Received: 16 April 2022 | Accepted: 3 October 2022

DOI: 10.1111/theo.12439

ORIGINAL ARTICLE

A new bridge principle for the normativity of logic

Francesco Orilia

Dipartimento di Studi Umanistici, Sezione di Filosofia e Scienze Umane, University of Macerata, Macerata, Italy

Correspondence
Dipartimento di Studi Umanistici, Sezione di Filosofia e Scienze Umane, via Garibaldi 20, Macerata 62019, Italy.
Email: orilia@unimc.it

Funding information
Italian Ministry of Education, University and Research, Grant/Award Number: prot. 2017ZNWW7F_004

Abstract

Logic appears to be normative for rational belief. The thesis of the normativity of logic holds that indeed logic has such a normative status. Gilbert Harman has questioned it, thereby giving rise to what has been called "Harman's skeptical challenge". MacFarlane has clarified that in order to answer this challenge and support the normativity of logic, one needs a "bridge principle" that appropriately connects logical entailments and norms for belief, as well as relevant desiderata for the evaluation of candidate bridge principles. Steinberger has identified a list of desiderata, on the basis of which he has proposed his own bridge principle and shown the inadequacy of previous proposals. This paper argues that Steinberger's list is in need of revision and his principle is unsatisfactory, and then puts forward a revised list of desiderata and a new bridge principle in support of the normativity of logic.

KEYWORDS

belief, logic, normativity, rationality

1 | INTRODUCTION

Logic appears to be normative for thought and reasoning, or, we may say, rational belief. That is, logical entailments (or implications) appear to provide norms regarding our beliefs; obligations, or at least permissions or reasons, such that failing to comply with them casts doubt upon one's epistemic rationality. For example, if Tom believes both (i) *P* and (ii) *if P*, then *Q*, then, at least to the extent that Tom realises that (i) and (ii) entail *Q*, it seems Tom ought to believe *Q* as well, or, at the minimum, Tom is permitted, or has a reason, to believe it. The thesis of the normativity of logic holds that indeed logic has such a normative status.

The thesis in question has a "venerable pedigree" and "intuitive force" (Steinberger, 2020: introductory part). However, Gilbert Harman (1984, 1986, 2002) has questioned it; in his words, logic has no "special relevance" for the "theory of reasoning" (1986, p. 11). Steinberger (2019, 2020) calls this *Harman's skeptical challenge*. In essence, Harman builds up this challenge by considering and then criticising some prima facie plausible principles that are meant to connect logical entailments and norms for belief. Principles of this sort are now called, in the terminology of John MacFarlane (2004), *bridge principles*. Harman considers (more or less explicitly) the following ones:

Cr+. If $A_1, \ldots, A_n \vDash C$, then S has a reason to believe C, if S believes $A_1, \ldots, A_n$.
Cr + k. If S knows that $A_1, \ldots, A_n \vDash C$, then S has a reason to believe C, if S believes $A_1, \ldots, A_n$.
Co+. If $A_1, \ldots, A_n \vDash C$, then S ought to believe C, if S believes $A_1, \ldots, A_n$.
Cp+. If $A_1, \ldots, A_n \vDash C$, then S is permitted to believe C, if S believes $A_1, \ldots, A_n$.[1]

As we can see from these examples, a bridge principle is a conditional, in which the antecedent and the consequent may be aptly called a logical condition and a doxastic recommendation, respectively. Any such conditional is a specific way to flesh out, at least tentatively, the normativity of logic. For this thesis to be true, there must be some bridge principle that is true, or, to put it otherwise, the truth of the thesis is grounded on the truth of some bridge principle. Thus, the thesis can be undermined by arguing against prima facie plausible bridge principles, which is what Harman in effect does.[2]

MacFarlane (2004) has taken up Harman's challenge systematically by (i) proposing a complex taxonomy of candidate bridge principles, and (ii) setting up a set of criteria of adequacy, or desiderata, largely derived from Harman himself, in order to evaluate the different candidates. As expected, Cr+, Co + r, and Co + do not meet such desiderata. However, the taxonomy provides a huge number of other candidates, and MacFarlane explores it in a search for a principle that complies with all the desiderata, or at least that fares better than the rival candidates. He thus tries to provide an answer to Harman's sceptical challenge and rescue the thesis of the normativity of logic. In the end, MacFarlane lands on a couple of bridge principles that he finds promising but does not definitely settle on one of them. Following MacFarlane's lead, Steinberger has offered a further investigation of this issue: (a) he has clarified the sense of "norm" that should be assumed in this context (presupposed by Harman and relevant in meeting his challenge), namely "directive" or "first-personal guidance in the process of practical or doxastic deliberation", as opposed to third-personal "evaluation" or "appraisal" (Steinberger, 2019, §5; 2019a); (b) he has insisted on following Harman in taking doxastic attitudes to be "binary states, as opposed to states that admit of degrees" (Steinberger, 2019, p. 318); (c) he has provided, guided by such assumptions, his own list of desiderata; (d) he has argued, on the basis of this list, against the bridge principles discussed by Harman and MacFarlane, as well as those proposed by other authors (Broome, 2013; Field, 2009, 2015; Milne, 2009; Sainsbury, 2002; Streumer, 2007; cf. Steinberger, 2020, §4.1, 2019, §§3–6)[3]; (e) finally, Steinberger has proposed his own favourite bridge principle, which is meant to answer Harman's challenge, namely:

[1]Here and below, I follow Steinberger in using the variable "S" as ranging over typical reasoning agents, the variables "A_1," …, "A_n," "C" as ranging over propositions, and "$\vDash$" as expressing a logical consequence relation, which by default is taken to be classical (Steinberger, 2020, §1, 2019, n.5, p. 309). In formulating these and the following bridge principles, I follow closely the terminology of MacFarlane's and Steinberger's. Harman himself relies on different wordings, however, without significant differences in content (see Harman, 1986, p. 11 for Cr+, 1986, p. 17 for Cr + k, 2002, p. 173 for Co + and Cp+). The labels for the bridge principles are based on a nomenclature devised by MacFarlane (2004), which I explain in an Appendix.
[2]Harman builds up his sceptical challenge by also considering and criticising principles connecting logical inconsistency and norms for beliefs. We shall discuss such principles in the following.
[3]For an interesting critical analysis of these bridge principles, see also Celani, 2015.

> Wr + b*. If S believes that $A_1, \ldots, A_n \vDash C$, and S has an interest in C, then S has reasons to: believe C, if S believes $A_1, \ldots, A_n$.[4]

It is important to emphasise here that this principle relies on a (pro tanto) *reason* deontic operator, expressed by the locution "has reasons to", weak or loose in providing a norm, rather than on a stricter obligation deontic operator, expressed by "ought".

As regards points (a), (b), and (d), I take for granted Steinberger's line. As regards points (c) and (e), however, I demur. I shall argue for revisions and additions in Steinberger's list of desiderata; in particular, I shall add to the list a crucial desideratum, which has to do with explosion, that is, the construction of "explosive" deductive arguments, in the light of which an agent could see any proposition whatsoever as logically implied by her beliefs.[5] My revised list of desiderata will lead me to two main problems with Steinberger's bridge principle (or analogous ones): First, any bridge principle of this sort is problematic in the light of this crucial Explosion desideratum and of related matters. Second, any bridge principle that, like Steinberger's, relies on the reason deontic operator is too weak to appropriately defend the thesis of the normativity of logic. I shall then propose my own bridge principle, which, given appropriate conditions, resorts to the obligation deontic operator. In the light of it, I claim, the normative status of logic can be rescued and Harman's challenge appropriately answered.

The rest of this paper is organised as follows. In Section 2, there is a brief review of Steinberger's list of desiderata, and of Steinberger's reasons in favour of his Wr + b*. In Section 3, the list is modified as a result of focusing on the tension between two desiderata pulling in different directions. In Section 4, the Explosion desideratum is put forward. In Section 5, my bridge principle is introduced; and in Section 6 it is tested by considering some case studies. In Section 7, a final summary highlights the main results.

