

Network Calibration and Metamodeling of a Financial Accelerator Agent Based Model

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Abstract

We introduce a simple financially constrained production framework in which heterogeneous firms and banks maintain multiple credit connections. The parameters of credit market interaction are estimated from real data in order to reproduce a set of empirical regularities of the Japanese credit market. We then pursue the metamodeling approach, i. e. we derive a reduced form for a set of simulated moments $h(\theta, s)$ through the following steps: 1. we run agent-based simulations using an efficient sampling design of the parameter space Θ ; 2. we employ the simulated data to estimate and then compare a number of alternative statistical metamodels. Then, using the best fitting metamodels, we study through sensitivity analysis the effects on h of variations in the components of $\theta \in \Theta$. Finally, we employ the same approach to calibrate our agent-based model (ABM) with Japanese data. Notwithstanding the fact that our simple model is rejected by the evidence, we show that metamodels can provide a methodologically robust answer to the question “does the ABM replicate empirical data?”.

1 Introduction

The relationship between Agent Based Models (ABMs) and empirical evidence is a widely discussed topic among scholars in the field. On the one hand, ABMs provide a more faithful representation of economic reality, introducing more realistic behavioral assumptions than mainstream models.

Thus, they should potentially provide a better agreement with empirical data. Indeed numerous contributions have underlined the success of ABM in replicating “stylized facts” thanks to the introduction of agent heterogeneity, bounded rationality and learning, and decentralized out-of-equilibrium interactions¹. On the other hand, there is still little consensus in the field on how to evaluate the agreement between models and facts. Some ABM scholars claim that calibration or validation, not to speak of estimation or forecasting, are neither possible nor desirable (Valente, 2005). Most researchers underline instead that empirical evidence imposes a much needed discipline on model building, and that ABMs should accept the challenge of a stringent comparison with this evidence (Fagiolo *et al.*, 2007).

In particular, a growing number of contributions tackle the issue of econometric estimation of agent-based models, although these exercises are confined at the moment to relatively simple models of financial markets (Alfarano *et al.*, 2005; Manzan and Westerhoff, 2007).

One key characteristic of ABMs is that the mathematical form of the relationship between endogenous and exogenous variables is unknown. Generally speaking, if y is a time series generated from the ABM, h is a vector function defined over y , whose components are usually called *moments*, and θ is the vector of parameters of the model, y and $h(y)$ are typically unknown, possibly non-linear, random functions of θ with unknown likelihood. Thus maximum-likelihood estimators, or standard approximations of the likelihood function, are of no use. Instead, we can employ a class of methods defined as “simulated minimum distance” (Grazzini and Richiardi, 2015), which can be stated as follows

$$\theta^* = \arg \min_{\theta \in \Theta} F(h(x), h(y(\theta, s))) \quad (1)$$

where F is a criterion function, s is a label for a fixed generator of pseudo-random numbers, Θ is the domain of variation of parameters and x is a real time series. If the model is overidentified, a frequent choice for F is a quadratic loss function with an optimal weighting matrix, i.e. one that minimizes the uncertainty of estimation. If h results from the estimation of the same “auxiliary” statistical model over real and simulated data, this approach is usually termed “indirect inference”; if h stands for a set of moments computed over x and y , we obtain the method of simulated moments

¹For a review see Chen *et al.* (2012)

(MSM) (Gouriéroux C. and Monfort, 1996).

In general we cannot exclude that ABMs are unidentifiable. Indeed, even linear or linearized DSGE models exhibit a number of pathologies in estimation due to the flatness of the objective function or to the existence of multiple maxima (Canova and Sala, 2009). ABMs entail additional difficulties due to the possible non linearity of moments in the parameters. On the other hand, the existence of non linearities in the model is usually assumed but not proved, so we cannot exclude either that the ABM might be characterized by linear relationships between variables.

In order to address the identification issue, the parameter space should be explored systematically before any estimation exercise. In this context, it's useful to estimate the influence of θ on $h(y(\theta, s))$ by means of a *metamodel*, i.e. a statistical auxiliary model of the following form:

$$h(y(\theta, s)) = \beta' f(\theta) + u_s \quad (2)$$

where $f(\theta)$ is a deterministic, possibly non linear, vector function of θ , β is a vector of coefficients, and u_s is a second-order stationary, zero mean, potentially heteroskedastic, random term with given covariance matrix. This approach is widely used for ABM metamodeling in various fields (see e.g. Salle and Yildizoglu (2014), Dancik *et al.* (2010) and references therein). The metamodel is estimated from a sample of points in the parameter space, which still represents a computationally costly exercise for ABMs that can be made more efficient by an appropriate choice of evaluation points, e.g. with latin hypercubes or other parsimonious sampling designs (see Appendix A). Furthermore, the parameter space may be eventually restricted through the calibration of at least some of them, following the suggestion of Brenner and Werker (2007).

The result obtained from the estimation of a metamodel represents the analogue, for a simulated model, of the reduced form of an analytically solvable model. We can employ this reduced form, if its fitness compared to the original ABM is good enough, for a variety of purposes, like sensitivity analysis (Campolongo *et al.*, 2000), calibration and estimation.

Given the general framework outlined above, in this paper we proceed as follows:

- we introduce a model of financially constrained production with a credit network composed of heterogeneous firms and banks;

- we calibrate some parameters of the model *before* simulations (*input calibration*), in particular we calibrate the network of the model with Japanese real credit network data;
- we simulate the ABM using an efficient sampling scheme of Θ ;
- we estimate different specifications of eq. (2), i.e. different metamodels, on simulated data;
- we choose the best metamodel performing some goodness-of-fit analysis on its predictions \hat{h} ;
- we employ \hat{h} for sensitivity analysis, quantifying the effect of each parameter on the components of $h(\theta, s)$;
- we identify the parameters of the ABM matching \hat{h} with a set of empirical moments \bar{h} ;
- we verify that at the optimal parameter values θ^* the response of the ABM is consistent with the predictions of the metamodels
- we check if at the optimal parameter values θ^* the ABM is able to replicate the empirical moments \bar{h}

The model we introduce is parsimonious in terms of parameters if compared to others of a similar vein (Ricetti *et al.*, 2013), since we wish to focus on the methodological novelty of the metamodelling approach. Indeed, the simplifications we have introduced limit the scope of the economic analysis and a more complicated model should lead to more realistic results. However, we underline that the proposed metamodeling methodology can be extended to more complex frameworks with no additional qualifications, since no a priori restrictions are imposed on ABMs for its application. In particular, this methodology is especially fit for large scale models with a large number of parameters since, especially when combined with optimal sampling schemes, it allows to reduce dramatically the number of simulations required for calibration or estimation (Barde and van der Hoog, 2017).

The main novelty of our model is that we allow firms to have multiple credit suppliers (see e.g. Bargigli *et al.* (2014)). In particular we opt for a representation of credit market interactions by means of a random network model where both firms and banks can have multiple connections. We make

this choice mainly because it is less expensive in terms of computational time. In order to allow for multiple credit connections, standard AB simulations should go through all the potential links, i.e. cycle over $n \times m$ steps, where n and m are the number of firms and banks respectively, but this is a slow, inefficient, solution. Using a random network model, we perform, instead, two faster operations: firstly we compute the parameters of a set of $n \times m$ probability distributions; secondly, we draw $n \times m$ random variables from these distributions.

The additional advantage of this choice is that we can easily calibrate the parameters of credit market interactions with real data. The main motivation for pursuing calibration is that credit markets represent a typical example of a sparse network². If a network is sparse, its topological properties³ become non trivial. For instance, the size of the neighborhood of a node, i.e. her *degree*, becomes very important. Nodes with a high number of neighbors, called *hubs*, are typically conducive of large systemic effects, in particular they can potentially trigger bankruptcy avalanches, if affected by external shocks, through balance-sheet effects on many other agents (Shin, 2008).

Since real credit markets display a high fraction of hubs, their degree distributions are typically right-skewed. We wish to replicate this property in our model. A well known solution for this task is to build a statistical ensemble of random networks for which the average degree of each node is equal to the degree of the same node in the real network (Park and Newman, 2004). Random networks drawn from this ensemble trivially replicate the degree distribution of the original network. Here we follow a different route, because we wish to connect the degree distribution with the economic variables of the model, net worth in particular. At the same time, we wish to replicate the debt and loan size distributions of the real market. In order to control for topological properties and assign loan amounts at the same time, we need to follow an approach in two stages: firstly, a couple of firms and

²A network involving n firms and m banks connected by l links is said to be sparse when $l \ll n \times m$, otherwise it is said to be dense. The Japanese credit market studied in Bargigli and Gallegati (2011), whose most recent data are employed in this paper, had $l = 21,811$ connections over a maximum of $n \times m = 2,674 \times 182 = 486,668$ in 2005.

³By topological property we mean any observable which is defined on a binary network or on the binary representation of a weighted network. The latter is obtained from the binary representation of its weighted links, which is defined, for each couple of nodes (i, j) , as $a_{ij} = \mathbf{1}(w_{ij} > 0)$, where $\mathbf{1}(\cdot)$ is the Indicator function and w_{ij} is the strength of the relationship between i and j .

banks activate a credit relationship with a given probability; secondly, if the link is activated, the loan amount is determined.

The paper is organized as follows. In Sec. 2 we introduce our model. In Sec. 3 we calibrate the parameters required for credit market interactions using Japanese data. In Sec. 4, after having specified a suitable sampling design for the remaining parameters of the model and after having defined the components of h , we turn to agent-based simulations. Then, using simulated data, we compare a number of alternative metamodels which could serve as reduced form of $h(\theta, s)$ and, after having selected the most fitting metamodels, we quantify the effect of each parameter on the components of h using the corresponding predictions \hat{h} . Finally, in Sec. 5 we employ the same approach to identify the parameter values of the ABM compared to a set of moment conditions. Section 6 provides some conclusions along with considerations regarding the long standing issue of aggregation.

