A didactical tetrahedron supporting co-disciplinary design, development and analysis of mathematical e-learning situations

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In this paper we propose a comprehensive model, conceived as a heuristic to support a co-disciplinary approach to the design, development and analysis of the didactical system, in particular in the case of mathematical e-learning situations. It has been developed by expanding the classical didactical triangle into a tetrahedron, and including within it a mediatory sphere whose intersection points with the tetrahedron can shed some light on the impact of technology within the didactical system. We explain how the model could address the need to take into account, in a co-disciplinary mode, different theoretical and empirical perspectives, within and beyond mathematics education.

Keywords: e-learning, didactical tetrahedron, co-disciplinary approach

INTRODUCTION

At a first level, this paper focuses on the interactions between students and teachers with the content and with digital resources, in the context of mathematical e-learning situations. First of all, we need to specify what we mean by e-learning. There is no doubt that we move into a web based environment, but the Web has changed throughout the years. The three web generations can be described by the following verbs: read, write/communicate, collaborate, each of them intended to be added to the previous ones (Hussain, 2012; Miranda et al., 2014). This means that educational technology has allowed the learners more and more engaged, from a passive role to an active one and finally to a social one. Just as knowledge was delivered in the era of e-learning 1.0, then it was co-constructed by the learners with the advent of e-learning 2.0 and now it is socially constructed by communities of learners. Note that the main difference between the 2.0 and 3.0 eras depends on the kinds of interaction among learners: in the former case, learners can write resources and share them, in the latter case, learners can collaborate in writing resources. Moreover, we point out a further feature of the Web 3.0, that is mobility: nowadays we can access technology anywhere and anyhow, e.g. by any device. Some contend that e-learning 3.0 should also be considered "intelligent", as well as "collaborative" (Rubens et al., 2014), but in this paper we neglect the subject on artificial intelligence. Herein, we will draw our attention to the teaching/learning process which occurs in an e-learning 3.0 environment, taking into consideration general purposes teaching platforms, integrated with social apps, eventually added to online mathematical software. Note that the most popular platform, Moodle, is already available in mobile version, and it can be used in mobile learning together with other social apps, such as Whatsapp.

From now on, with the term mathematical e-learning we will refer to this kind of teaching/learning process, focused on mathematics education, concerning both a distance and blended setting. In order to study mathematical e-learning situations we believe that we need a comprehensive model which could firstly take into account research results on the use of tools in mathematics education, without disregarding input coming from other research fields beyond it. We consider worth of note, for instance, that in the General Didactic research field it has been underlined that learning is a process that can occur only with teaching mediation: thanks to the interaction with didactic mediators, that facilitate the transition from the specific experience to the generalization of it, pupils organize and conceptualize their own experience during the learning process. In particular, the key role of every learning activity is played by a system of didactic mediators: the educational action makes use of functional multiples mediators that follow each other. In this perspective what becomes important to understand, in the case of mathematical e-learning situations, is what does change if in the mediators' system there are also digital tools. As far as mathematics education is concerned, instead, tools were used long before new technology entered the classroom and have always played an important role. Here, the term "tools" is used in a broad meaning as means, incorporated in mathematical activities. Moreover, it is noteworthy that tools always have affordances and impose constraints on the user, and that teachers need to understand and to be aware of the implications of the use of tools in mathematics classrooms. And this is true, in particular, in every teaching/learning situation involving new technologies, and thus also in e-learning situations.

In this paper we intend to propose a comprehensive model which expands the classical didactical triangle into a tetrahedron. Indeed, in order to become aware of the impact of technology on the relationships between the Teacher (T), the Students (S) and the Mathematics (M) in a e-learning situation, as will be explained better below, we believe it is important to consider also the Designer (D). A fundamental characteristic of the model we are going to present is that, unlike what happened in other studies, the technology is not a vertex, but it is embedded in a mediatory sphere immersed in the tetrahedron, whose vertices are T, S, M and D.