2 | STEINBERGER'S DESIDERATA

Steinberger builds up on contributions by Harman, MacFarlane, and Broome, and thus arrives to five desiderata, which I present in streamlined fashion as D1–D5′ below (see Steinberger, 2019, §3 for D1-D4, and §4 for D5′).[6] The prime mark in the label "D5′" is meant to signal that in my opinion the corresponding desideratum should not really be accepted, for reasons to be explained. In the end, I shall propose in its stead a desideratum, D5, in its vicinity, after considering an intermediate step, D5″, which will also be superseded. Here are the desiderata:

> D1. *Belief revision* (Harman, 1986, p. 12). Suppose that S believes $A_1, \ldots, A_n$ and these propositions entail C. Despite this, and even assuming that S is aware of it,

[4]Cf. Steinberger (2019, p. 325). Actually, he ends up accepting a variant of Wr + b*, which differs from the latter in replacing the clause "S believes that $A_1, \ldots, A_n \vDash C$" in the antecedent with the clause "according to S's best estimation at the time, S takes it to be the case that $A_1, \ldots, A_n \vDash C$" (Steinberger, 2019, p. 325). However, this does not change the picture much, as it simply specifies that the agent's belief in the entailment $A_1, \ldots, A_n \vDash C$ is grounded on the agent's "best estimation" (I assume "taking P to be the case" is equivalent to "believing P"). I think we may take it for granted that an agent's belief that a certain entailment holds is always grounded on the agent's best estimation, but this explicit consideration of the estimation does not change the nature of the bridge principle; hence, we may just consider Wr + b*. In any case, my criticism of this principle does not hinge on this point. It should also be noted that, where I use the locution "S has an interest in C", Steinberger puts things as follows: "S considers C or has subjective reasons to consider C". My formulation is meant to be a more concise and appropriate way to make essentially the same point, as I shall clarify in the following. Similarly, in desideratum D5′ below, I speak in terms of having an interest in a proposition, rather than in terms of having a reason to consider a proposition, as Steinberger does.

[5]Steinberger (2016) considers explosion in order to critically assess an argument to the effect that explosion should motivate the adoption of a paraconsistent logic, but he does not take it up as providing a desideratum that constraints the choice of a bridge principle, as I shall do here.

[6]Steinberger also discusses two further desiderata proposed by MacFarlane, namely *Obtuseness*, and *Priority Question*, but argues that they should not be taken into account (2019, p. 316, n.16, and p. 317). I side with Steinberger on this, and I have thus neglected them.

S is not obliged to believe *C*, since there could be good reasons to reject (stop believing) one of the premises.

D2. *Excessive demands* (Harman, 1986, p. 17). Proving that a given proposition *C* is entailed by *S*'s beliefs could be extremely complex, possibly beyond *S*'s cognitive capacities. Since ought implies can, *S* is not obliged to believe any proposition *C* logically entailed by *S*'s beliefs, as if *S* were logically omniscient.

D3. *Clutter avoidance* (Harman, 1986, p. 12). Every proposition *P* entails infinitely many other propositions, most of which are useless or redundant, if one already believes *P*. For instance, Camberra is in Australia entails Camberra is in Australia or there are flying donkeys. However, our cognitive resources are limited and it would then be irrational to engage them by coming to believe new propositions in this way—clutter propositions, as we may call them. In short, *S* is not obliged to believe any clutter proposition *P* logically implied by *S*'s beliefs.[7]

D4. *Rational inconsistency* (Harman, 1986, pp. 15–16; MacFarlane, 2004, p. 11). *S* may have inconsistent beliefs (have an inconsistent belief set) and even be aware of it, without being irrational. Here we may assume that the belief set of an agent is inconsistent when it is possible to deduce via inference rules accepted by *S* two opposite propositions, say *A* and its opposite $\sim A$ (this is a reasonable assumption, typically left implicit in this debate).[8]

D5′. *Strictness test* (MacFarlane, 2004, p. 12; Broome, 2000, p. 85). At least for obvious entailments, if *S* has an interest in the conclusion of the entailment, then it is not rational for *S* to believe the premises and disbelieve the conclusion.[9]

These five desiderata lead Steinberger to reject several candidate bridge principles by other authors (see references above), and to finally pick Wr + b*. I shall now briefly review how Steinberger motivates this. As we shall see, the selection of a candidate such as Wr + b* involves selecting options at choice points regarding specific aspects of a principle (see the Appendix for a clarification of the options and their corresponding labels, which show up in the acronym "Wr + b*").[10]

[7]It is worth emphasising that the notion of clutter proposition is context-sensitive, as it depends on the epistemic goals of the agent.
[8]I use the name *Rational Inconsistency* for D4, since it conveys more generality than the term used by Steinberger and most people in the debate, namely *Preface Paradox*.
[9]In formulating D5′, Steinberger (2019, p. 315) uses "ordinary, readily recognisable" where I have put "obvious". This is justified by the fact that the discussion of this desideratum at p. 323 in his 2019 (see below) implies that obviousness is what Steinberger seems to have in mind. Moreover, Steinberger's formulation differs from mine in that he says "there is something amiss about an agent who endorses the premises and yet disbelieves the conclusion". Broome and then MacFarlane (who quotes Broome) rather say that, if you are such an agent, "you are definitely not entirely as you ought to be". I think that my formulation in terms of an agent not being rational more explicitly conveys what the problem is here.
[10]In a recent paper, Bradley (2021) has proposed several bridge principles from the point of view of his "'ought-contextualism'," according to which a deontic operator can be relativized in various ways depending on the context. Thus, Bradley proposes bridge principles regarding not only first-personal guidance, which is what interests us here, but also third-personal evaluation and appraisal. As far as I can see, as regards guidance, what Bradley (2021, § 8.2) proposes amounts, in the terminology of this paper, to this: if (i) $A_1, \ldots, A_n \vDash C$, and (ii) S believes that $A_1, \ldots, A_n \vDash C$, then *S* ought$_A$ to believe *C*, if *S* believes $A_1, \ldots, A_n$, where the index "'A'" should be read as "'given that *S* believes $A_1, \ldots, A_n$ and believes $A_1, \ldots, A_n \vDash C$'." It seems to me that this principle fails to take care of *Clutter Avoidance*, but this could be easily remedied by inserting the clause that *S* has an interest in *C*. More importantly, in proposing a principle with an ought operator such as this, Bradley appears to neglect, contrary to Steinberger, the preface paradox (Bradley 2021, §5) and thus *Rational iInconsistency*. This principle also fails to take care of the *Explosion* desideratum, to be considered in the following.

Let us consider first the selection of an option that is not based on the above desiderata; it regards the choice between a positive and a negative doxastic attitude in the doxastic recommendation (believing vs. disbelieving): Steinberger does not see any clear advantage in preferring the latter and thus definitely buys the former, that is, option + (Steinberger, 2019, p. 321). The selection of the other options relies on the desiderata.

In order to meet *Belief revision*, Steinberger selects *option W*; that is, the deontic operator must take wide scope. This is because, given this choice, the normative requirement in the doxastic recommendation is conditional, and thus the agent can discharge it either by coming to believe the conclusion of the entailment in the logical condition, or by rejecting at least one of the premises of such an entailment.[11] In order to meet *Excessive demands*, Steinberger argues that the entailment must be attitudinally constrained by a doxastic operator; that is, *option b* must be chosen. To fulfil *Clutter avoidance*, Steinberger (2019, pp. 320–321) proposes that an additional constraint regarding the antecedent of the bridge principle must be in play.[12] The constraint is that the agent, in Steinberger's words, "considers or has subjective reasons to consider *C*". I have expressed the constraint with the more concise locution "has an interest in *C*". I think in fact that, besides being more concise, it is also more appropriate, in the light of the considerations regarding *Clutter avoidance* that I shall offer in a moment. It is worth noting that, in line with this constraint, Steinberger inserts in *Strictness test* the clause regarding having an interest in the conclusion of the entailment (MacFarlane and Broome do not have this clause).[13] Surely, this is done in order to make *Strictness test* sensitive to the demands coming from *Clutter avoidance*.

As regards *Rational inconsistency* and *Strictness test*, there is a tension in Steinberger. On the one hand, *Strictness test* pushes him toward *option o*, that is, an obligation operator in the doxastic recommendation. This suggests, given the other choices already considered, this principle:

> Wo + b*. If *S* believes that $A_1, \ldots, A_n \vDash C$ and *S* has an interest in *C*, then *S* ought to: believe *C*, if *S* believes $A_1, \ldots, A_n$.