2 Model Description

Our economy is populated by firms and banks which interact in the credit market. Firms are indexed by $f = 1, 2, \dots, F$, and banks are indexed by $b = 1, 2, \dots, B$. The initial conditions of the model are the equity of firms (E_f) and banks (E_b). Firms produce a single homogeneous good according to a linear production function:

$$Y_f = \alpha N_f \tag{3}$$

where Y_f is the quantity of good produced by firm f , α is a fixed productivity parameter, which we set to be equal across firms, and N_f is the number of employed workers. We assume that firms are equity constrained, so that they need to borrow in order to fund their production. The total disposable financial resources of each firm, which are given by equity E_f and debt D_f , cover the wage bill $WB_f = wN_f$, where w is a fixed parameter representing the wage rate.

Firms set their production at the maximum level allowed by their liabilities. Using the balance sheet constraint $D_f + E_f = WB_f = wN_f$ and the production function, this level is determined as follows

$$Y_f = \frac{\alpha}{w} (D_f + E_f) = \frac{\alpha}{w} (1 + \lambda_f) E_f \tag{4}$$

where $\lambda_f = \frac{D_f}{E_f}$ is the firm leverage. We see that Y_f depends on two fixed parameters (α, w) , and on E_f and D_f which are endogenous and time varying. In order to determine the latter variable, we set $D_f = \sum_b L_{fb}$, where L_{fb} is the amount of loan extended from bank b to firm f . This quantity is defined as follows:

$$L_{fb} = a_{fb} \times W_{fb} \quad (5)$$

where for simplicity of estimation we assume that a_{fb} and W_{fb} are mutually independent random variables. In particular, a_{fb} is a random binary variable which is equal to one if the link between firm f and bank b is activated, and W_{fb} is a random variable providing the amount of loan conditioned to the activation of the same link. The probability distribution of W_{fb} is specified below (see Sec. 3.2). In other terms we set:

$$L_{fb} = \begin{cases} 0 & \text{if } a_{fb} = 0 \\ W_{fb} & \text{if } a_{fb} = 1 \end{cases} \quad (6)$$

where L_{fb} and a_{fb} , contrary to W_{fb} and a_{fb} , are not independent. Thus we can observe a positive amount of loans between the couple (f, b) only if a link between the same couple has been previously established. We further suppose that the expectations of a_{fb} and W_{fb} are distinct functions of the net worth of the couple (f, b) :

$$\mathbb{E}[a_{fb}] = F(E_f, E_b) \quad (7)$$

$$\mathbb{E}[W_{fb}] = G(E_f, E_b) \quad (8)$$

The exact form of these expectations, together with the distribution of W_{fb} , will be determined by means of the network calibration of Sec. 3. Indeed, we introduce the two stages of interaction (firstly, activate a connection between firm f and bank b ; secondly, determine the amount of the loan) in order to control for a set of empirical properties of the real credit markets.

Summarizing, firms and banks are connected based on their respective net worth, and the amount of debt D_f is a consequence of this matching. For example, a big firm and a big bank have a high probability of being matched and for a high amount. Starting from this matching, each firm ends up with a certain debt and, consequently, this gives rise to a given leverage ratio. The interest rate paid by firms to banks, as we will see in a

while, is an increasing function of the leverage ratio according to the financial accelerator mechanism. So, the leverage ratio is the connecting part between the empirical calibration of the network and the economic model.

We assume that firms sell all the produced output on the goods market at a stochastic price (Greenwald and Stiglitz , 1993):

$$p_f = \frac{w}{\alpha} (1 + \epsilon_f) \quad (9)$$

where ϵ_f is a shock assumed to be normally distributed with mean μ and variance σ , that is $\epsilon_f \sim N(\mu, \sigma)$. In general, we may view the distribution of price shocks as reflecting demand conditions, within a framework of price adjustments to market imbalances. Thus, when a firm picks a higher ϵ_f , it represents a stronger final demand and vice versa.

The interest rate charged from banks to firm f is set in the following manner:

$$r_f = r_{cb} (1 + \delta \lambda_f) \quad (10)$$

where r_{cb} is the benchmark policy rate and $\delta \geq 0$ is a parameter which reflects the sensitivity of lenders to the creditworthiness of borrowers. When leverage increases, the firm is riskier and the banks charge a higher risk premium. This leads to a financial accelerator mechanism as in Riccetti *et al.* (2013). The profit of firms π_f is given by the following equation:

$$\pi_f = p_f Y_f - W B_f - r_f D_f \quad (11)$$

The net worth of firms is updated according to profits, assuming that no dividends are distributed:

$$E_f^{t+1} = E_f^t + \pi_f \quad (12)$$

When the net worth of a firm becomes negative, it goes bankrupt and is replaced by a new entrant with the median net worth of survived firms ⁴. As a consequence, banks' profits π_b are given by the sum of interest on loans received from survived firms, minus the losses due to non performing loans.

⁴Admittedly, with this choice we introduce potentially a small survivor bias in the model, since surviving firms are typically larger. However, the number of firm defaults is very limited over the parameter space we use for simulations and we choose the median (instead of the mean) in order to minimize the bias.

The net worth of banks is updated according to profits, assuming that no dividends are distributed:

$$E_b^{t+1} = E_b^t + \pi_b \quad (13)$$

When the bank's net worth becomes negative, it defaults and is replaced by a new entrant with the median net worth of survived banks (see footnote 4 above). To summarize, in each time period the following steps are taken in the model:

1. L_{fb} is determined for each couple (f, b) using eqs. (6)-(8) and the distribution of W_{fb} specified in sec. 3.2
2. Y_f is determined for each firm using eq. (4)
3. N_f is determined for each firm using the labour demand function $N_f = \alpha^{-1}Y_f$
4. p_f is determined for each firm using eq. (9)
5. r_f is determined for each firm using eq. (10)
6. firms' profits are computed using eq. (11)
7. the net worth of firms is updated according to eq. (12)
8. bankrupt firms are replaced
9. banks' profits are computed taking into account loan losses on bankrupt firms
10. the net worth of banks is updated according to eq. (13)
11. bankrupt banks are replaced

It's possible to show that firms profits do not depend on the productivity parameter α and the wage rate w . In fact, using eq. (4) we can write

$$\begin{aligned}
\pi_f &= \left(p_f - \frac{w}{\alpha} - r_f \frac{D_f}{Y_f} \right) Y_f = \\
&= \left(\epsilon_f \frac{w}{\alpha} - r_f \frac{w}{\alpha} \frac{\lambda_f}{1 + \lambda_f} \right) \frac{\alpha}{w} (1 + \lambda_f) E_f = \\
&= \left(\epsilon_f - r_f \frac{\lambda_f}{1 + \lambda_f} \right) (1 + \lambda_f) E_f = \\
&= [\epsilon_f + \lambda_f (\epsilon_f - r_f)] E_f \\
&= (\epsilon_f - r_f)(1 + r_f + \lambda_f) E_f
\end{aligned} \tag{14}$$

We see that λ_f has the effect of magnifying either earnings or losses depending on the sign of the difference between ϵ_f and r_f , also recalling that the latter is increasing in λ_f . Provided that the dynamics of the model, according to eq. (14), depends on price shocks and the consequent fluctuations of firm profits, in sections 4 and 5 we focus only on the parameters which influence profits through ϵ_f (μ, σ) and r_f (δ, r_{cb}).

3 Network calibration

The aim of this Section is to calibrate the credit network before proceeding with simulations. The purpose of the estimations is to align our simulated credit market with real data taking as reference a set of properties of choice, namely the credit network degree and debt distributions. For this exercise we employ the dataset for the banks-firms lending-borrowing relationships in Japan, maintained by the Econophysics Group at the University of Kyoto. The dataset includes balance sheet data on commercial banks and other credit institutions, as well as on listed companies, from 1980 to 2012⁵. In subsection 3.1 we calibrate the credit network degree, that is the activation of credit relationships between firms and banks. In subsection 3.2, given the presence of a credit relationship, we calibrate the amount of bank loans.

3.1 Links

In the model we presume that connections between banks and firms on the credit market are binary random variables whose expectation depend on

⁵For more details see http://www.econophysics.jp/foc_kyoto/.

respective equity, see eq. (7). From network theory (Park and Newman, 2004) we know that the maximum entropy distribution of the value of the link between nodes (f, b) , in a statistical ensemble of binary networks \mathcal{G} , is associated with an expectation of the following form:

$$\mathbb{E}[a_{fb}] = \frac{1}{1 + \exp(-H_{fb})} \quad (15)$$

where H_{fb} embodies a set of constraints imposed on \mathcal{G} . Then the most natural candidate for calibration is logistic regression. In practice, we make the following specification of (15):

$$H_{fb} = \alpha_A + \beta_A \log E_f + \gamma_A \log E_b \quad (16)$$

where f, b stand for firms and banks indexes respectively.

