BANK ĊENTRALI TA' MALTA
EUROSISTEMA
CENTRAL BANK OF MALTA

CENTRAL BANK OF MALTA
WORKING PAPER

MaCGE-MOD:

Malta's Computable General Equilibrium Model*

Ian P. Cassar† Nathaniel Debono‡ Stefano Deriu§ Rosita Pretaroli¶

Noel Rapa‖ Francesca Severini** Claudio Socci††

WP/02/2023

*The authors would like to thank Governor Scicluna, Deputy Governor Demarco, Deputy Governor Bonello and Dr. Aaron George Grech for comments and suggestions raised throughout the development of this project. This work has also greatly benefited from comments raised by various participants in an internal seminar. Any remaining errors are the authors' own. The views expressed in this paper are the authors' own and do not necessarily reflect the views of the Central Bank of Malta.

Correspondence: `publicrelations@centralbankmalta.org`.

†Ian P. Cassar is a Senior Lecturer in Economics at the University of Malta and a Visiting Senior Lecturer in Economics at the University of Macerata.

‡Nathaniel Debono is corresponding author and a Senior Research Economist in the Modelling Office, within the Economic Research Department of the Central Bank of Malta.

§Stefano Deriu is assistant professor of Economic Policy at the University of Macerata - Department of Economics and Law.

¶Rosita Pretaroli is associate professor of Economic Policy at the University of Macerata - Department of Political Science, Communication and International Relations.

‖Noel Rapa is the Head of the Economic Research Department of the Central Bank of Malta.

**Francesca Severini is assistant professor of Economic Policy at the University of Macerata - Department of Education, Cultural Heritage and Tourism.

††Claudio Socci is full professor of Economic Policy at the University of Macerata - Department of Economics and Law.

Abstract

This paper introduces the latest addition to the modelling toolkit of the Central Bank of Malta: a static Computable General Equilibrium model for Malta named MaCGE-MOD. Developed through a collaboration with the University of Macerata, the model is a multi-input, multi-output and multi-sector model, calibrated on a Social Accounting Matrix which illustrates the income flows among production processes and institutional sectors in Malta. The model constitutes a set of equations which describe the circular flow of income in the economy and the behaviour of economic agents. It is a useful tool to determine *ex ante* the impacts of policy decisions or exogenous shocks once all the associated direct, indirect and induced effects have propagated through the economy. As it is based on a Social Accounting Matrix, the model can provide disaggregated sectoral analyses to complement analyses of the aggregate effects of policy-making scenarios or exogenous shocks. Thus, MaCGE-MOD can potentially be an important tool for policy makers to identify effects not easily discernible from the other aggregated models currently available.

JEL Classification: C61; C68; D58; E16; E17; H30

Keywords: Computable General Equilibrium, Social Accounting Matrix, Malta, simulations, sectoral analyses.

Contents

1 Introduction

The need for economic tools to anticipate and/or evaluate the effects of policy measures or economic shocks in a timely and precise manner has seen the development of different classes of models over time. In the local context for example, the Central Bank of Malta [CBM] has in recent years developed its main macro-econometric model, STREAM (Grech et al., 2013), and a Dynamic Stochastic General Equilibrium model, MEDSEA (Rapa, 2016), with both models being continually and substantively updated. The latest model to be introduced within the modelling toolkit of the CBM is a Computable General Equilibrium (CGE) model calibrated on the Social Accounting Matrix (SAM) for Malta. A CGE model comprises a set of equations that describe the functioning and the relationships occurring within the whole economy. The solution to a CGE model necessitates the equilibrium of all markets under consideration and the compliance with budgetary and resources' constraints to be classified as a "general equilibrium" model (Dixon and Parmenter, 1996; Wing, 2004; Burfisher, 2021). This class of models feeds off actual economic data for a base year, and numerically "compute" a solution to quantify the effects of a shock or policy decision on prices and quantities (Berck et al., 1991; Kancs and Kremers, 2002). The output from the model can be expressed by all macroeconomic variables under consideration and the policy effects found by comparing this solution with the benchmark data (Böhringer and Löschel, 2006).

CGE models are rooted in the concept of Walrasian general equilibrium and the circular flow of income in the economy (Wing et al., 2007). In principle, the Walrasian general equilibrium prevails at the set of prices and quantities which clear perfectly competitive markets by equating demand to supply (Walras, 1874; Robson et al., 2018). CGE models developed over time have preserved many of the Walrasian features although some have been relaxed. In this light, Willenbockel (1994) distinguishes between two classes of CGE models. The first set of CGE models maintain Walrasian principles, assuming that the decisions of the agents in the economy are based on optimisation decisions, price movements clear all markets, and optimal demand remains unchanged if prices and income change by equivalent proportions. Apart from relaxing some of the Walrasian features, the second group of models - described as "less orthodox" (Bellù, 2014) - allow for the possibility of the formation of involuntary unemployment and imbalances in the Government's account, among others (e.g. Socci et al. (2021)).

The equations that feature in a CGE model convey the circular flow of income captured by the SAM which underpins the CGE model (Robinson, 1991; Thiele et al., 1993). The SAM is an evolution of the Input-Output (I-O) tables since it provides additional detail about flows involving institutional sectors (Reinert and Roland-Holst, 1997). In fact, the SAM conveniently illustrates in the same framework the multi-sectoral and circular income flow in its various stages represented by the production, the value-added generation, the primary income allocation, the secondary income distribution, the income utilisation, and the capital accumulation (Ciaschini et al., 2012). By solving the system of simultaneous equations in the presence of a particular shock or policy in place, SAM-based CGE models generate a new set of disaggregated macroeconomic variables, with differences relative to the baseline ones allowing the quantification of the economic impact of the shock. Therefore, SAM-based CGE models can be considered attractive tools in determining the effects of policy decisions. In fact, CGE models have been extensively applied in various contexts, including but not limited to the study of tourism benefits (Narayan, 2004; Pratt, 2012), costs associated with sickness (Smith et al., 2011), and the economic effects of gender (Severini et al., 2019), employment (Zotti et al., 2020), environmental and climate change policies (Allan et al., 2014; Carbone and Rivers, 2017; Li and Masui, 2019), and other fiscal reforms (Guo and Shi, 2021).

MaCGE-MOD, the CGE model developed for Malta through a collaboration between the CBM and the University of Macerata is a multi-input, multi-output and multi-sector SAM-based CGE model. It deviates from the traditional Walrasian features, allowing for the formation of involuntary unemployment and the presence of rigidities in consumption decisions for the Government, and thus in the Government's closing balance. The model rests on a SAM which includes information about 44 production activities in the Maltese economy which make use of intermediate goods, labour and capital to generate their domestic production, which in turn can be disaggregated into 44 different commodities. It also encompasses the income transactions linking the income generated by the production processes and 6 institutional sectors, allowing for a comprehensive analysis of the transmission mechanism of shocks hitting the Maltese economy. The highly-disaggregated economic data provided by the SAM is one distinct feature by which MaCGE-MOD complements the other models already forming part of the modelling toolkit of the CBM. This detailed information enables disaggregated analyses of the policy-induced effects which, despite clearly complementing other aggregate analyses of the main macroeconomic effects of a shock, are not easily attainable through the other models currently available. In addition, being a *static* (rather than *dynamic*) model, simulation results on the basis of the CGE model capture all direct, indirect and induced effects of the underlying shock or policy in the medium run (Socci et al., 2021).

This paper proceeds as follows. Section 2 gives a detailed description of the Maltese SAM which underpins the CGE model for Malta. Section 3 describes the model before the calibration strategy is outlined in Section 4. Finally, Section 5 illustrates the macroeconomic and sectoral effects derived from applying the model to simulate hypothetical macroeconomic and sectoral shocks which might hit the Maltese economy.

2 The Social Accounting Matrix for Malta

The Social Accounting Matrix, henceforth referred to as "SAM", stands at the basis of the CGE model as it serves as the source of data for the calibration of the model parameters. It records the value of all transactions involving commodities, activities and institutional sectors that take place in a particular economy during a given year. Starting from the Supply and Use Tables, the SAM conveniently illustrates the circular flow of income in the economy (Mainar-Causapé et al., 2018; Zotti et al., 2020). It is a square table whereby economic agents are represented by a set of rows and columns identifying the receipts and the expenditure of their respective accounts. In general terms, an element x found in a row account i and a column account j, denoted by $x_{i,j}$, represents the receipts of account i from account j. By definition, the SAM is a "balanced" matrix whereby for every account, the total receipts (i.e. total row value) must equal its total expenditures (i.e. total column value). This follows from the concept of *value conservation* ingrained in the Walrasian general equilibrium, which necessitates that total expenditures must equate to total revenues (Wing, 2004).[1]

The SAM underpinning the current version of MaCGE-MOD is the most recently available and covers the year 2015. By definition, since the CGE model is in baseline calibrated on the values documented in the SAM, the model's solution in the absence of shocks must replicate the exact same flows documented in the SAM. A template of the SAM built to represent the income circular flow in Malta is presented in Figure 1. It comprises a disaggregation of the accounts, and thus of the economic flows, into a set of: Commodities, Activities, Primary Factors, Trade and Transportation Margins, Taxes, Institutional Sectors, and Capital Formation.

[1]For more general details about the construction and principles of the SAM, see Mainar-Causapé et al. (2018).

Figure 1: A template of the Social Accounting Matrix for Malta

		Commodity (g = 1,…,44)	Activity (a = 1,…,44)	Value Added at factor cost (f): Compensation of employees	Value Added at factor cost (f): Gross operating surplus	Taxes (t): Trade margins	Taxes (t): Net Taxes less Subsidies	Taxes (t): Other taxes on activities minus subsidies	Taxes (t): Taxes on income	Taxes (t): Other current taxes	Institutional Sectors (k): Non-financial corporations Nfc	Institutional Sectors (k): Financial corporations Fc	Institutional Sectors (k): Households Hh	Institutional Sectors (k): Non-profit institutions Npi	Institutional Sectors (k): Government Gov	Institutional Sectors (k): Rest of the world RoW	Capital formation
Commodity (g = 1,…,44)			$U_{(g,a)}$										$Chh_{(g)}$	$Cnpi_{(g)}$	$Cgov_{(g)}$	$Crow_{(g)}$	$Inv_{(g)}$
Activity (a = 1,…,44)		$S_{(a,g)}$															
Value Added at factor cost (f)	Compensation of employees		$COE_{(a)}$														
	Gross operating surplus		$GOS_{(a)}$														
Taxes (t)	Trade margins	$MRG_{(g)}$															
	Net Taxes less Subsidies	$NTAX_{(g)}$															
	Other taxes on activities minus subsidies		$OTAX_{(a)}$														
	Taxes on income										$TINC_{(Nfc)}$	$TINC_{(Fc)}$	$TINC_{(Hh)}$	$TINC_{(Npi)}$	$TINC_{(Gov)}$		
	Other current taxes												$TOTH_{(Hh)}$			$TOTH_{(Row)}$	
Institutional Sectors (k)	Non-financial corporations Nfc				$GOS_{(Nfc)}$						$TR_{(Nfc,Nfc)}$	$TR_{(Nfc,Fc)}$	$TR_{(Nfc,Hh)}$	$TR_{(Nfc,Npi)}$	$TR_{(Nfc,Gov)}$	$TR_{(Nfc,Row)}$	
	Financial corporations Fc				$GOS_{(Fc)}$						$TR_{(Fc,Nfc)}$	$TR_{(Fc,Fc)}$	$TR_{(Fc,Hh)}$	$TR_{(Fc,Npi)}$	$TR_{(Fc,Gov)}$	$TR_{(Fc,Row)}$	
	Households Hh			$LAB_{(Hh)}$	$GOS_{(Hh)}$						$TR_{(Hh,Nfc)}$	$TR_{(Hh,Fc)}$	$TR_{(Hh,Hh)}$	$TR_{(Hh,Npi)}$	$TR_{(Hh,Gov)}$	$TR_{(Hh,Row)}$	
	Non-profit institutions Npi				$GOS_{(Npi)}$						$TR_{(Npi,Nfc)}$	$TR_{(Npi,Fc)}$	$TR_{(Npi,Hh)}$	$TR_{(Npi,Npi)}$	$TR_{(Npi,Gov)}$	$TR_{(Npi,Row)}$	
	Government Gov				$GOS_{(Gov)}$		$NTAX_{(Gov)}$	$OTAX_{(Gov)}$	$TINC_{(Gov)}$	$TOTH_{(Gov)}$	$TR_{(Gov,Nfc)}$	$TR_{(Gov,Fc)}$	$TR_{(Gov,Hh)}$	$TR_{(Gov,Npi)}$	$TR_{(Gov,Gov)}$	$TR_{(Gov,Row)}$	
	Rest of the world RoW	$M_{(g)}$		$LAB_{(Row)}$			$NTAX_{(Row)}$	$OTAX_{(Row)}$			$TR_{(Row,Nfc)}$	$TR_{(Row,Fc)}$	$TR_{(Row,Hh)}$	$TR_{(Row,Npi)}$	$TR_{(Row,Gov)}$		
Capital formation											$SAV_{(Nfc)}$	$SAV_{(Fc)}$	$SAV_{(Hh)}$	$SAV_{(Npi)}$	$SAV_{(Gov)}$	$SAV_{(Row)}$	

Starting from the production side, the SAM for Malta encompasses a disaggregation of 44 different commodities.[2] Reading the associated flows column-wise shows the sources of supply of each good g, where $g \in \{1, ..., 44\}$. The matrix S, containing the elements $s_{(a,g)}$, shows the domestic production of each good g by each activity a, such that summing these elements across a yields the total domestic production of good g. The imports value of good g is in turn represented by the element $m_{(g)}$ in the (1×44) vector M, which amount is received by the Rest of World (RoW). Trade and transportation margins ($mrg_{(g)}$) and any taxes levied on products ($ntax_{(g)}$) are recorded separately. As the SAM is itself the representation of macroeconomic balance, total supply of any good g (identified by the 'column' total) equals the total demand, the sources of which are observed from the row entries of the account of good g. The demand components comprise the intermediate use of the good by each activity a, denoted by $u_{(g,a)}$, the final demand for consumption by households and non-profit institutions ($Chh_{(g)}$ and $Cnpi_{(g)}$), Government ($Cgov_{(g)}$), Rest of World ($Crow_{(g)}$) and the demand of goods for investment purposes, denoted by $Inv_{(g)}$. To provide a concrete example of the interpretation of the flows in the SAM, we can focus on table U, which collects the flows accruing from the 'activities' to the 'goods' account. Each element in $u_{(g,a)}$ records the payments by activity a into the account of good g in order to acquire commodity g to be used for intermediate consumption.