On the other hand, *Rational inconsistency* pushes Steinberger toward *option r*, that is, a reason operator in the doxastic recommendation. This, given the other choices already considered, suggests Wr + b*. At last, Steinberger resolves the tension by preferring option r, and thus he ends up endorsing Wr + b*. I shall discuss in the next section how this tension arises, how Steinberger resolves it, and how I think it should be resolved. Before doing this, some remarks on the other choices are in order.

I completely concur with Steinberger regarding options + and b, and thus I subscribe them without further ado. I also think that option W is a good way to take care of *Belief revision*,

[11]Curiously, Steinberger (2016, p. 409) fails to realise this, in discussing how option W behaves when explosive arguments enter the picture. He considers the situation of an agent who rationally believes some inconsistent premises, wherefrom any conclusion *C* explosively follows via *Ex Contradictione Quodlibet*, and argues thus: from the fact that the agent believes all of the premises (she cannot do otherwise, since the inconsistency is rational), it merely follows that the agent also believes *C*, but it does not follow that the agent ought to believe *C*. This is in stark contrast with what he says on behalf of option W in discussing *Belief Revision* (2016, p. 406, 2019, p. 320). In this case, he assumes that the agent can be absolved from the *obligation* to believe the conclusion by dropping at least one of the beliefs that work as premises in the relevant argument; that is, the agent has an obligation to believe the conclusion, if she does not stop believing at least one of the premises. It seems that in the latter case Steinberger accepts the following: given option W (i.e., *S* ought to: believe the conclusion, if she believes the premises), if *S* believes all of the premises, then *S* ought to believe the conclusion. In contrast, in the former case, he accepts this other principle: given option W (i.e., *S* ought to: believe the conclusion, if she believes the premises), if *S* believes all of the premises, then *S* believes the conclusion. However, the former principle seems correct and the latter wrong. Incidentally, this problem troubles Steinberger's (2016) analysis of the argument from explosion to paraconsistency. I myself think that this argument can be resisted, since explosion can be taken seriously, without thereby accepting a paraconsistent logic, as the bridge principle to be proposed in the following suggests.

[12]This is signalled in the acronym for the bridge principle by appending an asterisk to the acronym obtained by following MacFarlane's conventions, explained in the Appendix.

[13]Steinberger makes this addition with his different terminology: "having a reason to consider" is preferred to "having an interest in".

and thus I shall also follow Steinberger in making this choice.[14] Moreover, I basically agree with Steinberger's way of meeting *Clutter avoidance*. What Steinberger is proposing is in line with the interest condition that Harman himself (1986, p. 56) has put forward in connection with this desideratum. The crucial point is that the use of logic in inferring a proposition from believed propositions must be constrained by the agent's goals, which determine whether the agent has an interest in taking a stand on the truth value of the proposition in question (see also Michael, 2016, § 6), that is, in brief, whether the agent has an interest in such a proposition.

3 | THE INFERENTIAL TENSION

Harman articulated his sceptical challenge about the normative role of logic not only by criticising some candidate bridge principles but also by arguing against these two principles regarding inconsistency:

> *Logical Inconsistency Principle*. Logical inconsistency is to be avoided. (Harman, 1986, p. 11);
>
> *Recognised Inconsistency Principle*. One has a reason to avoid believing things one recognises to be inconsistent (Harman, 1986, p. 18).

Harman attacked them by putting forward *Rational inconsistency*. He argued for it by noting that an agent may have evidence, or good reasons, in favour of all the propositions in her belief set while not having enough evidence, or good reasons, to decide which propositions should be rejected and replaced, in such a way as to eliminate the inconsistency (Harman, 1986, p. 15). As often noted by supporters of paraconsistent logic (see, e.g., Priest et al., 2022, § 2.1), in science one may happen to be in a predicament of this sort, since it could be rational for a scientist to accept an inconsistent theory on a given subject, as long as no better theory is available (Vickers, 2013). The preface paradox even suggests that, in the light of a reasonable principle of epistemic modesty, every rational agent with limited resources as we are should be in this predicament (Harman, 1986, p. 16, p. 23, 1984, pp. 108–109).[15] For, on the one hand, we obviously believe each of our beliefs, A_1, …, A_n, which jointly entail their conjunction A_1 & … & A_n; however, on the other hand, given epistemic modesty, we also believe that not all of our beliefs are true, and thus believe the disjunction $\sim A_1 \vee \ldots \vee \sim A_n$, which is logically equivalent to $\sim(A_1$ & … & $A_n)$. If so, each of us has an inconsistent belief set.[16] In these *preface scenarios*, as we may call them, the agent is rationally inconsistent and also knowingly so, to the extent that she is aware of the arguments sketched above. Similarly, the scientist who endorses an inconsistent theory for lack of a better alternative is rationally inconsistent, and knowingly so inasmuch as she is aware of the inconsistency.

[14]Option W takes care of Broome's (2000) *Bootstrapping* problem, since this is a special case of *Belief Revision*. Broome notes that any proposition, even an absurd proposition A, logically implies itself. Yet, we do not want to grant that an agent S, who believes A, ought to believe A, if A is absurd. However, given option W, S has a merely conditional obligation to believe the absurd proposition A (to the extent that S believes that A entails A and S has an interest in A), and this obligation can be discharged by dropping A. Steinberger thinks that in order to take care of this problem, additional considerations must be put forward, which have to do with the fact that the proposition *S believes A if S believes A* is a tautology. In contrast, I think that this is irrelevant.

[15]Harman (1986, p. 16) hints at this problem without mentioning the preface paradox explicitly.

[16]The problem is usually presented in this restricted form: an author believes all of the propositions A_1, …, A_n in the non-fiction book she has written with great care and expertise, and yet in the preface she makes her epistemic modesty explicit by declaring that the book surely contains an error. We may also put things as follows. An agent S may believe that, for every proposition P, if P has property G, where G is being believed by S, or being contained in the book written by S, then P is true. This entails that it is not the case that there is a P such that P has property G and P is not true. However, by epistemic modesty S also believes the opposite of this, namely that there is a P such that P has property G and P is not true.

Let us now focus on a point hardly noted, despite its significance, in the current debate on the normativity of logic, namely that one can also argue for *Rational inconsistency* as follows. We made the default assumption that a typical rational agent implicitly accepts classical logic (CL). However, plausibly, she also implicitly accepts what we could call the truth rule, and the predication rule. The truth rule allows one to consider equivalent, and thus inter-substitutable *salva veritate*, a sentence A, and the sentence $S(A)$ *is* true, where $S(A)$ is a singular term referring to A. For instance, a typical agent would agree that the proposition Rome is in Italy, and the proposition "Rome is in Italy" is true, are equivalent. The predication rule allows one to consider equivalent a proposition resulting from predicating a certain complex predicate of some term, and a corresponding complex proposition resulting from this term's filling the argument places in this predicate. For example, a typical agent would agree that the proposition John is both rich and unkind is equivalent to the proposition John is not kind and John is rich.[17] Let us call *CL+* the system of logical rules involving both the rules of CL and the truth and predication rules. As is well known, with CL+ available one can get contradictions, for example, via the Liar (considered by Harman) or Russell's paradox.

In sum, we can agree with Harman that both the Logical inconsistency principle and the Recognised inconsistency principle should be rejected. And in fact, both MacFarlane and Steinberger agree with Harman in rejecting them. However, they do not see this as in itself a reason to discard the thesis of the normativity of logic, as Harman seems to have it. For them, to have an appropriate bridge principle seems to be enough to answer Harman's challenge. Nevertheless, they acknowledge that the reason behind the rejection of these principles, namely *Rational inconsistency*, constitutes a crucial desideratum, to be taken into account in the search for the appropriate bridge principle.

