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Chapter 10

Firms Clustering In Presence Of Technological Renewal Processes

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Abstract

This chapter aims at exploring companies' profit maximization in presence of a hierarchical organization among firms and when technological renewal processes take place. The introduction of a hierarchical structure among firms allows us to describe the reality of the industrial districts. In this respect, some policies for the management of the renewal process in the district are derived.

Keywords: Technological renewal, Aggregate productivity, Firms size.

10.1 Introduction

The role of technology in firms is continuously growing, and competitive small as well as big firms must adequate their technology in order to survive on the market. An evident example is about the speed of the need of renewal of technology connected to the computer science: also small firms need the use of at least a computer and the use of new software requires the renewal of hardware. Even jobs based on human skills that improve through

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time, like law matter, can't avoid the retrieval of information through fast computer, fast database, and fast communication systems. Investments in technology renewal are higher in the case of industrial plants and in their conversion for supporting new technological processes. The need of substantial up front investment of capital may force to delay the technological upgrade.
In this chapter we construct a very simple model to merge the analysis of firms clustering and hierarchical organization, referring to the case of the industrial districts and the importance of technological renewal in the industrial management strategies. In particular, we propose the analysis of the optimal renewal time leading to the maximum profit, in the context of a network of companies.
The aggregated profit of the district has been obtained by using as unit measures the growth rates and the sizes of the companies. Therefore, an analysis of some results on the distribution of the firms depending on their size and growth rate is also presented. In particular, the relationships between the distribution of the firms and the internal structure of the district is evidenced. The next section reviews empirical results on the distribution of firms depending on their size and growth rate. In section 3, the hierarchical structure of the district is described. Section 4 and 5 analyze, respectively, the size of the district and the costs of technological renewal, section 6 is devoted to the analysis of the profit of the district. In the last section some conclusions and suggestions for economic policies are proposed.

10.2 Distribution of Firms Depending on Their Size and Growth Rate

The studies about size and growth rate of firms differ for the hypotheses tested and for the data sets that were used. Most literature studies analyze data got from Census and COMPUSTAT data bases. Census data give more information about small firms, that are crucial for the understanding the impact of social dynamics at the individual level. The volume of sales is used as a proxy for firm size, and in some studies other fundamental variables like as total assets, sales and the number of employees are used as a complementary variable so as to check the validity and robustness of the results.
The discussion is not purely academic exercise. Right skewness implies that the biggest number of firms has a size just below the mean size, while there are a few huge, and some more very small. The detection of the proper distribution allows to explain differences of reaction of the market to external shocks, like as natural catastrophes, or the impact on some economy of exogenous economic factors. This kind of study can help both for driving the best policies for economy development and for detecting the maximum charge of bad events (taxes, wars, natural catastrophes etc.) that can be beared without a complete crash.

Several researches have been performed about the detection of skew distribution of firms, depending on their growth rates. Firms sizes in industrial countries are highly skew, such that small numbers of large firms coexist alongside larger numbers of smaller firms. On some data set, skewness has been shown to be robust over time [1]. It has even survived large-scale demographic transitions within work forces and widespread technological change. Finer analyses have shown skewness to grow during growing phases of the economy and to decrease during recessions [2], thus being an indicator of such economic cycles.

A characteristic that emerges is that although the position of individual firms in a size distribution does depend on the definition of size, the shape of the distribution does not. The main concern is to select the best fit to data histograms.

The pioneer work in the direction of the analysis of the distribution of the companies w.r.t. size and growth rate is [3], where Gibrat's law has been formalized. The law of Gibrat assumes three key hypotheses: firstly, the growth rate of a company is independent of its size; secondly, the successive growth rates of a firm are uncorrelated in time; thirdly, there is a lack of interaction among companies.