Corresponding one-for-one to the disaggregation of commodities, the Maltese SAM also includes transactions involving 44 activities which produce commodities. While information from the Maltese SAM shows that most of each activity's production is predominantly devoted to one principal commodity, some activities also produce other secondary goods in smaller quantities.[3] The row entry of the 'activities' account features the table of production containing the elements $s_{(a,g)}$, showing the amount of each commodity g produced by activity a, where $a \in \{1, ..., 44\}$, evaluated at basic prices (Mainar-Causapé et al., 2018). To produce its output, each activity a uses commodities as intermediate inputs ($u_{(g,a)}$), employs labour and capital, and also incurs taxes (less subsidies) denoted by $otax_{(a)}$. The Value Added by activity can be obtained as the sum of compensation of employees, gross operating surplus and production taxes less subsidies. As regard to the components of Value Added, the activities pay compensation for employees and gross operating surplus as the compensation for the use of primary factors, which inflows are allocated to the institutional sectors according to their property share.[4]

[2] The classification of commodities is provided in Appendix A.

[3] Within the framework of the SAM, this is illustrated by the non-necessarily-zero off-diagonal elements in the domestic production matrix, S.

[4] As evidenced by the SAM, compensation of employees is mostly received by households, although small amounts are received by RoW, the latter representing labour income sent abroad. On the other hand, all institutional sectors with the exception of RoW are endowed with capital, such that Gross Operating Surplus is

Trade and transportation margins are incurred in the process of delivering the commodities produced by activities to the markets. As a result, they are recorded in the column of the 'Commodities' account. The demand for commodities is valued at purchaser prices and thus is already inclusive of the trade and transportation margins, explaining why the 'Trade Margins' account's column contains no entries. The 'taxes' account in the Maltese SAM comprises four different sets of taxes, respectively levied on commodities ($Ntax$), activities ($Otax$) and Institutional Sectors ($Tinc$, $Toth$). The SAM shows that the total tax revenue generated in the economy - calculated as the sum of these tax payments - is predominantly collected by Government, although smaller amounts pertaining to $Ntax$ and $Otax$ are also directed to RoW.

Turning to the institutional sectors, their source of primary income derives from their respective endowment of primary factors (labour and capital). In fact, they receive compensation of employees and gross operating surplus by activity. As described previously, the income taxes and other current taxes incurred by institutional sectors on the basis of their primary income are collected by Government, alongside the other taxes levied on commodities and activities, some of which are also collected by RoW. In addition, the RoW account receives income flows M from the imports. The flows represented by TR in Figure 1 represent the secondary income distribution, which captures the transfers that take place among the institutional sectors themselves. All these flows determine the disposable income of each institutional sector, which in turn can either be allocated to present consumption or be set aside to be possibly consumed in the future.[5] The demand of each institutional sector for good g is represented by the flows recorded in the block of elements headed by the commodities g (from the row side) and the Institutional Sectors k (from the column side). Savings by Institutional Sectors are recorded in the capital accumulation account and indicated by $SAV_{(k)}$.

3 Model Setup

This section documents the main properties of MaCGE-MOD and exhibits the equations that characterise the model. The general setup of the production side of the model is graphically shown in Figure 2. The lower part shows the production function by describing the formation process of output by activity. To produce its output Y_a, each activity optimally combines intermediate goods and value added via a Constant Elasticity of Substitution (CES) function.[6]

directed to all institutional sectors, except RoW.

[5]The only exceptions are the financial and non-financial corporations who do not consume any commodities but devote all their disposable income to savings.

[6]While this is stated in general terms, it is recognised that in Malta's CGE model, intermediate goods and value added are assumed to be straightforwardly combined by a Leontief function. Further details are available

In turn, the total intermediate consumption of activity a is some composite of the goods g that are used as intermediate products, whereas value added by activity is produced by optimally combining capital and labour resources in a CES production function. Each activity is assumed to be able to produce more than one good, such that good g may be produced by more than one activity. As a result, the amount of good g that is produced domestically from the activities' operations, denoted by Y_g^D, is related to Y_a as some aggregate of the amount of good g produced by each activity a separately, which quantity is denoted by $Y_{g,a}$. In turn, the total quantity of domestic output of commodity g, Y_g^D, combined with the quantity imported, Y_g^M, yield the total supply of good g in the economy, denoted by Y_g. This can then either be consumed domestically or exported, in which case it is consumed by RoW. The amount consumed locally is split into different uses: by activities as intermediate products in their production process, by households, NPIs or Government as goods destined to final consumption, and demanded for investment purposes.

We now proceed to determine the optimal decisions of the respective economic agents by solving the optimisation problems which characterise the model as illustrated in Figure 2. The optimal decisions of the respective economic agents in the model are derived from the solutions to a series of nested problems. We start from the nested problems characterising the production function by activity, before proceeding to consider the production function by commodities.

3.1 Production Process

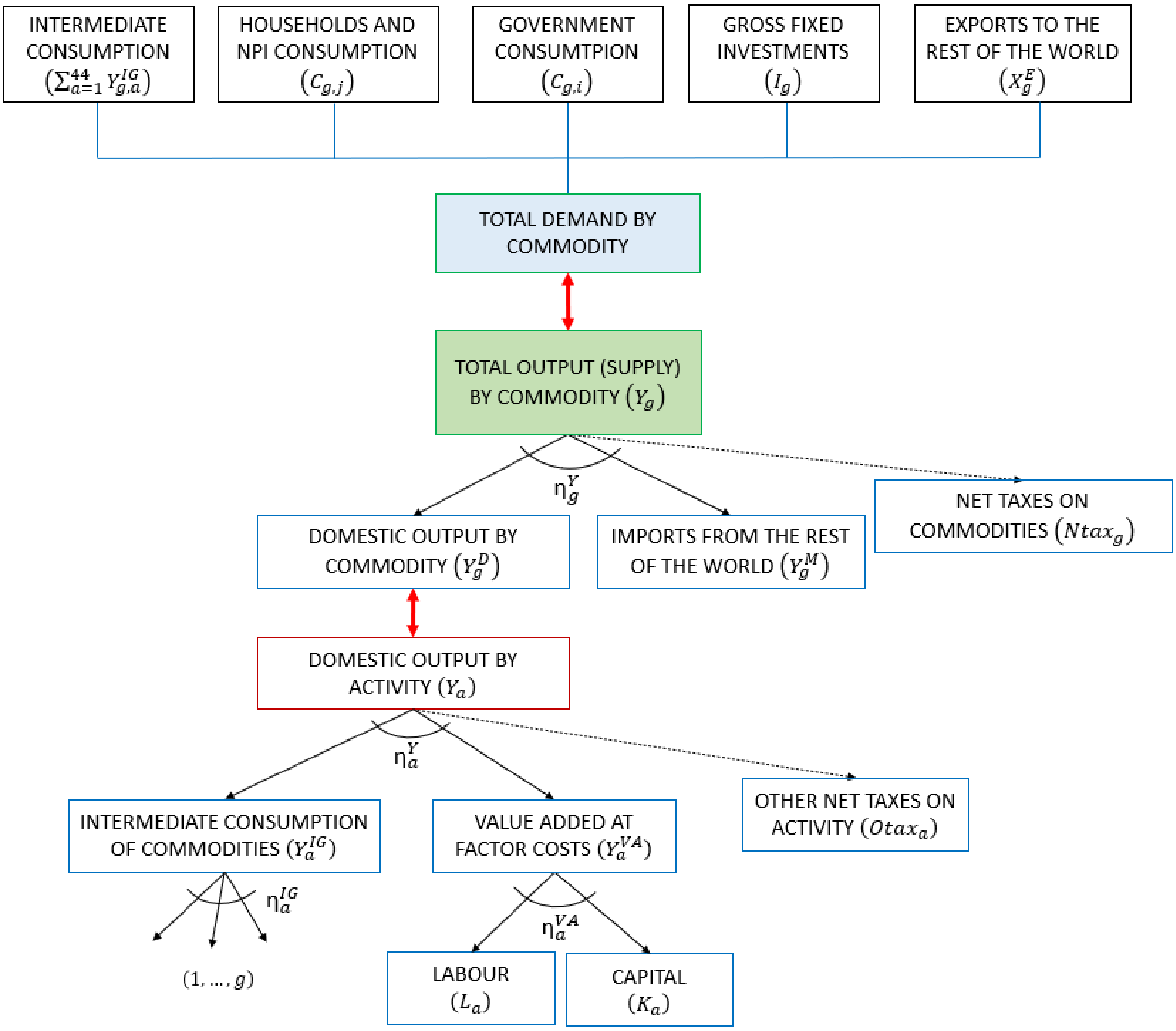
3.1.1 Output by activity

Output by activity, denoted by Y_a, is produced by an infinite amount of perfectly competitive firms operating in each activity $a \in (1, 44)$. To produce their output, firms within each activity combine a composite intermediate good, Y_a^{IG}, with value added, Y_a^{VA}, in a CES production function. The respective optimal quantities of Y_a^{IG} and Y_a^{VA} are chosen as firms seek to minimise their cost of production, C_a^Y, subject to a CES production function combining intermediate goods and value added. Total costs are obtained by summing the costs associated with the two inputs, evaluated at their respective prices, P_a^{IG} and P_a^{VA}. This optimisation problem can be written as:

$$\min_{Y_a^{IG}, Y_a^{VA}} \mathcal{C}_a^Y = P_a^{IG} Y_a^{IG} + P_a^{VA} Y_a^{VA} \tag{1}$$

in Section 4.4.

Figure 2: Description of the Production in the model (supply and demand side)



st.

$$Y_a = \left[(\iota_a^{IG})^{\frac{1}{\eta_a^Y}} Y_a^{IG\,\frac{\eta_a^Y - 1}{\eta_a^Y}} + (1 - \iota_a^{IG})^{\frac{1}{\eta_a^Y}} Y_a^{VA\,\frac{\eta_a^Y - 1}{\eta_a^Y}} \right]^{\frac{\eta_a^Y}{\eta_a^Y - 1}} \tag{2}$$

where ι_a^{IG} is the share of total intermediate goods used by activity a in producing its output and η_a^Y is the elasticity of substitution between intermediate goods and value added. Forming the Lagrangian we can write:

$$\min_{Y_a^{IG}, Y_a^{VA}} \mathcal{L}_a^Y = P_a^{IG} Y_a^{IG} + P_a^{VA} Y_a^{VA} + \kappa_a^Y \left[Y_a - \left[(\iota_a^{IG})^{\frac{1}{\eta_a^Y}} Y_a^{IG\,\frac{\eta_a^Y - 1}{\eta_a^Y}} + (1 - \iota_a^{IG})^{\frac{1}{\eta_a^Y}} Y_a^{VA\,\frac{\eta_a^Y - 1}{\eta_a^Y}} \right]^{\frac{\eta_a^Y}{\eta_a^Y - 1}} \right]$$

where κ_a^Y is the Lagrange multiplier associated with the production function, representing the marginal costs of production. In view of the perfectly competitive market structure assumed within the context of the CGE model, κ_a^Y corresponds to P_a^Y, defined as the price of output of activity a.[7]