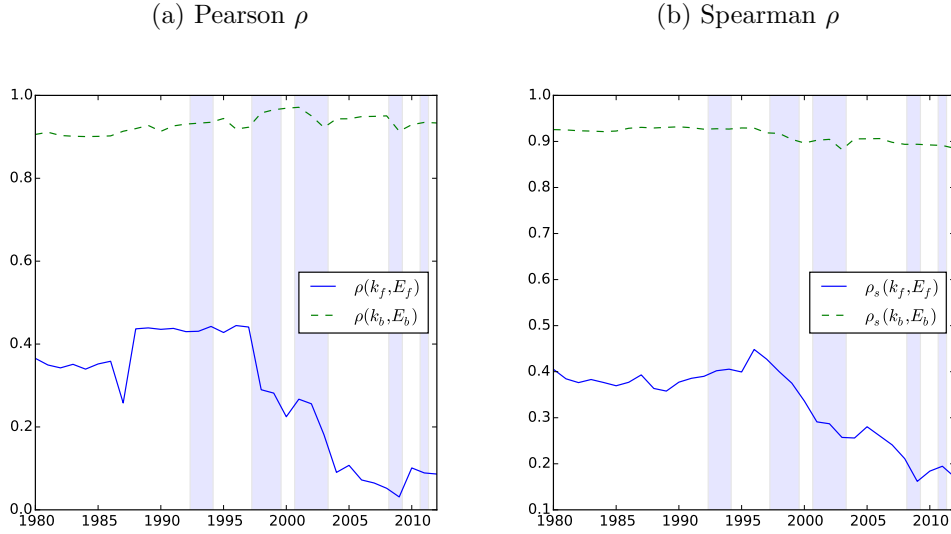
From the Japanese dataset we see that the two regressors behave differently over time. Fig. 1 highlights that there is a strong and stable linear relationship between the degree of banks k_b and their equity E_b . The relationship between companies' equity E_f and degree k_f is significant but weaker, less stable. This suggests that our specification does not include some relevant variables on the firms' side, like sector classification, which are not available in the Japanese dataset. Since we expect the logistic model to be misspecified, at least on the firms' side, we try to improve its fitness by means of random effects.

We estimate three models using the most recent data available in the dataset (2011): model A1 is given by eqs. (15) - (16); model A2 adds firm-specific random effects; model A3 includes both firm and bank-specific random effects⁶. We opted for random effects instead of fixed effects because the conditional log-likelihood estimation method of Chamberlain (1980) for logistic models with fixed effects does not provide the coefficients of the latter, which are needed for simulations. From Tab. 1 we see that the coefficients of the three models are always significant and their magnitude is similar. From the goodness-of-fit measures in the table we see that the introduction of random effects improves the estimation.

Fig. 2 compares the distribution of degrees in actual data with the one obtained from a sample of 1,000 random networks simulated using the estimated parameters. We see that model A1 provides a poor fit for the degree

⁶In detail, we employ random intercepts in a generalized linear mixed model estimated with the R (2015) package `lme4`.

Figure 1: Correlations between degree and equity, Japanese dataset (Shaded areas are recessions)



distribution of firms, while model A3 provides the best fit for the degree distribution of banks. In order to test this similarity, we perform the two-sample KS test comparing the actual and simulated degree distributions. From Tab. 2 we see that the null hypothesis of equal distributions cannot be rejected for models 2 and 3, and that the latter provides the best approximation for both distributions. For this reason we select model A3 for our simulations. This means that eq.(7) is specified as follows:

$$\mathbb{E}[a_{fb}] = \mathbb{E}[a_{fb} | u_f, u_b] = \frac{1}{1 + \exp(4.35155 + u_f + u_b) E_f^{-1.60026} E_b^{-0.18615}} \quad (7\text{bis})$$

where u_f and u_b are respectively firm and bank specific random terms which are fixed as initial conditions of the simulations.

3.2 Loans

We want to employ the same regressors of the previous Section in order to explain the value of loans *conditioned* to the existence of a link between a

Table 1: Logistic estimation on the Japanese credit market data (2011), ***($p < 0.01$)

	A1	A2	A3
(Intercept)	-3.95951***	-4.20761***	-4.35155***
$\log E_f$	1.53145***	1.62358***	1.60026***
$\log E_b$	0.19733***	0.17885***	0.18615***
Firms RE	No	Yes	Yes
Banks RE	No	No	Yes
Null Dev.	75,004		
Resid. Dev.	53,576	51,686	49,750
AIC	53,582	51,694	49,760
Pseudo R^2	0.286	0.312	0.337
Note:	* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$		

firm f and a bank b , see eqs. (6) and (8). From Fig. 3 we see that E_f and E_b are correlated in the Japanese dataset respectively with corporate bank debt D and the loan assets of banks S , i.e. the sum of loans extended to firms. In particular, the correlation between E_b and S is very high and stable over time.

We estimate the following model firstly with OLS (W1):

$$\log W_{fb} = \alpha_W + \beta_W \log E_f + \gamma_W \log E_b + v_{fb} \quad (17)$$

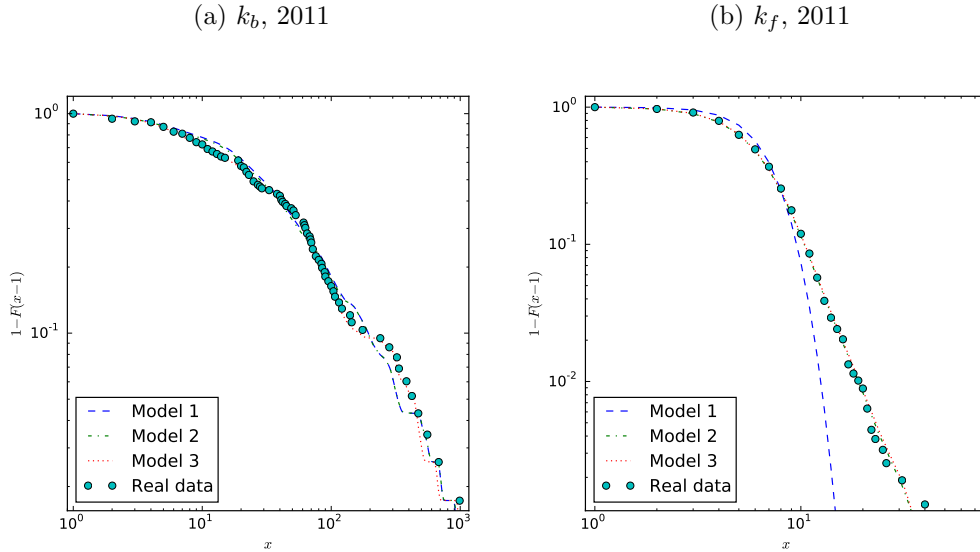
where v_{fb} is a zero mean random term with finite variance. With standard tests we detect both firm and bank specific effects in the data. In order to take these into account, we estimate distinct models with clustered errors at the firm and bank level (W2, W3, W4).

From Tab. 3 we see that the coefficients are always significant and with

Table 2: Kolmogorov-Smirnov 2-sample test for real and simulated degree distributions

		A1	A2	A3
k_f	KS stat.	0.111	0.021	0.016
	p -value	0.000	0.460	0.804
k_b	KS stat.	0.081	0.067	0.035
	p -value	0.410	0.643	0.998

Figure 2: Simulated *vs.* real degree distribution (artificial network sample size $R = 1,000$)



the expected sign. The inclusion of random effects improves the fitness of the estimation, as shown by the decrease of the AIC measure, and by the increase of the conditional R^2 proposed by Nakagawa and Schielzeth (2013), which is equal to the proportion of variance explained by both the fixed and random factors, while the marginal R^2 accounts for the variance explained by fixed factors alone.

In Fig. 4 we compare real data with simulated values obtained from models by randomly drawing from the residuals. We see that all models provide at first sight a good approximation to the distributions of D and S across firms and banks respectively. From Tab. 4 we see that the hypothesis of equal distribution (empirical *vs.* simulated) of credit demand D cannot be rejected for models W3 and W4, showing that the inclusion of random effects at the firm level is essential to reproduce the distribution of D .

Overall, we opt for model W4 in our AB simulations. This means that eq.(8) is specified as follows:

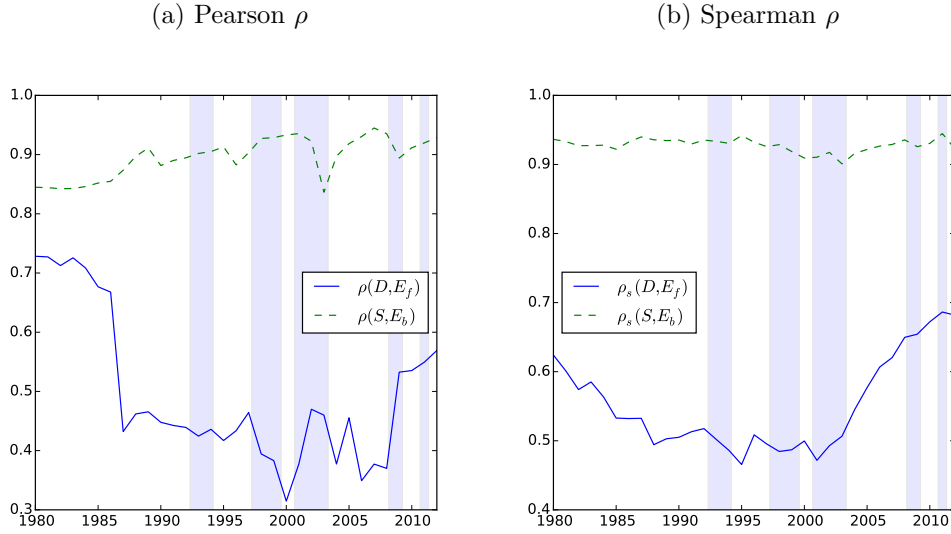
Table 3: Results

	W1	W2	W3	W4
$\log E_b$	0.280***	0.288***	0.268***	0.271***
$\log E_f$	0.617***	0.612***	0.642***	0.646***
Const.	-3.336***	-3.380***	-3.402***	-3.485***
Firm RE	N	N	Y	Y
Banks RE	N	Y	N	Y
marg. R^2	0.374	0.372	0.381	0.383
cond. R^2	0.374	0.391	0.655	0.670
AIC	32,817	32,689	30,118	29,935
Note:	* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$			