By a mathematical point of view, consider $t,u \geq 0$ and let S_t be the size of a firm at time t. For each firm Gibrat's law states that

$$S_{t+u} = S_t(1+g_u) \tag{10.1}$$

where g_u is the rate of growth, and it is assumed to be an i.i.d stochastic process with bounded distribution and small variance (usually assumed to be Gaussian).

Assume now $t = 0$. Taking the logarithm of both sides of (10.1) and solving back recursively, it is straightforward to obtain

$$log(S_u) = \sum_{i=1}^{u} log(1+g_i) + log(S_0).$$

Formula (10.1) and the related assumptions stated in Gibrat's law have two main consequences: first of all, for u large enough, the growth rate $g_u = S_{t+u}/S_t - 1$ are log-normally distributed; secondly, if the companies have the same initial size and are born at the same time, then also the distribution of firm size is log-normal.

Some econometric analyses support Gibrat's law.

In [4, 5], the authors inquiry the independence between growth rate and size. In [4] it is shown that the lognormal distribution hypothesis holds for UK firms larger than eight employers. Later, the same authors report that the size of the distribution of UK companies is shown is close to the lognormal, although the hypothesis of lognormality can be rejected statistically [5]. Other studies report that the fit of the log-normal distribution to size data is good close to the mean, but it performs less on the tails, and families of functions that include as a particular case the log-normal and that take into account a power-law decay of tails in the general case have been developed. The goodness of statistical fit allows for some compromise: Gibrat's law has been shown to be compatible with power law under further hypotheses. As an example, in Simon's model [6] Gibrat's law is combined with an entry process to obtain a Levy distribution for firm's size.

A different part of the literature on firms size aims at showing the limits of the Gibrat model, and new growth rates and firms size distribution are proposed by fitting data. Amongst the others, some evidence against Gibrat's assumptions can be found in [7], where it is shown that large firms are more diversified and the growth rate fluctuations decreases w.r.t. size. This result has been confirmed in [8–13]. Moreover, it is well established the presence of interactions among companies.

In a series of recent papers the power law for firm size and Laplace law for firms growth rates is proposed (see [6]). In this respect, let us consider S_0 as the size at time 0 and S_1 the size at time 1. From (10.1) the growth rate is $g_1 = \frac{S_1}{S_0} - 1$. Using the approximation

$log(x+1) \approx x$ we have that

$$g_1 = \frac{S_1}{S_0} - 1 \approx log\left(\frac{S_1}{S_0}\right) \approx log(S_1) - log(S_0). \tag{10.2}$$

It can be shown that the logarithm of a Pareto random variable follows an exponential distribution, and that the difference of two exponential random variables obeys a Laplace distribution. The first result can be proved considering the monotonic property of the logarithmic function and the rule of transformation of random variables. Assuming that X follows a Pareto distribution with parameter α it is possible to derive the probability distribution of $Y = log(X)$:

$$Pr(Y \geq y) = Pr(log(X) \geq y) = Pr(X \geq exp(y)) \propto (exp(y))^{-\alpha} = exp(-\alpha y) \tag{10.3}$$

that is an exponential distribution with parameter α [6]. The second result uses more sophisticated mathematical methods, and it is not reported here. Due to this relationship, the Laplace law for firms growth rate should be a consequence of power law for firm size and not succeed from log-normal one, thus invalidating the Gibrat hypotheses. Studies on the Laplace law for firms growth rate then give indirectly results on the Pareto law for firms sizes. Therefore, literature focuses at most on the Pareto distribution as an alternative to log-normal distribution for firms size. In particular, computer aided simulations of economic systems show that in the case of log-normally distributed data shocks are absorbed, whilst in the case of Pareto distribution correlation internal to the system can amplify the external shocks leading to strong oscillations of the entire system and risking the collapse [6].