The resulting F.O.Cs are:

$$\begin{aligned} \frac{\partial \mathcal{L}_a^Y}{\partial Y_a^{IG}} = 0 \Leftrightarrow P_a^{IG} - \kappa_a^Y \left(\frac{\eta_a^Y}{\eta_a^Y - 1} \right) & \left[(\iota_a^{IG})^{\frac{1}{\eta_a^Y}} Y_a^{IG\,\frac{\eta_a^Y - 1}{\eta_a^Y}} + (1 - \iota_a^{IG})^{\frac{1}{\eta_a^Y}} Y_a^{VA\,\frac{\eta_a^Y - 1}{\eta_a^Y}} \right]^{\frac{1}{\eta_a^Y - 1}} \\ \left(\frac{\eta_a^Y - 1}{\eta_a^Y} \right) (\iota_a^{IG})^{\frac{1}{\eta_a^Y}} Y_a^{IG\,\frac{-1}{\eta_a^Y}} = 0 & \end{aligned} \tag{3}$$

$$\begin{aligned} \frac{\partial \mathcal{L}_a^Y}{\partial Y_a^{VA}} = 0 \Leftrightarrow P_a^{VA} - \kappa_a^Y \left(\frac{\eta_a^Y}{\eta_a^Y - 1} \right) & \left[(\iota_a^{IG})^{\frac{1}{\eta_a^Y}} Y_a^{IG\,\frac{\eta_a^Y - 1}{\eta_a^Y}} + (1 - \iota_a^{IG})^{\frac{1}{\eta_a^Y}} Y_a^{VA\,\frac{\eta_a^Y - 1}{\eta_a^Y}} \right]^{\frac{1}{\eta_a^Y - 1}} \\ \left(\frac{\eta_a^Y - 1}{\eta_a^Y} \right) (1 - \iota_a^{IG})^{\frac{1}{\eta_a^Y}} Y_a^{VA\,\frac{-1}{\eta_a^Y}} = 0 & \end{aligned} \tag{4}$$

which after substituting $\kappa_a^Y \equiv P_a^Y$ and simplifying return the following optimal demand functions for Y_a^{IG} and Y_a^{VA}:

$$Y_a^{IG} = \iota_a^{IG} \cdot \left(\frac{P_a^{IG}}{P_a^Y} \right)^{-\eta_a^Y} \cdot Y_a \tag{5}$$

[7] P_a^Y represents the respective "final" price of activity a, i.e. inclusive of the taxes levied on activities' production at the effective rate τ_a^{Otax}, where the latter is derived from the SAM as the share of the amount paid by activity a in $Otax$ in the total value of activity a's output. See equation (71) for a mathematical description of τ_a^{Otax}.

$$Y_a^{VA} = (1 - \iota_a^{IG}) \cdot \left(\frac{P_a^{VA}}{P_a^Y}\right)^{-\eta_a^Y} \cdot Y_a \tag{6}$$

These conditions imply that the optimal ratio of intermediate goods to value added employed by activity a depends on the relative prices of intermediate goods and value added and their respective shares in total output. Since firms operate in a perfectly competitive environment, the overall price of output can be obtained by imposing a zero-profit condition equating the total revenues of activity a (net of applicable taxes) to its total costs, and substituting for the F.O.Cs above:

$$P_a^Y(1 - \tau_a^{Otax})Y_a = P_a^{IG}\left[\iota_a^{IG} \cdot \left(\frac{P_a^{IG}}{P_a^Y}\right)^{-\eta_a^Y} \cdot Y_a\right] + P_a^{VA}\left[(1 - \iota_a^{IG}) \cdot \left(\frac{P_a^{VA}}{P_a^Y}\right)^{-\eta_a^Y} \cdot Y_a\right] \tag{7}$$

This zero-profit condition determines the final price of activity a's output, where P_a^Y is the price of output, inclusive of the relevant payable taxes:

$$P_a^Y(1 - \tau_a^{Otax}) = \left[\iota_a^{IG}(P_a^{IG})^{(1-\eta_a^Y)} + (1 - \iota_a^{IG})(P_a^{VA})^{(1-\eta_a^Y)}\right]^{\frac{1}{1-\eta_a^Y}} \tag{8}$$

3.1.2 Intermediate Consumption by Activity

The demand of intermediate goods required by each activity for the formation of output Y_a may comprise $g \in (1, 44)$ goods. The cost share of good g in the total cost of intermediate goods used by activity a is represented by $\iota_{g,a}^{IG}$. Each activity decides on the optimal quantity of good g to be used as an intermediate product, which quantity is denoted by $Y_{g,a}^{IG}$. Activities minimise their costs subject to a function that aggregates the different goods used by activity a into aggregate intermediate consumption. At this stage, firms are assumed to take the price at which they source the intermediate goods, P_g, as given.[8] This problem can be expressed as:

$$\min_{Y_{g,a}^{IG}} \mathcal{C}_a^{IG} = \sum_{g=1}^{44} P_g Y_{g,a}^{IG} \tag{9}$$

st.

$$Y_a^{IG} = \left[\sum_{g=1}^{44} (\iota_{g,a}^{IG})^{\frac{1}{\eta_a^{IG}}} (Y_{g,a}^{IG})^{\frac{\eta_a^{IG}-1}{\eta_a^{IG}}}\right]^{\frac{\eta_a^{IG}}{\eta_a^{IG}-1}} \tag{10}$$

where η_a^{IG} is the elasticity of substitution among intermediate goods; $Y_{g,a}^{IG}$ is the quantity of

[8] P_g is the "final" price of good g, i.e. inclusive of the taxes levied on the goods at the effective rate τ_g^{Ntax}, where the latter is derived from the SAM as the share of the amount paid by the account of good g in $Ntax$ in the total value of good g. See equation (70) for a mathematical description of τ_g^{Ntax}.

commodity g used by activity a as an intermediate good; and $\sum_{g=1}^{44} \iota_{g,a}^{IG} = 1$. The solution to this problem yields the optimal demand of activity a for good g to be used in its production process:

$$Y_{g,a}^{IG} = \iota_{g,a}^{IG} \cdot \left(\frac{P_g}{P_a^{IG}}\right)^{-\eta_a^{IG}} \cdot Y_a^{IG} \tag{11}$$

where $\kappa_a^{IG} \equiv P_a^{IG}$. Then, changes in the quantity of commodity g demanded by activity a as an intermediate good depend on the aggregate quantity of intermediate goods used by activity a and on the price of good g, relative to the aggregate price of the intermediate goods by activity a. The latter can be obtained by setting a zero-profit condition equating the total cost of activity a's intermediate consumption to the summative cost of using g goods as intermediate goods:

$$P_a^{IG} = \left[\sum_{g=1}^{44} \iota_{g,a}^{IG} (P_g)^{(1-\eta_a^{IG})}\right]^{\frac{1}{1-\eta_a^{IG}}} \tag{12}$$

3.1.3 Value Added by Activity

The value added of each activity a in producing its output is a composite of the labour (L_a) and capital (K_a) it demands. Activities choose the quantities of labour and capital which at their respective costs P^L and P^K minimise the total cost, subject to a CES function that combines the two factors to generate value added.

$$\min_{L_a, K_a} \mathcal{C}_a^{VA} = P^L L_a + P^K K_a \tag{13}$$

st.

$$Y_a^{VA} = \left[(\iota_a^L)^{\frac{1}{\eta_a^{VA}}} L_a^{\frac{\eta_a^{VA}-1}{\eta_a^{VA}}} + (1-\iota_a^L)^{\frac{1}{\eta_a^{VA}}} K_a^{\frac{\eta_a^{VA}-1}{\eta_a^{VA}}}\right]^{\frac{\eta_a^{VA}}{\eta_a^{VA}-1}} \tag{14}$$

where Y_a^{VA} is the value added of each activity; ι_a^L is the share of labour in the generation of value added of activity a; and η_a^{VA} is the elasticity of substitution between labour and capital. The F.O.Cs for this problem yield each activity's optimal demand for L_a and K_a:

$$L_a = \iota_a^L \cdot \left(\frac{P^L}{P_a^{VA}}\right)^{-\eta_a^{VA}} \cdot Y_a^{VA} \tag{15}$$

$$K_a = (1-\iota_a^L) \cdot \left(\frac{P^K}{P_a^{VA}}\right)^{-\eta_a^{VA}} \cdot Y_a^{VA} \tag{16}$$

where the marginal cost of production is equivalent to the price of value added, P_a^{VA}. These two conditions suggest that each activity's optimal decision on the allocation of the primary factors depends on its quantity of value added and the respective factor prices, relative to the aggregate price of value added. The latter can be derived from a zero-profit condition which equates the total cost of value added to the summative cost of labour and capital resources:

$$P_a^{VA} = \left[\iota_a^L (P^L)^{(1-\eta_a^{VA})} + (1-\iota_a^L)(P^K)^{(1-\eta_a^{VA})}\right]^{\frac{1}{1-\eta_a^{VA}}} \tag{17}$$

3.1.4 Total Output (Supply) by Commodity

Turning to the production function by commodity, the total supply of good g, Y_g, is a combination of the total output produced domestically and imports, where the two are assumed to be imperfect substitutes by the Armington (1969) assumption. We assume that for each good g there are an infinite number of aggregators, all operating in a perfectly competitive market that optimally combine imports Y_g^M and the domestic output Y_g^D such as to minimise the costs of producing total output of any commodity Y_g taking the respective prices P_g^D and P_g^M as given. Aggregators face a constraint in the form of a CES function that combines Y_g^D and Y_g^M to supply good g. In mathematical notation, this optimisation problem can be expressed as follows:

$$\min_{Y_g^D, Y_g^M} \mathcal{C}_g^Y = P_g^D Y_g^D + P_g^M Y_g^M \tag{18}$$

st.

$$Y_g = \left[(\iota_g^D)^{\frac{1}{\eta_g^Y}} {Y_g^D}^{\frac{\eta_g^Y-1}{\eta_g^Y}} + (1-\iota_g^D)^{\frac{1}{\eta_g^Y}} {Y_g^M}^{\frac{\eta_g^Y-1}{\eta_g^Y}}\right]^{\frac{\eta_g^Y}{\eta_g^Y-1}} \tag{19}$$

where ι_g^D is the share of the value of good g output that is produced domestically, and η_g^Y is the elasticity of substitution between domestic and imported commodities. The optimal quantities are expressed in (20) and (21), respectively, where in light of the perfectly competitive nature of the goods market in the model, the marginal cost of production is necessarily equivalent to the price of good g, denoted by P_g.

$$Y_g^D = \iota_g^D \cdot \left(\frac{P_g^D}{P_g}\right)^{-\eta_g^Y} \cdot Y_g \tag{20}$$

$$Y_g^M = (1-\iota_g^D) \cdot \left(\frac{P_g^M}{P_g}\right)^{-\eta_g^Y} \cdot Y_g \tag{21}$$

Since the market for commodities is perfectly competitive, the aggregate price of each good g can be derived from the substitution of these F.O.Cs in a zero-profit condition equating the total cost of supplying the good (net of applicable taxes) to the cost of domestic output and imports. When simplified, this yields the aggregate price of good g as a composite of the price of domestic output and the price of imports, where P_g represents the price of commodity g inclusive of the tax levied on commodities at the effective rate τ_g^{Ntax}:

$$P_g(1-\tau_g^{Ntax}) = \left[\iota_g^D (P_g^D)^{(1-\eta_g^Y)} + (1-\iota_g^D)(P_g^M)^{(1-\eta_g^Y)}\right]^{\frac{1}{1-\eta_g^Y}} \tag{22}$$

Similar to the MACGEM-IT model for Italy (Socci et al., 2021) and the I3E model for Ireland (de Bruin and Yakut, 2021), MaCGE-MOD operates under a small-country assumption whereby domestic activity has no bearing on foreign prices. As a result, the price of imports is taken to be exogenously determined by an aggregate foreign price of goods, denoted by P_g^f, and the nominal exchange rate, e, both of which are considered as exogenous.

$$P_g^M = \frac{P_g^f}{e} \tag{23}$$

3.1.5 Domestic Output by Commodity

The total domestic output produced by activities, denoted by Y_g^D is an aggregate of the production of the good g by each activity a, denoted by $Y_{g,a}$. This in turn implies that while each activity generally devotes its production efforts to one principal good, it could also produce other secondary goods. We therefore assume that there are a number of aggregators for each good g, operating in a perfectly competitive market, that optimally choose the quantity of output of each activity a to be devoted for the production of good g, denoted by $Y_{g,a}$. The aggregator minimises the costs associated with the production of good g by optimally choosing $Y_{g,a}$ subject to a CES function and taking the demand for the domestically-produced good g as given.

$$\min_{Y_{g,a}} C_g^D = \sum_{a=1}^{44} P_a^Y Y_{g,a} \tag{24}$$

st.

$$Y_g^D = \left[\sum_{a=1}^{44} (\iota_{g,a}^D)^{\frac{1}{\eta_g^D}} (Y_{g,a})^{\frac{\eta_g^D-1}{\eta_g^D}}\right]^{\frac{\eta_g^D}{\eta_g^D-1}} \tag{25}$$

where $\iota_{g,a}^D$ is the share of the total domestic output of good g that is produced by activity a, such that $\sum_{a=1}^{44} \iota_{g,a}^D = 1$, and η_g^D represents the elasticity of substitution among activities in the formation of domestic output of good g. It is also assumed that each good g produced by activity a is priced at the aggregate price of the activity, such that $P_a^Y = P_{g,a}^Y$, further implying that $Y_a = \sum_{g=1}^{44} Y_{g,a}$. The resulting F.O.C. with respect to $Y_{g,a}$ yields the optimal quantity of good g produced domestically by each activity:

$$Y_{g,a} = \iota_{g,a}^D \cdot \left(\frac{P_g^D}{P_a^Y}\right)^{\eta_g^D} \cdot Y_g^D \tag{26}$$

where the assumption of perfect competition in the model implies that the marginal cost of domestic production is equivalent to the price of domestically-produced good g, denoted by P_g^D. Then, an expression for the price of the domestic output of good g can be obtained through the setting of a zero-profit condition which equates the value of good g that is produced domestically to the total cost incurred by all activities to produce good g, which when simplified yields:

$$P_g^D = \left[\sum_{a=1}^{44} \iota_{g,a}^D (P_a^Y)^{(1-\eta_g^D)}\right]^{\frac{1}{1-\eta_g^D}} \tag{27}$$

3.2 Final Demand by Commodity

The total demand of good g, Y_g, is given by a market clearing condition that aggregates demand for good g raised by each activity a as intermediate good, and final demand of good g raised by the institutional sectors k. The optimal demand for intermediate consumption by activity is given by equation (11), whereas the quantity demanded by each institutional sector k is determined as explained below.