However, as anticipated, *Rational inconsistency* is in tension with *Strictness test*, which both MacFarlane and Steinberger find compelling, and this complicates their search for a bridge principle. Let us see how the problem arises. Given *Strictness test*, it seems that any agent who has beliefs that entail in an obvious way a proposition in which the agent has an interest (Steinberger, 2019, p. 323) ought to believe this proposition, unless the agent gives up the beliefs in question; otherwise, the agent would not be fully rational. Setting aside qualms about the vagueness of "obvious", this may be too strong. For no matter how obvious the entailment is, there may be room in a specific circumstance to miss it, and when this is the case, it is questionable that the agent ought to believe the conclusion of the entailment, given that the oughtness is understood as a first-person directive. However, if the agent is aware of the entailment, this obligation appears to subsist, in the sense that rationality demands it, and failure to comply with it may result in irrational behaviour, that is, behaviour that conflicts with intentions and desires. For example, suppose that Tom has the disjunctive belief that Mary is either in London or in Paris. He is in London, and he is interested in the truth value of the proposition that Mary is in London, since he has a strong desire to visit her. He comes to believe that Mary is not in Paris, since a friend that he trusts completely informs him of this. He realises that the proposition in question is entailed by this new belief and his previous disjunctive belief. And yet he fails to believe that Mary is in London, and accordingly does not look for her, despite his desire. Clearly, Tom′s behaviour is not fully rational. On the other hand, if we grant there are such obligations, we open the way, given *Rational inconsistency*, for a rational agent′s being obliged to believe opposite propositions A, and $\sim A$, or even a contradiction A & $\sim A$. For *Rational inconsistency* makes room for the possibility that a rational agent has inconsistent beliefs, and this agent may well realise that there are obvious entailments leading from her (rationally) inconsistent beliefs to opposite conclusions, A, and $\sim A$, which in turn obviously entail the

[17]In a formal language with a quotation device and a truth predicate, "T", the truth rule can be conveyed with an axiom scheme such as T'A' $\leftrightarrow$ A. In a formal language with the lambda operator, and a predication predicate, "p", the predication rule can be conveyed as p ([λx A], a) $\leftrightarrow$ $A(x/a)$, or more generally, as p([$\lambda x_1 \ldots x_n$ A], t_1, …, t_n) $\leftrightarrow$ $A(x_1/t_1, \ldots, x_n/t_n)$, where $A(x_1/t_1, \ldots, x_n/t_n)$ is the wff resulting from simultaneously replacing each x_i in A with t_i (for $1 \leq i \leq n$), provided t_i is free for x_i in A.

contradiction A & $\sim A$. For example, an agent in a preface-like situation, who believes both A_1, …, A_n, and (for epistemic modesty) $\sim A_1$, $\vee$ … $\vee \sim A_n$, may also believe $\sim(A_1$ & …& $A_n)$, since it is logically equivalent to $\sim A_1 \vee$ … $\vee \sim A_n$, and may recognise the obvious inference from A_1, …, A_n to A_1 & … & A_n, which is the opposite of the already believed $\sim(A_1$ & … & $A_n)$. Both MacFarlane and Steinberger, however, agree that this agent is not obliged to believe A_1 & … & A_n, despite her awareness of the obvious inference.

They want to make a general point, of course; and the point is, I take it, an implicit acceptance of a further desideratum along these lines: no rational agent ought to believe a proposition and its opposite. Actually, Steinberger (2019, pp. 309–10) comes quite close to endorse this desideratum explicitly, when he says that he makes these assumptions: "(i) that one ought not believe and disbelieve one and the same proposition, and (ii) that disbelieving a proposition is tantamount to believing its negation". Of course, believing an explicit contradiction such as A & $\sim A$ is not any better than believing both A and $\sim A$, and thus I think that, more accurately, the desideratum in question is: for no proposition A, S ought to believe both A and $\sim A$, or their conjunction A & $\sim A$. This seems reasonable; one thing is to have an inconsistent set of beliefs and realise that one has it, and another thing is to believe two opposite propositions or even their conjunction; this may be considered irrational or perhaps even impossible to achieve. Here, dialetheists such as Priest (1987) would disagree. They would say that in some special cases, for example, when facing logical paradoxes, to believe that some contradictions are true, that they are *dialetheias*, might be the most rational thing to do (provided it is accompanied by the rejection of classical logic and the endorsement of a paraconsistent logic). If we want to take this into account, the desideratum takes this final form:

> D6. *No contradictions*. For any proposition A, S ought not to believe both A and $\sim A$, or their conjunction A & $\sim A$ (unless S takes A to be a dialetheia and is willing to endorse an appropriate paraconsistent logic).[18]

In view of issues such as these, MacFarlane cannot choose one definite bridge principle (MacFarlane, 2004, pp. 14 ff.),[19] and Steinberger opts for a principle with a reasons operator, namely Wr + b*. I agree that *Rational inconsistency* and D6 should be accepted. As regards *Strictness test*, I manifested a qualm, viz. that it is too strong, since an agent may fail to notice even an obvious entailment, and, when this is the case, the agent can hardly have an obligation, understood as a first-person directive. On the other hand, in the light of the above discussion, we may add that it is in another respect too weak, since it merely addresses obvious entailments, and thus fails to consider what one should make of non-obvious entailments that one comes to acknowledge. Shouldn′t they also condition the beliefs of an agent? In the light of this, we should replace *Strictness test* as follows:

> D5″. *Acknowledged entailments*. If S acknowledges that there is an entailment leading to a conclusion C, which an agent S has an interest to consider, then it is not rational for S to disbelieve C, if S believes the premises.

Of course, D5″ is in tension with *Rational inconsistency* and *No contradictions* just like *Strictness test*, indeed, even more so. We may in general speak of an *inferential tension*, which

[18]Note that an agent may well be rationally inconsistent, in line with D4, and yet comply with D6. For example, in a preface-like scenario, an agent, let us suppose, believes A_1, …, A_n, as well as the disjunction $\sim A_1 \vee$ … $\vee \sim A_n$, wherefrom both of the opposites A_1 & … & A_n and $\sim(A_1$ & … & $A_n)$ can be derived. This makes the agent rationally inconsistent, which D4 allows for. However, if the agent does not come to believe both A_1 & … & A_n and $\sim(A_1$ & … & $A_n)$, she complies with D6. The agent may find it impossible to endorse a contradiction, even if she realises that it can be inferred from her beliefs.

[19]As Steinberger (2019, p. 323) puts it, "[a]ccording to MacFarlane, we simply must reconcile ourselves to the existence of an ineliminable normative conflict".

MacFarlane and Steinberger individuate on the basis of *Strictness test*, and which, in my opinion, had better emerge by focusing on D5″ rather than *Strictness test*.[20] We shall see that, in the end, it will seem more appropriate to replace D5″ with a more cautious desideratum, which takes into account the nature of the entailment in question.

4 | EXPLOSION

As we saw, the inferential tension leads Steinberger to a bridge principle with a reason operator. On the one hand, this may seem too weak, for of course it cannot do full justice to D5″ (or for that matter to D5′). On the other hand, resorting to a reason operator, as in Steinberger, may seem too strong, in the light of the well-known fact that there are explosive arguments, as we shall now see.

We made the default assumption that a typical rational agent implicitly accepts CL, and this means that such an agent also implicitly accept its *Ex Contradictione Quodlibet* rule (ECQ), which allows one to infer any proposition whatsoever from two opposite propositions: A and $\sim A$. We have also seen that an agent may be in a position to deduce two opposites from her rationally inconsistent belief set. Hence, the subject may also be in a position to deduce from A and $\sim A$, via ECQ, any proposition whatsoever. If so, we can imagine a situation such as this. Tom is thirsty in a country where fountains often dispense undrinkable water, which can cause dangerous diseases. He runs into a fountain, and it is important for him to know whether its water is drinkable or not. In sum, he has an interest in the proposition, WD, asserting that the water from that fountain is drinkable; it is a goal for him to take a stand on this proposition. Tom then appeals to what we may call an *ECQ argument* to deduce WD from his (inconsistent) beliefs. That is, he first derives both A and $\sim A$ from his beliefs and then appeals to ECQ to deduce WD.

Alternatively, Tom may get the conclusion WD via the notorious Curry's paradox, for example, as follows. Tom writes on a sheet, which he calls *s1*, just one sentence, "if the longest sentence on s1 is true, then WD", and thus he comes to believe that the following is true:

> (CP) the longest sentence written on s1 is "if the longest sentence on s1 is true, then WD".

He then reasons as in Curry's paradox to infer WD from the premise (CP) (see Shapiro and Beall, 2021, §1.1). Or Tom may resort to a predicational version of Curry's paradox in order to prove WD from no premises. That is, he first considers the Curry property, C, of being something such that, if self-predicated then WD,[21] and then considers the proposition that predicates C of itself, $\mathrm{p}(C, C)$,[22] whereby he derives WD (see Shapiro and Beall, 2021, §2.1).