The power law behavior seems to be common also to parameters that involve the most industrialized countries. The results reported in [14] can be interpreted as the existence of a significant range of the world GDP distribution where countries share a common, size-independent average growth rate. Further particular hypotheses like entry and exit of companies from the market give results that contradicts the Gibrat's law. As an example in [13,15,16] the exponential distribution for the growth rate of firms has been found to hold for the 20 years 1974-1993 of COMPUSTAT publicy-traded United States manufacturing firms, whilst the variance of the growth rate should grow with the size of the firm. More specifically, the size S_0 of the firm at the time 0 is measured through the sales. Since the law of proportionate effect implies a multiplicative process for the growth of companies, it is defined $s_0 = log(S_0)$. Rather than a gaussian, as expected from Gibrat's law, the best fit procedure provides an exponential distribution:

$$p(r_1 \mid s_0) = \frac{1}{\sqrt{2}\sigma_1(s_0)} exp\left(-\frac{\sqrt{2} \mid r_1 - \bar{r}_1(s_0) \mid}{\sigma_1(s_0)}\right), \text{ where } r_1 = log\left(\frac{S_1 - S_0}{S_0}\right) \tag{10.4}$$

and $\sigma_1(S_0) \sim S_0^{-\beta}$, σ_1 is the size of the standard deviation of (10.4). The estimated value of β is $\beta = 0.20 \pm 0.03$ [16], that is similar to $\beta = 0.18 \pm 0.03$, obtained when the size of the firms is measured by using as a proxy the number of employers [13]. We point out that (10.4) can be used for a uniperiodal model, from 0 to $t \neq 1$. We will prefer this prospective to conduct our discussion, as we shall see.

To sum up, the presence of right skewness supports both the Gibrat's law and the Pareto distribution. Particular assumptions like the one of the validity of the detailed balance, that states the time-reversal symmetry for the growth rate, show that Gibrat's law and Pareto-Zipf's law hold for firms bigger than a fixed threshold [17]. This property is not valid in general [18], but the behavior of biggest companies is important because determines the most part of economy. Therefore, such kind of analysis is useful for driving economic policies at the Country level.

Districts constitute small worlds with a prevalence of small sized industries, so policies for district developments will be different from those based on the common behavior of big firms, and need a finer analysis. In this chapter, we deal with the analysis of the optimal profit of an industrial district. At this aim, we refer to S_t as the aggregate size of the district at time t.

10.3 A Hierarchical Organization of Firms

The present section shows a model for a management structure of the district. Such model depends on parameters that allow to describe the entire range of organizations, from the complete anarchy of units composing the system to their perfect control. The aim is twofold: by one side the most suitable parameters for the economic system under observation are discussed; on the other side this approach allows to consider the variation of parameters that improve the profit. Moreover to enter in managing details opens the way to the understanding of the process of the spreading of the renewal of technology giving an insight in the system, instead that considering it as a black box and observing the results only at the external level.

Let us consider the district to be composed by N firms. Following the approach outlined in [16] we model the internal managerial organization of the district through a hierarchical managing tree. For simplicity of modeling we suppose that the tree has n levels and that at every level other than the lowest one each node is connected exactly to z units in the next lowest level. Therefore $N = z^n$. Managers in the hierarchy of firms share common production targets, but they also maintain individual freedom in management units. This implies, that decisions of the head of the hierarchy are propagated through the tree, but followed with a probability Π, and disobeyed with probability $(1-\Pi)$. Reasons of deviation from the main district policy encompass the entire range of evaluation of projects, from subjective evaluation of managers, to budget constraint.

This type of system description is very flexible, and the distance between Π and $1/2$ provides a measure of the organization of the district. More precisely, if $z = N$ and $\Pi = 1/2$ then there are no levels, and random disobedience, so the system is completely anarchic and not organized. If $z = 1$ and $\Pi = 1$ ($\Pi = 0$) then managers' decisions are systematically taken for good (disobeyed), and the system is deterministically ordered.