3.2.1 Consumption of commodity by Households and NPIs

The optimal amount of good g consumed by households and NPIs derives from the solution to a utility-maximisation problem in which their utility is maximised subject to their disposable income, which can be used either to consume in the present or be set aside to possibly be consumed in the future (Böhringer et al., 2005). The utility function of households and NPIs, grouped by a set $j \in \{Hh, Npi\}$, is assumed to be a CES function of their present consumption, C_j, and possible future consumption, which we denote by S_j. The institutional sector j chooses the optimal quantity to consume today and how much to leave for the future so as to maximise their utility, U_j, subject to satisfying their budget constraint, which is in turn determined by their disposable income YD_j.

$$\max_{C_j,S_j} \mathcal{U}_j = \left[(\iota_j^C)^{\frac{1}{\eta_j^u}} C_j^{\frac{\eta_j^u - 1}{\eta_j^u}} + (1-\iota_j^C)^{\frac{1}{\eta_j^u}} S_j^{\frac{\eta_j^u - 1}{\eta_j^u}} \right]^{\frac{\eta_j^u}{\eta_j^u - 1}} \tag{28}$$

st.

$$YD_j = P_j^C C_j + P^I S_j \tag{29}$$

where ι_j^C is the weight of present consumption in the utility function by institutional sector; P_j^C is the price of the consumption basket to be consumed by j in the present; P^I represents the price of investment (common for all institutional sectors) and η_j^u is the elasticity of substitution between present and possible future consumption in the utility function by institutional sector j. This optimisation problem is solved by defining the Lagrangian as follows:

$$\max_{C_j,S_j} \mathcal{L}_j^U = \left[(\iota_j^C)^{\frac{1}{\eta_j^u}} C_j^{\frac{\eta_j^u - 1}{\eta_j^u}} + (1-\iota_j^C)^{\frac{1}{\eta_j^u}} S_j^{\frac{\eta_j^u - 1}{\eta_j^u}} \right]^{\frac{\eta_j^u}{\eta_j^u - 1}} + \kappa_j^u (YD_j - P_j^C C_j - P^I S_j)$$

where the specification of the optimisation problem in this way implies that $\kappa_j^u \equiv \frac{1}{P_j^U}$, where P_j^U is the price of utility by institutional sector. The respective F.O.Cs with respect to present and possible future consumption can be derived as follows:

$$\frac{\partial \mathcal{L}_j^U}{\partial C_j} = 0 \Leftrightarrow \left(\frac{\eta_j^u}{\eta_j^u - 1} \right) \left[(\iota_j^C)^{\frac{1}{\eta_j^u}} C_j^{\frac{\eta_j^u - 1}{\eta_j^u}} + (1-\iota_j^C)^{\frac{1}{\eta_j^u}} S_j^{\frac{\eta_j^u - 1}{\eta_j^u}} \right]^{\frac{1}{\eta_j^u - 1}} \left(\frac{\eta_j^u - 1}{\eta_j^u} \right) (\iota_j^C)^{\frac{1}{\eta_j^u}} (C_j)^{\frac{-1}{\eta_j^u}} - \kappa_j^u P_j^C = 0 \tag{30}$$

$$\frac{\partial \mathcal{L}_j^U}{\partial S_j} = 0 \Leftrightarrow \left(\frac{\eta_j^u}{\eta_j^u - 1} \right) \left[(\iota_j^C)^{\frac{1}{\eta_j^u}} C_j^{\frac{\eta_j^u - 1}{\eta_j^u}} + (1-\iota_j^C)^{\frac{1}{\eta_j^u}} S_j^{\frac{\eta_j^u - 1}{\eta_j^u}} \right]^{\frac{1}{\eta_j^u - 1}} \left(\frac{\eta_j^u - 1}{\eta_j^u} \right) (1-\iota_j^C)^{\frac{1}{\eta_j^u}} (S_j)^{\frac{-1}{\eta_j^u}} - \kappa_j^u P^I = 0 \tag{31}$$

Substituting for $\kappa_j^u \equiv \frac{1}{P_j^U}$ and simplifying yields the optimal quantity for households and NPIs to consume today, and, by default, how much to be allocated to possibly be consumed in the future:

$$C_j = \iota_j^C \cdot U_j \cdot \left(\frac{P_j^U}{P_j^C} \right)^{\eta_j^u} \tag{32}$$

$$S_j = (1-\iota_j^C) \cdot U_j \cdot \left(\frac{P_j^U}{P^I} \right)^{\eta_j^u} \tag{33}$$

We can derive an expression for the price of utility by institutional sector j by equating the total value of utility to the sum of the values of present and possible future consumption:

$$P_j^U U_j = P_j^C \left[\iota_j^C \cdot U_j \cdot \left(\frac{P_j^U}{P_j^C} \right)^{\eta_j^u} \right] + P^I \left[(1 - \iota_j^C) \cdot U_j \cdot \left(\frac{P_j^U}{P^I} \right)^{\eta_j^u} \right]$$

which when simplified results in the price of utility by institutional sector j as follows:

$$P_j^U = \left[\iota_j^C (P_j^C)^{(1-\eta_j^u)} + (1 - \iota_j^C)(P^I)^{(1-\eta_j^u)} \right]^{\frac{1}{1-\eta_j^u}} \tag{34}$$

Households and NPIs have the possibility to spread their current consumption over all the goods produced in the economy. This implies that the total consumption of each institutional sector, C_j, is an aggregate of the demand for all the commodities demanded by j. Taking the final price of commodities, P_g, as given, households and NPIs optimally choose their demand for each commodity g, which demand is denoted by $C_{g,j}$, in order to minimise the cost of their current consumption, subject to a transformation technology that aggregates the demand for each good by institutional sector j into aggregate present consumption. In mathematical terms, this optimisation problem is set up as follows:

$$\min_{C_{g,j}} CST_j^C = \sum_{g=1}^{44} P_g C_{g,j} \tag{35}$$

$st.$

$$C_j = \left[\sum_{g=1}^{44} (\iota_{g,j}^C)^{\frac{1}{\eta_j^c}} (C_{g,j})^{\frac{\eta_j^c - 1}{\eta_j^c}} \right]^{\frac{\eta_j^c}{\eta_j^c - 1}} \tag{36}$$

where η_j^c is the elasticity of substitution among the commodities presently consumed by j, and $\iota_{g,j}^C$ represents the share of total current consumption of j that is devoted to good g, thus $\sum_{g=1}^{44} \iota_{g,j}^C = 1$. The demand for each good g meant for the present consumption of institutional sector $j \in \{Hh, Nip\}$ is given by the F.O.C. below:

$$C_{g,j} = \iota_{g,j}^C \cdot C_j \cdot \left(\frac{P_j^C}{P_g} \right)^{\eta_j^c} \tag{37}$$

where the aggregate price of the consumption basket to be consumed by institutional sector j in the present is given by P_j^C and is equal to the marginal cost associated with the basket C_j. Based on this expression, we can determine the price of the consumption decisions of institutional sector j as a price index obtained by combining the prices of goods consumed by institutional sector j in the present:

$$P_j^C = \left[\sum_{g=1}^{44} \iota_{g,j}^C P_g^{(1-\eta_j^c)} \right]^{\frac{1}{1-\eta_j^c}} \tag{38}$$

3.2.2 Consumption of Commodities by Government

The real consumption of Government, denoted by C_i, is assumed to be fixed. Having said that, the Government chooses the optimal level of demand for each good g that makes up C_i, denoted by $C_{g,i}$, so as to minimise its total consumption expenditure, subject to an aggregator which transforms the demand for each good by Government into aggregate consumption:

$$\min_{C_{g,i}} CST_i^C = \sum_{g=1}^{44} P_g C_{g,i} \tag{39}$$

st.

$$C_i = \left[\sum_{g=1}^{44} (\iota_{g,i}^C)^{\frac{1}{\eta_i^c}} (C_{g,i})^{\frac{\eta_i^c - 1}{\eta_i^c}}\right]^{\frac{\eta_i^c}{\eta_i^c - 1}} \tag{40}$$

where $\iota_{g,i}^C$ represents the share of Government's demand attributed to good g, thus $\sum_{g=1}^{44} \iota_{g,i}^C = 1$, and η_i^c represents the elasticity of substitution among the goods consumed by Government. The Government's optimal consumption of each good g can be derived from the following F.O.C.:

$$C_{g,i} = \iota_{g,i}^C \cdot C_i \cdot \left(\frac{P_i^C}{P_g}\right)^{\eta_i^c} \tag{41}$$

The price of the Government's consumption basket can then be obtained by equating the aggregate consumption expenditure of Government to the total cost incurred by Government to consume the goods g:

$$P_i^C = \left[\sum_{g=1}^{44} \iota_{g,i}^C P_g^{(1-\eta_i^c)}\right]^{\frac{1}{1-\eta_i^c}} \tag{42}$$

3.2.3 Investment

The total investment in the economy, denoted by I, is a CES aggregate of the demand of each good g for investment purposes, I_g. The optimal demand of each good for investment is determined from a cost-minimisation problem optimised subject to an aggregation function:

$$\min_{I_g} CST^I = \sum_{g=1}^{44} P_g I_g \tag{43}$$

st.

$$I = \left[\sum_{g=1}^{44} (\iota_g^I)^{\frac{1}{\eta^I}} (I_g)^{\frac{\eta^I - 1}{\eta^I}}\right]^{\frac{\eta^I}{\eta^I - 1}} \tag{44}$$

where ι_g^I represents the share of investment demand for good g in the total demand for investment in the economy, thus $\sum_{g=1}^{44} \iota_g^I = 1$ and where η^I is the elasticity of substitution among the commodities demanded for investment. The optimal demand for each good g for investment purposes is obtained as follows:

$$I_g = \iota_g^I \cdot I \cdot \left(\frac{P^I}{P_g}\right)^{\eta^I} \tag{45}$$

implying that the optimal investment demand for each commodity depends on the price of investment and the price of the commodity. The price of investment can now be easily determined as follows:

$$P^I = \left[\sum_{g=1}^{44} \iota_g^I (P_g)^{(1-\eta^I)}\right]^{\frac{1}{1-\eta^I}} \tag{46}$$

3.2.4 Exports

The quantity of commodity g demanded by RoW, representing the exports of the Maltese economy and denoted by X_g^E, depends on foreign income, Y_{RoW}, and foreign prices, P_g^f, relative to the domestic price of good g, P_g, taking into consideration also the nominal exchange rate, e:

$$X_g^E = \iota_g^E \cdot Y_{RoW} \cdot \left(\frac{P_g^f}{e \cdot P_g}\right)^{\eta_g^E} \tag{47}$$

where Y_{RoW}, P_g^f, and e are all exogenous; ι_g^E is the share of exports of good g in the total exports, thus $\sum_{g=1}^{44} \iota_g^E = 1$, and η_g^E is the elasticity of substitution among exported commodities.

3.3 The derivation of Disposable Income by Institutional Sector

In the model, the disposable income of each institutional sector is determined by the flows that make up the primary income distribution, the flow of taxes in the economy, and the transfers received and paid as part of the secondary distribution of income. This notwithstanding, the items constituting inflows and outflows vary by institutional sector.