We may call arguments of this sort *explosive*. They can be characterised in a way that makes it clear how irrational it would be to rely on them to acquire new beliefs. That is, any argument D of this sort is such that it is possible for an agent S who accepts D to construct a *counterbalancing* argument, that is, an argument D' with the opposite conclusion, with premises that S believes just as much as S believes the premises of D (if D has any premises), and such that D' is isomorphic to D; where two deductions D and D' are *isomorphic* when D' is obtained from D by replacement of propositions $P_1, \ldots, P_n$ in D with propositions $Q_1, \ldots, Q_n$ in D' (so that

[20]Something like this inferential tension leads Celani (2015) to favour like Steinberger the reason operator, while partly siding with Harman on the sceptical challenge in admitting that "the normativity of logic is of a weak sort" (p. 173).
[21]More formally, $[\lambda x\ \mathrm{p}(x, x) \rightarrow \mathrm{WD}]$.
[22]More formally, $\mathrm{p}([\lambda x\ \mathrm{p}(x, x) \rightarrow \mathrm{WD}], [\lambda x\ \mathrm{p}(x, x) \rightarrow \mathrm{WD}])$.

D and *D*′ have the same number of steps, and each step is justified in the same way).[23] To illustrate, focus again on the above arguments leading to the conclusion WD. Consider first those that used ECQ after having derived two opposites. In all these cases, there is an isomorphic argument with the same premises and with conclusion ∼WD obtained by simply replacing WD with ∼WD in the very last step. Turn next to the argument that leads to WD by relying on a predicational version of Curry's paradox. This argument does not rely on any premise, and an isomorphic argument with the opposite conclusion can be obtained from it by replacing in all its steps WD with ∼WD. Finally, consider the argument that leads to WD by relying on the truth-theoretical version of Curry's paradox and the premise (CP). In this case one obtains a counterbalancing argument, for example, as follows. By writing an appropriate sentence on another sheet, s2, one first brings about the truth of a proposition analogous to (CP) but involving s2 and ∼WD rather than s1 and WD, namely,

> (CP′) the longest sentence written on s2 is "if the longest sentence on s2 is true, then ∼WD".

Then, one replaces (CP) with (CP′) in the original argument.

Thus, Tom has an interest in WD, and believes that there is an entailment leading to WD, either from premises that he believes and cannot stop believing, or from no premises at all. And yet it does not seem right that Tom ought to believe WD on this basis. It does not also seem right, moreover, that Tom has, on this basis, even a reason to believe WD. It is simply irrational to rely in any way on an explosive argument to reach any conclusion at all. We should then consider the following further desideratum.

> D7. *Explosion*. The fact that there is an explosive argument leading from the beliefs of a subject *S* to a proposition *P* does not constitute a reason, let alone an obligation, for *S* to believe *P*, even if *S* has an interest in *P* and believes that there is such an argument.

It is important to note here that the situation in explosive scenarios is quite different from that in ordinary cases of belief revision, which can be dealt with by resorting to the wide scope option W. For in the latter case, we assume that the agent can rationally reject some of the premises; she ought to either believe the premises or the conclusion of a certain argument, and she can discharge this obligation by rejecting some of the premises, thereby foregoing any obligation to believe the conclusion. In explosive scenarios, in contrast, there are no premises that the agent can rationally reject; either there are no premises, as in the predicational version of Curry's paradox, or it is impossible for the agent not to assent to the premises, in the light of the evidence for it, as in the case of the premise (CP) of the other version of Curry's paradox. Again, given option W, she ought to either believe the premises or the conclusion of a certain

[23]For instance, the following two deductions are isomorphic in that the latter is obtained from the former by replacement of proposition *F* with proposition ∼*F*:

1. $F \rightarrow G$ (premise)
2. $G \rightarrow H$ (premise)
3. F (hypothesis)
4. G (from 3 and 1 by MP)
5. H (from 2 and 4 by MP)
6. $F \rightarrow H$ (from 3–5 by hypothetical reasoning).

1. $\sim F \rightarrow G$ (premise)
2. $G \rightarrow H$ (premise)
3. $\sim F$ (hypothesis)
4. G (from 3 and 1 by MP)
5. H (from 2 and 4 by MP)
6. ∼F → H (from 3–5 by hypothetical reasoning).

argument; but this time she cannot rationally reject any of the premises, and thus it seems that the only way to discharge the obligation is by coming to believe the conclusion.

We saw that the inferential tension suggested to Steinberger a bridge principle with a reason operator. However, given *Acknowledged entailments*, a reason operator appears too weak, and, given *Explosion*, it seems too strong. Thus, after D7 is in stock, the inferential tension is even more troublesome than before. How can we face it? Steinberger makes a distinction between "preface-like contexts" in which the defeasible nature of the reason operator "makes itself felt", and "ordinary contexts" in which this operator "behaves like the corresponding ought would", and this leads him to suggest that "we would like instructions telling us how to understand the reasons invoked by [or+b*]" (Steinberger, 2019, pp. 324–25). In other words, Steinberger would want a criterion telling us how to distinguish preface-like contexts and ordinary contexts. Surely something of this sort is precisely what is needed here, but unfortunately Steinberger does not provide it. Actually, an appropriate criterion must separate ordinary contexts not only from preface-like contexts, but, more generally, from dangerous contexts, as we may call them, that is, those that involve rational inconsistency or explosion. We need a criterion that tells us when we are not in such a context. This can be done by providing requirements that must be met, if damage from inconsistency and explosion is to be avoided. If they are not met, it would not be rational for an agent to rely on an entailment that the agent has acknowledged. Let us say that an entailment that does not meet such requirements is dismissible for the agent. I shall characterise this notion of dismissibility more precisely below.

For the time being, it should be noticed that, in the light of this discussion, D5″ is really too demanding, for it does not take into account that some entailments could be dismissible. We should thus admit that D5″ had better be replaced by this:

> D5. *Non-dismissible acknowledged entailments*. If *S* acknowledges that there is an entailment leading to a conclusion *C*, in which an agent *S* has an interest, then it is not rational for *S* to disbelieve *C*, if *S* endorses the premises, *unless the entailment is dismissible*.

Similarly, in the light of *Clutter avoidance*, Steinberger has inserted the interest condition in *Strictness test*.

Now, if a criterion based on dismissibility allows us to separate dangerous contexts from ordinary contexts, why should we rest content with a bridge principle relying on a reason deontic operator such as Wo + b*? We can rather settle on a bridge principle with an obligation operator, as long as this criterion is taken into account. Let us see how we can go in this direction.

5 | A NEW BRIDGE PRINCIPLE

If we grant the notion of dismissibility, a bridge principle can successfully resort to an obligation operator, by making the strict demand introduced by such an operator a conditional demand: it subsists only if the entailment considered by the agent is not dismissible for the agent in question. In sum, we can arrive to the following bridge principle:

> Wo + b*#. If (i) *S* believes that $A_1, \ldots, A_n \vDash C$, (ii) *S* has an interest in *C*, (iii) it is not dismissible for *S* that $A_1, \ldots, A_n \vDash$ C, then *S* ought to: believe C, if *S* believes $A_1, \ldots, A_n$.

As regards the desiderata *Belief revision*, *Excessive demands* and *Clutter avoidance*, this bridge principle in essence meets them just like Steinberger's principle Wr + b*, even though the former picks option o (hence, the "o" in the acronym), whereas the latter picks option r. The crucial differences with Wr + b* have to do with the other desiderata. In order to deal

appropriately with *Non-dismissible acknowledged entailments*, Wo + b*# favours option o. Moreover, *Rational inconsistency*, *No contradictions*, and *Explosion* are behind the addition of a third clause in the antecedent of Wo + b*#, namely that it is not dismissible for S that $A_1, \ldots, A_n \vDash C$. This enrichment (signalled by the hashtag in the acronym) may be called *dismissibility clause*. We shall see below how it takes care of the latter three desiderata in the presence of option o. At this juncture, I wish to specify more precisely when an entailment is dismissible for an agent. This will be done by the definitions *Blocker* and *Dismissibility*, to be provided at the end of this section. Before arriving to them, some preliminary ideas should be marshalled, and to them I now turn.