The organization structure of the district mirrors on its size, since the total sales of a district is given by the sum of the sales of the companies composing it. In this respect, as we shall see in details in next section, we are interested at examining the amount of district size S_t at time $t > 0$ and at comparing it to the initial district size S_0, also by taking into account the presence of a technological renewal in the economic environment.

Considering the entire district as a unique, composed structure of size S_0 means that its growth rate is expected to be modeled by (10.4).

Therefore, [16] suggests that a deeper analysis can be made on the contribution given to the growth rate by each unit composing the district through the decision of the managing organization. Indeed, parameters Π and z describe the hierarchical structure of organization, and they can be calibrated in order to achieve composition of management hierarchy that allows the district to access the same growth rates as it were a unique big firm. More precisely, [16] shows that the parameter β related to the variance of growth rate can be written as follows:

$$\beta = \begin{cases} -\ln \Pi / \ln z & \text{if } \Pi > z^{-1/2} \\ 1/2 & \text{if } \Pi < z^{-1/2} \end{cases} \tag{10.5}$$

The calibration of Π and z allows to calibrate β, in order to reach a fixed economic target. We stress that (10.4) and (10.5) imply that the variance is co-monotone with Π, in accord with the very agreeable remark that the most the control is direct, the most the growth is uniform.

10.4 Variation of the Size in Technological Improvement

We denote as ξ_i the contribution of the size of company i to the total ssize of the district at time 0. Let

$$\bar{\xi} := \frac{1}{N} \sum_{i=1}^{N} \xi_i$$

be the mean size at time 0 of the companies composing the district. The total size of the district is

$$S_0 = \sum_{i=1}^{N} \xi_i = N\bar{\xi}.$$

We suppose that the firms output depends linearly on the time. This assumption allows to model industrial processes that never stop, even it does not take into account some interesting aspects, like the seasonality properties of some production processes, as it is mandatory in agricultural sectors. We are conscious of the limits imposed by this restriction, made to obtain a less cumbersome algebra, and the analysis of a more general firms output evolution rule is already in our research agenda.

Furthermore, in our model it is assumed the existence of a date $T > 0$ where each firm of the district must take a position with respect to the adoption of a new technology. We also suppose that the old technology is abandoned by the district when the new one is adopted, at time T.

The introduction of a new technology changes naturally the productivity of a district, and the contribution of firm i to the size variation of the district is denoted with δ_i. The change δ_i can be either positive or negative. In fact some firms can become bigger, as an example due to cost/quality ratio improvement, but this can also mean that some firms satellite of the main production could bear some reduction. Therefore, $\delta_i \in \mathbf{R}$. The total size of firm i at time t is:

$$r_i(t) = \xi_i t 1_{\{t \le T\}} + [\xi_i t + \delta_i (t - T)] 1_{\{t > T\}}, \tag{10.6}$$

and the evolution of the size of the district is then

$$S_t = \begin{cases} S_0 t, & \text{for } t \le T; \\ S_0 t + \sum_{i=1}^{N} \delta_i (t-T), & \text{for } t > T. \end{cases} \tag{10.7}$$

10.5 Costs of Renewal of Technology

In order to model the costs of renewal we follow the approach purposed in [19]. We measure the size as sales, and denote as C is the district marginal cost for producing a unit of output, and introduce the parameter $0 < \alpha \le 1$, that is a measure of the learning rate of the district. In particular the production costs for each unit of sales, free from renewal of technology, is given by $C\alpha^t$. Further, we denote as $\rho \in [0,1]$ a parameter that indicates how well the marketplace reduces the manifacturing costs. We assume $\rho < \alpha$, since the technological environments go faster than the internal learning.