Consider first the national private institutional sectors, grouped in a set h which incorporates households (Hh), non-profit institutions (Npi), financial corporations (Fc), and non-financial corporations (Nfc), i.e. $h \in \{Hh, Npi, Fc, Nfc\}$. Their receipts from the primary income distribution comprise the respective flows received as payment for their supply of labour and capital as primary factors.[9] The total amount that these institutional sectors receive depends

[9] While this is stated in general terms, according to the Maltese SAM, domestic labour resources in Malta supplied by national private institutions are only provided by households. As a result, Nfc, Fc and Npi receive primary income only in respect of their endowment of capital.

on their respective share of the total domestic endowment of the factor f; $f \in \{L, K\}$ in the economy, which share is denoted by ϕ_h^f. Each institutional sector h pays taxes on their respective income, alongside (possibly) other current taxes, which amounts can be expressed in terms of the effective tax rates τ_h^{Tinc} and τ_h^{Toth} levied on their primary income. As part of the secondary distribution of income, each institutional sector h receives transfers from the other institutional sectors k, while on the other hand also transfers some monetary flows to the other sectors itself. The amount of transfers that h receives from the other institutional sectors k can be expressed in terms of an effective rate of transfers $\psi_{h,k}^{Tr}$, applied to a transfers base TB_k, with the latter varying by the institutional sector k.[10] Similarly, the amount that is transferred from h to the other institutional sectors k can be expressed as the application of a rate of transfers $\psi_{k,h}^{Tr}$ to the primary factor income received by h. Then, the disposable income of institutional sector h, denoted by YD_h, can be expressed as in equation (48), where $\phi_h^L = 0$ for $h \neq \{Hh\}$.

$$\begin{aligned} YD_h = {} & \phi_h^L\left(p^L \sum_{a=1}^{44} L_a\right) + \phi_h^K\left(p^K \sum_{a=1}^{44} K_a\right) \\ & - \tau_h^{Tinc}\left[\phi_h^L\left(p^L \sum_{a=1}^{44} L_a\right) + \phi_h^K\left(p^K \sum_{a=1}^{44} K_a\right)\right] - \tau_h^{Toth}\left[\phi_h^L\left(p^L \sum_{a=1}^{44} L_a\right) + \phi_h^K\left(p^K \sum_{a=1}^{44} K_a\right)\right] \\ & + \sum_k (\psi_{h,k}^{Tr} TB_k) - \sum_k \psi_{k,h}^{Tr}\left[\phi_h^L\left(p^L \sum_{a=1}^{44} L_a\right) + \phi_h^K\left(p^K \sum_{a=1}^{44} K_a\right)\right] \end{aligned} \tag{48}$$

Turning to the disposable income of Government, this institution receives its share of capital (ϕ_{Gov}^K) income. Moreover, it collects a proportion δ of the total $Ntax$ (δ_{Ntax}) and $Otax$ (δ_{Otax}) paid in the economy, respectively levied on the total output by commodity and activity at the effective rates τ_g^{Ntax} and τ_a^{Otax}.[11] The Government also receives all income taxes and other current taxes paid by private national institutional sectors h and RoW on their respective primary income. RoW is only endowed with labour and therefore no Gross Operating Surplus is directed to it (i.e. $\phi_{RoW}^K = 0$). The government also pays income taxes and other current taxes on the basis of its capital endowment, but since these payments are made to itself, they carry a net-zero effect on the Government's disposable income. Finally, similar to the case of the national private institutional sectors h, the government receives transfers from, and pays transfers to other institutional sectors k at the rates $\psi_{Gov,k}^{Tr}$ and $\psi_{k,Gov}^{Tr}$, respectively. The resulting disposable

[10] Further detail about the construction of each institutional sector's transfers base is provided in Section 4.3.

[11] The total output, as given by the SAM, of commodity g and of activity a are denoted by $\bar{Y}_g$ and $\bar{Y}_a$, respectively.

income is then used by Government to consume commodities, which despite being fixed in real terms, nominal consumption expenditure changes due to changes in commodity prices. Any resulting difference between the Government's disposable income and its consumption expenditure represents the Government's savings. In turn, any differences between this level of savings and the benchmark savings found in the SAM represent an increase/decrease of the Government's surplus/deficit.

$$
\begin{aligned}
YD_{Gov} = {} & \phi^{K}_{Gov}\left(p^{K}\sum_{a=1}^{44}K_{a}\right) + \delta_{Ntax}\sum_{g=1}^{44}(\tau^{Ntax}_{g}\bar{Y}_{g}) + \delta_{Otax}\sum_{a=1}^{44}(\tau^{Otax}_{a}\bar{Y}_{a}) \\
& + \tau^{Tinc}_{h}\left[\phi^{L}_{h}\left(p^{L}\sum_{a=1}^{44}L_{a}\right) + \phi^{K}_{h}\left(p^{K}\sum_{a=1}^{44}K_{a}\right)\right] + \tau^{Tinc}_{RoW}\left[\phi^{L}_{RoW}\left(p^{L}\sum_{a=1}^{44}L_{a}\right)\right] \\
& + \tau^{Toth}_{h}\left[\phi^{L}_{h}\left(p^{L}\sum_{a=1}^{44}L_{a}\right) + \phi^{K}_{h}\left(p^{K}\sum_{a=1}^{44}K_{a}\right)\right] + \tau^{Toth}_{RoW}\left[\phi^{L}_{RoW}\left(p^{L}\sum_{a=1}^{44}L_{a}\right)\right] \\
& + \tau^{Tinc}_{Gov}\left[\phi^{K}_{Gov}\left(p^{K}\sum_{a=1}^{44}K_{a}\right)\right] - \tau^{Tinc}_{Gov}\left[\phi^{K}_{Gov}\left(p^{K}\sum_{a=1}^{44}K_{a}\right)\right] \\
& + \tau^{Toth}_{Gov}\left[\phi^{K}_{Gov}\left(p^{K}\sum_{a=1}^{44}K_{a}\right)\right] - \tau^{Toth}_{Gov}\left[\phi^{K}_{Gov}\left(p^{K}\sum_{a=1}^{44}K_{a}\right)\right] \\
& + \sum_{k}(\psi^{Tr}_{Gov,k}TB_{k}) - \sum_{k}(\psi^{Tr}_{k,Gov}TB_{Gov})
\end{aligned}
\tag{49}
$$

Turning to the disposable income of RoW, this institutional sector receives payment for the local economy's imports. This payment can be quantified as the product of the quantity of each good imported, Y^{M}_{g}, and their respective price, P^{M}_{g}, summed across all goods g. The RoW also receives its share of income from labour (ϕ^{L}_{RoW}), alongside any taxes paid on commodities and activities' production that are due to itself, denoted by $(1 - \delta_{t})$. On the other hand, it pays taxes on its labour income at the effective rate τ^{Tinc}_{RoW} and τ^{Toth}_{RoW}, while also receiving transfers from, and effecting transfers to, the other institutional sectors k, where $k \neq$ RoW. In general, the RoW receives transfers at the rate $\psi^{Tr}_{RoW,k}$, whereas transfers to other institutional sectors are assumed fixed at the respective SAM values.

$$YD_{RoW} = \left(\sum_{g=1}^{44} P_g^M Y_g^M\right) + \phi_{RoW}^L \left(p^L \sum_{a=1}^{44} L_a\right) + (1 - \delta_{Ntax}) \sum_{g=1}^{44} (\tau_g^{Ntax} \bar{Y}_g) + (1 - \delta_{Otax}) \sum_{a=1}^{44} (\tau_a^{Otax} \bar{Y}_a)$$
$$- \tau_{RoW}^{Tinc} \left[\phi_{RoW}^L \left(p^L \sum_{a=1}^{44} L_a\right)\right] - \tau_{RoW}^{Toth} \left[\phi_{RoW}^L \left(p^L \sum_{a=1}^{44} L_a\right)\right]$$
$$+ \sum_k (\psi_{RoW,k}^{Tr} TB_k) - \sum_k (TR_{k,RoW}) \tag{50}$$

3.4 Market Clearing Conditions and Closure Rules

The description of the model concludes with the definition of market clearing conditions and a number of macroeconomic closure rules designed to ensure equilibrium in the respective markets.

3.4.1 The Markets

The market for each commodity g is assumed to be perfectly competitive and the total supply Y_g must be fully absorbed either by domestic demand or as exports (Figure 2). Any discrepancy between the total supply and the total demand for a commodity is cleared by movements in the price of the good, denoted by P_g, such that the market for good g returns to an equilibrium where the demand equates its supply. The value of total supply of goods comprises the domestic output, any trade and transportation margins and net taxes on production, and imported commodities. On the other hand, the demand for goods derives from the activities that require the commodities as intermediate consumption, final demand by domestic institutions, demand of goods for investment, and exports:

$$Y_g = \left(\sum_{a=1}^{44} Y_{g,a}^{IG}\right) + \left(\sum_j C_{g,j}\right) + C_{g,i} + I_g + X_g^E \tag{51}$$

In a similar vein to commodities, the market clearing condition for activities necessitates that the total output of activity a, denoted by Y_a, be equal to the total amount of commodities produced by the same activity a.

$$Y_a = \sum_{g=1}^{44} Y_{g,a} \tag{52}$$

Moving on to the markets of labour and capital, the total demand for factor f by local activities has to be equal to the respective total factor endowment made available domestically by institutional sectors, which amounts are denoted by f_k^s, where $f \in \{L, K\}$. The demand for factor f by activity is determined endogenously by the equations (15) and (16), whereas factor supply is assumed to be exogenous for both factors.[12] In the case of the market for labour, the unemployment rate is allowed to vary in order to guarantee balance between total labour supply and demand (as in Socci et al. (2021)), whereas balance on the (perfectly competitive) market for capital is obtained through changes in the price of capital.

$$\sum_{a=1}^{44} L_a = (1-u)\sum_k L_k^s \tag{53}$$

$$\sum_{a=1}^{44} K_a = \sum_k K_k^s \tag{54}$$

3.4.2 Savings-Investment Condition

The level of investment in the economy is assumed to be determined by the savings of the respective institutional sectors. More specifically, total investment equates to the sum of savings of the national private institutional sectors, Government and RoW. Having said this, it is further assumed that the Government does not use any savings that exceed the benchmark level recorded in the SAM (S_i^{bench}). Instead, these "extra" savings are assumed to be set aside and later used for consumption purposes or to balance Government's books, similar to Socci et al. (2021). Additionally, the savings of RoW that are allocated to capital formation are assumed to remain fixed at the quantity given by the SAM, which we denote by $\bar{S}_{RoW}$, such that changes in real investment are driven solely by changes in the savings of households, NPIs and corporations.

$$\sum_{g=1}^{44} I_g = \sum_h S_h + S_i^{bench} + (\bar{S}_{RoW} \cdot e) \tag{55}$$

3.4.3 Government Balance Condition

As explained earlier, the real consumption of Government is considered fixed whereas its nominal consumption expenditure is deemed flexible by virtue of changes in commodity prices. The balance in the Government's account - representing the Government's savings/deficit - originates endogenously as the difference between its disposable income and its total consumption expenditure, all expressed in nominal terms. When the change in Government's disposable income exceeds the change in its consumption expenditure, such that total Government savings increase,

[12]The domestic supply of labour by institutional sector k, denoted by L_k^s, is obtained from the SAM as $LAB_{(k)}$, whereas the domestic supply of capital by institutional sector k, denoted by K_k^s, is represented by $GOS_{(k)}$.

the Government records a positive balance on its account. These resources can be destined to different uses: they can be stored or used to raise Government's consumption expenditure. Conversely, if the nominal consumption expenditure of Government rises by more than its disposable income, the balance on the Government's account turns negative.

$$Bal_i = YD_i - \left(\sum_{g=1}^{44} C_{g,i} \cdot P_g \right) - S_i^{bench} \tag{56}$$

3.4.4 Rest of the World Balance Condition

The net debit or credit flows of the domestic economy with the rest of the world, which we denote by S_{RoW}, are determined by the difference between the disposable income of RoW as determined in equation (50) and its nominal expenditure on exports.

$$S_{RoW} = YD_{RoW} - \left(\sum_{g=1}^{44} X_g^E \cdot P_g \right) \tag{57}$$

4 Calibration

This section documents the calibration of key parameters and quantities that serve as the main link between the theoretical model derived above and the SAM for Malta. Model parameters are all calibrated based on the values recorded in the SAM.[13]

4.1 Production

The share of the total intermediate goods used by activity a in producing its total output net of production taxes corresponds to ι_a^{IG}. From the template of the SAM illustrated in Figure 1, the total output produced by activity a is determined as $\bar{Y}_a = \sum_{g=1}^{44} u_{g,a} + COE_a + GOS_a + OTAX_a$, where $\bar{Y}_a$ is the total output value of activity a derived from the SAM and $u_{g,a}$ is an element within the U matrix showing the use of commodity g as intermediate good by activity a. ι_a^{IG} is therefore estimated as:

$$\iota_a^{IG} = \frac{\sum_{g=1}^{44} u_{g,a}}{\bar{Y}_a - OTAX_a} \tag{58}$$

The problem solving for the optimal quantity of good g that activity a should use in its intermediate consumption (i.e. solving for the entries of the U matrix) requires the estimation of the

[13]The calibrated values of all parameters discussed in this section are available from the authors upon request.

good-specific bias in intermediate consumption by activity. In this respect, $\iota^{IG}_{g,a}$ represents the use of good g as an intermediate consumption by activity a, expressed as a proportion of the total intermediate goods used by a.

