shocks are shown:

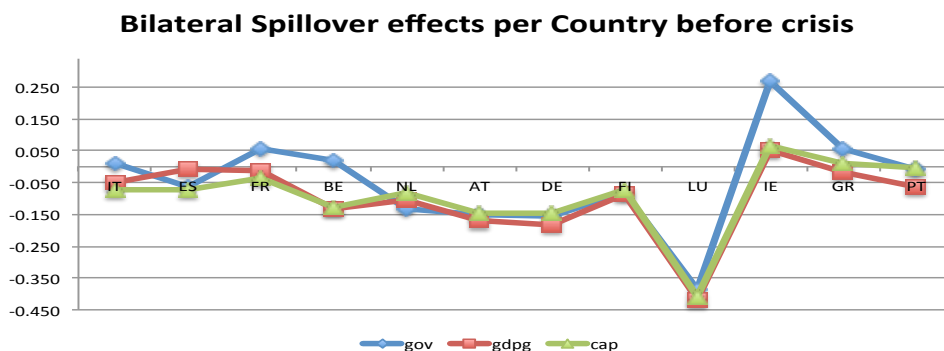


Figure 9: The Figure shows bilateral net spillover effects for real dimension for the period 1999 - 2006. The index is computed for each real variable in the system by equation (64).

The variable *gov* shows higher values for the strong interdependence with directly policy guidance. About the half of countries are net sender of the system (Italy, France, Belgium, Ireland, and Greece); while, the rest of EA12 countries are net receiver. The variable *gdp* and *cap* show similar path in Netherlands, Finland and some large economies as Germany and France due to the presence of co-movements in real dimension. Bilateral net spillover effects are also computed

for the period 1999 - 2006 ('before crisis'). In Figure 10, the variable *cap* is strongly affected by the presence of economic and institutional factors interacting over time. For example, in 2000 - 2002 according to the replacement of the eurozone currencies by the Euro. In the beginning of 2007, the variable seems to increase approaching with the current crisis. The variable *gdp* draws lower trends with respect to *gov* and *cap*. This result holds important cause-effect relationship affecting growth's path in an approximate future. The variable *gov* seems to be simply affected by direct and/or indirect fiscal actions in Eurozone's governments.

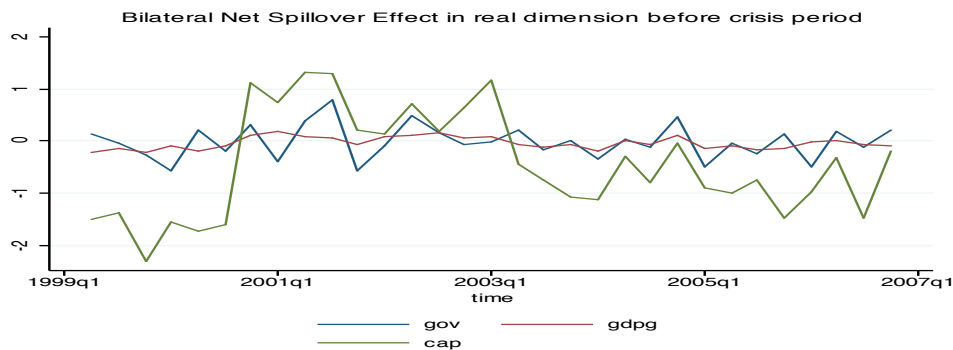


Figure 10: The Figure shows bilateral net spillover effects accounting for the selected period 1999 - 2006. The index is computed for each real variable in the system by equation (64).

## Financial Dimension

According to financial dimension, Figure 11 shows bilateral spillover effects for the period 1999 - 2006 per country. Mostly countries are net sender of the system and, hence, unexpected shocks directly affect own output growth and real economy because of interdependencies.

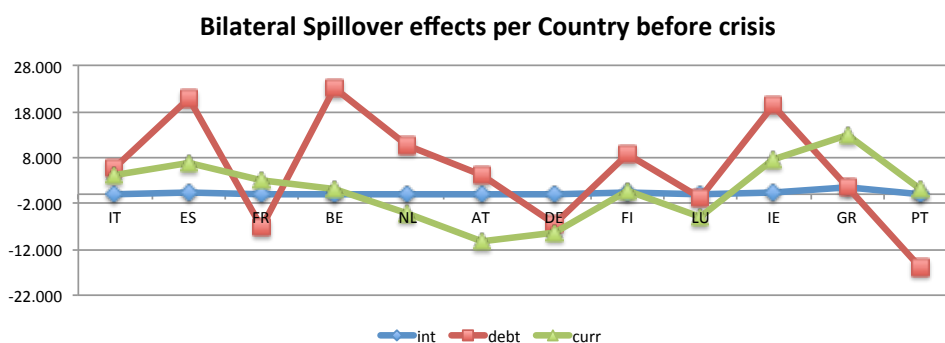


Figure 11: The Figure shows bilateral net spillover effects for real dimension for the period 1999 - 2006. The index is computed for each real variable in the system by equation (64).

France, Germany, Luxembourg and Portugal are net receiver of the system in the variable *debt* since sensitive to growth shocks in other countries (see Table 3). Germany and Austria are net receiver of the system in the variable *curr*. Mostly countries are net sender of the system



due to *austerity* measures in order to keep under own current account. In the variable *int*, Germany and Luxembourg show negative values. This result would confirm the replacement of the eurozone currencies by the Euro. Bilateral net spillover effects are also computed for the period 1999 - 2006. In Figure 12, countries are either net receiver or sender over time relative to euro convergence criteria and the official launch of the euro (on 1 January 1999 until 2002 with the total replacement of all national currencies). The variables *debt* and *curr* show higher values since potentially strongly affected by policy commitment. Positive output effects are larger in financial dimension proving that consolidations occurred simultaneously.

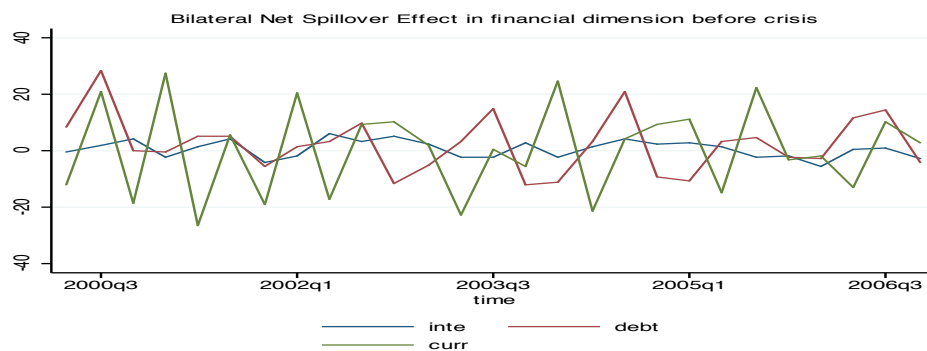


Figure 12: The Figure shows bilateral net spillover effects accounting for the selected period 1999 - 2006. The index is computed for each real variable in the system by equation (64).

### 8.3 Systemic Contribution and Contagion Index

In recent years, successive consolidations have depressed growth in the Euro Area. Output effects are significantly larger as consolidations occurred simultaneously, which led to significant spillovers across the Euro Area. Shocks spill over in a heterogeneous way across countries. Moreover, financial variables show higher amplification of spillover effects which can be seen as a result of increased interdependence between variables. The transmission is faster and deeper between financial than real variables. Positive output effects are larger in financial dimension proving that consolidations occurred simultaneously. A first approach in order to study size and dimension of spreading of spillovers' effects , two index are discussed in this chapter: the systemic contribution and the contagion index.

The systemic contribution is defined as the ratio between the total net contagion effects and the total net positive spillover of the system:

$$SC_{y_i} = \frac{TEC_{NET,y_i}}{TNP_{spillover}} \quad (68)$$

The total contagion index of the system is introduced as the average potential spillover effects in the system. There, the cumulative impulse responses are restricted in the interval  $[0, 1]$  and the (individual) spillover effects are restricted in the interval  $[-1, +1]$  so that the index will be bound between 0 and 100 (or between -100 and 0 if negative effects occur). There are several computational forms. In this analysis, the below formula is used in order to account each variable and its *contagion effect* in both real and financial dimension.

$$CI_{fin} = \frac{100}{N(N-1)} \cdot IR_{y_i \rightarrow y_j} \quad (69)$$

where,  $IR_{y_i \rightarrow y_j}$  denotes individual (out) spillover effects. The equation 69 is used for real and financial dimension.

The above-mentioned indices are able to observe and, hence, answer to the following questions: how do economic and institutional events affect real economy and financial dimension? How was effect changed over time? What component is more sensitive to unexpected shocks? In which component have unexpected changes had larger impact?

## Real Dimension

The Figure 13 shows the systemic contribution index for each real variables during 'before crisis' period. The variables *gdp* and *cap* observe similar path over time because of strict existing correlation between real GDP growth rate and gross capital formation (or Investments). They show negative values until the quarter 2001q1 keeping similar trends for all period in relation to euro convergence criteria. Later, the variables showed positive impact for the following two years. In the previous years to the current crisis, the variables have again observed a negative impact; but, nevertheless, they showed smaller values than oen in the quarter 2001q1.

The variable *gov* show a different trend with respect to *gdp* and *cap*. This result would confirm the strong impact of pressures of fiscal policy (*austerity*) in relation to financial measures in order to keep a sustainable growth and to guarantee the respect of stablished euro convergence criteria.

The non smooth responses of the real variables, given an unexpected change in real economy, would confirm the strong divergence across countries yet. This divergence become more stringent

accounting for public and private factors (see Section 10). The Figure 13 shows this findings.

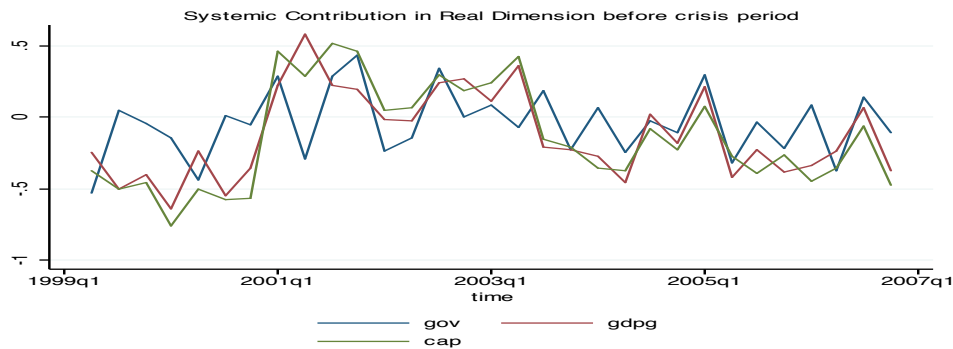


Figure 13: The Figure shows the systemic contribution index for the period 1999 - 2006 in real dimension. The index is computed for each real variable in the system by equation (68).