The unitary production costs for each firm before and after the innovation date T can be resumed by

$$k(t) = \begin{cases} C\alpha^t & t < T \\ \\ C\rho^T \alpha^{t-T} & t \ge T \end{cases} \tag{10.8}$$

The total cost of the firm i at time t will be indicated as $k_i(t)$, and it has to take into account the presence of the technological renewal date T. We write

$$k_i(t) = \xi_i t \alpha^t C 1_{\{t \le T\}} + C[\xi_i t + \delta_i (t-T)] \rho^T \alpha^{t-T} 1_{\{t > T\}}. \tag{10.9}$$

10.6 Analysis of the Profit of the District

This section is devoted to the analysis of the impact of the renewal process on the profit function, with a particular analysis of the hierarchical structure of the district.

Firstly, we fix a hierarchical structure of the district and discuss the behavior of the profit as a function of the technological renewal time. By aggregating and discounting with a factor $\gamma \in (0,1)$, we obtain:

$$\Phi(T) = \int_0^{+\infty} \sum_{i=1}^{N} [r_i(t) - k_i(t)] \gamma^t \mathrm{d}t, \tag{10.10}$$

where k_i and r_i are given, respectively, by (10.9) and (10.6).

The following result states:

Proposition 10.1 *Fix* $\bar{T} \in \mathbf{R}^+$. *If* $\rho\gamma > e^{-1}$, *then* Φ *decreases in* $[0, \bar{T}]$.

Proof. By (10.6) and (10.9), the aggregate profit in (10.10) can be written as

$$\Phi(T) = \frac{CS_0 (\alpha\gamma)^T}{log(\alpha\gamma)} \cdot \left[T - \frac{1}{log(\alpha\gamma)} \right] \cdot \left[1 + \left(\frac{\rho}{\alpha} \right)^T \right]$$

$$+ S_0 \cdot \left[\frac{1}{log^2(\gamma)} + \frac{C}{log^2(\alpha\gamma)} \right] + \gamma^T \left[\sum_{i=1}^{N} \delta_i \right] \cdot \left(\frac{1}{log^2(\gamma)} - \frac{C\rho^T}{log^2(\alpha\gamma)} \right). \tag{10.11}$$

A straightforward computation gives that, if $\rho\gamma > e^{-1}$, then it results $\Phi'(T) < 0$, for $T \in [0, \bar{T}]$. ■

Proposition 10.1 admits an easy interpretation. If the technological renewal can take place up to a fixed date $\bar{T}$ and under a certain condition, that will be discussed below, the best strategy that a district can adopt is to renew its technology as soon as possible. The required condition to let this result be true involves the discount rate γ and the parameter ρ: substantially, the attitude of the marketplace to reduce the manifacturing costs should be greater than a threshold depending on the discount rate applied to evaluate the profit of the district.

Let us observe now the influence of the district structure in profit maximization. The hierarchical structure of the district is set free to vary within the profit function. The growth rate of the district can be explored by the knowledge of β, that intervenes in the decay of the variance of r_t as function of S_0 (see formula (10.4) and following arguments). Furthermore, the number β can be estimated by using the parameters Π and z, that describe the structure of the district (see formula (10.5)).

To explore this issue, we proceed by performing a simulation analysis of the profit function Φ. Simulations have been made to analyze the dependence of the profit Φ as a function of Π and z. We got the value of the parameters used to calculate the function Φ from empirical literature. We used $\gamma = 0.97$. From [19] we have that $\rho < \alpha < 1$. We used intermediate values $\alpha = 0.6$ and $\rho = 0.5$. Also the value $\beta = 0.02$ was suggested by empirical analyses [13]. Fixed $t > 0$, $(S_t - S_0)/S_0$ was calculated through random sampling of r_t basing on (10.4). The value of Π represents the percentage of obedience/disobedience, so it ranges in $(0, 1]$. We included $\Pi = 1$ in the analysis as a special case of perfect obedience. The value of z represents the number of companies connected with each unit at the previous hierarchical level in the district. In our simulation it ranges from 2 to 10. For each couple of values of Π and z we calculated β by using (10.5) and we sampled 100 random variables r_t through a Laplace distribution $p(x) = \lambda/2 * exp(-\lambda * abs(x))$ with $\lambda = \frac{\sqrt{2}}{\sigma_t(S_0)}$, $\sigma_t(S_0) \sim S_0^{-\beta}$. We insert r_t in the profit function and we calculated $max_{t\in[0,T]}\Phi(t)$.