First of all, let us dwell on the distinction between an entailment, $A_1, \ldots, A_n \vDash C$, and the deductive argument or simply deduction, that, we may say, backs up the entailment, in that it leads from the premises $A_1, \ldots, A_n$ to the conclusion C, via various steps and logical rules such as Modus Ponens, Disjunctive Syllogism, or Conjunction Introduction. Let us indicate with "$D[A_1, \ldots, A_n / C]$" a deduction D with $A_1, \ldots, A_n$ as premises and C as conclusion. As is familiar from introductory logic textbooks, there may be different deductions, say $D[A_1, \ldots, A_n / C]$ and $D'[A_1, \ldots, A_n / C]$, that back up the same entailment; despite having the same premises and conclusion, they may differ in that they involve different steps and different logical rules.[24]

Let us adopt for the sake of conciseness the following terminology. A subject S accepts an entailment when S believes that a certain entailment holds; moreover, S strongly accepts the entailment when it is also the case that S believes every premise of the entailment. Similarly, S endorses a deduction when S is aware of D and takes it to be valid; that is, S takes all the rules of inference employed in D to be logically valid and correctly applied; moreover, S strongly endorses D when S, in addition to endorsing D, believes all its premises. It seems appropriate to assume that S (strongly) accepts an entailment, $A_1, \ldots, A_n \vDash C$, inasmuch as S, more or less explicitly, (strongly) endorses at least one deduction $D[A_1, \ldots, A_n / C]$, which may be called a deductive ground for the entailment, relative to S.

We should admit that an agent may have more than one deductive ground for the same entailment.[25] It should not be assumed, however, that an agent who endorses a deduction has necessarily explicit beliefs about the validity of the rules employed in the deduction in question, or that she even explicitly possesses the very notion of rule of inference or of deduction. This may be the case if the agent is sophisticated enough, as in the extreme case of the professional logician. In less sophisticated agents, there may simply be the construction of deductive arguments that lead via a number of steps from premises to conclusion, where the steps can be seen as applications of certain logical rules, without it being necessarily the case that the agent is aware that she has applied the rules in question and that she has constructed a deduction.

We typically make the default assumption that deductive validity is an absolute notion, so that, for any two arguments, if both deductively valid, they are equally valid. However, in the light of the logical paradoxes, we should make room for the idea that an agent may regard an argument as more valid than another argument. The following considerations lead me to this. Let us reflect on how a typical agent may react to the preface paradox or to deductions leading to logical paradoxes such as the ones we discussed above. Wittgenstein (1956, part I, appendix I, 11–14; part II, 77–90; part V, 27–28) has well captured the situation here, in a way nicely summarised by Nozick (1981, p. 408) as follows:

[24]For example, the entailment $F \to G, G \to H \vDash F \to H$ could be backed up by the first deduction of note 23, or by the following simpler deduction:

1. $F \to G$ (premise)
2. $G \to H$ (premise)
3. $F \to H$ (from 1–2 by hypothetical syllogism).

[25]For example, an agent may (strongly) accept the entailment $F \to G, G \to H \vDash F \to H$, because she (strongly) endorses both the first deduction of note 23 and the deduction of note 24.

> Many of us have had the experience of explaining or showing a paradox to someone, for instance, Russell's paradox or the paradox of the liar. Often the person merely will smile in amusement; he does not think the paradox is important, even if we show he accepts each of the components that give rise to the paradox. It is philosophers who get excited about paradoxes and spend much time trying to resolve them. Most other people would embrace the possibility broached by Wittgenstein: pigeonhole the paradoxes, segregate them off harmelessly, and then go about regular business. Most people are not distressed to discover their beliefs exhibit the inconsistency of the paradox of the preface, or of the logical or semantical paradoxes.

In the light of the paradoxes, logicians have proposed different logical systems, which drop or circumscribe one or another of the rules of CL+, on the basis of a diagnosis of the paradoxes, according to which the dropped or circumscribed rules are held to be responsible for the paradoxes. For example, there are approaches that circumscribe the truth and predication rules (Kripke, 1975; Gupta and Belnap, 1993; Orilia, 2000) and other approaches that drop or circumscribe rules of CL (Priest, 1987; Field, 2008). As Priest (1987, ch. 8) has emphasised, the dropped or circumscribed rules may then be regarded as default rules, which hold in most circumstances, but which also admit of exceptions in those cases where they lead to logical paradoxes. Since Priest endorses a paraconsistent logic, which rejects some of the rules of CL, the rejected CL rules are for him the ones to be regarded as merely default rules. In contrast, Harman (1986, p. 16) takes for granted that it is the truth rule that should be treated as a default rule[26]:

> In practise the best solution may be to retain the Biconditional Truth Schema and yet avoid contradiction by interpreting the schema not as something that holds without exception but rather as someting that holds "normally" or "other things being equal". It is then a "default assumption".

Harman also seems to assume that this is the implicit response of the typical agent exposed to the truth paradoxes.

However, there is no consensus among logicians on which among these many approaches is the correct response to the paradoxes. Moreover, the above Wittgeisteinian remarks voiced by Nozick strongly suggest that no logical rules in particular would be explicitly regarded by a typical agent, or the majority of agents, as the culprits to be treated as merely default rules. It seems to me that at most we can say that, for some agent, there is an implicit hierarchy of logical rules, on the basis of which the rules that are lower in the hierarchy would be regarded as merely default rules by the agent, or, as we may put it, as less valid than the rules which are higher in the hierarchy. That is, the agent may be more disposed to regard some of the rules as responsible, or more responsible, for the paradoxes, thus treating such rules as merely default rules (presumably, the presence of dispositions of this sort in the agent could be revealed by appropriate probing). For example, for some agents, all rules of CL, such as Modus Ponens or Conjunction Elimination, could be at the top of the hierarchy, whereas the truth rule and the predication rule would have a lower rank; and, contrariwise, for some other agents, some of the rules of CL could be lower in the hierarchy, whereas the other rules of CL and the truth and predication rules would be together at the top of the hierarchy. Or there could be, for still other agents, more complex and articulated hierarchies; and, in contrast, for some agents there could be no hierarchy at all (Orilia, 2014, §9.10).

Be this as it may, we should acknowledge that, for an agent who implicitly accepts a hierarchy of rules, a given deduction D may be more valid than another, D', to the extent that D' uses

[26]The truth rule is in essence what Harman calls "the Biconditional Truth Schema".

rules which are lower in the hierarchy, whereas D does not; and two deductions D and D' may be equally valid in that D and D' use rules that have the same rank in the hierarchy. For example, for an agent endorsing a simple two-level hierarchy with all the rules of CL on top and the truth and predication rules with a lower rank, a deduction that uses only rules of CL is more valid than a deduction that uses both rules of CL, and the truth or predication rules. Of course, for an agent who endorses no hierarchy of rules, all deductions are equally valid.

Let us now define what, for a given agent, counts as a blocker of a deduction that the agent strongly accepts.

> *Blocker*. A blocker for S of a deduction D with conclusion C and strongly accepted by S is either (1) the fact that D is explosive, or (2) the fact that there is a non-explosive deduction D' such that: (2a) the conclusion of D' is the opposite of C, (2b) D' is strongly accepted by S, and (2c) D' is for S at least as valid as D.

To illustrate, consider an agent who is exposed to Curry's paradox and thus believes a Curry premise such as (CP) and strongly accepts a deduction D that uses CL and the truth rule to reach a certain conclusion C. Since D is explosive, D has a blocker, by clause (1) of *Blocker*. Moreover, consider an agent S who is rationally inconsistent, since she believes A_1, …, A_n, which by rules of CL entail C, and she also believes B_1, …, B_n, which by rules of CL lead to $\sim C$. Suppose S strongly accepts a deduction D that leads from A_1, …, A_n to C. This deduction D has a blocker, by clause (2) of *Blocker*, if S also strongly accepts a deduction D' leading from the premises B_1, …, B_n to the conclusion $\sim C$, and S takes D' to be at least as valid as D.

We are finally in a position to specify when an entailment is dismissible for S:

> *Dismissibility*. An entailment E that is strongly accepted by S is dismissible for S when every deductive ground that S has for E has a blocker for S.