The Contagion Index (CI) would confirm the presence of deeper co-movements in real dimension. In Figure 14, in the top left box, the CI is observed for each real variable. The variable *cap* show greater value than variables *gov* and *gdp*. This result seems to prove the strict relationship of the variable *cap* with financial sector. The top right box draws the index per countries in the variable *gov*. There are co-movements across countries like so in the variable *gdp* and *cap*. Luxembourg shows higher contagion because of small size and little trade and capital flows with the rest of the Stated Memebers. It attended by Ireland and other smaller open economies (Netherlands, Belgium, Austria, and Finland). In the two bottom boxes, the variables *gdp* and *cap* show the same trend over time. Nevertheless, the variable *gdp* shows larger impact of the index than the variable *gov*. These findings would confirm the low independence in responding to shocks to real economy (*wide-ranging austerity measures*). In addition, the variable *cap* shows larger index than variables *gov* and *gdp* due to the strong relationship between investments and trade flows. This result would confirm Euro Area imbalances can be traced back to competitiveness factors rather than *catching-up* relationship.

## Financial Dimension

Accounting for financial dimension, Figure 15 shows financial variables differently react to shocks. For example, the variable *int* observe the maximum negative value in the quarter 1999q1 soon after the strong depreciation happened in participant countries. The variables *debt* and *curr* show different trends over time observing positive and negative values during all period in which Euro (€) adopted as common currency.

The responses, given an unexpected change in macroeconomic-financial dimension, are larger

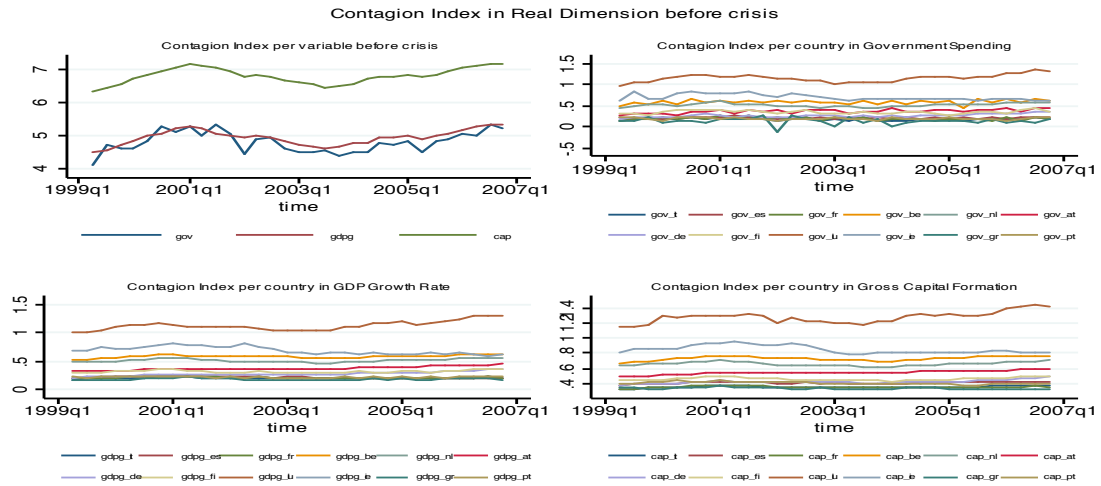


Figure 14: The Figure shows the contagion index for the period 1999 - 2006 in real dimension. The index is computed for each real variable in the system by equation 69.

than one observed in real economy and, hence, the presence of deeper interdependencies across countries.

Financial component has greater weight than real component; but there is deeper heterogeneity in responding to unexpected shocks in economy. This latter is because of potentially strong relationship with public and private factors. The same relationship turn out to be inverse following a high degree of divergence across countries.

Larger systemic contributions in financial dimension would confirm the prominent role of coordinated fiscal actions across Members; but, at the same time, deeper and faster consolidations depressed growth across countries.

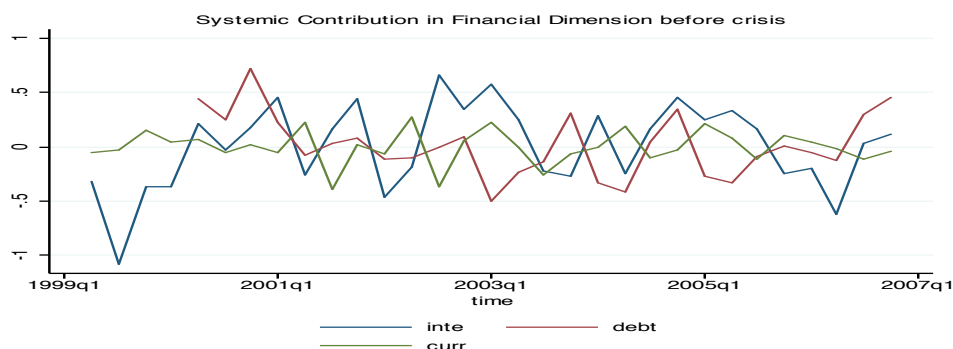


Figure 15: The Figure shows the systemic contribution index for the period 1999 - 2006 in financial dimension. The index is computed for each financial variable in the system by equation (68).

In financial dimension, the CI shows larger values than one in real dimension. This latter confirms consolidations occurred simultaneously behind more coordinated fiscal actions across members. A proof of this result is the plot observed in the top right box. Here, the contagion index per countries in the variable *int* is drawn. The index is homogeneously distributed across

countries confirming the prominent role of coordinated fiscal actions for the presence of deeper interdependencies. Nevertheless, in the bottom boxes, the contagion index for the variables *debt* and *curr* is plotted. The variables show larger heterogeneity in their trend over time and, hence, lower co-movements than one in real economy. In the variable *debt*, Italy shows greater effect following Greece and Belgium. Moreover, it shows larger values than variables *int* and *curr*. This result would confirm imbalances in the EA are traced back to competitiveness factors and divergence in respecting euro convergence criteria. In the variable *cap*, trends are non smooth than the others because of latent factors behind economic and institutional implications. To be more precise, the financial measurements in keeping the government surplus and deficit at the imposed level combine to bring about deeper divergence across countries given a shock to real and financial economy. In the top left box, the variable *debt* is badly larger than *int* and *curr* because of more accomodating tolerance allowance.

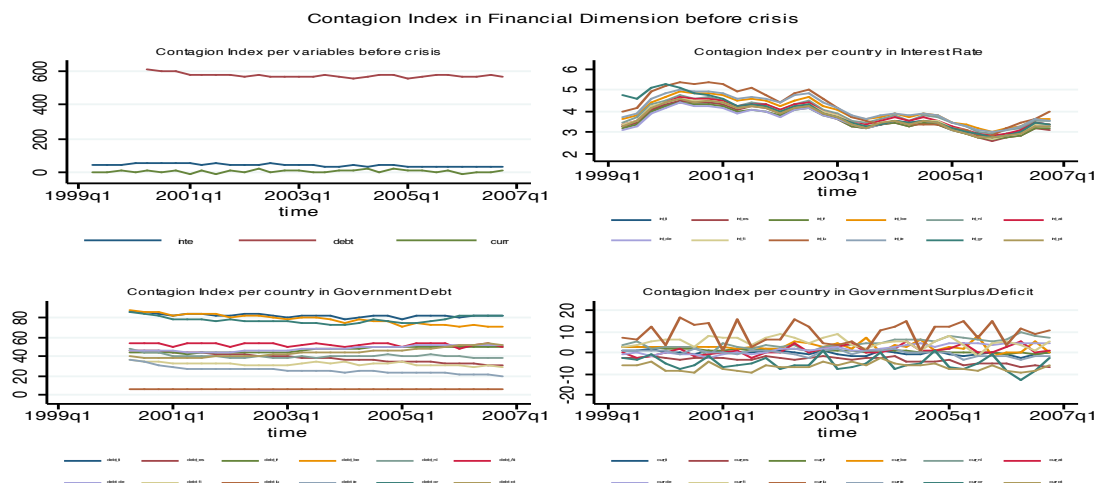


Figure 16: The Figure shows the contagion index for the period 1999 - 2006 in financial dimension. The index is computed for each financial variable in the system by equation 69.

## 9 How did common and cross-country factors change during the last recession?

### 9.1 Net Spillover matrix during crisis period and fiscal consolidation

In this Section, real and financial dimension have shown the largest shocks for the crisis period. At the same time, inward growth spillovers across countries seem to have been as sizeable as in recent fiscal consolidations (2011 - 2014), but with mainly synchronized feedback given a shocks.

To be more precise, the analysis is consistent with the possibility that larger co-movements or macroeconomic-financial linkages observed in the last recession could be more related to the size of the shocks than to the intensification of their transmission to previous recessions<sup>17</sup>. In Section 10.1, these findings are examined in detail.

