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VALUE, PRICES AND DISTRIBUTION

COMPUTABLE METHODS FOR THE INVESTIGATION OF LONG-PERIOD ECONOMIC STRUCTURES

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INTRODUCTION

The research intends to analyze the fundamental relationships between the production of value and the distribution of income inside an economic system from the perspective of the Classical political economists like Adam Smith, David Ricardo and Karl Marx. This methodological perspective is recovered by the economic literature after the publication of Piero Sraffa (1960) with which the analytical knots left unresolved by Ricardo, on the invariant measure of value, and by Marx, on the transformation of values into production prices are dissolved. The work is organized into three independent essays that address the theoretical, methodological and empirical characteristics of the economic analysis based on the surplus approach.

The first essay, *From Pasinetti: backwards, to the analytical connections between Marx and Sraffa; and forward, to the macro-monetary perspectives of the labour theory of value,* reconstructs the framework of the labour theory of value by following the logical passages with which Pasinetti arrives at the solution of the Marxian transformation problem. After Sraffa's contribution, the conclusion was reached that production prices can be obtained regardless of the labour-value of the commodities and that therefore the labour theory of value is superfluous in determining the distribution of income with the exception of the particular case where the surplus is entirely distributed to the wage workers. However, according to Marx, the labour theory of value is a law of capitalist reproduction which links the production of use-values with the realization of exchange-values whatever the law that rules the distribution of income. The question, therefore, turns on the need to articulate the two levels of the analysis of capitalist systems: the production of the surplus and its composition between wage and profit goods, that can be represented in terms of labour embodied, and the distribution of income, for which production prices must be introduced for evaluating the economic aggregates consistently with the prevailing competitive assumptions put subsequently. Pasinetti (1977) succeeds in this aim starting from a formalization of the Marxian theoretical framework according to the same logic through which Sraffa elaborates the standard system. Pasinetti highlights that any procedure for transforming labour-values into prices must consider the choice of a single or composite commodity whose value does not change in the passage from the value system to the price system, this commodity must therefore be chosen as the *numéraire* of prices. However, in order to have a complete coincidence between values and prices in Marx's sense, each *numéraire* of prices must be chosen consistently with a certain definition of the real wage and vice versa. In this way, it is possible to obtain a double equivalence between the monetary value added and the total direct labour employed and between the aggregate profit and the global surplus-labour extorted in production. From this methodological movement derives the modern macro-monetary approach, notably the *Price of Net Product–Unallocated Purchasing Power Labour Theory of Value* (PNP–UPP LTV), through which the labour theory of value can be reappraised in the investigation of economic structures.

The second essay, *Production prices, income distribution and the value structure inside the Italian economy: An input-output investigation*, offers an application of the macro-monetary approach presented in the previous essay for the analysis of the Italian economy (2018) based on an input-output table of order 63. The investigation moves following three analytical passages: that of the technical conditions of production captured by the labour values (direct prices), that of the institutional conditions that guarantee the uniformity of the profit rates represented by the prices of production, and finally the one in which the real-world institutional settings lead to the formation of the observed market prices of the economy. The differences of production prices and labour values (direct prices) from the observed market prices in the economy are evaluated and the corresponding wage-profit curve is estimated. The survey shows that direct prices and production prices are able to explain a good percentage of market prices, thus confirming the results of previous research. Some simulations of the effects of the variations in income distribution on the ratios between production prices and labour-values and on key indicators such as the sectoral capital-labour and output-capital ratios are then performed. The analysis of the price changes focuses on the two effects through which the variations in income distribution impact the changes in production prices: the capital intensity effect and the price effect. The former consists of the impact on prices coming from the distance of the sectoral capital-labour ratios from the capital intensity of the commodity-*numéraire*, the second consists of the impact on prices coming from the simultaneous revaluation of the price of the means of production. Thanks to the use of Sraffian standard commodity, that when chosen as *numéraire* of prices has the advantage of deleting the disturbances effects deriving from the price variation of the *numéraire*-commodity, we observe how the trajectories followed by the prices to the variations in income distribution are sufficiently predictable to the extent that the capital intensity effect prevails over the price effect in most cases, while the feedback effect of the revaluation of the means of production are often negligible.

The third essay, *Self-replacing prices with alternative market structures and the methodological reappraisal of the Sraffian standard system*, focuses on the rules of self-replacing price formation in the framework of non-uniform rates of profit. Existing literature agrees that the role of the standard commodity can be neglected because, when the assumption of uniformity of profit rates is removed, its properties lose their usefulness of making the distributive relation transparent, i.e. independent of prices. between wages and the rate of profit. This contribution puts forward the hypothesis of the possibility that the standard system can regain methodological success for the descriptive purposes of economic analysis in a self-replacing price system with non-uniform profit rates. A computational procedure is developed to construct the standard system as an "equivalent system" showing the necessary distribution of income that allows the self-replacement of the system while remaining independent of the level and structure of prices, profit rates and any institutional mechanisms or constraints that influence their variations. Notably, we highlight the fact that the general rate of profit, based on the hypothetical situation of perfect competition, acquires a deeper theoretical meaning than the ex post measure of the average of sectoral profit rates and can be obtained within the standard system to carry out comparative assessments on the observed market structure inside the economy.

CHAPTER ONE

From Pasinetti: Backwards, to the analytical connections between Marx and Sraffa; and forward, to the macro-monetary perspectives of the labour theory of value

Abstract

The contribution investigates the relationships between the production of value and the distribution of income within the theoretical framework of the Classical political economy. In this regard, we will follow the methodological approach inaugurated by Luigi Pasinetti (1977) starting from the re-reading of the theories of value of Adam Smith, David Ricardo and Karl Marx carried out in the analytical language of John von Neumann (1945), Wassily Leontief (1951) and Piero Sraffa (1960). It will be argued that Pasinetti's contribution plays the role of a bridge between Classical political economy and the modern theoretical and empirical approaches to structural economic investigation. The discussion will focus on two directions: backwards, to the "transformation problem of values into prices of production", brought by Marx in volume III of *Capital*, to which Pasinetti provides a generalizable solution according to the two fundamental propositions of the Marxian analysis; and forward, to the macro-monetary perspectives of the modern analysis of value and distribution.