At a second level, according to results coming from a recent study (Faggiano *et al*., 2017) involving educationalists and experts in mathematics education, we argue that every didactical intervention is shaped by a complex space/time device in which knowledge is consolidated and conceptualization is fine tuned. The interactions between teacher and student allow a sort of alignment between the student's experiences and the scientific knowledge. It has been made possible also thanks to the presence of artefacts/mediators with structural and structured role with respect to the mediation between teacher and student, student and knowledge, but also teacher and knowledge. These considerations call for the need to study mathematical teaching/learning processes from a wider perspective, considering and taking advantage of results coming from other research fields, such as general didactic or the educational technology. For this purpose, we adopted a co-disciplinary perspective

(Blanchard-Laville, 2000) in which the prefix "co", which means "with", is about evoking the construction of a co-thinking research space fostered by a certain empathic understanding and acquaintance with the points of view of the other researchers about the same object of study.

FROM THE DIDACTICAL TRIANGLE TO THE TETRAHEDRON

The didactical triangle can be considered as a heuristic that identifies the fundamental components of any didactic system: teacher, student and content. The idea to expand the didactical triangle to a tetrahedron in order to consider the role of technological artefacts in mathematics education is not new (see e.g. Tall, 1986). In particular, Rezat and Sträßer (2012) proposed a socio-cultural tetrahedron, in which the fourth vertex is the mediating artefact. It offers an important representation of the complexity of the system that affords, in particular, a level of detailed reflection on the didactical role of the tasks. Our approach starts from the assumption that the full exploitation of e-learning environments requires a well design didactical intervention, not only in terms of contents and tasks (didactical transposition) to be arranged and eventually included in a platform, but also of the environment to be set up, the structure of the teaching/learning activities to be organized, the technology to be selected, the methodologies through which the interactions can be allowed and fostered and so on (didactical engineering). We argue, indeed, that an e-learning platform has a role of aggregator and that the true and meaningful sense of the didactical action lies in the complex system architecture of the learning environment rather than only in the content materials. For this reason, in the attempt to model the situations, we contend the need to introduce as a new vertex the designer (D). This allows us to highlight the role, performed mainly through a-priori rather than situational choices, of a further actor or, more precisely, "scriptwriter", in charge of that complex designing activity. In an ordinary situation it is often the teacher who assumes, on one hand the role of the designer, when he/she is involved in the selection and/or design of the resources, the construction of the tasks and the planning of the activities, and on the other the role of teacher/tutor during the development of the teaching/learning process. In more complex situations, however, it could also be the case that a collective entity, with different professional skills, needs to act as designer, while a (eventually further) collective entity acts as teacher/tutor during the development of the activities.

As far as mathematical e-learning is concerned, Borba, Clarkson, and Gadanidis (2013) already noted the importance of teamwork inside the collective designer. They claimed that the low design and pedagogical quality of online interactive mathematics contents can be avoided by the simultaneous work of various experts, such as mathematics educators and human-computer designers, which can take into account and integrate both didactic objectives and design principles. We argue that, a co-disciplinary team, especially involving educationalist, can take care not only of the design of the content materials but also of all the other choices which impact on the teaching/learning process: the comparison, the discussion, the co-thoughts that can occur among the different experts can affect decisions about the whole didactical architecture with

respect to a fixed didactic goal. This point of view also seems to be highlighted by Schoenfeld (2009) who wishes for a synergy between educational researchers and educational designers. He claims that the richness of the designer enables the creation of varied scenario of pedagogical expectations concerning knowledge, of professional or ideological beliefs, of implicit philosophies that provides an enrichment of the e-environment and carries out robust and well-engineered products made available to the targeted learners.