## Real Dimension

Accounting for the 'crisis period' (2007 - 2011) and 'fiscal consolidation' (2011 - 2014), in gov, mostly countries are net senders of the system due to unexpected shocks deriving from financial crisis, such as Italy, France, Belgium, Netherlands, Austria, Greece, and Portugal. Germany follows by showing negative outward growth spillovers and, hence, less sensitivity to shocks in other Eurozone countries. Germany is particularly sensitive to shocks in Belgium, France, and Italy according to size. France becomes a net sender of unexpected shocks on growth in Germany and in Spain (possibly due to large trade exposures). Positive inward growth spillovers proves the presence of co-movements across countries for a given shock affecting real component. Focusing on the last period (2011 - 2014), all inward growth spillovers are negative and, hence, net sender of the system. According to size, higher component is observed in Greece, France, Spain, Italy, and Portugal. Germany and Austria show relatively less extent with respect to the other EA countries. Thus, restrictive fiscal measures on financial dimension heavily affect real component for the presence of common features and inter-linkages between two selected sectors. This result holds observing negative outward growth spillovers and, hence, the mostly countries are net receiver of unexpected shocks, except Germany and an another smaller economy as Belgium and peripheral countries as Ireland and Portugal. Graphically,

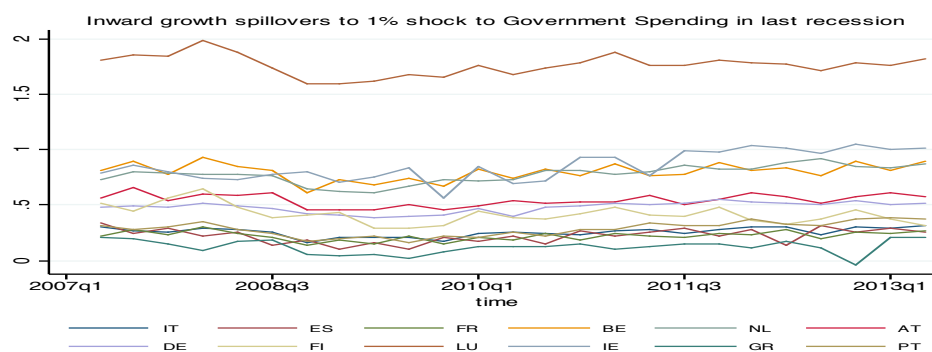


Figure 17: The Figure shows inward growth spillover effects to 1% shock to government spending accounting for the crisis period 2007 - 2014.

<sup>17</sup>See e.g., Stock and Watson, 2012.

Accounting for the variable *gdp<sub>g</sub>*, during crisis period, inward growth spillovers are negative in majority of countries and, hence, net receiver of the system. Germany's growth is sensitive to other large countries as France and Spain, but less sensitive than Spain is to Germany. This latter is also sensitive to Ireland, Greece, and France. France is less sensitive to Spain than Spain is to France. Greece, Portugal, Italy and other smaller economies (Netherlands, Austria, and Belgium) are sensitive to unexpected shocks negatively affecting outward growth spillovers.

In Figure 18, the responses to an unexpected shocks in Germany, France and Spain, and other smaller open economies (Netherlnads, Austria, and Finland) show higher size. Nevertheless, the increasing in *gdp<sub>g</sub>* is smaller than one observed in *gov* for the presence of omitted factors affecting the GDP growth rate.

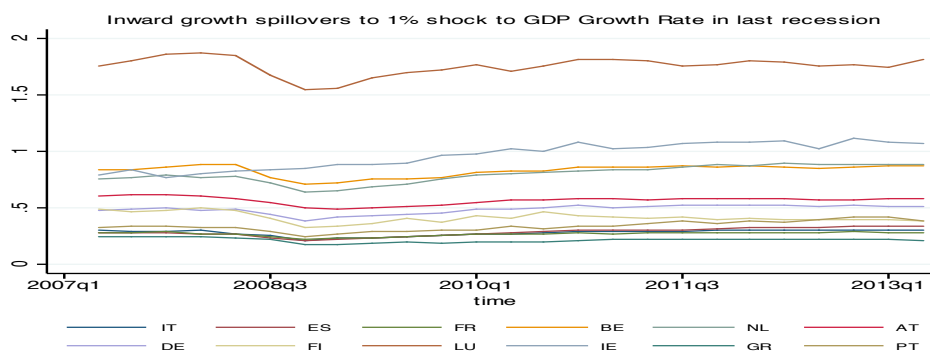


Figure 18: The Figure shows inward growth spillover effects to 1% shock to GDP growth rate accounting for the crisis period 2007 - 2014.

In variable *cap*, inward growth spillovers are negative in the majority of countries. Nevertheless, Germany, attended by Italy, Spain and France, is affected by unexpected shocks to other countries (e.g., crashing into trade exposures). Germany's growth is sensitive to large economies as France and Spain, and to smaller open economies as Austria. However, the former is less sensitive than France and Spain are to Germany. Greece, Ireland, and Portugal (GIP) are net receiver with respect to the rest of Eurozone countries.

In Figure 19, impulse responses are smoother than *gov* and *gdp<sub>g</sub>* for the presence of large trade exposures between countries, as Germany, France and Spain, and other smaller open economies (Netherlands, Austria, and Finland). The same trade channels will be important future researches for international and non-EA trade flows.

## Financial Dimension

Accounting for the 'crisis period' (2007 - 2011) and 'fiscal consolidation' (2011 - 2014), in variable

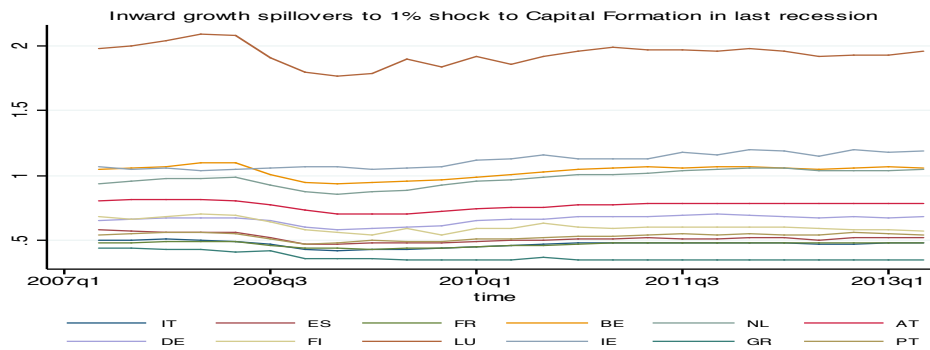


Figure 19: The plots show inward growth spillover effects to 1% shock to gross capital formation accounting for the crisis period 2006 - 2014.

*int*, impulse responses increased during financial crisis (2007 - 2011). To be more precise, either inward or outward growth spillovers are higher than the 'before crisis' period. Inward growth spillovers are always positive proving the strong impact of fiscal actions are on Eurozone countries. According to outward growth spillovers, some countries (Belgium and Netherlands) becomes net sender of the system affecting other countries through spillovers' transmission channels. Germany's growth is sensitive to growth shocks in Italy and other smaller economies (Netherlands and Finland). Nevertheless, the former affects Spain, France, Austria, and Portugal. Italy's growth is sensitive to growth shocks in Germany, Finland, and Ireland. France's growth is sensitive to a shock in Italy, Germany and other smaller economies (Netherlands, Belgium, and Finland). Moreover, Germany is potentially strongly affected by growth shocks in other countries by own growth independence. Focusing on the last period (2011 - 2014), Italy, Greece, and Portugal are bigger than one in 'crisis period' due to pressing fiscal-recovery actions. During last years, mostly countries become net sender of the system (Germany, France, Netherlands, and Ireland). This findings would highlight the high degree of heterogeneity across countries given a fiscal spillover effect and the presence of stronger accomodating policy (*austerity*).

In Figure 20, Portugal, Greece, Ireland show higher inward growth spillovers attended by Italy, Spain, and France. The other countries seem to show coordinated responses to a fiscal shocks; thus, the degree of heterogeneity could be affected by potential latent factors in public and private sectors.

Accounting for the variable *debt*, during crisis period, inward growth spillovers are positive in all countries and, hence, they are net receiver of the system. This result seems to two important findings proved in the described SUR model. Accounting for size, Italy, Belgium, and Austria show greater outward growth spillovers in respect to debt load. France, Ireland, and other smaller



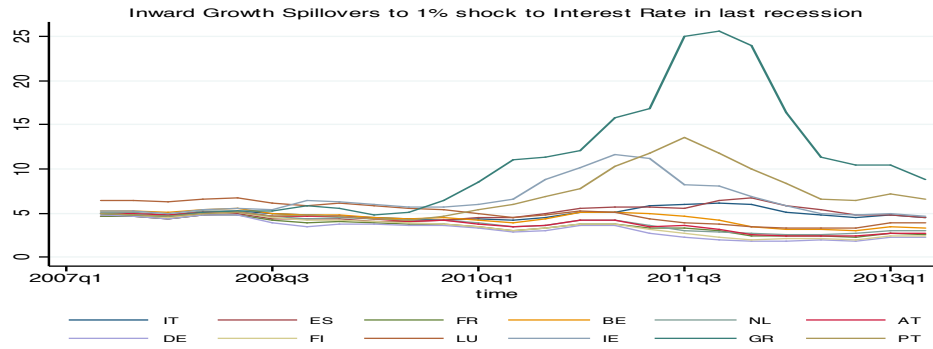


Figure 20: The Figure shows inward growth spillover effects to 1% shock to interest rate accounting for the crisis period 2007 - 2014.

economies (Netherlands and Finland) are net receiver of the system and, hence, affected by growth shocks in other countries. During fiscal consolidation (2011 - 2014), Greece, Portugal, and Ireland, in order of size, are net sender of the system affecting other countries given a shock in own current account. Mostly countries are net receiver and, hence, potentially strongly sensitive to growth shocks due to recovery fiscal actions (*austerity*). Accounting for size, Belgium, Austria, and France show greater inward growth spillovers than others. Mostly countries are sensitive to unexpected growth shocks (negative spillovers). According to outward growth spillovers, Belgium, Ireland, Greece, and Portugal are net receiver of the system and, hence, potentially sensitive to unexpected growth shocks.

In Figure 21, Greece, Italy, Portugal, and Ireland show bigger inward shocks spillovers than others.

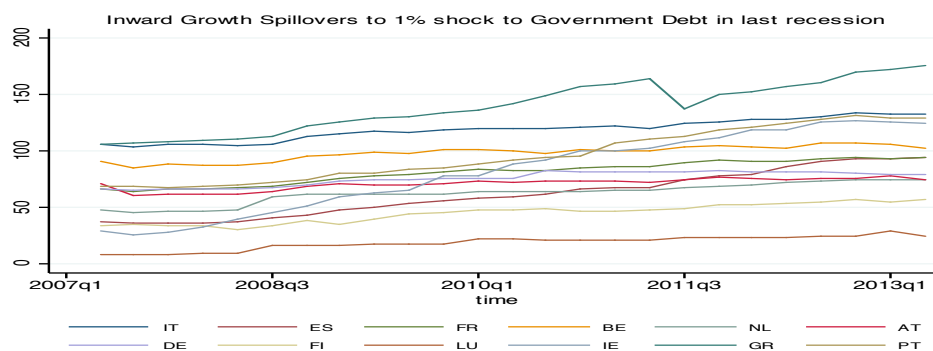


Figure 21: The Figure shows inward growth spillover effects to 1% shock to government surplus/deficit accounting for the crisis period 2007 - 2014.