Keywords: Marx, Sraffa, Pasinetti, Labour theory of value, Production price systems.
JEL classification: B51, C67, E11.

1. Introduction

The contribution investigates the relationships between the production of value and the distribution of income within the theoretical framework of the Classical political economy. In this regard, we will follow the methodological approach inaugurated by Luigi Pasinetti (1977) starting from the re-reading of the theories of value of Adam Smith, David Ricardo and Karl Marx carried out in the analytical language of John von Neumann (1945), Wassily Leontief (1951) and Piero Sraffa (1960). It will be argued that Pasinetti's contribution plays the role of a bridge between Classical political economy and the modern theoretical and empirical approaches to structural economic investigation. The discussion will focus on two directions: backwards, to the "transformation problem of values into prices of production", brought by Marx in volume III of *Capital*, to which Pasinetti provides a generalizable solution according to the two fundamental propositions of the Marxian analysis; and forward, to the novel macro-monetary perspectives that advance into the methodological and empirical sphere of modern analyses of value and distribution.

The work is organized as follows. In section 2 a simple linear scheme of production is presented through the lenses of the Classical economists. In section 3 we present the basic rules for the formation of the prices inside a pre-institutional analytical framework relating, namely where the equality between labour embodied and labour commanded holds. In section 4 we enter the analysis of the capitalist mode of production in the terms of a labour-value system as elaborated by Marx after the manner of Ricardo. In section 5 we deal with the formation of production prices inside a capitalist economy, namely inside an institutional framework where capitalist appropriate a fraction of the surplus in the form profit and pays the workers with a subsistence wage. Consequently we show the reasons why the transformation problem arises. In section 6 we do an excursus of the solution that we can obtain when dealing with Sraffa's standard system due to its peculiar properties. In section 7 we provide a simple explanation of the procedure elaborated by Pasinetti (1977, appendix to chapter 5) for the formation of Marx's production prices. In section 8 we discuss the elaboration of modern macro-monetary method to value and distribution. In section 9 we provide an explanatory comparison among the different approaches considered in previous sections. In section 10 we translate the concepts discussed into numerical examples, where the main propositions explained are verified. Concluding remarks follow.

2. Vertically integrated self-replacing subsystems

Consider an economic system with a number of industries that produce goods necessary for both production and consumption using means of production that are replaced in each period. The technique adopted by the system can be expressed with a square matrix of multi-sectoral coefficients $\mathbf{A} = [a_{ij}]$ which we assume to be viable (i.e. has a dominant eigenvalue less than unity, $\lambda_m^{\mathbf{A}} \leq 1$) and with a row vector of coefficients of direct labour employed in the production of each commodity $\mathbf{l}^{\mathbf{T}} = [l_j]$. The system of physical quantities can be expressed as follows:[1]

$$\begin{bmatrix} (1-a_{11}) & -a_{12} & \cdots & -a_{1n} & -y_1 \\ -a_{21} & (1-a_{22}) & \cdots & -a_{2n} & -y_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -a_{n1} & -a_{n2} & \cdots & (1-a_{nn}) & -y_n \\ -l_1 & -l_2 & \cdots & -l_n & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \\ L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

that in a more compact form, can be written as:

$$[\mathbf{I} - \mathbf{A}]\mathbf{x} = \mathbf{y}L \tag{2}$$

$$\mathbf{l}^{\mathbf{T}}\mathbf{x} = L \tag{3}$$

$$\mathbf{A}\mathbf{x} = \mathbf{M} \tag{4}$$

where $\mathbf{x} = [x_i]$ is the column vector of gross finished product, $\mathbf{y} = [y_i]$ the column vector of the net product per worker, $\mathbf{M} = [m_i]$ the column vector of the means of production and L the scalar of the quantity of labour required in the economic system.

The condition of the existence of economically meaningful (non-zero) solutions requires that the determinant of the system of physical quantities is equal to zero, hence:

$$\det[\mathbf{I} - \mathbf{A}] = 0 \tag{5}$$

Suppose that the technology defined by the technical coefficients of production remains constant, as well as the level of the employment that for simplicity we consider equal to total labour force, namely the system runs in a stationary state. Given the net product required for final consumption, we obtain the following solutions for quantities, labour and means of production:

$$\mathbf{x} = [\mathbf{I} - \mathbf{A}]^{-1}\mathbf{y}L \tag{6}$$

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[1] All vectors are vertical, while the horizontal vectors are transposed and therefore marked with superscript **T**. Vectors are indicated in bold lowercase letters, while bold uppercase letters indicate matrices. The scalars are signed with italic letters.

$$L = \mathbf{l^T}[\mathbf{I} - \mathbf{A}]^{-1}\mathbf{y}L = \mathbf{v^T}\mathbf{y}L \tag{7}$$

$$\mathbf{M} = \mathbf{A}[\mathbf{I} - \mathbf{A}]^{-1}\mathbf{y}L = \mathbf{H}\mathbf{y}L \tag{8}$$

$$L = \bar{L} \tag{9}$$

where matrix $[\mathbf{I} - \mathbf{A}]^{-1}$ is the well-known Leontief inverse matrix whose columns contain the quantities of goods that are directly and indirectly required in the economic system to obtain a physical quantity of final good (Leontief 1951). We define with $\mathbf{v^T} = \mathbf{l^T}[\mathbf{I} - \mathbf{A}]^{-1}$ the vector of vertically integrated labour coefficients, that in Classical terms are defined as the quantities of labour embodied in each commodity (what Marx calls labour-values), representing the quantity of labour directly and indirectly required to produce one unit of final good, and with $\mathbf{H} = \mathbf{A}[\mathbf{I} - \mathbf{A}]^{-1}$ the matrix of vertically integrated production capacity coefficients, representing the means of production directly and indirectly required to make each product available for final consumption (Pasinetti 1973).