$$\iota^{IG}_{g,a} = \frac{u_{g,a}}{\sum_{g=1}^{44} u_{g,a}} \tag{59}$$

In the determination of the optimal demand for labour and capital, ι^{L}_{a} measures the labour intensity in the value added production of activity a and is calibrated as the share of labour in the generation of value added. Thus:

$$\iota^{L}_{a} = \frac{COE_a}{COE_a + GOS_a} \tag{60}$$

Turning to the problem aggregating domestic production by commodity and imports, we estimate ι^{D}_{g}, interpreted as the domestic production bias in the total supply of goods net of output taxes. This is calculated as the share of domestic output by good (including trade margins) in the total supply of commodities, excluding net taxes levied on commodities. The total supply of good g can be expressed as $\bar{Y}_g = \sum_{a=1}^{44} s_{a,g} + MRG_g + NTAX_g + M_g$, where $\bar{Y}_g$ is the value of total supply of commodity g obtained from the SAM and $s_{a,g}$ is an element within the S matrix, representing the production of commodity g by activity a. In mathematical terms, ι^{D}_{g} can then be expressed as:

$$\iota^{D}_{g} = \frac{\sum_{a=1}^{44} s_{a,g} + MRG_g}{\bar{Y}_g - NTAX_g} \tag{61}$$

The total value of good g that is produced domestically depends on the domestic production of the activities in the economy. More specifically, the domestic production of good g is determined by the share of domestic production of commodity g that is produced by each activity a separately. This share, represented by $\iota^{D}_{g,a}$, is calibrated as follows:

$$\iota^{D}_{g,a} = \frac{s_{a,g}}{\sum_{a=1}^{44} s_{a,g}} \tag{62}$$

4.2 Final Demand

In the maximisation of households and NPIs' utility, ι^{C}_{j} represents the share of consumption in the utility of institutional sector j, with the latter comprising the present consumption and savings of institutional sector j. Then, ι^{C}_{j} can be calibrated using the information from the SAM as follows:

$$\iota_j^C = \begin{cases} \frac{\sum_{g=1}^{44} Chh_g}{\sum_{g=1}^{44} Chh_g + SAV_{Hh}} & \text{if } j \in \{Hh\} \\ \frac{\sum_{g=1}^{44} Cnpi_g}{\sum_{g=1}^{44} Cnpi_g + SAV_{Npi}} & \text{if } j \in \{Npi\} \end{cases} \tag{63}$$

On the other hand, $\iota_{g,j}^C$ represents the weight of commodity g in the consumption basket of institutional sector j. This parameter is calibrated from the SAM as the share of consumption expenditure on good g in the total consumption expenditure of institutional sector j:

$$\iota_{g,j}^C = \begin{cases} \frac{Chh_g}{\sum_{g=1}^{44} Chh_g} & \text{if } j \in \{Hh\} \\ \frac{Cnpi_g}{\sum_{g=1}^{44} Cnpi_g} & \text{if } j \in \{Npi\} \end{cases} \tag{64}$$

As regards the Government's consumption expenditure, $\iota_{g,i}^C$ carries an analogous definition to $\iota_{g,j}^C$. In words, $\iota_{g,i}^C$ represents the Government's expenditure on commodity g, expressed as a proportion of its total consumption expenditure:

$$\iota_{g,i}^C = \frac{Cgov_g}{\sum_{g=1}^{44} Cgov_g} \tag{65}$$

Turning to the investment demand for commodities, ι_g^I represents the proportion of total investment demand attributed to good g. The parameter ι_g^I is therefore calibrated as:

$$\iota_g^I = \frac{Inv_g}{\sum_{g=1}^{44} Inv_g} \tag{66}$$

4.3 Disposable Income

Information from the SAM is used to calibrate the parameters entering equations (48), (49) and (50). In general, the parameter ϕ_k^f, where $f \in \{L, K\}$, represents the share of the total domestic supply of factor f that is endowed by institutional sector k, such that $\sum_k \phi_k^f = 1$. Therefore, this parameter is in general calibrated as:

$$\phi_k^f = \begin{cases} \frac{LAB_k}{\sum_k LAB_k} & \text{if } f \in \{L\} \\ \frac{GOS_k}{\sum_k GOS_k} & \text{if } f \in \{K\} \end{cases} \tag{67}$$

While this is stated in general terms for all institutional sectors k, the Maltese SAM shows that only households and RoW supply labour, whereas RoW does not receive any gross operating surplus. These patterns imply that $\phi_k^L = 0$ for $k \in \{Nfc, Fc, Npi, Gov\}$ and $\phi_{RoW}^K = 0$.

τ_k^{Tinc} and τ_k^{Toth} respectively represent the effective rates of tax on income and other current taxes paid by institutional sector k. For all institutional sectors k, both rates are calculated as the share of the total amount of the respective tax paid by k in their total primary income (i.e. the income received from their endowment of primary factors). Thus:

$$\tau_k^{Tinc} = \frac{TINC_k}{LAB_k + GOS_k} \tag{68}$$

$$\tau_k^{Toth} = \frac{TOTH_k}{LAB_k + GOS_k} \tag{69}$$

The effective rate of tax levied on the production of good g, denoted by τ_g^{Ntax}, and on the activity a, denoted by τ_a^{Otax}, are obtained from the SAM as follows:

$$\tau_g^{Ntax} = \frac{NTAX_g}{\bar{Y}_g} \tag{70}$$

$$\tau_a^{Otax} = \frac{OTAX_a}{\bar{Y}_a} \tag{71}$$

A share δ_t of the total amount paid in $Ntax$ and $Otax$, where $t \in \{Ntax, Otax\}$, is payable to Government, with the remaining amount payable to RoW. The respective shares of $Ntax$ and $Otax$ paid to Government are calibrated from the SAM as follows:

$$\delta_{Ntax} = \frac{NTAX_{Gov}}{\sum_g NTAX_g} \tag{72}$$

$$\delta_{Otax} = \frac{OTAX_{Gov}}{\sum_a OTAX_a} \tag{73}$$

As regards the calibration of transfers flowing among the institutional sectors in the economy, in general the effective rate of transfers from institutional sector k to m is equivalent to the corresponding amount of transfers documented in the SAM expressed as a share of a transfers base of the payer, denoted by TB_k:

$$\psi_{m,k}^{Tr} = \frac{TR_{m,k}}{TB_k} \tag{74}$$

While this is stated in general, in practice the determination of the flow of transfers varies depending on the economic agents involved in the transaction. Firstly, the flow of transfers from RoW to all other institutional sectors (i.e. the national private institutional sectors and

Government) are considered fixed in nominal terms at the amounts documented in the SAM, as are the flows transferred from Government to RoW. On the other hand, the transfers from Government to the national private institutional sectors depend on the factor and tax revenues received by Government. In fact, the amount transferred by Government to the national private institutional sectors depends on the application of the (fixed) $\psi^{Tr}_{h,Gov}$ to the following transfers base:

$$\begin{aligned}
TB_{Gov} = {} & \phi^K_{Gov}\left(p^K \sum_{a=1}^{44} K_a\right) + \delta_{Ntax} \sum_{g=1}^{44} (\tau^{Ntax}_g \bar{Y}_g) + \delta_{Otax} \sum_{a=1}^{44} (\tau^{Otax}_a \bar{Y}_a) \\
& + \sum_h \tau^{Tinc}_h \left[\phi^L_h\left(p^L \sum_{a=1}^{44} L_a\right) + \phi^K_h\left(p^K \sum_{a=1}^{44} K_a\right)\right] + \tau^{Tinc}_{RoW}\left[\phi^L_{RoW}\left(p^L \sum_{a=1}^{44} L_a\right)\right] \\
& + \sum_h \tau^{Toth}_h \left[\phi^L_h\left(p^L \sum_{a=1}^{44} L_a\right) + \phi^K_h\left(p^K \sum_{a=1}^{44} K_a\right)\right] + \tau^{Toth}_{RoW}\left[\phi^L_{RoW}\left(p^L \sum_{a=1}^{44} L_a\right)\right] \\
& + \tau^{Tinc}_{Gov}\left[\phi^K_{Gov}\left(p^K \sum_{a=1}^{44} K_a\right)\right] - \tau^{Tinc}_{Gov}\left[\phi^K_{Gov}\left(p^K \sum_{a=1}^{44} K_a\right)\right]
\end{aligned} \tag{75}$$

The national private institutional sectors transfer flows to other national private institutional sectors, Government and RoW on the basis of their own primary factor income. In other words, the amount of transfers flowing from institutional sector h to k, where $k \in \{h, Gov, RoW\}$, is determined on the basis of the following transfers base:

$$TB_h = \phi^L_h\left(p^L \sum_{a=1}^{44} L_a\right) + \phi^K_h\left(p^K \sum_{a=1}^{44} K_a\right) \tag{76}$$

4.4 Elasticities of Substitution

To conclude the calibration process, we document the parameterisation of the elasticities of substitution which feature in the current version of the model. By construction, the model assumes that intermediate goods and value added are aggregated to produce output via a Leontief production function (i.e. $\eta^Y_a = 0$). This implies that each activity combines intermediate goods and value added in fixed proportions, with no substitutability between them. Similarly, constructing $\eta^{IG}_a = 0$ implies that the share of commodity g demanded by activity a as intermediate good is set to remain fixed and not be affected by relative price changes. In a similar vein, the model assumes that the export bundle is fixed by constructing $\eta^E_g = 0$, while the elasticity of substitution among activities in the domestic production is also set to 0 by construction (i.e. $\eta^D_g = 0$). By setting η^u_j to be equal to 0, the model also assumes that the utility of households and NPIs depends on their present and possible future consumption in fixed proportions. In other words, changes in utility (or disposable income) are assumed to not affect the decision of these institutional sectors about present and possible future consumption in relative terms. All other elasticities are free to take any value, but in the current version of the model are calibrated to the values found in Table 1. In the absence of information for Malta, the elasticity of substitution between labour and capital is set to 0.5218 for all activities, following the findings for Italy by Van der Werf (2008). Imported commodities are assumed to be imperfect substitutes to domestically-produced goods, with η_g^Y taking a value of 0.3 for all commodities. The consumption bundle of households, NPIs and Government is assumed to remain fixed and not be affected by changes in the relative prices of goods ($\eta_j^c = \eta_i^c = 0$). Similarly, the elasticity of substitution among the commodities which comprise the investment bundle is also calibrated to 0 (i.e. $\eta^I = 0$), implying that the investment bundle is considered fixed in terms of the quantities of each good demanded for investment purposes.

Table 1: Elasticity of Substitution Parameters: Calibration

Parameter	Description	Calibration	Remarks
η_a^Y	Elasticity of substitution between intermediate goods and value added	0	Leontief *
η_a^{IG}	Elasticity of substitution among the goods used for intermediate consumption	0	Leontief *
η_a^{VA}	Elasticity of substitution between labour and capital	0.5218	Van der Werf (2008) **
η_g^Y	Elasticity of substitution between domestic and imported commodities	0.3	
η_g^D	Elasticity of substitution among activities in domestic production	0	Leontief *
η_j^u	Elasticity of substitution between present consumption and possible future consumption in utility function	0	Leontief *
η_j^c	Elasticity of substitution among commodities consumed by households and NPIs	0	Leontief
η_i^c	Elasticity of substitution among commodities consumed by Government	0	Leontief
η^I	Elasticity of substitution among commodities demanded for investment	0	Leontief
η_g^E	Elasticity of substitution among exported goods	0	Leontief *

Notes. * These parameters are hardcoded to 0 in the model, implying that a Leontief technology is assumed by construction. ** This parameterisation is based on the result for Italy documented in Van der Werf (2008).

5 Simulations

In this section, MaCGE-MOD is used to simulate the macroeconomic aggregated and disaggregated effects of a set of standard shocks that could (hypothetically) hit the Maltese economy. The shocks are a mixture of macroeconomic and sector-specific shocks, where each shock is considered permanent and the resulting impact is measured by the percentage deviation from the baseline. Also, because the model is static, the results show the total (direct, indirect and induced) effect of the shock on the economy once all markets readjust to a new equilibrium. In this light, results from the model are to be interpreted as materialising in the medium run.

5.1 An increase in Investment

The first simulation explores the effects of a 1% increase in the total demand for investment, simulated as a 1% increase in the investment demand of each commodity. The main macroeconomic effects of this shock are presented in Table 2. The original 1% shock to investment demand sees total investment rise by 1.19% in real terms. The difference between this effect and the original 1% shock reflects the indirect and induced effects that the shock generates within the economic system. In response to the additional demand for investment goods, domestic production increases, as do imports, thus stimulating employment and primary factors' remuneration. This additional income generated by the higher production is distributed to the institutional sectors, enlarging their disposable income. As a consequence, private consumption increases in real terms, as do prices, which in turn lowers the export competitiveness of the economy. At the same time, the level of savings by institutional sector increases as well, thereby driving further investments.[14] Overall, Real GDP increases by 0.09% in response to the original increase in investment demand. Among the main fiscal implications of the underlying shock, the Government's income increases primarily because of the additional receipts of income tax and taxes on production. However, the Government also incurs additional payments following the shock, predominantly due to the higher transfers to other institutional sectors and higher consumption expenditure, albeit only in nominal terms. The overall balance in the Government's account improves by 0.06% of GDP.

The sectoral impacts of the shock to investment demand are presented in Table 3. The products that would be most impacted by this hypothetical investment shock are those showing the highest proportions of total demand for investment in the SAM. They are (in descending order): (14) *Mining and quarrying and construction*, (11) *Manufacture of computer, electronic and optical products, electrical equipment*, (24) *Inputed Rent*, (27) *Architectural and engineering activities; technical testing and analysis*, and (10) *Manufacture of fabricated metal products, except machinery and equipment.* All the corresponding activities producing these goods as their primary production feature among the most impacted by the 1% increase in investment

[14] Results from this simulation show that real savings of all national institutional sectors rise following a positive shock to investment.