Inward growth spillovers given a shock in the variable *curr* are bigger than one during 'before crisis' period. Moreover, some countries (Spain, France, Greece, Portugal and other smaller economies as Netherlands and Austria) become from net receiver to net sender of the system. Therefore, these countries are sensitive to growth shocks and affect other countries through trade

and capital transmission channels. These findings increase interdependencies across countries and, hence, the size and magnitude on how shocks spill over.

In Figure 22, during fiscal consolidation, heterogeneity in inward growth spillovers across countries shrink. The maximum value is observed during 'crisis period' due to economic and institutional implications arisen from divergence in public and private sectors.

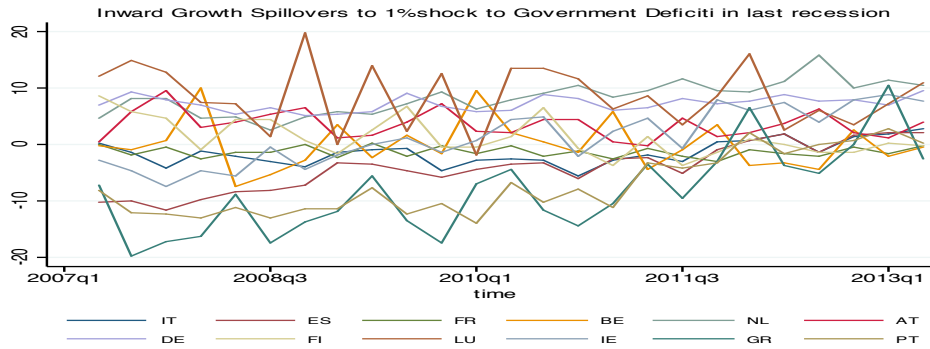


Figure 22: The plots show inward growth spillover effects to 1% shock to government surplus/deficit accounting for the crisis period 2007 - 2014.

## 9.2 Fiscal spillover effects and their main components

Bilateral Net spillover effects are drawn for 'crisis period' and 'fiscal consolidation' accounting for real and financial dimension.

### Real Dimension

The Figure 23 shows the variable *gdp* is negative for mostly countries and, hence, are net receiver of the system. *PIG* shows greater values than the rest of the EA12, following Spain, France, and Italy . The variable *cap* is almost constant and a net sender in the majority of countries. It seems to be due to strong interdependencies with external transmission channels. The variable *gov* is rather uneven across countries proving the existence of strong degree of heterogeneity between public and private sectors.

During fiscal consolidation, there is a sharp improvement on extent of transmission shocks across countries showing a smoother responses. Nevertheless, the degree of heterogeneity holds over time. The variable *gov* and *gdp* are again net receiver of the system in mostly countries as Italy, Spain, Belgium, Germany, Austria for the large trade exposures to the rest of Europe. The variable *cap* is otherwise a net sender in more than half countries just as it is an important

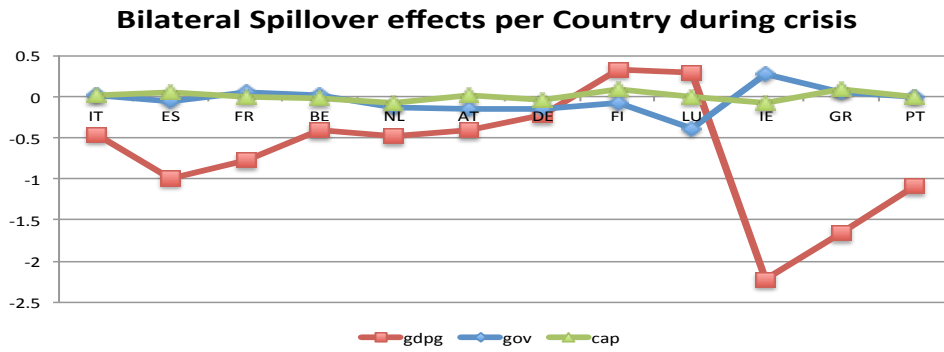


Figure 23: The plots show bilateral spillover effects per country during crisis period. The index is computed by equation 67

component for transmission trade channels.

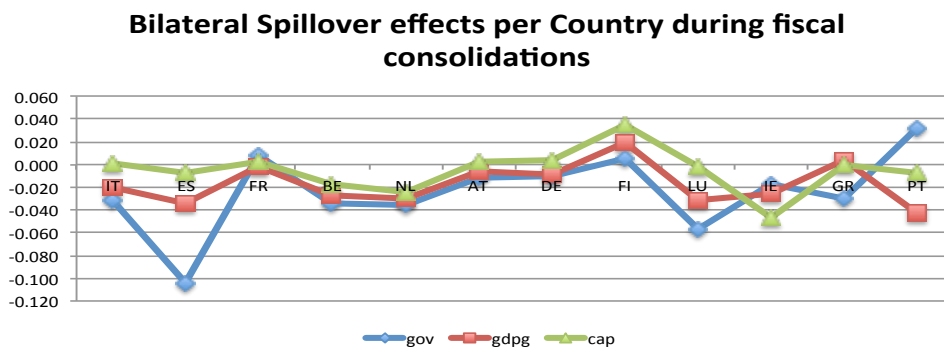


Figure 24: The plots show bilateral spillover effects per country during fiscal consolidation. The index is computed by equation 67

During crisis period, total bilateral spillover effects are negative (net receiver) and, hence, countries are sensitive to growth shocks given. To be more precise, an unexpected shock (given an economic/institutional occurrence) on real economy affects own output growth. The maximum value is observed during financial crisis for the period from 2009 to 2011. The variable *gdp* show bigger values than others the above-mentioned period. Accounting for size, it is attended by *gov* and *cap*. Unlike everyone else, the former is negative and reach positive value in the quarters 2011q4, 2012q3, and 2013q1 during fiscal consolidations. Therefore, there is large interdependence between real and financial sectors and economic/institutional events affect spillovers' trend over time.

## Financial Dimension

The Figure 26 shows the variable *int* is negative in Portugal, Ireland, Italy, Greece, and Spain (PIIGS) and, hence, are net receiver of the system being potentially sensitive to growth shocks in other countries. The remaining countries are net sender of the system because of financial mea-

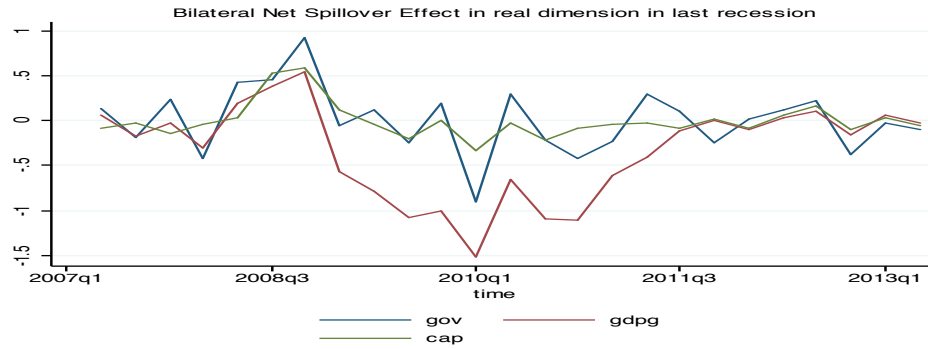


Figure 25: The plots show bilateral spillover effects for the period from 2007 to 2014. The index is computed by equation 67

asures. Germany observed the greater total bilateral net spillover effects (as *leader country*) and attended by Luxembourg, Belgium, France, and other smaller economies (Austria and Netherlands). The variable *debt* is negative in all countries and, hence, negatively affected by financial measures in order to keep a sustainable growth. The variable *curr* observe smoother responses to economic/institutional events than others. Spain, France, and other smaller open economies (Finland, Ireland, Greece, and Portugal) are net receivers of the system due to a greater worsening in own current account.

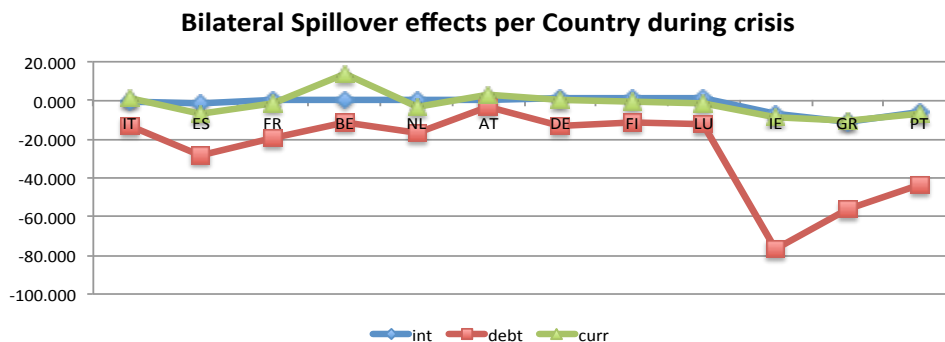


Figure 26: The plots show bilateral spillover effects per country during crisis period. The index is computed by equation 67

During fiscal consolidations, there is a sharp improvement in the variable *debt* observing smaller values than one during 'crisis period'. However, countries remain net receiver of the system, except Germany, because of strong worsening in debt accounts given recovery fiscal actions (*austerity*). The variable *curr* seems to be more monitored than *debt* by GSP's commitment. Finally, the variable *int* is positive in mostly countries sending impulses to other countries through inter-linkages across countries and to own output growth since tied under fiscal control actions.