Once condition (5) is satisfied, it introduces one degree of freedom into system (1). In other words, solution (6) determines only the relative quantities of the economic system. The necessary condition that fulfils (5) results to be the following:

$$\mathbf{v^T}\mathbf{y} = \mathbf{l^T}[\mathbf{I} - \mathbf{A}]^{-1}\mathbf{y} = 1 \tag{10}$$

Since we have considered the level of employment exogenous and constant, the *numéraire* of relative quantities can naturally be the unit of labour embodied (directly and indirectly required) by the bundle of commodities that constitute the net product per worker.

The analysis of value within Classical political economy does not involve the employment of a quantity system in the Keynesian, Leontevian or other approaches which imply variations of the quantities produced. The analysis concerns the mechanism that rules the production and the composition of the surplus in a certain period. Although it is useful to employ the Leontief inverse and the vertical integration of subsystems, there is no need to make hypotheses about the constancy of the given technique at different levels of output due to the simple fact that no hypotheses are put forward on the variations of quantities. The analysis aims, instead, to interpret the way in which the production of the period examined took place, and therefore, the meaning of the system of quantities changes completely. From the point of view of the Classical economists, condition (10) acquires a theoretical meaning that goes beyond the analytical need to obtain economically appreciable solutions. In fact, the latter states that the production process that gave rise to the net product (or surplus) took place by applying human labour activity to the composition of commodities used as means of production, which in turn were produced through the application of labour to other means of production etc. In other terms, the physical value added is produced (and therefore can be effectively measured) by the labour employed in the process of production of commodities by means of commodities (i.e. total direct or abstract labour). This idea derives from the macro-social perspective adopted by the Classical economists, who consider property rights and the capitalist forms of commodity production and exchange as nothing more than a few mechanisms, among many others, for organizing the human activity of production that is labour.

3. Pasinetti's "natural price system"

We can now move to the analysis of the mechanism that rules the exchanges and the distribution of the commodities produced. In a pre-institutional stage of economic analysis, that is by abstracting from any social rule that regulates the distribution of the surplus, we can presume that prices are exclusively determined by the division of labour, i.e. the exchange relations between commodities reflect their nature as products of organized labour. Hence, their realization allows nothing else than the replacement of the means of production and the individual consumption of the workers or independent producers. Such system of prices can be defined as follows:

$$\begin{bmatrix} (1-a_{11}) & -a_{21} & \dots & -a_{n1} & -l_1 \\ -a_{12} & (1-a_{22}) & \dots & -a_{n2} & -l_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -a_{1n} & -a_{2n} & \dots & (1-a_{nn}) & -l_n \\ -y_1 & -y_2 & \dots & -y_n & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

that in a compact form becomes:

$$\mathbf{p}^{\mathbf{T}} = \mathbf{p}^{\mathbf{T}}\mathbf{A} + w\mathbf{l}^{\mathbf{T}} \tag{12}$$

As with the quantity system, we also need an additional equation for the price system to define the unit of measure for relative prices. If we choose to consider the wage rate as the *numéraire* of the system, that is Smith's choice to measure prices in terms of labour commanded, hence with:

$$w = \bar{w} = 1 \tag{13}$$

we can obtain the following solution for the price system:

$$\mathbf{p}^{\mathbf{T}} = \bar{w}\mathbf{l}^{\mathbf{T}}[\mathbf{I} - \mathbf{A}]^{-1} = \mathbf{v}^{\mathbf{T}} \tag{14}$$

In the formation of prices, therefore, every characteristic of the economic process in the competitive sense plays no role and the prices express the relationships between the quantities of labour embodied in the commodities. Consequently, when profit is absent, the labour commanded (purchased) by the commodities corresponds to the respective quantity of labour embodied.

We arrive at what Pasinetti (1977, 1981, 1988) calls "natural price system": a configuration of relative prices that once realized channel to the workers exactly the purchasing power equivalent to the value added produced with their own labour.[2]

In the natural system, therefore, the labour theory of value prevails. This framework is often considered as an abstract representation of the "rude and primitive" stage of the economy, as elaborated by Smith, that therefore precedes the entire appropriation of land and the original accumulation of capital. Pasinetti refers instead to a pre-institutional (not pre-capitalist) setting of the analysis, meaning that the natural system abstracts from any possible rule of income distribution but focuses exclusively on the necessary and essential features of an economy of production of commodities by means of commodities.[3]

The condition that sets the wage rate equal to unity states that the wage can command (purchase) an amount of labour exactly equal to the labour embodied in it, which brings to conclude that the average wage rate is at the same level of the net

[2] Pasinetti (1981, 1988) defines with "natural system" a price structure emerging from growing economic subsystems where the role of profit is that to fund *ex post* the investment required to make the economic system grow at the same rate of the working population (natural growth rate), while the rest can be intended to wages. Natural prices are here proportional to the quantities of labour embodied, where the re-definition of labour embodied includes the part of labour employed to guarantee the growth of the economy (in his terms, vertically hyper-integrated labour coefficients). In that framework, the labour theory of value is not a rule of income distribution but is a physical rule of accumulation and full employment. In a situation of a stationary economy like the one that we are considering here, since new capital investment is absent, the natural price system simply provide the same results of the direct price system (i.e. prices proportional to Classical labour-values). This latter case is instead considered in Pasinetti (1977, appendix to chapter 5; see also Pasinetti and Garbellini 2015) where he discusses the Marxian problem of the transformation of values into production prices and he refers to direct prices as an "ideal price system", i.e. when the Ricardian socialist labour principle of income distribution prevails in the determination of exchange relationships. On Pasinetti's original formulation of the natural price system, see also Bellino (2021) and Garbellini (2010).