Table 2: Main macroeconomic effects of a 1% increase in Investment

	% deviation from Baseline
Price and Cost Developments	
Consumption Deflator	0.10
GDP Deflator	0.17
Economic Activity (*constant prices*)	
Real GDP	0.09
Private Consumption	0.12
Government Consumption*	0.00
Gross Fixed Capital Formation	1.19
Exports (goods and services)	-0.06
Imports (goods and services)	0.11
Real Disposable Household Income	0.12
Households' Saving Ratio**	-0.00
Fiscal Developments (*% of GDP*)	
Total Government Receipts	0.03
Total Government Expenditure	-0.03
Government Balance	0.06
Labour Market	
Employment	0.16

Notes. * Government consumption is not impacted in real terms as this is assumed fixed. ** Household savings are expressed as a % of Real Disposable Household Income and the resulting % deviation represents the p.p. deviation of this ratio from the baseline.

Table 3: Impacts on Real GVA by activity of a 1% increase in Investment

Nr.	Activity	% deviation from Baseline
14	Mining and quarrying and construction	0.84
8	Manufacture of other non-metallic mineral products	0.67
10	Manufacture of fabricated metal products, except machinery and equipment	0.56
27	Architectural and engineering activities; technical testing and analysis	0.46
24	Inputed Rent	0.44
11	Manufacture of computer, electronic and optical products, electrical equipment...	0.43
17	Retail trade, except of motor vehicles and motorcycles	0.37
6	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials	0.29
9	Manufacture of basic metals	0.27
40	Activities of membership organisations	0.17

demand. The other activities found in Table 3 are hit indirectly because they produce goods that are used as intermediate products by the activities which are directly hit by the investment shock. As the investment demand for the goods produced by these activities increases following the shock, their demand for intermediate goods increases as well, thereby creating an additional indirect effect of the investment shock on the activities providing the intermediate products.

5.2 An Appreciation of the Exchange Rate

In another scenario, we consider a context in which the nominal exchange rate appreciates by 1%. As shown in Table 4, this reduces the competitiveness of the goods produced locally, thus causing a fall in exports and consequently Real GDP (−0.07%). The exchange rate appreciation also makes foreign-produced goods relatively cheaper, thus leading to a (marginal) increase in imports.[15] The lower import prices, compared to domestic prices, drag prices downwards, as evidenced by both the consumption deflator and the GDP deflator which in part compensate the loss of local competitiveness. The deterioration in local competitiveness due to the exchange rate appreciation slightly shrinks the domestic production and labour demand, although the real disposable income of households experiences a net increase following the drop in prices. The higher real disposable income translates into higher consumption by households. Turning to Government finances, the net effect on the Government's balance, expressed as a percentage of GDP, is slightly positive at 0.01%.

As the fall in exports appears to be the main driver behind the economic downturn following an exchange rate appreciation shock, it is no surprise that activities producing export-oriented commodities are among those most impacted by the shock (Table 5). In fact, judging by the data in the SAM, seven of the eight goods with the highest export intensity are all produced primarily by the activities featuring in Table 5. The only clear outlier is activity (20) *Publishing activities, motion picture...*, whose primary good produced is not exported in large quantities (as a proportion of total demand). However, the SAM shows that the commodity (20), which is the main production of activity (20), is used as an intermediate product by sectors that are directly impacted heavily by the exchange rate appreciation (e.g. activity (38)). As a result, activity (20) suffers a sizeable indirect impact from the shock. On the other hand, sectors whose production is largely consumed locally rather than exported are impacted positively by the shock due to higher private consumption. Among others, these sectors include (25) *Real estate activities*, (15) *Wholesale and retail trade and repair of motor vehicles and motorcycles*, and (4) *Manufacture*

[15] The increase in imports is slightly muted because the elasticity of substitution between domestic and imported commodities is calibrated to (a relatively low) value of 0.3.

Table 4: Main macroeconomic effects of a 1% appreciation of the Exchange Rate

	% deviation from Baseline
Price and Cost Developments	
Consumption Deflator	-0.62
GDP Deflator	-0.30
Economic Activity (*constant prices*)	
Real GDP	-0.07
Private Consumption	0.32
Government Consumption*	0.00
Gross Fixed Capital Formation	0.09
Exports (goods and services)	-0.21
Imports (goods and services)	0.01
Real Disposable Household Income	0.33
Households' Saving Ratio**	0.00
Fiscal Developments (*% of GDP*)	
Total Government Receipts	0.01
Total Government Expenditure	0.00
Government Balance	0.01
Labour Market	
Employment	-0.18

Notes. * Government consumption is not impacted in real terms as this is assumed fixed. ** Household savings are expressed as a % of Real Disposable Household Income and the resulting % deviation represents the p.p. deviation of this ratio from the baseline.

Table 5: Impacts on Real GVA by activity of a 1% appreciation of the Exchange Rate

Nr.	Activity	% deviation from Baseline
16	Wholesale trade, except of motor vehicles and motorcycles	-0.24
38	Creative arts and entertainment activities; libraries, archives, museums...	-0.23
23	Activities auxiliary to financial services and insurance activities	-0.17
18	Land transport and transport via pipelines, water transport, air transport...	-0.13
26	Legal and accounting activities; activities of head offices...	-0.12
3	Fishing and aquaculture	-0.12
30	Rental and leasing activities	-0.11
20	Publishing activities, motion picture, video and television programme production, sound recording and music publishing activities...	-0.09
12	Repair and installation of machinery and equipment	-0.09
28	Advertising	-0.09

of food products, beverages and tobacco products.

5.3 An increase in Household Income Tax

Next we simulate a hypothetical increase in the income tax paid by households corresponding to 1% of GDP. The main macroeconomic effects of this shock, which implicitly results in a hike in the effective income tax rate paid by households, are presented in Table 6. As evident from the results, a policy which raises income tax inevitably constraints households' disposable income, directly causing a drop in consumption and hence price levels.[16] In turn, the fall in prices increases competitiveness, leading to higher exports, while the fall in aggregate demand also reduces the demand for imports and employment. The overall effects of the shock on economic activity are negative, with both investment (−1.28%) and Real GDP (−0.45%) falling. The Government's position also improves substantially, largely due to a 15% increase in the amount received in income taxes. This drives an increase in total receipts, despite the adverse effects of the shock on production causing the tax collected on output to fall. Payments by Government also increase as it transfers higher amounts to the other institutional sectors following the increase in its income. Overall, the additional income received following the shock exceeds the additional payments, leading to an improvement in the Government's balance. Turning to the disaggregated effects of this shock, these are reported in Table 7. As expected, the activities that feature among the most impacted by this shock are those producing commodities that are largely demanded by households. As an example, activity (25) *Real estate activities* only produces commodity (25) *Real estate* which is fully demanded by households.

5.4 An increase in Tax levied on Construction

As a final exercise, we simulate the macroeconomic aggregated and disaggregated effects of a hypothetical increase in the tax levied on the (14) *Construction* commodity. Similar to the previous simulation, this shock is calibrated to ensure that the total amount paid in tax by this commodity increases by an amount equivalent to 1% of GDP. Technically, this policy is modelled as an increase in the effective tax rate paid by the 'Construction' good and the main macroeconomic effects of this policy decision are presented in Table 8. The transfers paid by Government to the other institutional sectors are taken to be fixed in this simulation, such that no portion of the additional revenue received by Government is re-distributed in the economy.

[16] Since the labour market is modelled as purely exogenous, an increase in the disposable income of households does not have any supply-side effects that would suppress labour supply and exert upward pressures on the gross wage and thus prices.

Table 6: Main macroeconomic effects of an increase in Household Income Tax

	% deviation from Baseline
Price and Cost Developments	
Consumption Deflator	-0.62
GDP Deflator	-1.07
Economic Activity (*constant prices*)	
Real GDP	-0.45
Private Consumption	-2.71
Government Consumption*	0.00
Gross Fixed Capital Formation	-1.28
Exports (goods and services)	0.38
Imports (goods and services)	-0.41
Real Disposable Household Income	-2.65
Households' Saving Ratio**	0.05
Fiscal Developments (*% of GDP*)	
Total Government Receipts	2.05
Total Government Expenditure	1.25
Government Balance	0.80
Labour Market	
Employment	-0.84

Notes. * Government consumption is not impacted in real terms as this is assumed fixed. ** Household savings are expressed as a % of Real Disposable Household Income and the resulting % deviation represents the p.p. deviation of this ratio from the baseline.

Table 7: Impacts on Real GVA by activity of an increase in Household Income Tax

Nr.	Activity	% deviation from Baseline
25	Real estate activities	-2.49
17	Retail trade, except of motor vehicles and motorcycles	-2.35
42	Other personal service activities	-2.29
43	Activities of households as employers	-2.25
19	Accommodation and food service activities	-2.11
15	Wholesale and retail trade and repair of motor vehicles and motorcycles	-1.99
1	Crop and animal production, hunting and related service activities	-1.96
4	Manufacture of food products, beverages and tobacco products	-1.88
41	Repair of computers and personal and household goods	-1.75
5	Manufacture of textiles, wearing apparel and leather products	-1.70

The tax hike applied to the 'Construction' commodity increases the latter's price and the input prices for those activities which use it as an intermediate good. Moreover, being a commodity with a high share of investment demand (35% of all investment demand in the economy is attributed to the 'Construction' commodity), the additional tax raises the overall GFCF deflator, which in turn drives a higher GDP deflator. Economic activity declines by 0.28% in real terms following the implementation of the higher tax, with investment suffering most among the GDP components. This again reflects the high share of investment demand for the 'Construction' commodity. Lower domestic aggregate demand leads to a decline in imports and has negative implications on employment demand. This negatively impacts the disposable income of households who in turn consume less in real terms. Weakened economic activity is followed by a drop in the prices of commodities unrelated to 'Construction', which in turn see an improvement in their export competitiveness. All-in-all, prices for most commodities are expected to fall driven by the lower aggregate demand effect with only a few commodities (including the 'Construction' commodity) expected to experience higher prices. On aggregate, these effects are expected to reduce overall export prices leading to higher aggregate exports. Turning to fiscal developments, Government receipts are boosted by the additional tax revenue collected from the tax hike, with the net improvement in the Government's balance recorded at 0.91% of GDP.

Table 9 illustrates the activities most negatively impacted by such policy. Prices for the output of the 'Construction' activity increase due to the tax levied on its principal commodity leading to a reduction in the demand for the output of the 'Construction' sector. As expected, activities with very close links to the production of the 'Construction' commodity are affected the most through indirect and induced effects. Upstream activities, i.e. those which supply intermediate goods to the 'Construction' activity, experience a fall in the demand of their output as a consequence of the fall in the output of the 'Construction' sector. As evidenced from the SAM, such upstream activities include (8) *Manufacture of other non-metallic mineral products*, and (10) *Manufacture of fabricated metal products, except machinery and equipment*, among others. On the other hand, downstream activities, i.e. activities which use 'Construction' output as an intermediate input are faced with an increase in their marginal costs, leading to rising prices and lower demand. These activities include (8) *Manufacture of other non-metallic mineral products* (which besides supplying goods to the 'Construction' activity also make a substantial use of the 'Construction' output as an intermediate good - with overall effects on prices for this sector being positive) and (25) *Real estate activities.*

Table 8: Main macroeconomic effects of an increase in Construction taxes

	% deviation from Baseline
Price and Cost Developments	
Consumption Deflator	-0.01
GDP Deflator	0.71
Economic Activity (*constant prices*)	
Real GDP	-0.28
Private Consumption	-0.46
Government Consumption*	0.00
Gross Fixed Capital Formation	-3.00
Exports (goods and services)	0.10
Imports (goods and services)	-0.37
Real Disposable Household Income	-0.46
Households' Saving Ratio**	0.00
Fiscal Developments (*% of GDP*)	
Total Government Receipts	0.82
Total Government Expenditure	-0.09
Government Balance	0.91
Labour Market	
Employment	-0.47

Notes. * Government consumption is not impacted in real terms as this is assumed fixed. ** Household savings are expressed as a % of Real Disposable Household Income and the resulting % deviation represents the p.p. deviation of this ratio from the baseline.

Table 9: Impacts on Real GVA by activity of an increase in Construction taxes

Nr.	Activity	% deviation from Baseline
8	Manufacture of other non-metallic mineral products	-2.88
14	Mining and quarrying and construction	-2.83
10	Manufacture of fabricated metal products, except machinery and equipment	-1.81
27	Architectural and engineering activities; technical testing and analysis	-1.65
24	Inputed Rent	-1.44
11	Manufacture of computer, electronic and optical products, electrical equipment...	-1.19
6	Manufacture of wood and of products of wood and cork, except furniture, etc.	-1.01
9	Manufacture of basic metals	-0.93
25	Real estate activities	-0.90
17	Retail trade, except of motor vehicles and motorcycles	-0.86

6 Concluding Remarks

MaCGE-MOD constitutes a novel tool in the modelling framework of the Central Bank of Malta. This paper introduces and describes in detail this model and illustrates its usefulness as a means to determine the effects of a particular policy decision (or shock) through the simulation of three standard shocks and one sector-specific shock. Nevertheless, the information contained within the SAM underpinning the model makes it possible to consider a wide range of possible shocks or hypothetical policy decisions. In fact, the model can be used to determine the economic effects of potential policy decisions affecting one or more economic agents, or changes in exogenous factors. Since the SAM contains the flows among commodities, activities and numerous economic agents, the model can easily accommodate sector- or commodity-specific shocks and thereby quantify the effects on the rest of the economy.