6 | TESTING THE NEW BRIDGE PRINCIPLE

Let us now see how Wo + b*# works, by focusing on some case studies, C1–C4 below. I shall first consider some dangerous contexts, in which the dismissibility clause plays a crucial role, and then an ordinary context in which the dismissibility clause is bypassed. Let us start with the case of Tom, discussed in Section 4, in order to see how the dismissibility clause takes care of *Explosion*.

> C1. *Explosive arguments*. Consider an agent S, say the Tom of Section 4, who has an interest in a certain proposition P, and is aware of the existence of an explosive deduction $D[A/P]$, which thus constitutes for S a deductive ground for the entailment $A \vDash P$. Assume further that S believes A. Given this, the agent strongly accepts the entailment $A \vDash P$. However, let us assume, S has no other deductive ground for it. Since $D[A / P]$ is explosive, the entailment $A \vDash P$ has a blocker, by clause (1) of *Blocker*. Consequently, the entailment is dismissible for S. Hence, the dismissibility clause of Wo + b*# grants that S has no obligation to believe P, even though S believes the premise, A, of the entailment.[27]

Let us now consider two situations in which, as granted by *Rational inconsistency*, two opposites, C and $\sim C$, or even the contradiction C & $\sim C$, threaten to be propositions that an

[27]Thus, as remarked by a referee, one should never use ECQ to directly add any new proposition to one's beliefs, just because such beliefs host a contradiction, while this does not rule out that ECQ can be used in constructing proofs, in particular *reductio ad absurdum* proofs.

agent ought to believe. If an agent were obliged to believe such things, there would be a conflict with *No contradictions* (except in the special case of an agent who is willing to believe that there are dialetheias). We shall see, however, that in all these situations, the dismissibility clause of Wo + b*# grants that the agent (who by default is assumed not to be a dialetheist) has no obligation to believe two opposites, or a contradiction, in line with *No contradictions*.

C2. *Arguments with opposite conclusions*. Suppose S is rationally inconsistent, because she endorses for some good reason an inconsistent theory T. She has an interest in taking a stand on C or $\sim C$, which are not among S's beliefs. S strongly accepts an entailment E_1 with C as conclusion from a portion T_1 of T, on the basis of a non-explosive deductive ground D_1; and an entailment E_2 with $\sim C$ as conclusion from a portion T_2 of T, on the basis of a non-explosive deductive ground D_2. Despite this, in line with *No contradictions*, S is not obliged to believe both C and $\sim C$. To see this, we have to consider the two possible situations that might arise, and note that in each of them no such obligation is triggered: In Case 1, the deductive grounds D_1 and D_2 are equally valid for S; this implies, by clause (2) of blocker, that both entailments E_1 and E_2 have a blocker, so that both are dismissible for S. Thus, the dismissibility clause of Wo + b*# grants that neither is it the case that S ought to believe C, nor that S ought to believe $\sim C$.[28] In Case 2, One of the deductive grounds, say D_1, is for S more valid than the other, D_2. This implies, by clause (2) of *Blocker*, that E_2 has a blocker, so that it is dismissible for S. Hence, the dismissibility clause of Wo + b*# grants that it is not the case that S ought to believe $\sim C$.

C3. *Deriving a contradiction*. Suppose S constructs a deductive argument D with a contradiction, $C \,\&\, \sim C$, as conclusion, as follows. First, S derives two opposites C and $\sim C$ from a rationally inconsistent belief set, or via a paradox such as Russell's or the Liar, and then infers $C \,\&\, \sim C$ via Conjunction Introduction. We may then assume that S strongly accepts D and consequently strongly accepts an entailment E with conclusion $C \,\&\, \sim C$, for which D is the deductive ground. Nevertheless, given the dismissibility clause of Wo + b*#, S is not obliged to believe $C \,\&\, \sim C$: the entailment E is dismissible for S to the extent that its deductive ground D has a blocker for S. A deduction D' that S strongly accepts and has $\sim(C \,\&\, \sim C)$ as conclusion would be such a blocker. Assuming CL, there is of course a deduction with $\sim(C \,\&\, \sim C)$ as conclusion. We made the default assumption that a typical agent accepts CL, and by taking S to be typical, we can assume that S strongly accepts one such deduction. Of course, many other logics weaker than CL will also grant a deduction with $\sim(C \,\&\, \sim C)$ as conclusion, but if the agent accepts a paraconsistent logic that does not grant a deduction of $\sim(C \,\&\, \sim C)$, there is no blocker for the entailment E with conclusion $C \,\&\, \sim C$, and E is not dismissible for S. In this case, the dismissibility clause of Wo + b*# is ineffective and S ought to believe the contradiction C & $\sim C$. However, this is as it should be, if S is a dialetheist.

Finally, let us consider an ordinary context in which an agent S deductively infers from some previous beliefs a conclusion in which S has an interest, without exploiting an explosive argument or being affected by any rational inconsistency that she may have. We should expect that in this case, in line with *Non-dismissible acknowledged entailments*, the agent has an

[28]As noted by the above mentioned referee, this case highlights that clause (2) of *Blocker* embodies a principle of Pyrrhonian isosthenia such as this: if an agent S acknowledges that there is a reason, R_1, for believing A, and a Reason, R_2, for believing $\sim A$, and S deems the two reasons as equally strong, then S cannot rationally employ R_1 (R_2) for believing A ($\sim A$).

obligation to believe the conclusion in question, unless of course *S* gives up one or more of the previous beliefs that work as premises in the deduction. The expectation is fulfilled, because in such a case the dismissibility clause of Wo + b*# does not get in the way of the attribution of this obligation to the agent. For illustration, we can go back to the story of Tom who would like to see his friend Mary.

> C4. *Ordinary inference*. Tom has an interest in the proposition that (c) Mary in London. Tom believes that (a) either Mary is in London or Mary is in Paris, and he has also just come to believe that (b) Mary is not in Paris. Hence, he constructs a straightforward deduction *D*, granted by CL, that from the two premises (a) and (b) leads to conclusion (c) by Disjunctive Syllogism. Tom strongly endorses *D* and accordingly strongly accepts an entailment, *E*, with premises (a) and (b), and conclusion (c), for which *D* is the deductive ground. According to Wo + b*#, this implies, in line with *Non-dismissible acknowledged entailments*, that Tom ought to believe (c), assuming of course that he keeps believing the premises (a) and (b) of the entailment *E*.[29] For the dismissibility clause of Wo + b*# is ineffective in this case, since *E* is not dismissible for Tom; neither of the two reasons that could lead to a blocker for *E* is in play. First of all, the deductive ground *D* of *E* is not explosive. Second, we may assume, Tom does not have beliefs that could work as premises in a non-explosive deduction *D*′, with a conclusion opposite to the conclusion of *D*, strongly accepted by Tom, and which is for Tom at least as valid as *D*.

7 | CONCLUSION

The thesis of the normativity of logic can be supported by providing an appropriate bridge principle that connects entailments and norms of belief. Harman's sceptical challenge casts doubt on this thesis, given the difficulty in putting forward a bridge principle that complies with the relevant desiderata. I have put forward a list of desiderata that corrects and goes beyond the desiderata proposed by Steinberger, and I have proposed a bridge principle, Wo + b*#, that takes care of all the items in this list and thus fares well in supporting the thesis of the normativity of logic. With Wo + b*# at hand, this thesis can be upheld and Harman's sceptical challenge set aside.

ACKNOWLEDGMENTS

An earlier version of this paper was presented at the Department of Philosophy in the University of Florence in December 2017, where I received some useful feedback, in particular from Riccardo Bruni, and also in many subsequent discussions. An anonymous referee from *Theoria* (mentioned in the footnotes) offered some valuable comments. This work was supported by the Italian Ministry of Education, University and Research through the PRIN 2017 project "The Manifest Image and the Scientific Image" prot. 2017ZNWW7F_004. Open Access Funding provided by Universita degli Studi di Macerata within the CRUI-CARE Agreement.

ORCID

Francesco Orilia https://orcid.org/0000-0002-3556-6552

REFERENCES

Bradley, D. (2021) Ought-contextualism and reasoning. *Synthese*, 199, 2977–2999.

[29]Given option W, Tom could discharge his obligation by revising his beliefs in such a way as to drop at least one of these premises, but we are assuming here that he has no good reason to do so.

