

# Latent ties identification in inter-firms social networks

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**Abstract** Social networks are usually analyzed through manifest variables. However there are social latent aspects that strongly qualify the networks. This paper aims to propose a statistical methodology to identify latent variable in inter-firm social networks. A multidimensional scaling technique is proposed to measure this latent variable as a combination of an appropriate set of two or more manifest relational aspects. This method, tested on an inter-firm social network in the Marche region, is a new way to grasp social aspect with quantitative tools that could be implemented under several different conditions, using also other variables.

## 1 Introduction

According to Grandori and Soda [4], an inter-firm network can be defined as “a mode of regulating interdependence between firms, which is different from the aggregation of these units within a single firm and from coordination through market signals, and which is based on a cooperative game with partner-specific communication”. Particularly, social inter firm networks can be defined as social relations with no agreement. The first form of social networking is a personal network linking firms through contacts among their entrepreneurs and managers.

Members of inter-firm social networks aim to sharing core values, knowledge, and in some cases they also can share a vision. Topics like social capital, social cohesiveness, and the social embeddedness have a strong social component, that is not directly measurable but that can be discovered through the observation of their manifest variables.

In this paper we assume that the unobservable or latent variable can be defined as “basic social relational embeddedness”, and it is obtained as a function of the three

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chosen manifest variables. The main social characteristics of network ties are latent variables, as well as they just depend not on one manifest variable, but on an appropriate combination of two or more of them. Usually, social actors are heavily dependent on both their social structure [3, 5] and their motivation to engage in information exchange [11]. These two aspects represent the strength of the tie [5].

In our study, we propose to identify the basic relational embeddedness, that is a determinant component of the strength of tie and captures the quality of dyadic exchanges and the behaviour exchanges that parties exhibit, such as trust, confiding, and information sharing [6].

This paper focuses on a set of three manifest variables, that are significant to approach the study of the main relevant characteristics of the ties in social networks. These variables are *trust*, *frequency of exchanges among parties*, and *reciprocity*. Many scholars indicate trust as a fundamental ingredient for every typology of network or alliance and as a facilitator of knowledge transfer [9]. To have trust in relationships, it is necessary to build it in daily behaviour [7]. The frequency concerns how often parties have exchanges with each other [6]. Reciprocity “transforms a unilateral supply relationship into a bilateral one” and creates the perception of a similar “destiny” with greater “mutual interest” [6].

The paper is organised as follows: after this introduction, the presentation of the social inter-firm network model, then section three shortly describes the statistical methodology and, finally, section four illustrates a simple example.

## 2 Identification of the unique social-relational variable

In behavioural sciences, researchers are very familiar with the idea of “*latent variable*”. Latent variables refer to concepts that cannot be directly measured and are opposed to the observable variables. However, latent variables may be defined on the basis of properly identified sets of observable variables, which are also called *manifest variables* [1].

This paper introduces the idea of *latent tie* in the social network analysis. We assume that latent ties cannot be directly observed but they can be defined through combinations of appropriate sets of manifest relational aspects. This condition may occur in the case of networks that are based on social aspects. The tie is defined on the basis of a weighted sum of the manifest variables. Moreover, we assume that network membership evaluations can be registered in terms of proximity measures ([dis]similarities).

More formally we define the following data structure. Given a set of  $n$  statistical units and  $K$  manifest variables, the notation  $\delta_{ijk}$  indicates the general proximity measure between the statistical units  $i$  and  $j$  for the variable, which are arranged in  $K$  asymmetric  $n \times n$  matrices  $\{\Delta_1, \Delta_2, \dots, \Delta_k, \dots, \Delta_K\}$ , where  $(i, j) = 1, \dots, n$ . We remember that  $\delta_{iik} = \max \forall k$  (diagonal elements), and that  $\delta_{ijk} \geq 0$  and the value 0 indicates absence of any relationship, by definition.

The final aim of the paper is to identify and visualise latent ties of inter-firm social

networks; more specifically the problem consists of finding a good approximation of the 3-way proximity matrix by a  $n \times n$  distance matrix.