THE CO-DISCIPLINARY PERSPECTIVE

In light of the above, following Albano *et al*. (2013), we assume that the didactic system concerning mathematical e-learning situations can be modelled in a systemic way by a tetrahedron, which includes: some mathematical knowledge, that is Mathematics (M), someone who is expected to learn M, that is the Student (S), the Teacher/Tutor (T) and the Designer (D), in charge of planning, developing and managing the didactic organization. As a matter of fact, the present tetrahedron is an extended version of the cited, in particular with respect to the last vertex (that we have decided here to name "Designer" instead of "Author"): herein, indeed, we aim at assuming a wider approach considering the authoring of the content materials as part of a more complex role that is the one of the designer as described above.

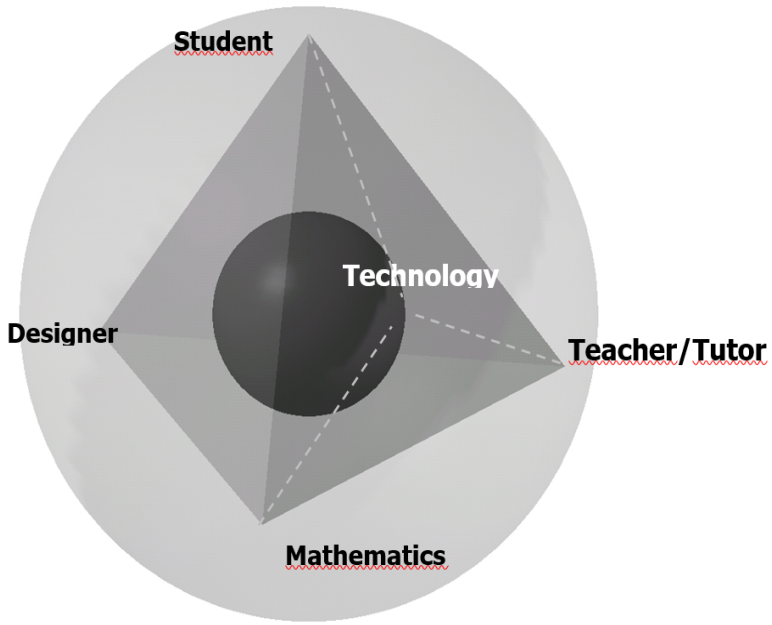

Figure 1: the didactical tetrahedron for mathematical e-learning

We claim that this model acknowledges the need to design, develop and analyse mathematical e-learning in a co-disciplinary approach, hence the tetrahedron with the internal mediatory sphere have to be considered in the whole complexity. However, an insight on each of the four faces allows us to focus on the various aspects on which the model can shed light, taking in account different perspectives on the interactions among the actors, with respect to the integration of technology in the teaching/learning situation.

The basis of the model still remains the classical didactical triangle students-teacher-mathematics, that is referred by the face STM. According to Rézeau (2001) teacher's action could be seen as a continuous balance/mediation between the didactical disciplinary oriented processes and the educational processes. The latters are intertwined with the formers and the teacher's expertise consists of the ability to realize this balance which varies according to the context, between scaffolding and fading, between autonomy and support. The intersection point between the mediatory sphere and this face of the tetrahedron is the technology, seen as a mediatory tool. The view of this face represents the focus on the didactical action and on the role of the teacher/tutor as "arranger".

In the perspective of DTM face, attention is focused on the processes, where design and enacting are intertwined, in which the Designer interacts with resources selecting, adapting, revising and reorganising them, by means of an a-priori analysis. It is worthwhile noting that these processes are ongoing processes which continue in usage. This face, hence, depicts the instrumented mediated activity of planning the mathematical experience. From this point of view, the Designer looks for resources, plans the activities, chooses the e-tools and defines the educational setting. The instrumental genesis takes place, that is the Designer defines how to use the artifact (instrumentalization) and at the same time the affordances and constraints of the particular chosen e-tools influence the design of the activities.

The view from the SDM face allows us to focus on the mediating role of technology with respect to the the role of the Designer in organizing the learning settings for the Student to learn Mathematics. From this perspective it can be useful to consider the Geiger's (2006) distinction of the four metaphors to describe the degree of sophistication in which technology can mediate learning: Technology as master, where the student is subservient to the technology and the relationship is induced by technological or mathematical dependence; Technology as servant, where the technology is subservient to the student, typically used as a reliable timesaving replacement for mental, or pen and paper computations; Technology as partner, where the technology is used creatively to boost student empowerment, treating the technology almost as a surrogate human partner; Technology as extension of self, where users draw on their technological expertise as an integral part of their mathematical thinking.