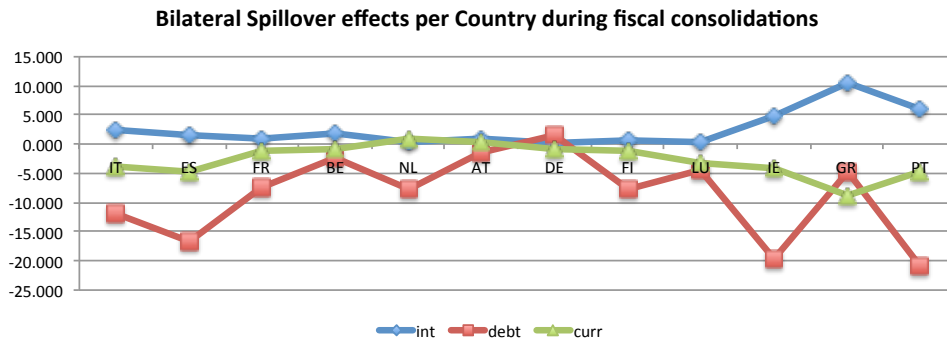


Figure 27: The plots show bilateral spillover effects per country during fiscal consolidation. The index is computed by equation 67

Accounting for the entire period from 2007 to 2014, total bilateral net spillover effects show greater size and magnitude than one in real dimension. However, there is heterogeneity on how shocks spill over across countries showing non smooth trends over time. In fact, BSEs are either positive or negative with relation to financial shocks and adopted fiscal measures. During fiscal consolidations, BSEs show unchanged trends, but they become net sender of the system affecting growth and real economy of Stated Members. This result seems to confirm the financial dimension has a common and an idiosyncratic component, but the former was larger during the more recent crisis in its financial dimension and even more in its real dimension. Positive output effects are larger in financial dimension proving that consolidations occurred simultaneously.

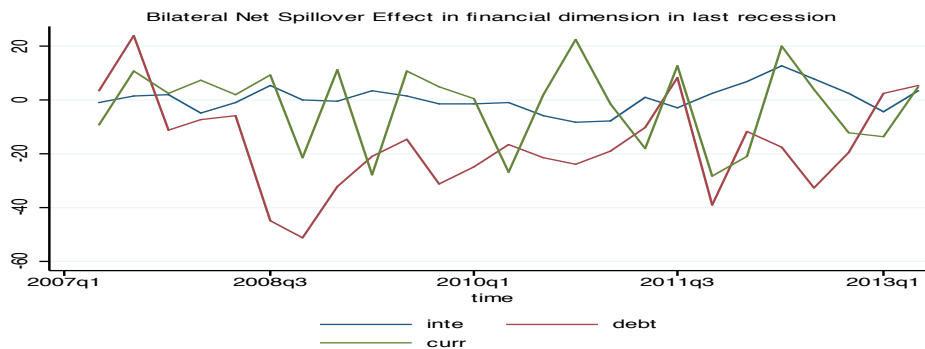


Figure 28: The plots show bilateral spillover effects for the period from 2007 to 2014. The index is computed by equation 67

The Section 9.2 show several important findings. In real dimension, the most countries are net receiver of the system for the entire selected *time-series* (1999 - 2014). This result would confirm the strong interdependence between real and financial dimension and real dimension has a greater common component highlighted by large trade exposures across countries and by the importance of *austerity* policies in the last period. Given an unexpected shock following extreme economic/institutional changes, financial dimension show higher size and magnitude in BSEs than

one in real dimension. The findings prove the greater incidence of financial sectors and the stronger interdependence across countries in financial component due to *austerity's* fiscal policy. Later, these changes affect real dimension through trade flows, which has a larger significance than capital transactions. The responses are larger in financial component proving the importance of the size of shocks than to the intensification of their transmission. Moreover, countries show greater heterogeneity in own financial accounts and larger co-movements in their real dimension. However, in this latter, countries typically are net receiver of the system and, hence, more sensitive to growth shocks in other countries than into their financial dimension. These inter-linkages across countries result into a worsening in output gap because of strong divergence in latent factors (public and private sectors) and the importance of the competitiveness in supporting their current account and unexpected changes in real economy to the detriment of *catching-up* and *causality* relationships.

### Systemic Contribution and Contagion Index

The Figure 29 shows the systemic contribution index for each real variables during crisis period and the recent fiscal consolidations. The variables show positive responses to shocks to real economy. Positive values confirm deeper pressure on the real variables, especially for the quarters from 2008q1 to 2010q2. During fiscal consolidations, there have been lower effects on real economy. Nevertheless, the variables show positive values with respect to one during 'before crisis' period in Figure 29. In the previous quarters to fiscal consolidations, real economy showed negative value because of no significant responses to shocks. Finally, the trend of the systemic contribution over time would confirm deeper common component in real dimension.

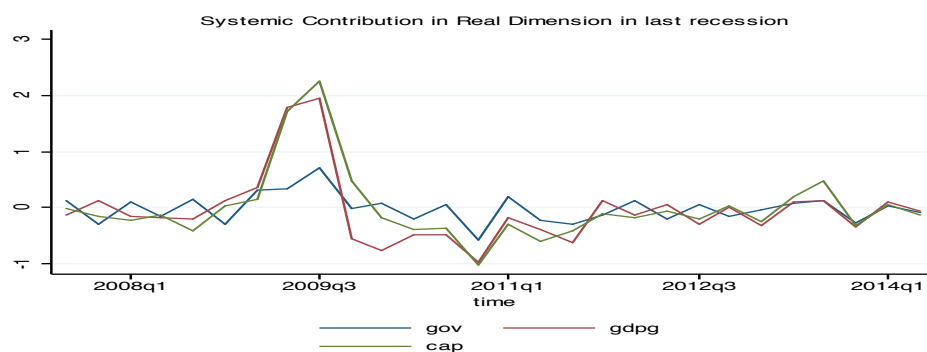


Figure 29: The Figure shows the systemic contribution index for the period 2007 - 2014 in real dimension. The index is computed for each real variable in the system by equation (68).

Accounting for financial dimension, Figure 30 shows financial variables differently react to

shocks. For example, the variable *int* observe the maximum value in the quarter 2013q4 soon after the strong depreciation happened in participant countries by fiscal measurements (*austerity*). Different trends confirm lower common component in financial dimension. Moreover, this result would confirm the prominent role of latent factors affecting financial variables responses and, hence, the impact on real economy. In fact, observing Figure 30, the variable *debt* show negative impact during crisis period because of more accomodating tolerance allowances. Conversely, the variable *cap* is less sensitive than other variables due to more stringent control on its tolerance level. Focusing on the current crisis, financial variables mostly react negatively to systemic events with respect to real dimension due to extreme fiscal-agreement measures (or deeper *wide-ranging austerity measures*). Therefore, the increasing of divergence across countries and deeper imbalances in the last recession can be traced back to important economic and institutional implications and to competitiveness factors affecting real economy trough large treade exposures across countries. Larger systemic contributions in financial dimension would confirm the prominent role of coordinated fiscal actions across Stated Members; but, at the same time, deeper and faster consolidations depressed growth across countries.

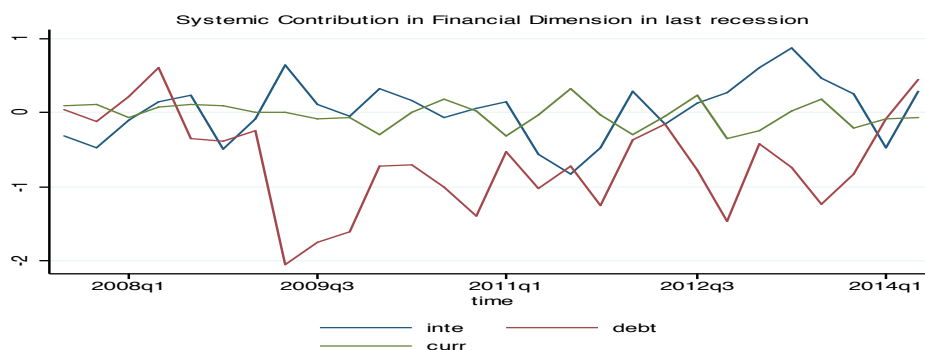


Figure 30: The Figure shows the systemic contribution index for the period 2007 - 2014 in financial dimension. The index is computed for each financial variable in the system by equation (68).

During crisis period, the Contagion Index (CI) in real dimension is greater than one for the period from 1998 to 2014. The variable *gdp* observes larger values during fiscal consolidation. This result would confirm the increasing of imbalances in responding to shocks to real economy across countries. The variables *gov* and *cap* increased during fiscal consolidations, but observing on average values lower than one during crisis period. These findings are shown in top left box of Figure 14. Generally, the index shows the common component has a strong impact over time. This result would confirm real dimension is affected by common factors. These latters differently spill over across countries for a large degree of heterogeneity in generating inward spillover effects. In addition, the same heterogeneity is increased due to a strong divergence existing across countries'

public and private sectors.

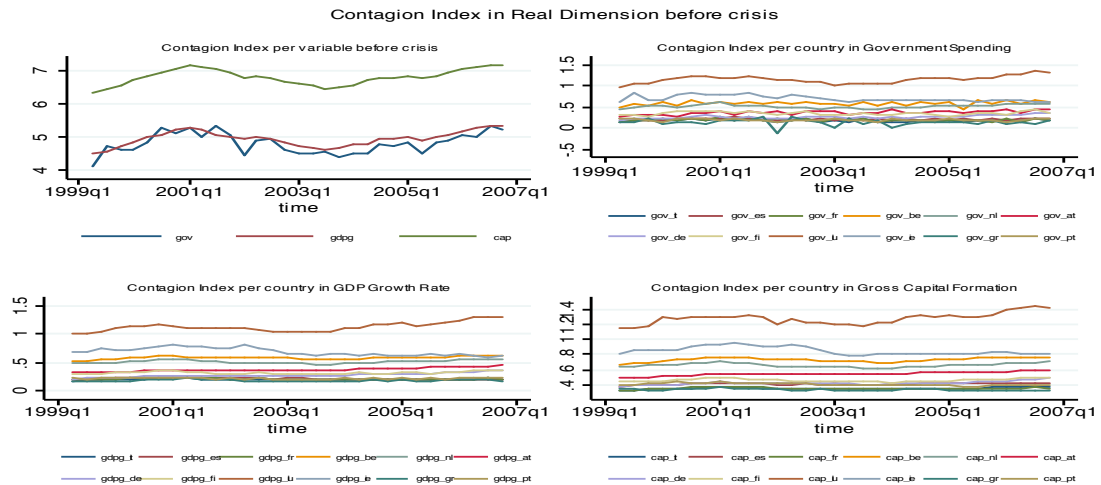


Figure 31: The Figure shows the contagion index for the period 1999 - 2006 in real dimension. The index is computed for each real variable in the system by equation 69.