### 3 MDS for latent ties identification

This section shortly presents the MultiDimensional Scaling (MDS) basic principles and then it motivates the choice of the PROXSCAL model with respect to other scaling models. For sake of space we cannot go in the PROXSCAL details; interested readers can refer to Borg and Groenen [2] for the MDS foundations and to [8] for the PROXSCAL model, more specifically.

The MDS aims at finding a configuration of  $n$  points into a  $p$  dimensional space. Generally,  $p$  is set equal to 2 in order to get graphic representation of the  $n$  points. The scaling transformation of the proximity matrix has two advantages: *i*) to summarise the different proximity measures into one single distance; *ii*) to permit graphical representations of the distances into 2 and 3 dimensional spaces. Moreover, it is worth noticing that there are many graphical approaches to display the network, if the connection strengths are transformed into metric measures.

In the simplest two dimensional case, where  $K = 1$ , the MDS aims at defining a function  $\varphi(\cdot)$  such that:

$$\varphi(\delta_{ij}) = d(x_i, x_j) + \varepsilon_{ij} \quad (1)$$

In other words,  $\varphi(\cdot)$  indicates a function that maps the dissimilarities  $\delta(ij)$  into a metric space, where the distance  $d(\cdot)$  (generally the Euclidean distance) is defined. The quantity  $\varepsilon_{ij}$  indicates the residual.

Some alternative models have been proposed to deal with 3Way data structures. In this paper we refer to the INDividual SCALing (INDSCAL) model, and more specifically to the PROXSCAL algorithm for 3Way dis(similarities) data structures. INDSCAL model is also referred to as weighted Euclidean model. Starting from  $K$  proximity matrices  $\Delta_1, \Delta_2, \dots, \Delta_k, \dots, \Delta_K$ , INDSCAL minimises the Stress (squared Euclidean distances) defined by the following equation:

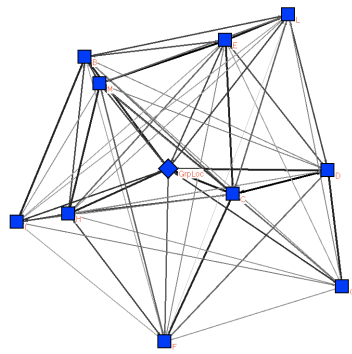
$$\sigma(\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_k, \dots, \mathbf{X}_K) = \sum_k \sum_{i < j} (\delta_{ijk} - d_{ij}(\mathbf{G}\mathbf{W}_k))^2, \quad (2)$$

where  $\mathbf{G}$  indicates the common space  $n \times n$  matrix and  $\mathbf{W}_k$  represents the generic weighting matrix. The point coordinates are then defined as  $\mathbf{X}_k = \mathbf{G}\mathbf{W}_k$  and the scaled distances as  $d_{ij}(\mathbf{X}_k)$ .

The main point in favour of PROXSCAL is that it works on the Euclidean distance and not on the squared distances. Avoiding the square transformation, it prevents putting more emphasis on large dissimilarities. The second, but not less important point, is the possibility to consider asymmetric (dis)similarity matrices. However, it is worth noticing that (dis)similarity matrices are transformed in symmetric ones. Alternative scaling models could be taken into account, and this surely represents a research direction for future works.

## 4 Empirical evidence and conclusive remarks

This section shortly presents the output obtained on a small real dataset. The data refer to a network of eleven firms in the Jesi (in the Ancona administrative district).



The variables taken into account refer to the *basic social embeddedness*, and they are: *Trust*, *Frequency of Exchanges Among Parties*, and *Reciprocity*. The same questionnaire was given to the eleven firms of the network in the case study; using a scale  $\{0, 2, \dots, 5\}$ , they have expressed their evaluation about the other firms, with respect to the three variables. By definition we assumed that the self evaluation is equal to 5. Network display has been obtained using Pajek<sup>©</sup>. Vertices of the net are positioned

into the 2D space according to the coordinates obtained by the MDS: this means that the distances among vertices can be interpreted in terms of reciprocal “*social closeness*”; at the same time the arcs thickness represents the basic social embeddedness as synthesis of the three considered variables.

The individual scaling model fits well to the data (standardised stress is equal 0.0223), however it reveals that the three variables have almost the same weight.

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