Table 4: Kolmogorov-Smirnov 2-sample test for real and simulated distributions

		W1	W2	W3	W4
S	KS stat.	0.114	0.103	0.079	0.077
	p -value	0.091	0.164	0.455	0.487
D	KS stat.	0.105	0.101	0.016	0.017
	p -value	0.000	0.000	0.810	0.748

Figure 3: Correlations between firms' debt and equity, Japanese dataset (Shaded areas are recessions)



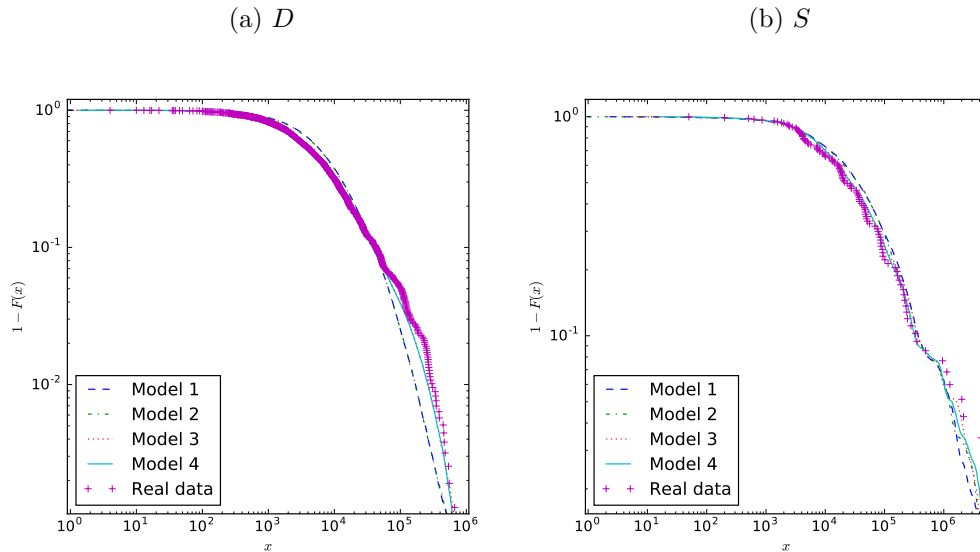
$$\begin{aligned} \mathbb{E}[W_{fb}] &= \mathbb{E}[W_{fb} | v_f, v_b] = \\ &= \exp(-3.485 + 0.646 \log E_f + 0.271 \log E_b + v_f + v_b) \end{aligned} \quad (8bis)$$

where v_f and v_b are respectively firm and bank specific random terms which are fixed as initial conditions of the simulations. From this expectation we obtain W_{fb} as follows:

$$W_{fb} = \mathbb{E}[W_{fb}] \times \exp(v) \quad (18)$$

where $\mathbb{E}[W_{fb}]$ is given by eq. (8bis) and v is a random term which is drawn, for each period of the simulation, from the residuals of model W4.

Figure 4: Loan estimation: models *versus* real data, 2011 sample size $R = 1,000$)



4 Metamodels: estimation, selection and sensitivity analysis

In this section we proceed as follows: 1. we run simulations for a sample of points in the space of parameters Θ ; 2. we estimate a set of alternative metamodels from simulated data, with the purpose of selecting the most fitting ones; 3. we employ the selected metamodels for sensitivity analysis.

Regarding simulation setup, the initial conditions of the model, that is the equity of firms and banks, come from the observed values of the Japanese dataset in March 2011, which includes $F = 1572$ firms and $B = 117$ banks. The expectation of eq. (7) is specified according to eq. (7bis), while the expectation of eq. (8) is specified according to eq. (8bis). The random terms u_f, u_b, v_f, v_b are fixed once and for all at the beginning of simulations using, for each firm and bank, the corresponding values obtained from models A3 and W4. W_{fb} is simulated using eq.(18): at each simulation step, the random terms v are drawn with replacement from the residuals of model W4.

The range of the free parameters of the model is presented in Tab. 5. In general, the dynamics of the model is determined by the earnings or losses of

firms and the resulting bankruptcies. For each firm the probability of facing losses depends on the one hand on the endogenous, firm specific, threshold r_f , which is dependent on r_{cb} , δ and λ_f , and on the other hand on the parameters of the distribution of price shocks (eq. 14). Indeed, the probability of losses increases if the probability mass of ϵ_f that falls below the endogenous threshold r_f increases. So if, for example, we decrease μ , the distribution of price shocks moves leftwards and losses / bankruptcies increase *ceteris paribus*. Otherwise, if we increase σ , the probability mass in the tails increases and the effect is the same. However, the interpretation is different, since in the former case the increase of bankruptcies is associated with a decrease of the expected profit at constant uncertainty, which we may call a “first order” effect of price shocks, while in the latter case uncertainty increases with expected profit unchanged, which we may call a “second order” effect of price shocks. Mixing the two effects would make the results more difficult to interpret. Thus, for sake of simplicity, we confine ourselves to first order effects of price shocks by varying μ while keeping $\sigma = 0.001$, and leaving the analysis of second order effects for the future.

Table 5: Range of parameters

r_{cb}	[0.0001, 0.05]
δ	[2, 5]
μ	[-0.001, 0.1]

We sample the range of Tab. 5 with the optimal design described in Appendix A. For each of the 33 points of the Nearly Orthogonal Latin Hypercubes (NOLH) scheme of Table A.1, we replicate 10 simulations over $T = 500$ periods, after an initial run of 200 periods.⁷

In order to proceed with the estimation of metamodels (step 2), we choose the following moments:

⁷The first 200 periods are discarded to get rid of transient dynamics that could introduce a bias in model statistics. Moreover, the long period of simulation does not represent a long-run analysis but a repeated business cycle analysis. In other words, we do not consider the presence of a trend in time-series by construction.

$$m(y) = \frac{1}{T-1} \sum_{t=1}^{T-1} \left[\log \left(\sum_{f=1}^N Y_f^{t+1} \right) - \log \left(\sum_{f=1}^N Y_f^t \right) \right] \quad (19)$$

$$v(y) = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T-1} \left[\log \left(\sum_{f=1}^N Y_f^{t+1} \right) - \log \left(\sum_{f=1}^N Y_f^t \right) \right]^2 - m(y)^2} \quad (20)$$

$$fb(y) = \frac{1}{TN} \sum_{t=1}^T \left[\sum_{f=1}^N \mathbf{1}(E_f^t < 0) \right] \quad (21)$$

where y is a vector of simulated data containing aggregate production values and the number of firm bankruptcies per period, and $\mathbf{1}(\cdot)$ is the indicator function which is used to count the number of bankrupt firms at t . Then the vector h of eq.(2) is specified as follows:

$$h(y) = (m(y), v(y), fb(y)) \quad (22)$$

The components of h are chosen among model outputs that are of interest from an economic viewpoint and at the same time are not highly correlated, in order to obtain independent equations. We exclude bank bankruptcies because, according to the available data, there were no events of this kind in Japan during the period under consideration (see Sec. 5) and the model is able to replicate this feature over all the parameter space of Tab. 5.

Since y depends on (θ, s) , the same holds true for the components of h . Taking this fact into account, we specify the right hand side of eq. (2) as follows:

$$m(\theta, s) = \beta'_m f(\theta) + u_s^m \quad (23)$$

$$\log(v(\theta, s)) = \beta'_v f(\theta) + u_s^v \quad (24)$$

$$fb(\theta, s) = \max\left(0, \beta'_{fb} f(\theta) + u_s^{fb}\right) \quad (25)$$

$$\theta = (r_{cb}, \delta, \mu) \quad (26)$$

$$f(\theta) = (1, r_{cb}, \delta, \mu, r_{cb}^2, \delta^2, \mu^2, r_{cb} \times \delta, r_{cb} \times \mu, \delta \times \mu) \quad (27)$$

where $\beta = (\beta_{fb}, \beta_m, \beta_v)$ is the matrix of coefficients which can be estimated equation by equation since we assume independence of errors across

(25)-(24). We use a logarithmic transformation of v in order to enforce the non-negativity constraint for this quantity. The higher order terms are introduced because we expect the ABM to display non linear behavior. In particular, we wish to capture the combined effect of parameters regulating credit cost (r_{cb}, δ) and demand conditions (μ) as explained in Sec. 2.

In a preliminary exercise, we compute a simple OLS regression with the specifications (23)-(24) for m and $\log(v)$, and we observe that the hypothesis of constant variance is rejected by the Breusch-Pagan test. Then we decide to estimate the same specifications with the following alternative assumptions: i) heteroskedasticity of errors, measured by the interquartile distance of simulated moments at fixed values of the parameters and accounted for by weighted least square regression (WLS); ii) heteroskedasticity and correlatedness of errors, represented by a set of Kriging correlation kernels as specified in Appendix B (K1-K5). Finally, fb is estimated by means of a Tobit regression in order to enforce the non-negativity constraint of this quantity.

The fitness of the alternative estimations of m and $\log(v)$ is computed by means of k -fold cross validation, i.e. the models are used to predict the response variables on k random sections of the experiment design after being estimated on the rest of it. In particular, we set $k = 5$. Fitness is compared through RMSE, MAE and Q^2 , which is a R^2 statistics computed out of sample (thus it can take negative values). The values of Tab. 6 are means over 100 replications of the procedure. We see that the weighted OLS regression always performs better than the alternative Kriging models.