[3] Cf. Garbellini and Wirkierman (2014).

product per worker. We can verify these sentences by computing the labour commanded by the aggregate net product:

$$\mathbf{p}^{\mathbf{T}}\mathbf{y} = \bar{w}\mathbf{l}^{\mathbf{T}}[\mathbf{I}-\mathbf{A}]^{-1}[\mathbf{I}-\mathbf{A}]\mathbf{x} = \mathbf{v}^{\mathbf{T}}\mathbf{y} \tag{15}$$

that states that the value of the net product (whatever the measure unit) divided by the wage rate (expressed in terms of the same measure unit) is equal to the labour embodied in the net product that, in turns, represents the total employment of the economic system. Finally, returning to condition (10), that excludes negative solutions for both the system of quantities (1) and that of the prices (11), we can replace the expression $\mathbf{v}^{\mathbf{T}}\mathbf{y}$ into equation (15) and obtain the following expression:

$$\mathbf{p}^{\mathbf{T}}\mathbf{y} = w \tag{16}$$

that coincides with the last equation of system (11). In the case in which the net product is entirely distributed to the workers, the labour embodied (determined by the system of quantities) and the labour commanded (determined by the price system) are equivalent.

4. Marx's labour-value system

The operations carried out up to now are of a purely accounting nature and are limited to a purely descriptive analysis of the production activity. The "arbitrary element" that we have introduced lies in the choice of labour as the entity in which to express the production process. This choice may seem arbitrary until we consider the process described by system (1) from a purely technical point of view. However, it ceases to be so when we consider production as a social phenomenon and becomes necessary in the theoretical framework of Classical political economy. From this perspective, labour is seen as abstract social energy that is considered homogeneous and distinct from the concrete forms of labour individually performed. As a use of social energy, labour needs to be periodically reproduced by society that therefore bears, as a community, the social cost of reproduction. This cost is represented by a basket of consumer goods necessary for the subsistence of workers that therefore defines the real wage:

$$\mathbf{b}^{\mathbf{T}} = [b_1 \quad b_2 \quad \dots \quad b_n] \tag{17}$$

In this sense, the economic system produces something more than what is consumed in production when $\mathbf{y}L \geq \mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x}$. According to Marx, the production of the value added, i.e. the neo-value,[4] is not only the consequence of the fact that the productive process is carried out through the division of labour but it is, above all, the necessary consequence of the capitalist mode of production in which labour is considered a commodity (labour power).[5] Therefore, the labour embodied in the real wage is the expression of the commodification of labour and it represents the value of the labour power as commodity, that must logically be less than the total labour employed in production, i.e. total direct or abstract labour:

$$\mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x} = \omega L, \qquad \text{where } \omega < 1 \tag{18}$$

For Marx, the exchange of the labour force contains an exploitative relationship to the extent that the working time (day, year, etc.) procured by the capitalists is longer than the working time necessary for the reproduction and maintenance of the labour power. In other terms, the abstract labour globally employed in production is greater than the labour embodied in the commodities that compose the real wage. In this formulation, ω represents the unit value of the labour power, i.e. the fraction of the working time during which each worker is employed to produce the subsistence goods, that is indeed "necessary labour". What remains, namely $(1-\omega)$, represents the fraction of the working time that is extorted by the capitalist class and that is required to produce the surplus value, that is the ultimate source of monetary profits, and is therefore "surplus-labour". This is the reason why Marx calls the relationship between these two labour magnitudes the rate of exploitation of the working class, i.e. the rate of surplus value:

$$\sigma = \frac{(1-\omega)\mathbf{l}^{\mathbf{T}}\mathbf{x}}{\omega\mathbf{l}^{\mathbf{T}}\mathbf{x}} = \frac{1}{\omega} - 1 \tag{19}$$

Marx's system of labour-values can therefore be formulated in the following way:

$$\begin{bmatrix} (1-a_{11}) & -a_{21} & \dots & -a_{n1} & -l_1 \\ -a_{12} & (1-a_{22}) & \dots & -a_{n2} & -l_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -a_{1n} & -a_{2n} & \dots & (1-a_{nn}) & -l_n \\ -b_1(1+\sigma) & -b_2(1+\sigma) & \dots & -b_n(1+\sigma) & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \\ \omega \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \tag{20}$$

that in compact form becomes:

$$\mathbf{v}^{\mathbf{T}}\mathbf{x} = \mathbf{v}^{\mathbf{T}}\mathbf{A}\mathbf{x} + \mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x}(1+\sigma) \tag{21}$$

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[4] The German term is *Neuwert* and is incidentally introduced by Marx to distinguish "the value added annually *ex novo* from the labour added *ex novo*" (Marx 1993, Ch. 50; cf. Gattei 2011).

[5] On the concept of mode of production, see Marx's *Foundations of a Critique of Political Economy* volume I and *Capital* volume III, Ch. 51.