In financial dimension, the CI shows the importance of economic and institutional factors in affecting inward growth spillovers. During fiscal consolidations, the variable *int* shows different indices. For example, Greece, Ireland, and Portugal observed greater values than other countries. Following Spain, France, and Italy in order of size. This result is drawn in the top right box of Figure 16. The variable *debt* observe higher value than *int* and *curr*. This result seems to derive on too much deeper tolerance allowances. The variable *curr* shows non smooth trends in the contagion index. These findings would confirm increasing of imbalances during last recession and even now in observing inward growth spillovers can be traced back to competitiveness and other economic/institutional factors triggering a *cause-effect* relationships rather than *catching-up* events.

## 10 Commonality vs. Heterogeneity

### 10.1 Evolution of group-specific and common factors

Accounting for common factors, systemic contribution and contagion index are shown for real and financial dimension. Transmission channels and selected latent factors are observed. In Figure 33, in bottom boxes, financial variables are observed. They keep the same feature of the data with low high difference. This result seems to confirm that growth shocks within the EMU are to a relatively larger extent transmitted via monetary and financial linkages. Focusing on financial



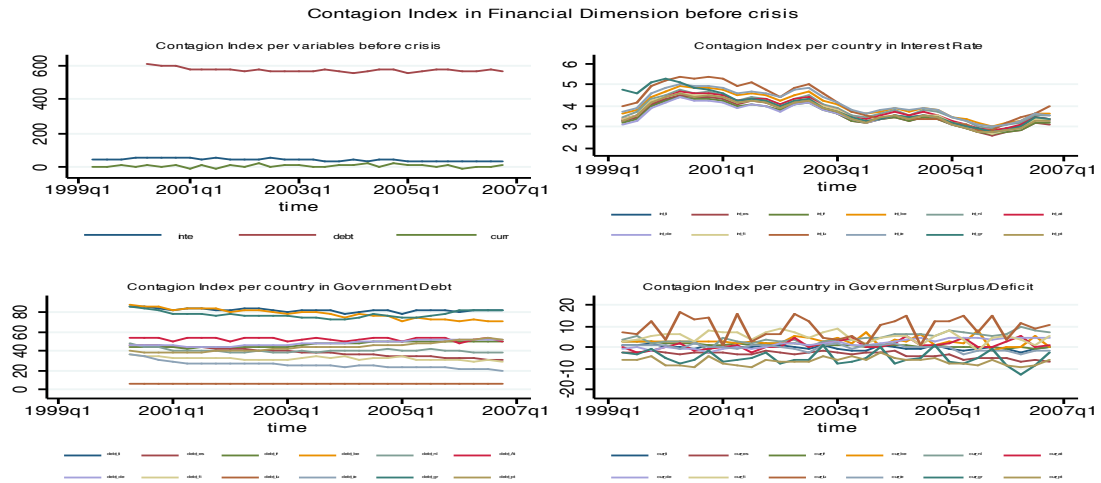


Figure 32: The Figure shows the contagion index for the period 1999 - 2006 in financial dimension. The index is computed for each financial variable in the system by equation 69.

component, the results confirm the prominent role of coordinated fiscal actions for the presence of deeper interdependencies. In fact, the contagion index decreased more than half during fiscal during spillover consolidations. Upper boxes draw real dimension showing different trend with respect to financial component. For example, during 'crisis period', the systemic contribution index keep positive and greater values than one in financial dimension. These findings would prove that trade channels and economic/institutional implications are very important in evaluating growth shocks. The latter appear to be relatively larger with respect to trade channels. In fact, in the top-right box, the systemic contribution observes lower values. Nevertheless, the index in real dimension follows to observe higher values during 'crisis period' than one in financial dimension. This result would prove the high degree of heterogeneity in spreading of spillover effects in real dimension and the presence of potential outliers within and outside the EMU. The latter appear to be predominantly transmitted via trade channels.

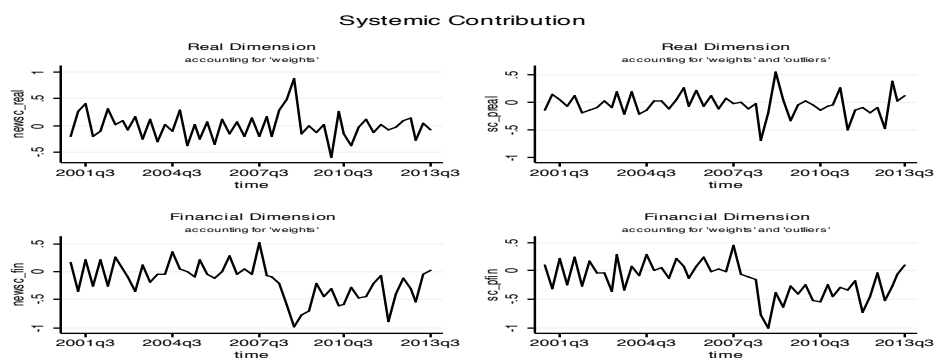


Figure 33: The Figure shows systemic contribution for real and financial dimension accounting for *weights* and *outliers*. The coefficient vectors observed in the analysis correspond to  $\chi_{1t}$ ,  $\chi_{2t}$ ,  $\chi_{3t}$ , and  $\chi_{4t}$  of equation 30.

Thus, the project aims at measuring whether there are significant co-movements among these countries and variables that simple summary statistics and bilateral spillover effects cannot identify in depth. The model estimated is described in equation 30. After estimating different specifications of this model, the highest marginal likelihood was found for the model including four country-specific components for each economy, four variable-type components, and two common components for all series. The first accounts for the coefficient vectors  $\chi_{1t}$ ,  $\chi_{2t}$ ,  $\chi_{3t}$ , and  $\chi_{4t}$ :  $\chi_{1t}$  and  $\chi_{3t}$  shared by real and financial variables, respectively, across countries accounting for *weights* and  $\chi_{2t}$  and  $\chi_{4t}$  shared by real and financial variables, respectively, across countries accounting for *weights* and *outliers*. The four variable-type components correspond to coefficient vector  $\chi_{5t}$ : one shared by all real variables with *weights*, another shared by all financial variables with *weights*, another shared by all real variables with *weights* and *outliers*, and another shared by all financial variables with *weights* and *outliers*. The two common components account for the coefficient vector  $\chi_{6t}$  shared by all series with *weights* and *outliers*, respectively. These common, country-specific and variable-type components quantify the relative contribution of common and heterogeneous factors in macroeconomic-financial linkages and help to address the following questions: Is there a significant common component in the real and financial interactions across eurozone members or do country-specific heterogeneities matter more? How did *weights* and *outliers* factors affect real economy and financial variables over time? What is the importance of transmission channels and latent confounding effects when studying growth shocks across countries within a common currency area? Despite the heterogeneous behaviour showed in Section 8, there is indeed a significant common component, especially in the last recession, in its financial dimension and even more in its real dimension. The result seems to confirm the existence of a statistically significant common factor linking these seemingly heterogeneous real and financial series across all countries and throughout several cycles. For example, focusing on Figures 34 and 35, evolution of the first and third country-specific factors over time is drawn. Real and financial dimension accounting for *weights* are estimated, respectively. Real variables, in Figure 34, show higher degree of heterogeneity than financial component (Figure 35). In fact, the box plot is comparatively large and it suggests that overall countries have a low level of *agreement* with each other. These findings would confirm the increasingly importance of capital flows in driving the spreading of spillover effects. The result is consistent with the more recent literature and empirical evidence of IMF (2014) and ECB (2013) recognizing growth shocks are predominantly transmitted via financial linkages. Moreover, box plots in Figure 34 show large difference between them and, hence, there

is high degree of divergence across countries to fiscal shocks originating within the EMU. Higher values in Figure 35 confirm more coordinated fiscal actions over time in financial dimension with respect to real dimension. Moreover, the median (the line that divides the box into two parts) observe negative values. These findings would confirm that country are by turns net receivers and net sender of the system over time, absorbing and generating, respectively, growth shocks (see Section 8 for more details). Stronger effects are observed in some smaller open economies (Netherlands, Austria, Belgium, and Finland) and, with the debt crisis, also to larger countries such as Spain, Italy, and France).

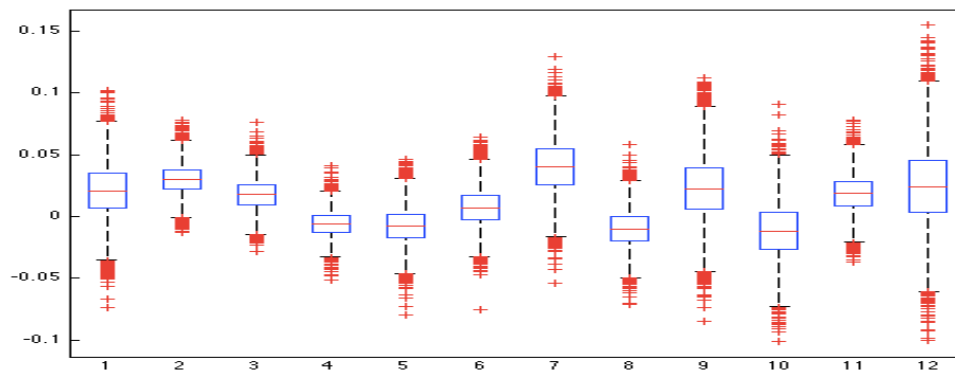


Figure 34: The Figure draws the country factors of all real variables with *weights*, expressed in standard deviation from the historical average of annual growth rates. These factors correspond to  $\chi_{1t}$  in the model described in 30. Box-plots are drawn vertically indicating variability outside the upper and lower quartiles. The spacings between the different parts of the box indicate the degree of dispersion (spread) and skewness in the data, showing outliers.