Looking at the face STD, the focus is on the classical relationship between the Teacher/Tutor and the Student, planned by the Designer. It can be represented by the word "conversation", referring to the Conversational Framework (Laurillard, 2001). The Designer models the learning experience as iterative interactions among two participants (e.g. Teacher and Student) at two levels, practice and communication, connecting the two levels by means of adaptation and reflection. This means that we can think of the whole face STD as cycles of designing for learning (D plans activities, also with the use of e-tools), doing for learning (S interacting with the e-environment), communicating for learning (starting from the practice, reflection and discussion

between S and T and among students), tuning for learning (D perfects the design with respect to T feedbacks). The previous cycles including peer learning, assuming both the participants are students.

DISCUSSION AND CONCLUSIONS

The didactical tetrahedron proposed above can operate as a heuristic not only acknowledging the need to analyse the relationships among the actors, but also drawing attention to the didactical system supporting the design, the development and the analysis of mathematical e-learning situations according to a co-disciplinary approach. An example can be given considering the Theory of Semiotic Mediation (Bartolini Bussi and Mariotti, 2008). From this perspective: the view of the STM face focuses on the role of the teacher within the process, which consists of fostering the social evolution of the emergent personal signs, coming from the artefact-use, into shared mathematical signs, through the orchestration of meaningful discussions; the view of the DTM face focuses on the design of both materials and activity phases, in order to foster the unfolding of the semiotic potential of the artefact in use and the construction of mathematical meanings through the guided evolution of signs, performed by the Designer; the view of the SDM face allows us to focus on the artefact sign production provoked by the use of technology thanks to the task that has been set up by the Designer; finally, it is with a co-disciplinary perspective that the analysis of the teaching-learning process can be broaden considering the STD face, taking into account some contributions coming beyond mathematics education thanks to which we can also focus on the didactical elements influencing the teaching-learning practices.

As a further example we can refer to a study in which the model has been applied to define, tune and analyse the design of a Digital Interactive Storytelling in Mathematics (Albano *et al*., 2018). One of the main features of the model is its systemic view of the actors involved, which has allowed us to reflect on the learning process in a non linear way, differing from the initial mode of learning in e-environments. Anyway, the model can be used also to conceive the actors in terms of played roles rather than of persons. In fact, this has led to thinking of them in a dynamic way, so imaging in some cases technology (suitably chosen from the internal sphere and shaped according to the intended use) as Tutor scaffolding specific learning goals. Analogously, the student can play the role of the Designer, who produces the resources needed to make the activities progress, or the role of the Tutor, in terms of expert among peers. This example refers to the design phase, but further work is on-going in order to use the model for analysing the output in terms of learning.

The design of teaching/learning activities in e-learning 3.0 environment is a complex work. In fact, such activities generally foresee the use of various e-tools, some general purposes and some domain specific, which should be pedagogically integrated among them, on the basis of the didactical objectives of the teaching/learning activity. The tetrahedron by Rezat & Sträßer (2012) that generalizes the classical didactic triangle including the “Artifact” as a new vertex, has, in our view, the merit to recognizes that the connections represented by the classical didactical triangle require mediation. It can

be seen as embedded in our model if we consider the face STM connected with the tangent point of the inside sphere of technology. However, we believe that the socio-didactical tetrahedron is not completely suitable to take care of the complexity of mathematical e-learning teaching-learning activities. This complexity is especially intrinsic in the non simultaneity of time and spaces of any interactions which requires a didactical orchestration not comparable to the face-to-face case. For this reason, we have considered the proposal of a new specific tetrahedron for mathematical e-learning worthwhile in which the vertex D is brought to the fore.

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