As it clearly emerges, the value of each commodity can be decomposed into three parts: (i) the replacement of the means of production, i.e. constant capital (indirect labour $\mathbf{v}^{\mathbf{T}}\mathbf{A}\mathbf{x}$); (ii) the replacement of consumer goods necessary for the subsistence of the workers, i.e. variable capital (necessary labour $\mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x}$); and (iii) the surplus value that by definition is equal to the surplus-labour ($\sigma\mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x}$). In this way Marx explains, in one time, the value of the labour power, the necessity of the production of surplus value (in order to support themselves, the workers must perform surplus labour inside production) and consequently the origin of the profit.[6]

The phases of the capitalist process can be defined as follows. Capitalist process begins in money when the capitalists bear the costs of production that include the means of production and the wage bill. On the one hand, the means of production then enter the actual production process as use values (all that requires a certain amount of coal, tools and machines). The value of these input-commodities is fully transferred into the value of the finished product, and in fact Marx calls their value *constant capital*. On the other hand, the wage goods necessary for the subsistence of the workers, that is the social costs of reproduction, do not enter production but instead they are consumed outside the factories and the countryside, expressing their use value in the particular needs of the workers. The wage goods represent the price of labour as commodity and in fact do not enter production directly as use values (bread, clothes and house rent are not strictly necessary for the production of value) but "are replaced by the actual human labour activity" (Perri 1998, p. 128, translated from the Italian) hence by the global abstract labour ($\mathbf{l}^{\mathbf{T}}\mathbf{x}$) that is divided between necessary and surplus labour. Upon payment of the monetary wage, the labour power is sold (and legally belongs) to the capitalists, who can dispose of it throughout the working day. This is the reason why Marx calls the value of the labour power (i.e. the wage bill) *variable capital*.

We have shown that the schema of labour-values is not a representation of technical relationships but a description of the social relationships between classes inside the capitalist economy. In fact, through this system, Marx not only observes that

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[6] Marx affirms: "Capitalist production is not merely the production of commodities, it is essentially the production of surplus-value. The labourer produces, not for himself, but for capital. It no longer suffices, therefore, that he should simply produce. He must produce surplus-value. That labourer alone is productive, who produces surplus-value for the capitalist, and thus works for the self-expansion of capital." (Marx 1990, Ch. 16).

production is carried out through the division of labour and that the surplus value is determined by deducting from the annual neo-value a share necessary to reproduce the labour power consumed, but he also observes that the capitalist production of commodities implies the production of exchange values, mediated by the production of use values, that can be expressed in terms of labour embodied.

Obviously, the solutions of the labour-values are the same as those obtained for the direct or natural prices (14) together with equations (17), (18) and (19) that define the real wage, the value of labour power and the rate of surplus value respectively. However, the same solutions can be obtained directly from equation (21) by doing the following algebraic manipulations:

$$\mathbf{v}^{\mathbf{T}}[\mathbf{I} - (\mathbf{A} + \mathbf{b}\mathbf{l}^{\mathbf{T}})] = \sigma \mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}} \tag{22}$$

$$\mathbf{v}^{\mathbf{T}} = \sigma \mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}[\mathbf{I} - (\mathbf{A} + \mathbf{b}\mathbf{l}^{\mathbf{T}})]^{-1} \tag{23}$$

$$\mathbf{v}^{\mathbf{T}}[\mathbf{I} - \sigma \mathbf{b}\mathbf{l}^{\mathbf{T}}[\mathbf{I} - (\mathbf{A} + \mathbf{b}\mathbf{l}^{\mathbf{T}})]^{-1}] = \mathbf{0}^{\mathbf{T}} \tag{24}$$

The necessary condition for the existence of non-negative solutions for the quantities of labour embodied is the following:

$$\det[\mathbf{I} - \sigma \mathbf{b}\mathbf{l}^{\mathbf{T}}[\mathbf{I} - (\mathbf{A} + \mathbf{b}\mathbf{l}^{\mathbf{T}})]^{-1}] = 0 \tag{25}$$

that can be derived as follows:

$$\mathbf{v}^{\mathbf{T}}\mathbf{b}(1 + \sigma) = 1 \tag{26}$$

which is equivalent to setting the quantity of one unit of labour embodied as the *numéraire* of the system of labour-values. We can notice that the value added (i.e. neo-value) is now defined as a composite commodity that has the same volume of the net product per worker ($\mathbf{y}$) but has the same proportions (i.e. the same structure) of the basket of the subsistence goods ($\mathbf{b}$). Hence, the composite commodity of the neo-value, that is here chosen as *numéraire* of labour-values, is defined here by the vector $\mathbf{b}(1 + \sigma)$ that accounts for a distinct separation between the components of the surplus or net product of the wage and the profit goods.

5. Production price system

The transition from the situation assumed in the natural system (i.e. a pre-institutional setting in which the division of labour is reflected inside the exchange relations) to a distribution of the net product in which other classes compete by virtue of a right of ownership of the means of production (i.e. a capitalist society) does not formally change the technical characteristics that determine the labour-values, although, as we have seen, it assigns a different meaning to the rules of product formation. Instead, the rules of price formation change. This is why Marx's system of labour-values cannot be grasped in the economic reality because the part of the net product that exceeds wages and is appropriated by the capitalists in the form of profits, is distributed in proportion to the capital invested and not in proportion to the labour employed in production, like it is instead for the surplus value. Since the production of commodities is finalized to capital valorisation, the prices of production must reflect the capitalist law of exchange, namely they express the nature of commodities as products of capital. Exchange relations between commodities are determined according to the competitive condition that guarantees the uniformity of the rate of profit realized by each industry in proportion to the capital invested ($\pi_j = \pi$). Thus, the prices of production will generally differ from the respective labour-values for any positive rate of profit.