With sensitivity analysis we can identify which parameters affect the components of h most. The general idea is that the variance of each of the latter is decomposed into additive terms which can be attributed to the variation of the parameter i in isolation (“main effect”) and to combined variations of parameters which include variations of i (“interactions”). Further details are provided in Appendix C.

From eqs. (C.1)-(C.3) we see that in order to compute S_i (main effect) and S_{I_i} (interaction) we need to know the conditional expectation of the components of h . We employ for this purpose the predictions obtained from the estimation of metamodels, which we denote generically as $\hat{h} = (\hat{m}, \hat{v}, \hat{fb})$. In this way we obtain the results of Fig. 5, from which we see that the variance of m and v is almost entirely dependent on the value of μ , i.e. on price shocks, although in the latter case there is a small role for credit costs which, as we see from panel (c), significantly affect the default rate of firms,

Table 6: Cross k -validation of Metamodels, $k = 5$

(a) m						
	WLS	K1	K2	K3	K4	K5
Q^2	0.9990 (0.0000)	0.9989 (0.0000)	0.9989 (0.0000)	0.9989 (0.0000)	0.9988 (0.0000)	0.9989 (0.0000)
RMSE	0.0008 (0.0000)	0.0009 (0.0000)	0.0009 (0.0000)	0.0009 (0.0000)	0.0010 (0.0000)	0.0009 (0.0000)
MAE	0.0005 (0.0000)	0.0006 (0.0000)	0.0006 (0.0000)	0.0006 (0.0000)	0.0006 (0.0000)	0.0006 (0.0000)
(b) $\log(v)$						
	WLS	K1	K2	K3	K4	K5
Q^2	0.9540 (0.0001)	0.8815 (0.0022)	0.8801 (0.0023)	0.8766 (0.0022)	0.8827 (0.0022)	0.8852 (0.0020)
RMSE	0.2416 (0.0003)	0.3795 (0.0034)	0.3816 (0.0035)	0.3878 (0.0034)	0.3776 (0.0034)	0.3746 (0.0031)
MAE	0.1721 (0.0001)	0.2727 (0.0021)	0.2705 (0.0022)	0.2755 (0.0020)	0.2687 (0.0021)	0.2660 (0.0020)

WLS = Weighted Least Squares; K1 = Kriging est., Matern(5/2); K2 = Kriging est., Matern(3/2); K3 = Kriging est., Gaussian; K4 = Kriging est., power-exponential; K5 = Kriging est., exponential. Standard errors in parentheses. For more details on Kriging regression see Appendix B.

mostly through the policy rate r_{cb} . This result is consistent with the fact that the replacement of bankrupt firms determines a discontinuity of the level of firm equity which affects the level of production through eq. (4). We see also that for fb the role of interaction effects is larger since a combination of low margins (i.e. low μ) and high cost of credit is required in order to push firms into loss according to eq. (14).

We are also interested to determine the sign of the effects of parameters on the components of h . In order to do so, we sample Θ again with a Latin Hypercube (LH) design of 1,000 data points and compute \hat{h} for each of them. The results that are shown in Fig. 6 vindicate those of sensitivity analysis, since the effect on both the conditional average (continuous line) and conditional variance (dashed line) of the components of h , estimated with a non parametric regression, are more pronounced when the weight of the same factors in Fig. 5 is larger. The direction of effects is consistent with expectations: the policy rate has a (weak) stabilizing effect at the cost of increasing firm bankruptcies; strong demand conditions (high μ) lead to strong aggregate growth and decreased volatility of production.

5 Model calibration

In this section we employ the metamodeling approach for the purpose of model calibration. The general approach we follow falls into the class of minimum distance techniques as defined by Grazzini and Richiardi (2015), although using metamodels in this context is relatively new to the ABM literature. In particular, we take the following steps: 1. compute a set of empirical moments \bar{h} from data which refer to the Japanese economy; 2. compute, within a given range, the value of a chosen loss function using the predictions derived from the metamodels \hat{h} and the empirical moments \bar{h} ; 3. identify the optimal values of the parameters θ^* , i.e. those that minimize the loss function; 4. compare the components of the simulated moment vectors $h(\theta^*, s)$ with the predictions of the metamodels $\hat{h}(\theta^*)$ and with the empirical moments \bar{h} . The purpose of step 2 is to verify that the parameters of the ABM are identified, i.e. that the loss function displays a single minimum in the chosen parameter range. The purpose of step 4 is twofold: i) to check if ABM simulations are able to replicate the empirical moments; ii) to check

Figure 5: Sensitivity Analysis

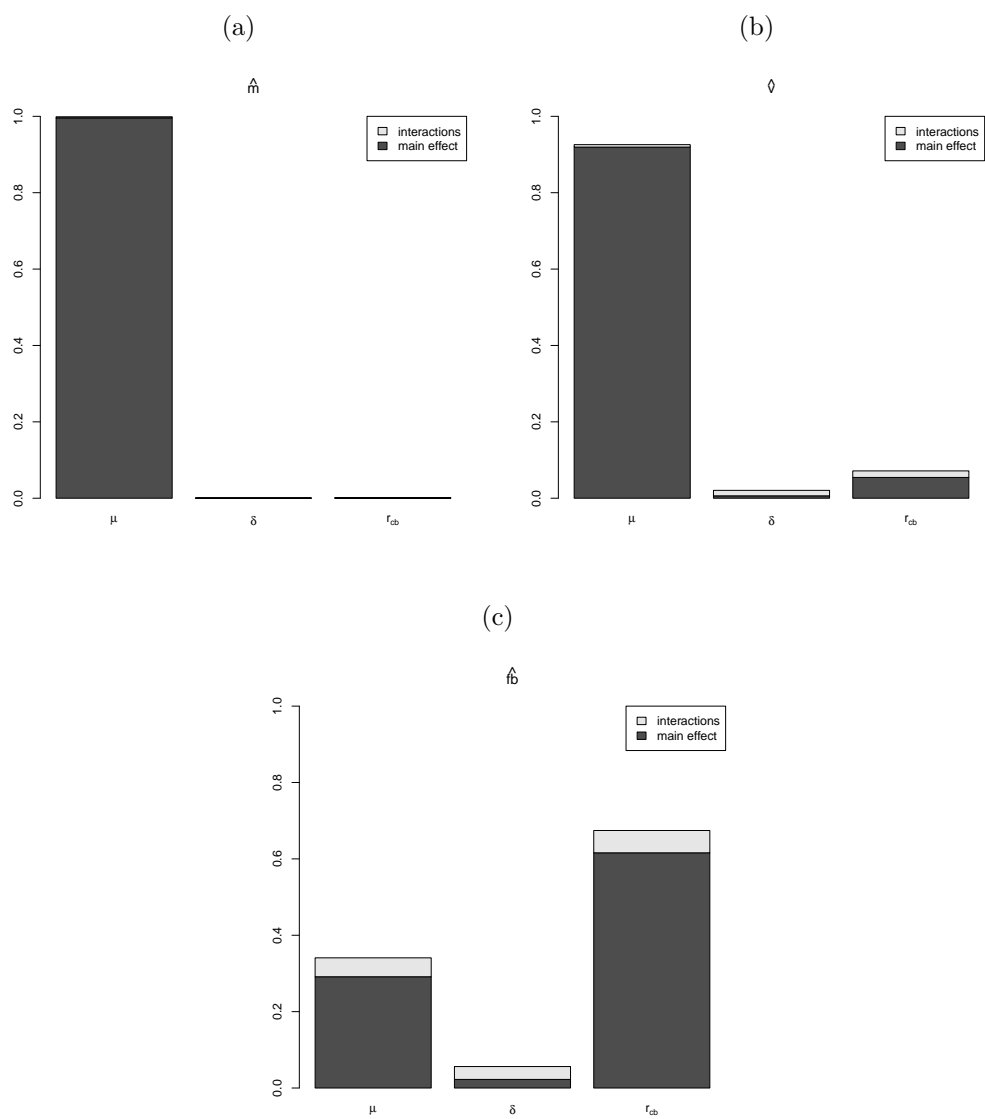
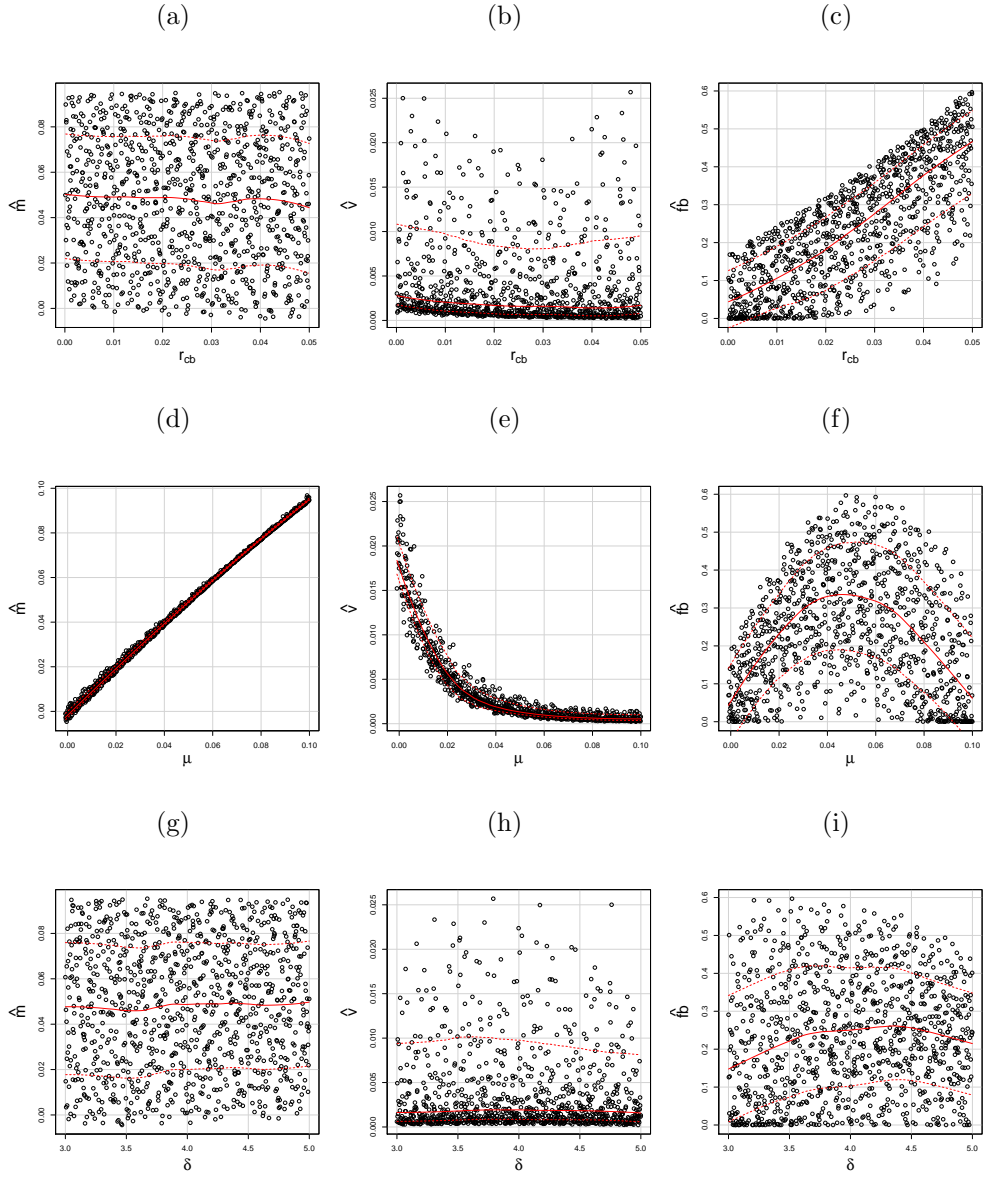