Having said that, MaCGE-MOD is not without its limitations. Firstly, even if the calibration is based on the actual economic data found in the SAM (for one specific year), the respective elasticities of substitution need to be assumed. This is particularly problematic in the absence of precise information about the Maltese context. The current version of MaCGE-MOD also includes a number of simplifying assumptions surrounding production functions and utility functions, which may not reflect accurately the behaviour of economic agents (Meng and Siriwardana, 2017). Moreover, the current model developed for Malta is static. This has important implications on the characteristics of the model, especially on the assumptions surrounding capital and technological change, both of which are assumed as exogenous in static models. Other limitations derive from the neoclassical assumptions underpinning the model as regards, for example, the perfectly competitive structure of markets, or exogenous endowments. For instance, the perfect competitiveness structure, which implies that there are no market failures in the economy, limits to a certain extent the model's ability to track the economy's dynamics over the business cycle. Also, the lack of rational expectations and stochastic processes make such models more prone to the Lucas critique especially when compared to New Keynesian models.

On the other hand, MaCGE-MOD is a very powerful tool that allows to evaluate intricate inter-sectoral linkages within a general equilibrium setting. This makes this class of models particularly helpful for policymakers that seek to understand the complex interactions in the economy that arise either from sector-specific or macro policy changes. Indeed, the results of the shocks simulated in this paper demonstrate the usefulness of the model in assessing the impact of shocks once the direct, indirect and induced effects have propagated through the economy.

In light of the disaggregated information contained within the SAM, it is also possible to trace the economic effects of an underlying shock to movements in the circular flow of income among the different agents in the economy. Moreover, the highly disaggregated sectoral information contained within the SAM allows for a precise sectoral analysis of the total effects generated by the underlying economic shock or policy implementation. This complements analyses of the main macroeconomic effects of an underlying shock and should therefore serve as an important tool for policy makers to identify effects obscured by more aggregate analyses.

References

Allan, Grant, Patrizio Lecca, Peter McGregor, and Kim Swales (2014), "The economic and environmental impact of a carbon tax for Scotland: a Computable General Equilibrium analysis." *Ecological Economics*, 100, 40–50.

Armington, Paul S (1969), "A theory of demand for products distinguished by place of production." *Staff Papers*, 16, 159–178.

Bellù, LG (2014), "Analyzing policy impacts and international price shocks: alternative Computable General Equilibrium (CGE) models for an aid-dependent less-industrialized country."

Berck, Peter, Sherman Robinson, and George Goldman (1991), "The use of Computable General Equilibrium models to assess water policies." *In: Dinar, A., Zilberman, D. (eds) The Economics and Management of Water and Drainage in Agriculture.*

Böhringer, Christoph, Stefan Boeters, and Michael Feil (2005), "Taxation and unemployment: an applied general equilibrium approach." *Economic Modelling*, 22, 81–108.

Böhringer, Christoph and Andreas Löschel (2006), "Computable General Equilibrium models for sustainability impact assessment: status quo and prospects." *Ecological Economics*, 60, 49–64.

Burfisher, Mary E (2021), *Introduction to Computable General Equilibrium models*. Cambridge University Press.

Carbone, Jared C and Nicholas Rivers (2017), "The impacts of unilateral climate policy on competitiveness: evidence from Computable General Equilibrium models." *Review of Environmental Economics and Policy*, 11, 24–42.

Ciaschini, Maurizio, Rosita Pretaroli, Francesca Severini, and Claudio Socci (2012), "Regional double dividend from environmental tax reform: An application for the Italian economy." *Research in Economics*, 66, 273–283.

de Bruin, Kelly and Aykut Mert Yakut (2021), "Technical documentation of the I3E Model, v4. 0."

Dixon, Peter B and Brian R Parmenter (1996), "Computable General Equilibrium modelling for policy analysis and forecasting." *Handbook of Computational Economics*, 1, 3–85.

Grech, Aaron George, Grech Owen, Micallef Brian, Rapa Noel, and Gatt William (2013), "A structural macro-econometric model of the Maltese economy." Technical report, CBM Working Papers.

Guo, Yue Mei and Yun Rui Shi (2021), "Impact of the VAT reduction policy on local fiscal pressure in China in light of the COVID-19 pandemic: a measurement based on a Computable General Equilibrium model." *Economic Analysis and Policy*, 69, 253–264.

Kancs, d'Artis and Hans Kremers (2002), "Assessing impacts of alternative renewable energy strategies." Technical report, EERI Research Paper Series.

Li, Gen and Toshihiko Masui (2019), "Assessing the impacts of China's environmental tax using a Dynamic Computable General Equilibrium model." *Journal of Cleaner Production*, 208, 316–324.

Mainar-Causapé, Alfredo Jose, Emanuele Ferrari, and Scott McDonald (2018), "Social accounting matrices: basic aspects and main steps for estimation." *Publications Office of the European Union: Luxembourg.*

Meng, Samuel and Mahinda Siriwardana (2017), *Assessing the economic impact of tourism: a Computable General Equilibrium modelling approach*. Springer.

Narayan, Paresh Kumar (2004), "Economic impact of tourism on Fiji's economy: empirical evidence from the Computable General Equilibrium model." *Tourism Economics*, 10, 419–433.

Pratt, Stephen (2012), "Tourism yield of different market segments: a case study of Hawaii." *Tourism Economics*, 18, 373–391.

Rapa, Noel (2016), "MEDSEA: a small open economy DSGE model for Malta." Technical report, CBM Working Papers.

Reinert, Kenneth A and David W Roland-Holst (1997), "Social accounting matrices." *Applied methods for trade policy analysis: A handbook*, 94.

Robinson, Sherman (1991), "Macroeconomics, financial variables, and Computable General Equilibrium models." *World Development*, 19, 1509–1525.

Robson, Edward N, Kasun P Wijayaratna, and Vinayak V Dixit (2018), "A review of Computable General Equilibrium models for transport and their applications in appraisal." *Transportation Research Part A: Policy and Practice*, 116, 31–53.

Severini, Francesca, Francesco Felici, Noemi Ferracuti, Rosita Pretaroli, and Claudio Socci (2019), "Gender policy and female employment: a CGE model for Italy." *Economic Systems Research*, 31, 92–113.

Smith, Richard D, Marcus R Keogh-Brown, and Tony Barnett (2011), "Estimating the economic impact of pandemic influenza: an application of the Computable General Equilibrium model to the UK." *Social science & medicine*, 73, 235–244.

Socci, Claudio, Francesco Felici, Rosita Pretaroli, Francesca Severini, and Renato Loiero (2021), "The Multisector Applied Computable General Equilibrium Model for Italian Economy (MACGEM-IT)." *Italian Economic Journal*, 7, 109–127.

Thiele, Rainer, Manfred Wiebelt, et al. (1993), "Modeling deforestation in a Computable General Equilibrium model." Technical report, Kiel Working Paper.

Van der Werf, Edwin (2008), "Production functions for climate policy modeling: an empirical analysis." *Energy Economics*, 30, 2964–2979.

Walras, Leon (1874), "Elements of Pure Economics, translated from the French by W. Jaffé, 1954."

Willenbockel, Dirk (1994), *Applied General Equilibrium modelling: imperfect competition and European integration*, volume 1. Wiley.

Wing, Ian Sue (2004), "Computable General Equilibrium models and their use in economy-wide policy analysis." *Technical Note, Joint Program on the Science and Policy of Global Change, MIT.*

Wing, Ian Sue et al. (2007), "Computable General Equilibrium models for the analysis of energy and climate policies." *Prepared for the International Handbook of Energy Economics.*

Zotti, Jacopo, Rosita Pretaroli, Francesca Severini, Claudio Socci, and Giancarlo Infantino (2020), "Employment incentives and the disaggregated impact on the economy. The Italian case." *Economia Politica*, 37, 993–1032.

A Accounts in the Maltese SAM - Definitions

Table A.1: Classification of Commodities and Activities in the Maltese SAM

Nr.	NACE	Description
1	A01	Crop and animal production, hunting and related service activities
2	A02	Forestry and logging
3	A03	Fishing and aquaculture
4	C10T12	Manufacture of food products, beverages and tobacco products
5	C13T15	Manufacture of textiles, wearing apparel and leather products
6	C16	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
7	C17-22	Manufacture of paper and paper products, printing and reproduction of recorded media, manufacture of coke and refined petroleum products, chemical products, basic pharmaceutical products and pharmaceutical preparations and rubber and plastic products
8	C23	Manufacture of other non-metallic mineral products
9	C24	Manufacture of basic metals
10	C25	Manufacture of fabricated metal products, except machinery and equipment
11	C26-32	Manufacture of computer, electronic and optical products, electrical equipment, machinery and equipment n.e.c., motor vehicles, trailers and semi-trailers, Other transport equipment and of furniture; other manufacturing
12	C33	Repair and installation of machinery and equipment
13	D35-E39	Electricity, gas, steam and air conditioning supply, water collection, treatment and supply, sewerage; waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services
14	B+F	Mining and quarrying and construction
15	G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
16	G46	Wholesale trade, except of motor vehicles and motorcycles
17	G47	Retail trade, except of motor vehicles and motorcycles
18	H49-53	Land transport and transport via pipelines, water transport, air transport, warehousing and support activities for transportation and postal and courier activities
19	I	Accommodation and food service activities
20	J58-63	Publishing activities, motion picture, video and television programme production, sound recording and music publishing activities; programming and broadcasting activities; Telecommunications, computer programming, consultancy and related activities; and information service activities
21	K64	Financial service activities, except insurance and pension funding
22	K65	Insurance, reinsurance and pension funding, except compulsory social security
23	K66	Activities auxiliary to financial services and insurance activities
24	L68A	Inputed Rent
25	L68B	Real estate activities
26	M69+70	Legal and accounting activities; activities of head offices; management consultancy activities
27	M71+M72	Architectural and engineering activities; technical testing and analysis
28	M73	Advertising
29	M74_75	Other professional, scientific and technical activities; veterinary activities; advertising and research
30	N77	Rental and leasing activities
31	N78	Employment activities
32	N79	Travel agency, tour operator reservation service and related activities
33	N80	Security and Investigation activities
34	O84	Public administration and defence; compulsory social security
35	P85	Education
36	Q86	Human health activities
37	Q87_88	Social work activities
38	R90T92	Creative, arts and entertainment activities; libraries, archives, museums and other cultural activities; gambling and betting activities
39	R93	Sports activities and amusement and recreation activities
40	S94	Activities of membership organisations
41	S95	Repair of computers and personal and household goods
42	S96	Other personal service activities
43	T97	Activities of households as employers
44	T98	Undifferentiated goods- and services-producing activities of households for own use and of extra-territorial organisations and bodies

B Aggregation Equations

This Appendix provides the expressions for the *real* and *nominal* Gross Domestic Product (GDP) laid out from the expenditure side. By definition, this approach quantifies GDP as the sum of private consumption, government expenditure, investment, and net exports. Nominal GDP, denoted by GDP^N, is calculated as follows:

$$GDP^N = \left\{ \sum_{g=1}^{44} P_g \left[\sum_j (C_{g,j}) + C_{g,i} + I_g + X_g^E \right] - \sum_{g=1}^{44} P_g^M Y_g^M \right\} \tag{77}$$

Real GDP, denoted by GDP^R, can be expressed analogously to Nominal GDP, with the sole difference being the non-consideration of commodity prices and imports prices:

$$GDP^R = \left\{ \sum_{g=1}^{44} \left[\sum_j (C_{g,j}) + C_{g,i} + I_g + X_g^E - Y_g^M \right] \right\} \tag{78}$$

The expressions for Nominal GDP and Real GDP can be used to estimate the GDP deflator, given by GDP^D:

$$GDP^D = \frac{GDP^N}{GDP^R} \tag{79}$$

We also calculate a consumption deflator, based on the nominal and real consumption of households and NPIs. More specifically, the *nominal* and *real* consumption expenditures of institutional sector $j \in \{Hh, Nip\}$ are respectively obtained as follows:

$$C^N = \left\{ \sum_{g=1}^{44} \left[P_g \sum_j C_{g,j} \right] \right\} \tag{80}$$

$$C^R = \left\{ \sum_{g=1}^{44} \left[\sum_j C_{g,j} \right] \right\} \tag{81}$$

where C^N and C^R represent nominal and real consumption, respectively. The consumption deflator, denoted by C^D, can then be calculated as:

$$C^D = \frac{C^N}{C^R} \tag{82}$$