Consider the case in which the wage, following Sraffa and unlike Marx, is paid at the end of the production period. The relative price system now becomes:

$$\begin{bmatrix} (1-(1+\pi)a_{11}) & -(1+\pi)a_{21} & \dots & -(1+\pi)a_{n1} & -l_1 \\ -(1+\pi)a_{12} & (1-(1+\pi)a_{22}) & \dots & -(1+\pi)a_{n2} & -l_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -(1+\pi)a_{1n} & -(1+\pi)a_{2n} & \dots & (1-(1+\pi)a_{nn}) & -l_n \\ -y_1 & -y_2 & \dots & -y_n & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad (27)$$

that can be written as:

$$\mathbf{p^T} = \mathbf{p^T A}(1+\pi) + w\mathbf{l^T} \quad (28)$$

Now suppose that, following Marx, the wage rate is defined with the price of the basket of physical wage goods specified above:

$$w = \mathbf{p^T b} \quad (29)$$

Hence, system (28) can be reformulated as follows:

$$\mathbf{p^T} = \mathbf{p^T}(\mathbf{A} + \mathbf{bl^T}) + \pi\mathbf{p^T A} \quad (30)$$

$$\mathbf{p^T}[\mathbf{I} - (\mathbf{A} + \mathbf{bl^T})] = \pi\mathbf{p^T A} \quad (31)$$

$$\mathbf{p^T} = \pi\mathbf{p^T A}[\mathbf{I} - (\mathbf{A} + \mathbf{bl^T})]^{-1} \quad (32)$$

where $\mathbf{A} + \mathbf{bl^T}$ is the augmented matrix, i.e. matrix that results from the material input matrix "augmented" by the necessary consumption coefficients required for the social reproduction (i.e. the subsistence) of the labour power.[7]

[7] For an examination of alternative rules for relative price determination, see Bródy (1970) and Mariolis and Tsoulfidis (2016).

In this way we obtain the following system of homogeneous equations:

$$\mathbf{p}^{\mathbf{T}}[\mathbf{I} - \pi\mathbf{\Theta}] = \mathbf{0}^{\mathbf{T}} \tag{33}$$

where $\mathbf{\Theta} = \mathbf{A}[\mathbf{I} - (\mathbf{A} + \mathbf{b}\mathbf{l}^{\mathbf{T}})]^{-1}$. These equations can lead to non-negative solutions only if the following condition is satisfied:

$$\det[\mathbf{I} - \pi\mathbf{\Theta}] = 0 \tag{34}$$

For the hypothesis of a viable technique advanced, the Perron-Frobenius theorem ensures that the Leontief inverse, and therefore also matrix $\mathbf{\Theta}$, is a non-negative matrix with a real and positive dominant eigenvalue that has a non-negative eigenvector (non-negative prices) associated with it. We can therefore obtain economically meaningful solutions for the relative price equations by deriving the rate of profit that satisfies condition (34) as follows:

$$\pi_{\mathbf{\Theta}} = \frac{1}{\lambda_m^{\mathbf{\Theta}}} \tag{35}$$

After substituting this rate of profit into equation (33) it only remains to add any equation that defines the *numéraire* of production prices so as to obtain all prices in terms of the chosen measure unit. What follows is a detachment of the determination of production prices and the rate of profit from the determination of labour-values and of the rate of surplus value. In fact, not only cannot prices be determined on the basis of the labour embodied in commodities, but the knowledge of the latter is not even necessary for their treatment. However, when instead we look at the price system from the point of view of the analysis of production, and therefore of value, the problem that arises is that of reconciling the fundamental propositions relating to capitalist production formulated in that sphere and the features proper of commodity exchanges and income distribution.

Marx's line of argument is based on the hypothesis that neo-value is extracted in production (proportionally to labour) by the capitalist class. After the production of commodities, the capitalists re-distribute the surplus value extracted from the global abstract labour in the form of profits (proportionally to capital). However, for the capitalists considered individually, what is interesting is the rate of profit, that is brought to uniformity by the competitive forces that drive the capitalists to invest in the most profitable sectors. In other words, the rate of profit is directly influenced by the surplus value and negatively by the technical composition of capital (the ratio between the means of production and the direct labour), as can be observed more clearly from expression (31). However, while the former is assumed to be uniform among the productive sectors, nothing guarantees that industries show uniform technical compositions of capital. Consequently, since, as stated above, prices generally diverge from their respective values ($\mathbf{p}^{\mathbf{T}} \neq \mathbf{v}^{\mathbf{T}}$), the two fundamental Marxian principles that state that the value added (neo-value) is equivalent to total direct labour and that the aggregate profit is equal to total surplus-labour cannot be satisfied except in special cases. Hence, in general we have:

$$\mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x}(1+\sigma) \neq \mathbf{p}^{\mathbf{T}}\mathbf{y}L \tag{36}$$

and

$$\sigma\mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x} \neq \pi\mathbf{p}^{\mathbf{T}}\mathbf{H}\mathbf{y}L \tag{37}$$

Of course, we can take advantage of the degree of freedom of the relative price system and choose the *numéraire* in such a way to reduce at least one of these two inequalities into an equality. However, before proceeding to such direction, we can consider once again the wage rate as the *numéraire* of production prices:

$$w = \overline{\mathbf{p}^{\mathbf{T}}\mathbf{b}} = 1 \tag{38}$$

that amounts to expressing the production prices in terms of labour commanded (purchased) by each commodity. Given the choice of the *numéraire*, system (28) can be reformulated as follows:

$$\mathbf{p}^{\mathbf{T}}[\mathbf{I}-\mathbf{A}] = \overline{\mathbf{p}^{\mathbf{T}}\mathbf{b}}\mathbf{l}^{\mathbf{T}} + \pi_{\mathbf{\Theta}}\mathbf{p}^{\mathbf{T}}\mathbf{A} \tag{39}$$

$$\mathbf{p}^{\mathbf{T}} = \mathbf{v}^{\mathbf{T}} + \pi_{\mathbf{\Theta}}\mathbf{p}^{\mathbf{T}}\mathbf{H} \tag{40}$$

from which it emerges clearly that each price is essentially composed by wages and profits. The solutions for prices are therefore:

$$\mathbf{p}^{\mathbf{T}} = \mathbf{v}^{\mathbf{T}}[\mathbf{I}-\pi_{\mathbf{\Theta}}\mathbf{H}]^{-1} \tag{41}$$