Figure 6: Global effect of parameters



that ABM simulations and metamodel predictions align in such a way that the latter can be used as a substitute for the former.

Regarding step 1, we look firstly at the index of monthly industrial production in Japan, provided by the Ministry of Economy, Trade and Industry ⁸. In particular, we compute the average and standard deviation of the time log-difference for the index in the period 01/2011 - 08/2016, which are respectively equal to $\bar{m} = 0.0015243$ and $\bar{v} = 0.0084397$. We choose this time span since we employ data referred to the first quarter of 2011 as initial conditions for the ABM simulations (see sec. 2). Secondly, we look at default rates. For this purpose we employ the information provided by Moody's concerning rated Japanese debt issuers in the period 1990-2013 ⁹. In this period the yearly default rates averaged at 0.2 % in the overall, while no bank defaults were recorded among this subset of companies. This reflects the general policy orientation of dealing with the ongoing banking crisis, which tried to avoid large bank failures. We opt to set the firm default rates at the observed monthly average of $\overline{fb} = 0.00166$. The set of empirical moments is thus given by the vector $\bar{h} = (\bar{m}, \bar{v}, \overline{fb})$.

We fix the policy rate at the average observed value in the period 01/2011 - 08/2016, which is equal to $\bar{r}_{cb} = 0.0006$ ¹⁰. We sample the remaining subspace of parameters $\theta = (\mu, \delta)$ by means of the efficient design of Tab. A.2 and simulate the ABM with the otherwise identical specifications of Sec. 4. We collect the simulated vector $h(\theta, s) = \{m(\theta, s), v(\theta, s), fb(\theta, s)\}$ and compute the metamodels following the same procedure of Sec. 4: $fb(\theta, s)$ is computed with a Tobit model which enforces the non negativity of the dependent variable; $\log(v(\theta, s))$ and $m(\theta, s)$ are computed through weighted least squares (WLS). All metamodels are specified as full second order polynomials

⁸See www.meti.go.jp/english/statistics/tyo/zenkatu/result-2.html

⁹https://www.moody.com/research/Moodys-Japanese-corporate-default-rates-remain-low-PR_310204

¹⁰See http://www.stat-search.boj.or.jp/ssi/html/nme_R031.27424.20161122195432.01.csv

in the parameters μ, δ . To summarize:

$$m(\theta, s) = \beta'_m f(\theta) + u_s^m \quad (28)$$

$$\log(v(\theta, s)) = \beta'_v f(\theta) + u_s^v \quad (29)$$

$$fb(\theta, s) = \max\left(0, \beta'_{fb} f(\theta) + u_s^{fb}\right) \quad (30)$$

$$\theta = (\delta, \mu) \quad (31)$$

$$f(\theta) = (1, \delta, \mu, \delta^2, \mu^2, \delta \times \mu) \quad (32)$$

From Fig. 7 we see that the correlation between simulated and predicted values is very high for m and fb .

Regarding Step 2, we choose a simple loss function in which we sum the Mean Absolute Percentage Error (MAPE) for each analyzed variable:

$$g(\theta, \bar{h}, N) = \left| (1/N) \sum_{s=1}^N \frac{m(\theta, s)}{\bar{m}} - 1 \right| + \left| (1/N) \sum_{s=1}^N \frac{fb(\theta, s)}{\bar{fb}} - 1 \right| + \left| (1/N) \sum_{s=1}^N \frac{v(\theta, s)}{\bar{v}} - 1 \right| \quad (33)$$

where N is the number of replications for each point of the design, which we set at $N = 10$ in our simulations. We take advantage of the metamodel estimation replacing g with a deterministic counterpart \hat{g} defined as follows

$$\hat{g}(\theta, \hat{h}) = \left| \frac{\hat{m}(\theta)}{\bar{m}} - 1 \right| + \left| \frac{\hat{fb}(\theta)}{\bar{fb}} - 1 \right| + \left| \frac{\hat{v}(\theta)}{\bar{v}} - 1 \right| \quad (34)$$

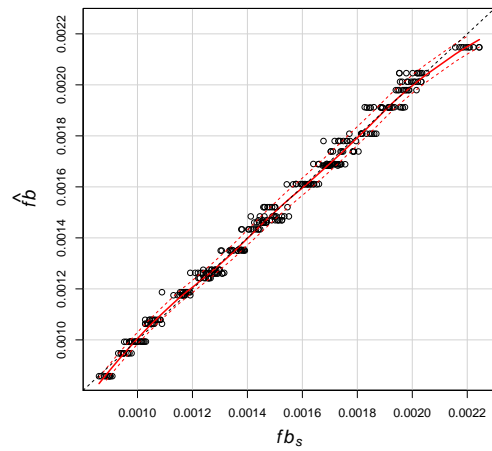
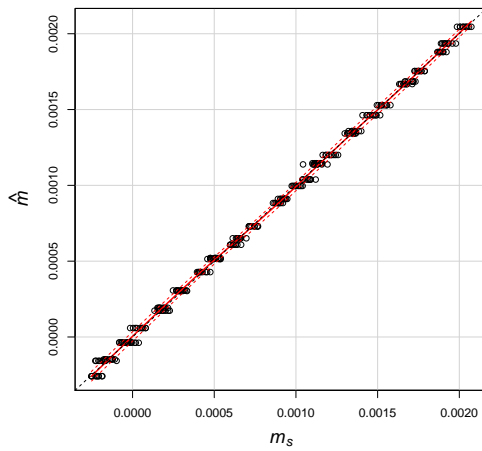
where $\hat{h} = (\hat{m}, \hat{fb}, \hat{v})$ are predictions obtained from the respective meta-models. From Fig. 8 we see that both parameters are well identified since \hat{g} has a single minimum in the chosen range, which is obtained for $\theta^* = (\mu^*, \delta^*) = (0.001655539, 4.75743086)$. In particular we obtain that $\hat{h}(\theta^*) = (0.00152310, 0.00166010, 0.01131811)$.

Finally, we compare the components of $h(\theta^*, s)$ with $s = 1, \dots, N$ and $N = 1,000$ with those of $\hat{h}(\theta^*)$ and of \bar{h} . From Fig. 9 we see that in all cases

Figure 7: Correlation between simulated (ABM) and predicted moments (metamodel). The red line is a non parametric kernel regression (colors online).