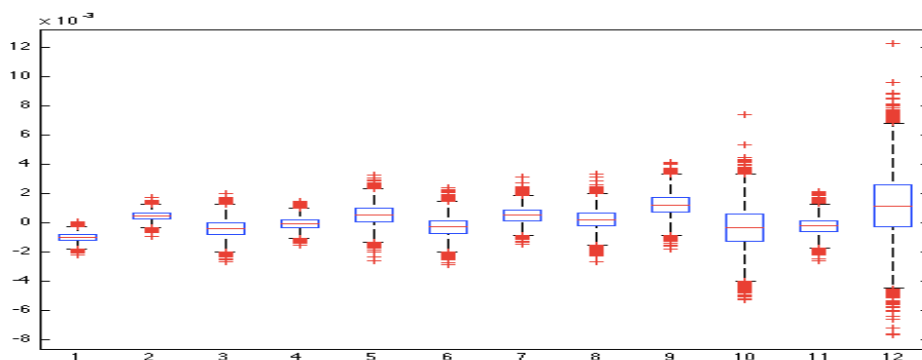


Figure 35: The Figure draws the country factors of all financial variables with *weights*, expressed in standard deviation from the historical average of annual growth rates. These factors correspond to  $\chi_{3t}$  in the model described in 30. Box-plots are drawn vertically indicating variability outside the upper and lower quartiles. The spacings between the different parts of the box indicate the degree of dispersion (spread) and skewness in the data, showing outliers.

Figures 36 and 37 show real and financial dimension, respectively, accounting for both *weights* and *outliers*. These findings would confirm the importance of accounting for other sectors such as labour and household's market and other *latent* factors such as competitiveness, evolution of consumption, investments, productivity across countries. Variability decreases in real dimension

proving the strong divergence across countries. Box plots in Figure 36 observe larger different distributions. Moreover, median is mostly positive and, hence, real economy is a net receiver of the system. This results would confirm potential unobserved variables strongly affect real economy. To be more precise, countries more strongly affected by outward growth shocks because of large trade exposures with other member states.

In Figure 37, box plots observe higher value than ones in Figure 35 and, hence, more coordinated fiscal action in financial dimension. However, those severe adjustment pressures have depressed output for the presence of a persistent divergence across countries in their real component.

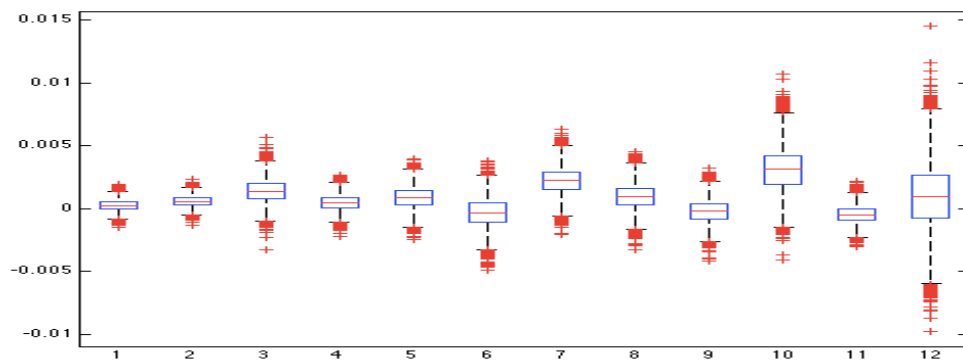


Figure 36: The Figure draws the country factors of all real variables with *weights* and *outliers*, expressed in standard deviation from the historical average of annual growth rates. These factors correspond to  $\chi_{2t}$  in the model described in 30. Box-plots are drawn vertically indicating variability outside the upper and lower quartiles. The spacings between the different parts of the box indicate the degree of dispersion (spread) and skewness in the data, showing outliers.

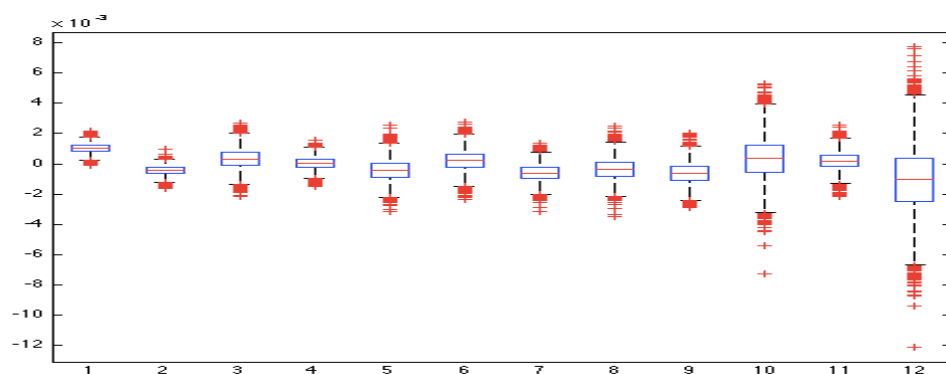


Figure 37: The Figure draws the country factors of all financial variables with *weights* and *outliers*, expressed in standard deviation from the historical average of annual growth rates. These factors correspond to  $\chi_{4t}$  in the model described in 30. Box-plots are drawn vertically indicating variability outside the upper and lower quartiles. The spacings between the different parts of the box indicate the degree of dispersion (spread) and skewness in the data, showing outliers.

Accounting for group-variable specific factors, Figure 38. draws real and financial dimension for all sample with and without *outliers*, respectively. Financial dimension without *outliers*(second box-plot) shows larger variability and, hence, more sensitive to inward growth shocks originating within the EMU. In the third and fourth box-plots, latent factors are accounted for other potential

inter-linkages across countries. Real dimension shows higher variability than financial dimension. This result seems to confirm that real economy is potentially strongly sensitive to outward growth shocks originating by changes and responses in other sectors.

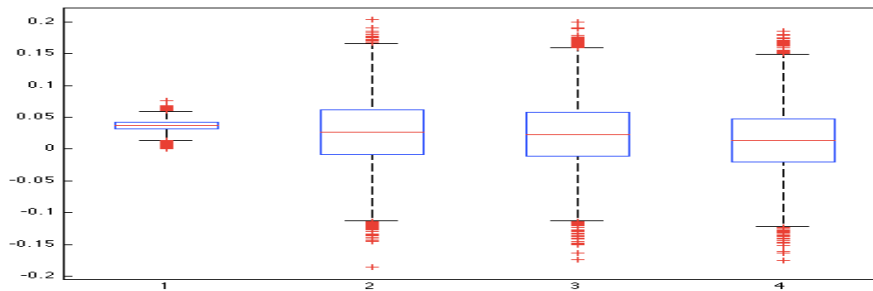


Figure 38: The Figure draws the variable factors across all countries for real and financial dimension, respectively, expressed in standard deviation from the historical average of annual growth rates. These factors correspond to  $\chi_{5t}$  in the model described in 30. Box-plots are drawn vertically indicating variability outside the upper and lower quartiles. The spacings between the different parts of the box indicate the degree of dispersion (spread) and skewness in the data, showing outliers.

Finally, common component accounts for all real and financial variables in an only coefficient vector observed with and without *outliers*. The result seems to prove the importance of accounting for both common and idiosyncratic components, which is large in its financial dimension and even more in its real dimension. Thus, real variables are strongly sensitive to outward growth shocks due to large interactions with financial and no-financial sectors.

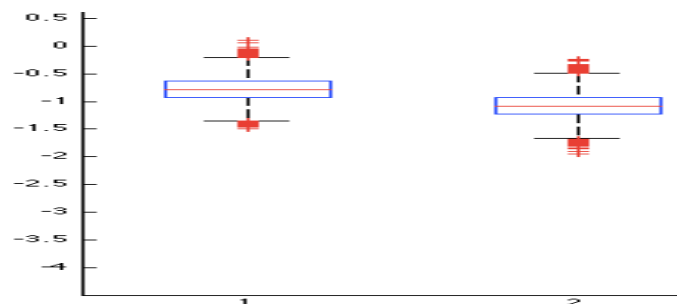


Figure 39: The Figure draws the country factors of all macroeconomic and financial variables with and without *outliers*, expressed in standard deviation from the historical average of annual growth rates. These factors correspond to  $\chi_{6t}$  in the model described in 30. Box-plots are drawn vertically indicating variability outside the upper and lower quartiles. The spacings between the different parts of the box indicate the degree of dispersion (spread) and skewness in the data, showing outliers.

## 10.2 Sign and dimension of cross-country spillovers from real and financial shocks

In order to highlight commonality and heterogeneity accounting for omitted economic/institutional implications, a 3D-dimensional graph will be plotted for real and financial dimension. The aim of the analysis is to observe magnitude and effect size of potential spillover effects over time in the Eurozone. For this latter, the following order of magnitude and size<sup>18</sup> will be used:

- *yellow* : empty sample corresponding to no significant impulse response,
- *orange* and *light green* : *small size* with  $0.1 \leq x \leq 0.3$ ,
- *sky blue* : *small & medium size* with  $0.4 \leq x \leq 0.5$
- *navy blue* : *high size* with  $0.6 \leq x \leq 0.7$
- *red* : *extreme size* with  $0.8 \leq x \leq 1$

The analysis draws surface plots in all EA12, accounting for real and financial dimension, and for transmission channels (as driven extent). The selected periods are: 1999 - 2007 and 2007 - 2014. The below surface of fitting objects complies with previous analysis. The effect size and magnitude of inward spillover effects appear limited over period, with values lower than 0.2 (*light green*). High values are shown when crisis is getting close with a magnitude of 0.8 (*navy blue*). Nevertheless, there is a considerable increasing in terms of heights reaching the minimum and maximum point in -1.5 and +1.5 respectively. Negative (net receiver country) and positive (net sender country) values confirm heterogeneity in transmission of growth spillovers across countries. In addition, the increasing of potential spillover although with low magnitude proves the presence of latent factors strictly correlated to real and financial linkages.