Since wages are paid in proportion to the labour performed in production, this means that wages can command only a fraction of the entire purchasing power expressed by commodity prices. The difference is in fact absorbed by the profits:

$$(\mathbf{p}^{\mathbf{T}}-\mathbf{v}^{\mathbf{T}})\mathbf{y} = \pi_{\mathbf{\Theta}}\mathbf{p}^{\mathbf{T}}\mathbf{H}\mathbf{y} \tag{42}$$

This formalization, as Pasinetti (1973) states, can be interpreted as a procedure for the Marxian transformation of labour-values into production prices. The price of production diverges from respective quantity of labour embodied as a consequence of the fact that the purchasing power of the commodities also accounts for the component of profit, represented by the linear operator $[\mathbf{I}-\pi_{\mathbf{\Theta}}\mathbf{H}]^{-1}$, that obviously disappears in the case where prices become proportional to labour embodied (i.e. natural prices). However, one could argue that here labour-values and production prices are expressed in terms of two totally different concepts of labour, which makes difficult the comparison between the two systems. In fact, in the general case in which the profit rate is positive, the prices will always diverge from (and be greater than) the respective labour-values. Moreover, the labour commanded depends on the distribution of income. It means that the ratio of profit to wages (i.e. income distribution) must remain constant in order to obtain an increase (decrease) in the labour commanded associated with an increase (decrease) in the labour embodied in the commodities. We can also notice that the right-hand side of expression (42) represents the labour that can be purchased by total profits and that therefore, according to Smith, can command both productive and unproductive labour.

6. Sraffa's standard system

The problem of the correspondence between the price system and the labour-value system can be tackled by considering Piero Sraffa's standard system. The standard system is defined as an economic system in which "the various commodities are represented among its aggregate means of production in the same proportions as they are among its products" (Sraffa 1960, p. 19). We can draw the standard system as follows:

$$\begin{bmatrix} (1-(1+R)a_{11}) & -(1+R)a_{12} & \dots & -(1+R)a_{1n} & -y_{s1} \\ -(1+R)a_{21} & (1-(1+R)a_{22}) & \dots & -(1+R)a_{2n} & -y_{s2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -(1+R)a_{n1} & -(1+R)a_{n2} & \dots & (1-(1+R)a_{nn}) & -y_{sn} \\ -l_1 & -l_2 & \dots & -l_n & 1 \end{bmatrix} \begin{bmatrix} x_{s1} \\ x_{s2} \\ \vdots \\ x_{sn} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad (43)$$

which in compact form becomes:

$$[\mathbf{I} - (1+R)\mathbf{A}]\mathbf{x}_s = \mathbf{0} \quad (44)$$

$$\mathbf{l}^{\mathrm{T}}\mathbf{x}_s = 1 \quad (45)$$

where R, i.e. the maximum rate of profit, establishes the (uniform) "standard ratio" between each industrial physical surplus and the respective commodities used as physical means of production.[8]

We take the standard net product as the *numéraire* of standard price system:

$$\mathbf{p}^{\mathrm{T}}[\mathbf{I} - \mathbf{A}]\mathbf{x}_s = \overline{\mathbf{p}^{\mathrm{T}}\mathbf{y}_s} = 1 \quad (46)$$

[8] For a deeper understanding of the standard commodity and the standard system, see also Pasinetti (1977, Ch. 5) and Bellino (2004; 2021, Ch. 8).

Given the normalization for total employment (45) introduced by Sraffa, it follows that the price of the standard net product and the labour embodied into it are equivalent:

$$\mathbf{p}^{\mathrm{T}}\mathbf{y}_s = \mathbf{v}^{\mathrm{T}}\mathbf{y}_s \tag{47}$$

Therefore, when we consider prices in terms of fractions of the standard net product we are also considering them in terms of labour embodied. In other terms, Sraffa chooses to measure the relative prices in terms of the standard net product and to measure the latter in terms of the total employment of the standard system, that is set equal to unity. Now if we assume that the rate of profit is less than the maximum associated with matrix $\mathbf{A}$ ($\pi < R$) and that the wages are paid in terms of standard commodity ($\mathbf{b} = \mathbf{b}_s$), which is equivalent of considering the wage rate w as the fraction of the standard net product that goes to labour, the latter also comes to represent the fraction of the total labour embodied in the standard net product that is necessary labour or, as defined above, the unit value of the labour power ω:

$$w_s = \mathbf{p}^{\mathrm{T}}\mathbf{b}_s = \mathbf{v}^{\mathrm{T}}\mathbf{b}_s = \omega \tag{48}$$

Now, making the necessary substitutions and calculating the production prices we can obtain the following relation:

$$\pi_s = R(1 - \omega) \tag{49}$$

which places the value of the labour power (the labour embodied in the real wage) in a linear and antagonistic relationship with the rate of profit emerging from the standard system when the (surplus) wage rate appears (π_s). It follows that we can similarly visualize a relationship between the rate of profit emerging from the standard system and the rate of surplus value:

$$\sigma = \frac{\pi_s}{R - \pi_s} \tag{50}$$

The last formalizations taken together offer a confirmation of the fundamental Marxian equivalences we have discussed above. In fact, given the normalizations for prices and employment, the equality between the neo-value extracted in production and the money value added is confirmed, as stated by the expression (47). Furthermore, since we have assumed that wages are paid in terms of standard commodity, then profit (calculated in prices) and surplus value (calculated in terms embodied labour) will both be proportional to the standard net product and equivalent to each other. Thus, in the standard system associated with matrix $\mathbf{A}$, we can turn the two inequalities above into equalities:

$$\mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x}(1+\sigma) = \mathbf{p}^{\mathbf{T}}\mathbf{y}_s \tag{51}$$

and

$$\sigma\mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x} = \pi_s\mathbf{p}^{\mathbf{T}}\mathbf{H}\mathbf{y}_s \tag{52}$$

The key assumptions that led to these results are two: that standard net product is measured in terms of total employment, i.e. the labour embodied in the standard net product, and that the basket of wage goods has the same composition of the standard product. In Marxian terms, thanks to the standard commodity, we have overcome the problem of the non-uniform technical composition of capital across productive sectors, being both wages and profits composed by the same commodity proportions.