(a) $\rho(\hat{m}, m_s) = 0.998$

(b) $\rho(\hat{fb}, fb_s) = 0.990$



(c) $\rho(\hat{v}, v_s) = 0.716$

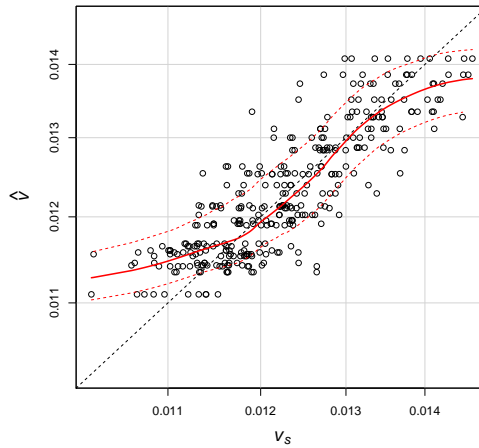
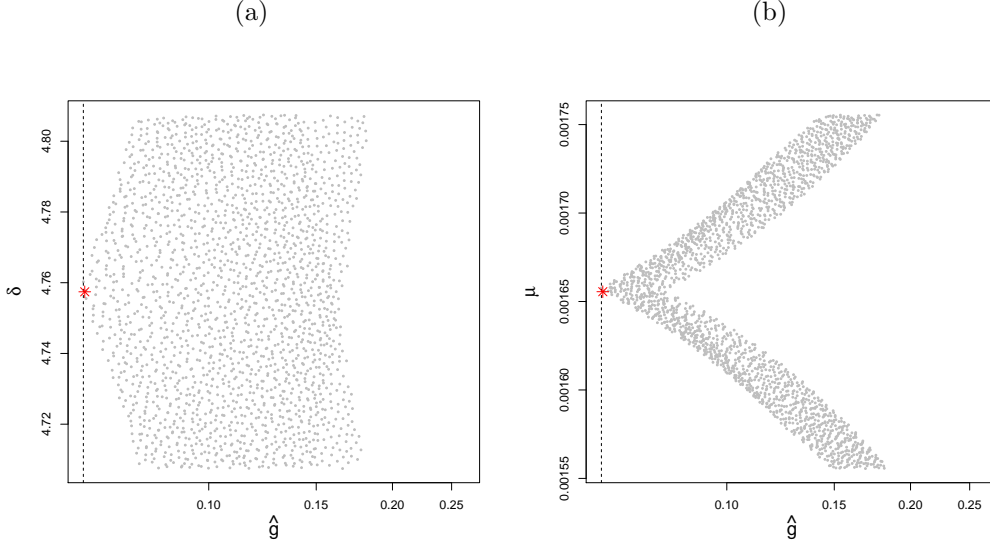


Figure 8: Minimization of \hat{g} and $\hat{g}(\theta^*, \bar{h}) = 0.06263874$ (red asterisk) with $\theta^* = (\mu^*, \delta^*) = (0.001655539, 4.75743086)$. Colors online.

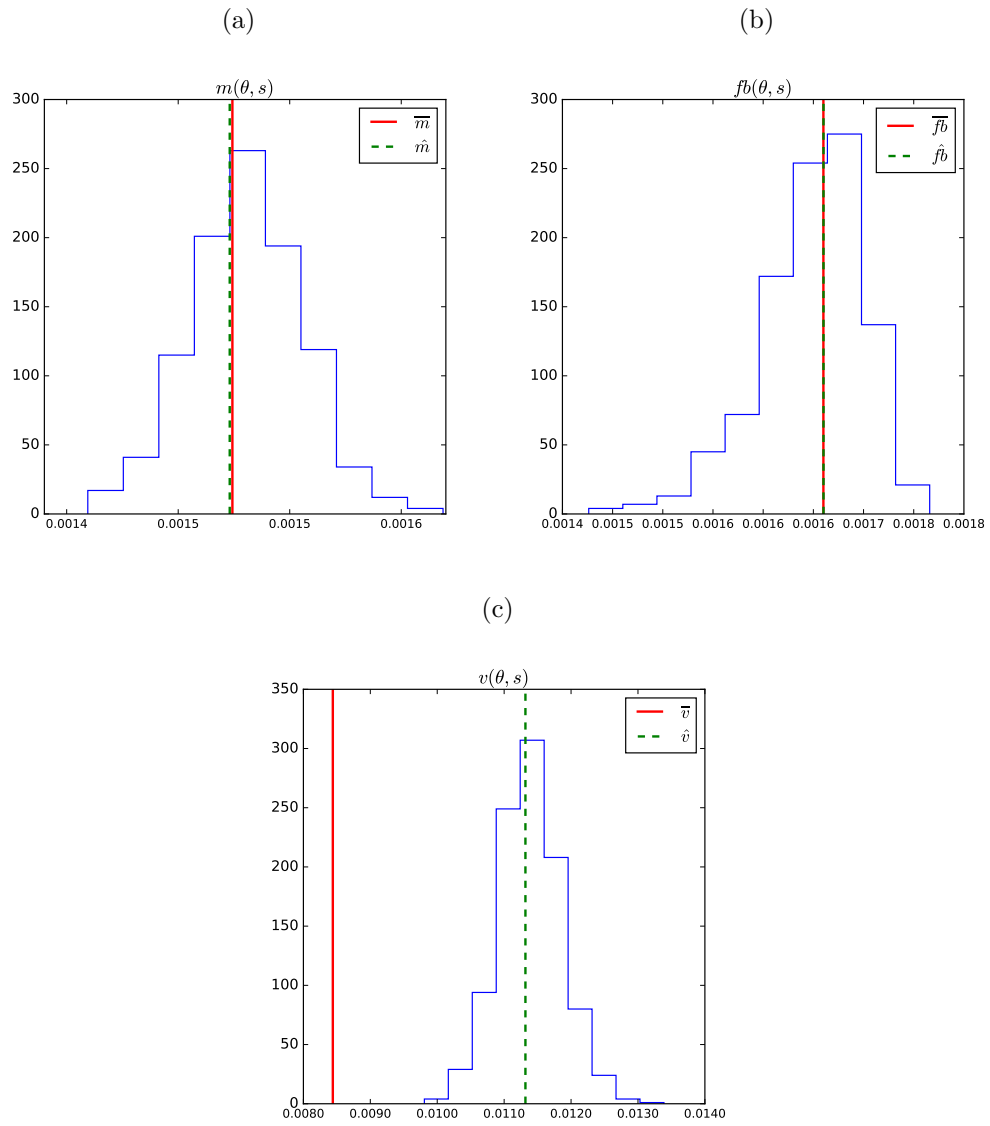


the predictions of the metamodels match the behavior of the ABM quite closely. Regarding the comparison with empirical moments, instead, we see that the ABM is able to replicate the value of two moments ($m(\theta, s), fb(\theta, s)$) out of three. Indeed, the ABM produces an excess of volatility compared to empirical data. Since the system (30)-(29) is overdetermined, this means that the restrictions over coefficients implied by the ABM are rejected from the empirical evidence. Given the simplicity of the model, a rejection is not unexpected. On the other hand, we prove that the methodology is indeed valuable since it provides a methodologically sound answer to the question “does the ABM replicate empirical data?”.

6 Conclusions

In this paper we extended the ABM of Riccetti *et al.* (2013) by allowing firms and banks to entertain multiple connections in a stylized credit market model. For this purpose we resorted to a random network model whose parameters

Figure 9: Comparison of the components of $h(\theta^*, s)$, $\hat{h}(\theta^*)$ and \bar{h} , with $s = 1, \dots, N$ and $N = 1,000$.



are calibrated with real data. The calibration of the credit network is set in order to reproduce the degree and strength (debt and loan) distributions of the Japanese credit market in 2011. At the same time, it allows to reduce the number of free parameters of the ABM, making it easier to systematically sample the parameter space with the efficient design proposed by Cioppa and Lucas (2007). Using simulated data we estimate and then compare a number of alternative statistical metamodels in order to select the best specification for the relationship between the ABM parameters θ and a set of simulated moments h . The selected metamodels, one for each component of h , allow to identify the effect of each parameter on the components of h , taking into account non-linearities and interaction effects.

Then we employ the metamodeling approach for the purpose of model calibration. In particular, we identify the optimal values of the parameters θ^* , i.e. those that minimize a loss function of choice \hat{g} which depends on the predictions of metamodels \hat{h} and on a set of empirical moments \bar{h} which refer to the Japanese economy. We compare the components of the simulated moment vectors $h(\theta^*, s)$ with the predictions of the metamodels $\hat{h}(\theta^*)$ and with the empirical moments \bar{h} . From this exercise we obtain on the one hand that the empirical evidence rejects the overidentifying restrictions implied by the ABM, on the other hand that the predictions of the metamodels match quite closely the behavior of the ABM. This shows that ABM simulations and metamodel predictions align in such a way that the latter can be used as a substitute for the former, which is the key point required for the application of the proposed calibration approach. Indeed, we prove that the methodology is valuable since it provides a clear-cut answer (although a negative one in this case) to the question “does the ABM replicate empirical data?”. Thus we believe that this approach, which does not impose restrictions on ABMs and is especially suited for large models with a high number of parameters thanks to the saving of computational time it allows, is of wide applicability in this field and makes it possible to obtain a strong insight into the characteristics of ABMs and their ability to replicate empirical data.

Some final considerations are necessary at this point. Aggregation is a long standing issue in economic theory. In the case of mainstream macro models, the gap between micro and macro is bridged by imposing ex ante strong theoretical restrictions, such as doing away with agent heterogeneity, which allow to derive a mathematical representation of the macro variables directly from the micro model. ABMs instead generally lack analytical solutions, and most ABM modelers are not in favor of selecting their assumptions

on the basis of analytical tractability.

As soon as heterogeneity is allowed for, the relationship between micro and macro variables becomes complex to handle, as underlined in many contributions (e.g. Kirman (1992), Stocker (1993) and Gallegati *et al.* (2006)). Our results show that we can bridge the gap between micro and macro variables through a rigorous statistical analysis of ABM simulations. Since we don't need to impose any a priori restrictions on ABMs apart from those required for the application of econometric techniques in general (like stationarity), the methodologies employed in this paper have a great flexibility and an extremely broad scope of application. This paper is a contribution aimed at explaining the potentiality of this approach, which is open to future applications in more complicated frameworks.