Broome, J. (2000) Normative Requirements. In: Dancy, J. (Ed.) *Normativity*. Oxford: Blackwell, pp. 78–99.
Broome, J. (2013) *Rationality through reasoning*. Oxford: Wiley-Blackwell.
Celani, L. (2015) *Logic in theory and in practice. The normative status of logic*. PhD thesis. University of St. Andrews: St. Andrews.
Field, H. (2008) *Saving truth form paradox*. Oxford: Oxford University Press.
Field, H. (2009) What is the normative role of logic? *Proceedings of the Aristotelian Society*, 83, 251–268.
Field, H. (2015) What is logical validity? In: Caret, C. & Hjortland, O. (Eds.) *Foundations of logical consequence*. Oxford: Oxford University Press, pp. 33–70.
Gupta, A. & Belnap, N. (1993) *The revision theory of truth*. Cambridge: MIT Press.
Harman, G. (1984) Logic and reasoning. *Synthese*, 60, 107–127.
Harman, G. (1986) *Change in view: principles of reasoning*. Cambridge: The MIT Press.
Harman, G. (2002) Internal critique: a logic is not a theory of reasoning and theory of reasoning is not a logic. In: Gabbay, D.M., Johnson, R.H., Ohlbach, H.J. & Woods, J. (Eds.) *Handbook of the logic of argument and inference: the turn towards the practical. Studies in logic and practical reasoning*, Vol. 1. Amsterdam: Elsevier, pp. 171–186.
Kripke, S. (1975) Outline of a theory of truth. *Journal of Philosophy*, 72, 690–716.
MacFarlane, J. (2004) In what sense (if any) is logic normative for judgements? Unpublished, presented at the *2004 Central Division APA symposium on the normativity of logic*, Available at: https://johnmacfarlane.net/normativity_of_logic.pdf.
Michael, M. (2016) On a 'most telling' argument for paraconsistent logic. *Synthese*, 193, 3347–3362.
Milne, P. (2009) What is the normative role of logic? *Proceedings of the Aristotelian Society*, 83, 269–298.
Nozick, R. (1981) *Philosophical explanations*. Cambidge: Harvard University Press.
Orilia, F. (2000) Property theory and the revision theory of definitions. *Journal of Symbolic Logic*, 65, 212–246.
Orilia, F. (2014) Degrees of validity and the logical paradoxes. In: Weber, E., Meheus, J. & Wouters, D. (Eds.) *Logic, reasoning and rationality*. Berlin: Springer, pp. 179–196.
Priest, G. (1987) *In contradiction: a study of the transconsistent*. Dordrecht: Martinus Nijhoff. 2nd edition, Oxford: Oxford University Press, 2006.
Priest, G., Tanaka, K. & Weber, Z. (2022) Paraconsistent logic. In: Zalta, E.N. (Ed.) *The Stanford encyclopedia of philosophy*. Spring 2022 edition, Availabe at: https://plato.stanford.edu/archives/spr2022/entries/logic-paraconsistent/
Sainsbury, R.M. (2002) What logic should we think with? In: O'Hear, A. (Ed.) *Logic, thought and language (Royal Institute of Philosophy supplement 51: 1–17)*. Cambridge: Cambridge University Press.
Shapiro, L. & Beall, J.C. (2021) Curry's paradox. In: Zalta, E.N. (Ed.) *The Stanford encyclopedia of philosophy* (Winter 2021 edition), Available at: https://plato.stanford.edu/archives/win2021/entries/curry-paradox/
Steinberger, F. (2016) Explosion and the normativity of logic. *Mind*, 125, 385–419.
Steinberger, F. (2019) Consequence and normative guidance. *Philosophy and Phenomenological Research*, 98, 306–328.
Steinberger, F. (2019a) Three ways in which logic might Be normative. *Journal of Philosophy*, 116, 5–31.
Steinberger, F. (2020) The normative status of logic. In: Zalta, E.N. (Ed.) *The Stanford encyclopedia of philosophy* (Winter 2020 Edition), Available at: https://plato.stanford.edu/archives/win2020/entries/logic-normative/
Streumer, B. (2007) Reasons and entailment. *Erkenntnis*, 66, 353–374.
Vickers, P. (2013) *Understanding inconsistent science*. Oxford: Oxford University Press.
Wittgenstein, L. (1956) *Remarks on the foundations of mathematics*. Oxford: Blackwell.

How to cite this article: Orilia, F. (2022) A new bridge principle for the normativity of logic. *Theoria*, 88(6), 1274–1292. Available from: https://doi.org/10.1111/theo.12439

APPENDIX—MACFARLANE'S TAXONOMY A

Here I summarise MacFarlane's taxonomy of bridge principles. There are various options regarding the formulation of the logical condition and the doxastic recommendation of a bridge principle and each of them is associated to a *bridge acronym*, as we may call it, obtained from the juxtaposition, in a conventional order, of symbols indicating the selected options. In describing the options, I shall indicate into parentheses the corresponding symbols, and I shall also specify which position such symbols are supposed to have in a bridge acronym. The logical condition could be either an entailment relation between some premises and a conclusion, A_1, …, $A_n \vDash C$, or a propositional attitude of an agent, S, regarding such an entailment relation, which is then "attitudinally constrained" (Steinberger, 2019, p. 313). The options here are of this sort (with the corresponding symbol in fourth position in the acronym): the entailment is constrained by an epistemic attitude: "S knows that A_1, …, $A_n \vDash C$" (k); the entailment is constrained by a doxastic attitude: "S believes that A_1, …, $A_n \vDash C$" (b); the entailment is not constrained by an attitude (this is signalled by the absence of a specific symbol; I shall speak of an *absent attitude* option in this case). The doxastic recommendation involves a deontic operator somehow associated to a conditional regarding the doxastic attitude of the agent about the premises and the conclusion, where the attitude could be either *positive*, *believing* (+), or *negative*, *not disbelieving* (−) (third position in the acronym). There are three options regarding the deontic operator: obligation (o), permission (p), and pro tanto reason (r) (third position in the acronym). Finally, there are three options regarding the scope of the deontic operator with respect to the conditional in question (first position in the acronym): wide scope (W), for example, it ought to be the case that: S believes C, if S believes A_1, …, A_n; the deontic operator has narrow scope with respect to the consequent (C), for example, if S disbelieves A_1, …, A_n, then S has a reason to believe C; the deontic operator is applied to both sides of the conditional (B), for example, if S is permitted to believe A_1, …, A_n, then S is permitted to believe C.

As an example, consider

> Wo + b. If S believes A_1, …, $A_n \vDash C$, then: S ought to believe C, if S believes A_1, …, A_n.

This is obtained by selecting (i) option W (deontic operator of the doxastic recommendation taking wide scope), (ii) option o (obligation deontic operator in the doxastic recommendation), (iii) option + (positive doxastic polarity in the doxastic recommendation), (iv) option b (entailment constrained by a doxastic attitude). The order (i)–(iv) corresponds to the conventional order to be used in generating the acronym by juxtaposing the symbols corresponding to the choices in question, hence the label "Wo + b" for this bridge principle. For further examples, turn to the three bridge principles considered by Harman and reported in Section 1.

Cr + is obtained by (i) selecting option C (deontic operator of the doxastic recommendation taking narrow scope), (ii) option r (reason deontic operator in the doxastic recommendation), (iii) option + (positive doxastic polarity in the doxastic recommendation), and (iv) absent attitude option (entailment not constrained by a doxastic attitude).

Cr + k is obtained in the same way as Cr+, except that option k (entailment constrained by an epistemic attitude) is selected instead of the absent attitude option.

Co + is obtained in the same way as Cr+, except that option o (obligation doxastic operator) is picked instead of option r.

It should be noted that the bridge principle accepted by Steinberger, namely Wr + b* (see Section 2), exceeds MacFarlane's taxonomy, since it involves an embellishment, the interest clause, not considered in the taxonomy. This is added to a principle in the taxonomy, namely Wr + b. The asterisk in the acronym "Wr + b*" signals the embellishment. Similarly, my

principle Wo + b*# (see Section 5) exceeds MacFarlane's taxonomy, in that it involves two embellishments not considered in the taxonomy, and added to a principle in the taxonomy, namely Wo + b. These are the interest clause, signalled as in Steinberger by an asterisk in the acronym, and the dismissibility clause, signalled by an hashtag.