Figure 10.1 shows the results of the simulation study. Generally, the profit function attains its maximum values when either Π and z are large. In particular it is shown that, fixed z, the profit function increases as Π increases. Conversely, it is not observed a growing behavior of the profit w.r.t. z when the probability Π assumes small values.

The interpretation of our findings is straightforward.

- When either Π and z are large, then the district is star like, and the strategies decided by the head of the hierarchy are followed by the lowest levels of the tree. In this case, the district seems to be powerful, well organized and with a strong managerial team. Therefore, it is reasonable that the profit is great, as it appears in our simulations.

- We can compare two districts with an identical connection structure. In this case, our results show that the bigger is the probability that the lowest level of the hierarchy will follow managerial decisions, the bigger is the profit. This finding is totally in agreement with the evidence that the comovement of the subjective choices of the companies allows to derive good financial performance of the entire district.

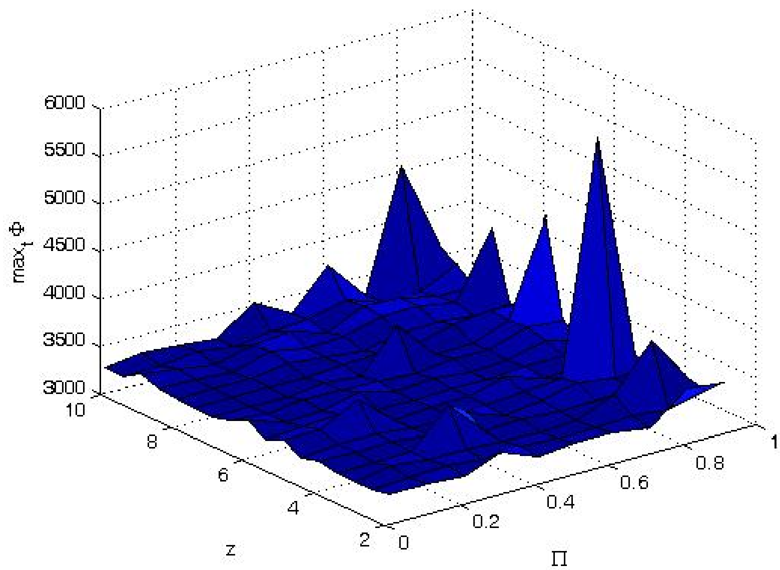

Figure 10.1:

10.7 Conclusion

The hierarchical framework we propose allows to derive the best strategies to optimize the profit of a district when a technological renewal takes place.

In our model, the leader company proposes a financial strategy, but it is followed by the firms in the lowest level with a certain probability Π. Moreover, z is a parameter describing connection level of the district.

We derive some interesting results.

- If the technological environment go fast enough, and the speed of the reduction of the manufacturing costs by the marketplace depends on the discount rate used to evaluate the profit, then the profit of the district is optimized by renewing immediately the technology (Proposition 10.1). In this respect, the managers of the districts should obtain a low discount factor from the financial institution in which the profit is invested. The natural policy, that the head of the hierarchy should apply, is a comparison between the offered discount factors related to the different financial institution, choosing the lowest one.

- The profit of a district can be optimized by letting Π become large. In particular, the maximum is obtained also when the parameter z is large. This means that the managerial team of a district should implement some techniques to let the subordinated companies follow their supervisor' policies. Simultaneously, the production of the district should be fragmented in a large number of interconnected companies. These goals can be achieved involving satellite firms in a proposed financial strategy, and making an exact distinction of the hierarchical roles in production within the district.

Methods to gain the consensus and rationalize the production system are then the study of managerial sciences.

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