Proceeding with the analysis, surface plot is also conducted in the last recession accounting for fiscal consolidation. The Figure 41 would prove the existence of higher values. Mostly magnitude keep values bigger than 0.8 with an improvement on the spillover heights. Financial crisis and fiscal consolidation have, hence, affected almost contemporaneously real and financial dimension. This results refers to the presence of strong interlinkages between sectors, which can be analyzed by considering trade and capital flows across countries. Moreover, higher values in terms of divergence (or not-smoother surface of fitting objects) would highlights the importance of economic and

<sup>18</sup>The analysis restricts the cumulative impulse responses in the interval [0,1]

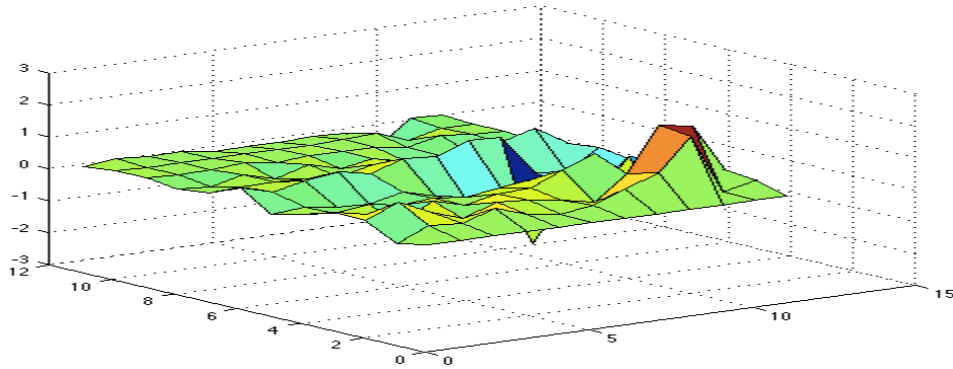


Figure 40: The Figure plots height and magnitude of potential growth spillovers given a 1% shock in real dimension for the period from 1999 to 2007. The plot is obtained creating a three-dimensional shaded surface from the z components in matrix  $Z$  (fitting objects), using  $x = 1:n$  and  $y = 1:m$ , where  $[m,n] = \text{size}(Z)$ . The height,  $Z$ , is a single-valued function defined over a geometrically rectangular grid.  $Z$  specifies the color data, as well as surface height, so color is proportional to surface height. The coefficient vector analyzed in this study is  $\chi_{6t}$  of the equation 30.

institutional factors<sup>19</sup> indirectly affecting the usual relationship between real and financial variables. Given high heterogeneity across countries in public and private sectors, spillovers differently affect countries creating divergence and no-coordinated impulse responses.

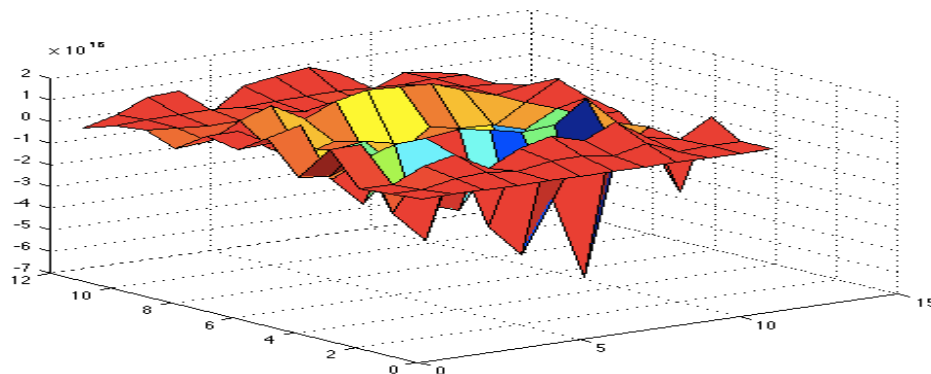


Figure 41: The Figure plots height and magnitude of potential growth spillovers given a 1% shock in real dimension for the period from 2007 - 2014. The plot is obtained creating a three-dimensional shaded surface from the z components in matrix  $Z$  (fitting objects), using  $x = 1:n$  and  $y = 1:m$ , where  $[m,n] = \text{size}(Z)$ . The height,  $Z$ , is a single-valued function defined over a geometrically rectangular grid.  $Z$  specifies the color data, as well as surface height, so color is proportional to surface height.  $\chi_{6t}$  of the equation 30.

## 11 Conclusion

The paper develops an approach to conduct inference in time-varying coefficients using a Bayesian multicountry VAR models with lagged cross-unit interdependencies and unit-specific dynamics. Bayesian computations are used to estimate and restrict the coefficients to have a low-dimensional

<sup>19</sup>In this analysis, the selected variables *unem*, *lab*, *cons*, *priv*, and *inv* are considered in data frame.

time-varying factor structure. The specification model uses a hierarchical prior for the vector of factors in order to permit exchangeability, time variations, and heteroskedasticity in the innovations in the factors. An overparametrized VAR is transformed into a parsimonious SUR model where the regressors are observable linear combinations of the right-hand side variables of the VAR, and the loadings are the time-varying coefficient factors. Generalized impulse response functions and conditional forecasts are obtained with the output of an MCMC routine.

The evidence would confirm the need to allow for cross-country and cross-factor interdependencies when analyzing macroeconomic-financial linkages. Net spillover matrices including real and financial variables for the EA12 are constructed to define total bilateral net spillover effects. They incorporate feedback effects from the impulse variables and temporary or persistent long-run effects of potential shocks that may lead to contagion. Analyzing the entire time-series period, shocks spill over in a heterogeneous way across countries, more intensive among financial variables. This finding accounts for higher amplification of spillover effects which can be seen as a result of increased interdependencies between variables. In this paper, spillovers are defined as the transmission of an unexpected but identified shock from one variable to receiving variables in the system. Accounting for cross-country and cross-variable interdependencies, conditional forecasts for bilateral trade and capital are computed. In this way, the model is able to investigate interactions between real and financial variables and to capture changes of interdependencies over time. Following the definition by Allen and Gale (2000), the contagion index proposed in this paper is defined as a consequence of excess spillover. Thus, extreme amplification of spillover effects can be seen as alarming levels which could lead to contagion. Optimal policy coordination in the Euro Area would have required a differentiation of consolidation efforts depending on the fiscal space to minimise the negative spillovers. Spillovers of fiscal consolidations are larger in financial dimension. Larger output effects prove that consolidations occurred simultaneously. The positive impact on outputs of most members in the financial dimension indicates the importance of coordinated fiscal actions in the Euro Area.

After estimating different specifications of this model, the highest marginal likelihood was found for the model including four country-specific components for each economy, four variable-type components, and two common components for all series. These common, country-specific and variable-type components quantify the relative contribution of common and heterogeneous factors in macroeconomic-financial linkages and help to address the following questions: Is there a significant common component in the real and financial interactions across eurozone members or do



country-specific heterogeneities matter more? How did *weights* and *outliers* factors affect real economy and financial variables over time? What is the importance of transmission channels and latent confounding effects when studying growth shocks across countries within a common currency area? Here, some considerations are in order. Country-specific factors remain very important explaining the presence of a heterogeneous pattern across members. However, interactions between real and financial dimension are important to understand co-movements in economic activity. Thus, bilateral trade and capital conduct a prominent role when analyzing foreign and domestic policies. Highly indebted countries were forced into taking wide-ranging austerity measures, having lost access to the financial markets. This has led to call for stronger cross-country differentiation and for temporary stimulus measures in countries not facing financial market pressure. Therefore, cross-border spillovers have exacerbated the negative effects of consolidations. This finding accounts for a substantial degree of heterogeneity in real dimension and a deeper interdependence in financial dynamic. These findings cast a new perspective for theoretical models of idiosyncratic business cycles and policy making.

From a modelling perspective, the analysis appears to favour models that assign an important role to *catching-up* and competitiveness factors in explaining current account imbalances and debt dynamics. Moreover, transmission channels suggest that trade channels matter relatively less than financial channels. Growth shocks appear to be predominantly transmitted via financial linkages. The interdependence is stronger in financial dimension, while real component shows higher degree of heterogeneity and it is mainly affected by latent confounding effects. The results are consistent with the recent literature which recognizes the importance of accounting for both country-specific and global factors when studying real and financial interactions. Moreover, the analysis is consistent with the premise that for countries to be an important source of growth spillovers, growth should rely to a greater extent on autonomous domestic sources. Nevertheless, testing for commonality and heterogeneity, the idiosyncratic components in driving fiscal shock transmissions is high, suggesting the necessity of accounting also for growth shocks outside the EMU that are to a relatively larger extent transmitted via trade. Finally the analysis is consistent with the possibility that larger co-movements or macroeconomic-financial linkages observed, mainly in the last recession, could be more related to the size and height difference of the shocks than to the intensification of their transmission.

From a policy perspective, several considerations can be displayed. First, despite high degree of heterogeneity, countries of the eurozone share common financial shocks and, hence, the analysis

is in line with rapidly increasing cross-border trade and financial linkages. Although early indications suggest that the imbalances have been reduced and the eurozone countries are weathering the current *storm* during current recession, without the appropriate adjustment of the private and public sector, euro area imbalances could pick up again if the macroeconomic conditions normalize. Second, despite a common monetary policy, national policies of fiscal policy, investments, and structural reforms in labour and complementary markets remain heterogenous across the euro area. This might have contributed to the emergence of different country-specific developments of competitiveness, consumption, investment, and production structures affecting national economy. Thus, national authorities may be tempted to design domestic policies so as to counteract world conditions, but those policies may be ineffective and counter-productive for the domestic economy. Third, structural differences among national policy may also be driven by idiosyncratic business cycles and, hence, the importance of accounting for transmission channels and latent confounding effects. Fourth and probably most importantly, divergence across countries were driven by different degrees of productivity growth. Thus, in the euro area, structural reforms without coordinated national fiscal actions affect the adjustment capacity of the currency union as a whole because of high degree of divergence.

Some policy implications in real dimension are: Germany as an important "transmitter of shocks" and, hence, trade channels seem to matter relatively more for German outward spillovers with respect to other large EA countries. It confirms Germany's own dependence on growth in the rest of the eurozone. France is sensitive to Spain and, to a lesser extent, Italy and much less sensitive to growth shocks in Germany than Germany is to growth shocks in France. Spain's growth is affected by growth shocks in France and to a lesser extent by growth shocks in Italy. Germany plays a less prominent role for the GIP than France and, to a lesser extent, than Italy. In financial dimension, policy implications would be: Germany tends to have more influence on Greece than Italy, but less than France. Germany's role in generating outward spillovers appears limited despite its large size and it is particularly sensitive to growth shocks in three large euro area countries (Italy, France and, to a lesser extent, Spain). During recent recession, there was no amplification of outward spillovers in Germany during financial crisis. Italy and France generate larger spillover effects to the Eurozone periphery Spain's growth is potentially strongly affected by growth shocks in France. Italy's growth reacts relatively similarly to growth shocks in other Euro area countries (e.g., France and Spain). Negative shocks of GIP as a group on other countries appears relatively small, consistent with their modest size.

These considerations raise interesting questions that could be addressed in future research. (i) The importance of fiscal and monetary policy interactions in a currency Union when analyzing macroeconomic-financial linkages. (ii) International business cycles play a prominent role with countries endogenously reacting to foreign impulses.

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