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Appendices

A Efficient sampling of Θ

In order to simulate the ABM, we have to specify the value of its parameter vector θ . In general, some parameters can be set at a value which comes from the literature, experimental studies and empirical data. For other parameters, we can define an appropriate range of variation and study the behavior of the model within that range using Montecarlo simulations.

Random sampling from a uniform distribution, which is a common choice in Montecarlo exercises, is inefficient because it generates a high number of redundant sampling points (points very close to each other), while leaving some parts of the parameter space unexplored. A common alternative is importance sampling, which however requires prior information. A proper “design of experiment” (DOE) delivers instead a parsimonious sample which is nevertheless representative of the parameter space. In particular, representative samples are said to be “space filling”, since they cover as uniformly as possible the domain of variation.

The sampling scheme we adopt for the subspace of free parameters $\theta = (r_{cb}, \delta, \mu)$, specified in Tab. 5, is the one suggested by Cioppa and Lucas (2007) and employed by Salle and Yildizoglu (2014). This scheme is based on Nearly Orthogonal Latin Hypercubes (NOLH). In the context of sampling theory, a square grid representing the location of sample points for a couple of parameters is a Latin square if there is only one sample point in each row and each column. A Latin hypercube is the generalization of this concept to an arbitrary number of dimensions, whereby each sample point is the only one in each axis-aligned hyperplane containing it. This property ensures that sample points are non collapsing, i.e. that the 1-dimensional projections of sample points along each axis are space filling. In fact, with this scheme, the sampled values of each parameter appear once and only once.

Basic Latin Hypercube schemes may display correlations between the columns of the $k \times n$ design matrix X , where k is the number of parameters and n is the sample size for each parameter, especially when k is lower but close to n . Instead, an orthogonal design is convenient because it gives uncorrelated estimates of the coefficients in linear regression models and improves the performance of statistical estimation in general. In practice, in orthogonal sampling, the sample space is divided into equally probable subspaces. All

sample points in the orthogonal LH scheme are then chosen simultaneously making sure that the total ensemble of sample points is a Latin Hypercube and that each subspace is sampled with the same density.

The NOLH scheme of Cioppa and Lucas (2007) improves the space filling properties of the resulting sample when $k \lesssim n$ at the cost of introducing a small maximal correlation of 0.03 between the columns of X . Furthermore, no assumptions regarding the homoskedasticity of errors or the shape of the response surface (like linearity) are required to obtain this scheme. The values of $\theta = (r_{cb}, \delta, \mu)$ obtained from this scheme and used for simulations of Sec. 4 are reported in Tab. A.1, while those employed for simulations of Sec. 5 are reported in Tab. A.2.

B Kriging regression

In the metamodeling selection exercise of Sec. 4, we estimate various Kriging models. These are generalized regression models, potentially allowing for heteroskedastic and correlated errors. The approach is widely used for ABM metamodeling in various fields (see e.g. Salle and Yildizoglu (2014), Dancik *et al.* (2010) and references therein). Using generalized regression is convenient since some of the parameters of our model are related to random distributions which naturally affect the variability of model output. The Kriging approach (Roustant *et al.*, 2012) resorts to feasible generalized least squares by assuming a stationary correlation kernel $K(h) = K(\theta_i - \theta_j)$, where θ_i, θ_j are distinct points in the parameter space Θ . $K(h)$ takes the following general form:

$$K(h) = \prod_{j=1}^d g(h_j, \lambda_j) \tag{B.1}$$

where d is the dimension of Θ , and $\lambda = (\lambda_1, \dots, \lambda_d)$ is a vector of parameters to be determined. In particular, we employ for g the specifications of Tab. B.1.

Since we work with noisy, potentially heteroskedastic observations, in our estimation the covariance matrix of residuals is determined as follows:

$$C = \sigma^2 R + \text{diag}(\tau) \tag{B.2}$$

Table A.1: Design of Experiment (DOE) for simulations of Sec. 4.

δ	r_{cb}	μ
3.0000	0.0406	0.0116
3.0625	0.0360	0.0558
3.1250	0.0391	0.0306
3.1875	0.0313	0.0527
3.2500	0.0422	0.0968
3.3125	0.0063	0.0842
3.3750	0.0001	0.0179
3.4375	0.0032	0.0243
3.5000	0.0157	0.0779
3.5625	0.0017	0.0653
3.6250	0.0375	0.1000
3.6875	0.0235	0.0274
3.7500	0.0048	0.0621
3.8125	0.0219	0.0400
3.8750	0.0297	0.0053
3.9375	0.0328	0.0085
4.0000	0.0251	0.0495
4.0625	0.0173	0.0905
4.1250	0.0204	0.0937
4.1875	0.0282	0.0590
4.2500	0.0453	0.0369
4.3125	0.0266	0.0716
4.3750	0.0126	-0.0010
4.4375	0.0484	0.0337
4.5000	0.0344	0.0211
4.5625	0.0469	0.0748
4.6250	0.0500	0.0811
4.6875	0.0438	0.0148
4.7500	0.0079	0.0022
4.8125	0.0188	0.0463
4.8750	0.0110	0.0684
4.9375	0.0141	0.0432
5.0000	0.0095	0.0874

Table A.2: Design of Experiment (DOE) for simulations of Sec. 5.

δ	μ
3.0000	0.0003
3.0625	0.0012
3.1250	0.0007
3.1875	0.0011
3.2500	0.0019
3.3125	0.0017
3.3750	0.0005
3.4375	0.0006
3.5000	0.0016
3.5625	0.0013
3.6250	0.0020
3.6875	0.0006
3.7500	0.0013
3.8125	0.0009
3.8750	0.0002
3.9375	0.0003
4.0000	0.0011
4.0625	0.0018
4.1250	0.0019
4.1875	0.0012
4.2500	0.0008
4.3125	0.0015
4.3750	0.0001
4.4375	0.0008
4.5000	0.0005
4.5625	0.0015
4.6250	0.0016
4.6875	0.0004
4.7500	0.0002
4.8125	0.0010
4.8750	0.0014
4.9375	0.0009
5.0000	0.0018

Table B.1: Correlation kernels (Roustant *et al.*, 2012)

K1	Matérn ($\nu = 5/2$)	$g(h) = \left(1 + \frac{\sqrt{5} h }{\lambda} + \frac{5h^2}{3\lambda^2}\right) \exp\left(-\frac{\sqrt{5} h }{\lambda}\right)$
K2	Matérn ($\nu = 3/2$)	$g(h) = \left(1 + \frac{\sqrt{3} h }{\lambda}\right) \exp\left(-\frac{\sqrt{3} h }{\lambda}\right)$
K3	Gaussian	$g(h) = \exp\left(-\frac{h^2}{2\lambda^2}\right)$
K4	Power-Exponential	$g(h) = \exp\left(-\left(\frac{ h }{\lambda}\right)^t\right)$
K5	Exponential	$g(h) = \exp\left(-\frac{ h }{\lambda}\right)$

where R is the correlation matrix with elements $R_{ij} = K(\theta_i - \theta_j)$ and $\tau = (\tau_1^2, \dots, \tau_n^2)$ is the vector containing the observed variance of model output at fixed points of the parameter space and n is the size of the NOLH design. ML estimation is performed on the “concentrated” multivariate Gaussian log-likelihood, obtained by substituting the vector of regression coefficients with their generalized least square estimator. The “concentrated” log-likelihood is a function of σ and λ , which are the optimization variables of the estimation. The solution is obtained numerically through the quasi-Newton algorithm provided by the DiceKriging R (2015) package (Roustant *et al.*, 2012).

C Sensitivity analysis

Campolongo *et al.* (2000) define sensitivity analysis (SA) as the study of how uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input. In this respect, SA techniques should satisfy the two main requirements of being global and model free. By global, one means that SA must take into consideration the entire joint distribution of parameters. Global methods are opposed to local methods, which take into consideration the variation of one parameter at a time, e.g. by computing marginal effects of each parameter. By model independent, one means that no assumptions on the model functional relationship with its inputs, such as linearity, are required.

Campolongo *et al.* (2000) propose a global approach based on the decomposition of variance:

$$\begin{aligned}
V(h) &= \sum_i^k V_i + \sum_{i<j} V_{ij} + \sum_{i<j<m} V_{ijm} + \cdots + V_{12\dots k} \\
V_i &= \mathbb{V}_{\theta_i} [\mathbb{E}_{\theta_{-i}}(h|\theta_i = x)] \\
V_{ij} &= \mathbb{V}_{\theta_{(i,j)}} [\mathbb{E}_{\theta_{-(i,j)}}(h|\theta_i = x, \theta_j = y)] - V_i - V_j \\
&\dots
\end{aligned}$$

where h is a generic vector of moments. We see that V_i represents the variance of the main effect of parameter i , while all the other terms are related to interaction effects. From this general formula we can obtain the contribution of interaction effects S_{Ii} involving the parameter θ_i as follows:

$$S_{Ii} = S_{Ti} - S_i \tag{C.1}$$

$$S_i = \frac{V_i}{V} \tag{C.2}$$

$$S_{Ti} = \frac{\mathbb{E}_{\theta_{-i}} [\mathbb{V}_{\theta_i}(h|\theta_{-i})]}{V} = 1 - \frac{\mathbb{V}_{\theta_{-i}} [\mathbb{E}_{\theta_i}(h|\theta_{-i})]}{V} = 1 - \frac{V_{-i}}{V} \tag{C.3}$$

The multidimensional integral of the last line can be evaluated numerically using the extended FAST method described in Campolongo *et al.* (2000). The results of Fig. 5 show, for each parameter in θ , the main effect (C.2) and the interaction effect (C.1) on the components of $h = (m, v, fb)$.