In the case in which the real wage has a different composition from that of the standard product ($\mathbf{b} \neq \mathbf{b}_s$) then the correspondence between the aggregates evaluated in prices and in labour-values requires a further normalization. In fact, the standard proportions associated with matrix $\mathbf{A}$ differs from those associated with the augmented matrix $\mathbf{A}+\mathbf{b}\mathbf{l}^{\mathbf{T}}$ (hence, $\mathbf{x}_s \neq \mathbf{x}_{(\mathbf{A}+\mathbf{b}\mathbf{l}^{\mathbf{T}})}$).[9] This the reason why the normalization of the price system considered by Sraffa is no longer available. In fact, while the employment normalization (45) still suggests that the net product is composed by wages and profits when evaluated in labour-values, the expression $\mathbf{p}^{\mathbf{T}}[\mathbf{I}-(\mathbf{A}+\mathbf{b}\mathbf{l}^{\mathbf{T}})]\mathbf{x}_{(\mathbf{A}+\mathbf{b}\mathbf{l}^{\mathbf{T}})}$ no longer represents the net product. The latter, represents instead the aggregate profit (i.e. the Classical notion of net product or surplus). Therefore, if we want to maintain the analytical correspondence between the two systems, we have to close the price system by setting the aggregate monetary profit equal to the quantity of labour embodied in the part of net product that does not include wages:

$$\mathbf{p}^{\mathbf{T}}[\mathbf{I}-(\mathbf{A}+\mathbf{b}\mathbf{l}^{\mathbf{T}})]\mathbf{x}_{(\mathbf{A}+\mathbf{b}\mathbf{l}^{\mathbf{T}})} = \mathbf{v}^{\mathbf{T}}[\mathbf{I}-(\mathbf{A}+\mathbf{b}\mathbf{l}^{\mathbf{T}})]\mathbf{x}_{(\mathbf{A}+\mathbf{b}\mathbf{l}^{\mathbf{T}})} \tag{53}$$

This equation substitutes the price normalization (46). Both sides of the expression will be less than unity. This means that the net product, that evaluated in labour-values is equal to unity, is no longer necessarily equal to unity when evaluated in

[9] The subscript $(\mathbf{A}+\mathbf{b}\mathbf{l}^{\mathbf{T}})$ refers to the matrix that the eigenvector is associated with, namely the augmented matrix, to distinguish it from the eigenvector associated with matrix $\mathbf{A}$ for which we have used the subscript $\boldsymbol{s}$.

production prices. Consequently, in this case only one of the two equalities is maintained, namely that between the surplus value and aggregate profit.[10]

We can conclude this brief excursus inside Sraffa's standard system by asserting that a correspondence between the fundamental macroeconomic aggregates evaluated in values and the same ones evaluated in prices can be achieved through the use of the standard commodity. Due to the particular properties of this system, the standard net product, when chosen as *numéraire*, makes the relationship between the wage rate and the rate of profit linear and transparent, i.e. independent of prices. It follows that, if we assume that both the actual net product and the standard net product are evaluated in terms of total employment and that the real wage is paid in terms of the standard commodity (that enters the wage in the same proportions in which it enters the profit) then we can trace the labour-value categories simply within the price system and obtain a transparent relationship between the rate of profit and the rate of surplus value.

7. Surplus value inside the price system

Following Pasinetti (1977, appendix to chapter 5), we can now consider the most natural way to close the Marxian price system that is to choose as *numéraire* the composite commodity of the neo-value defined above as a commodity that has the same structure of the real wage but has the same volume of the net product per worker:

$$\mathbf{p}^{\mathbf{T}}\mathbf{b}(1+\sigma)=1 \tag{54}$$

that is the symmetrical to equation (26) previously chosen to close the labour-value system. This is to say that the value system and the price system are both defined with the net product per worker composed exclusively by wage goods $\mathbf{b}$. At this point, substituting the (maximum) rate of profit associated with matrix $\boldsymbol{\Theta}$ (equation 35) inside the price system (33) and using the *numéraire* (54), the solutions for the Marxian production prices become:

$$\mathbf{p}^{\mathbf{T}}=\mathbf{v}^{\mathbf{T}}[\mathbf{I}-\pi_{\boldsymbol{\Theta}}\mathbf{H}]^{-1}\left(\frac{1}{1+\sigma}\right) \tag{55}$$

[10] Pasinetti (1977) provides further considerations on this particular case with reference to the Marxian price system, see also Pasinetti and Garbellini (2015).

Once again, we obtain a linear operator which translates the labour-values into production prices in which the relationship between prices and the rate of surplus value is made explicit. We can now verify that the wage rate that emerges from this solution is equivalent to the share of labour devoted to produce the subsistence goods consumed by the workers, i.e. the unit value of the labour power:

$$w = \mathbf{p}^{\mathbf{T}}\mathbf{b} = \frac{1}{1+\sigma} = \omega \tag{56}$$

The necessary considerations at this point are the following. The main difference between choosing to measure prices in terms of labour commanded, i.e. in terms of wage rate (38) and that of measuring them in terms of labour embodied in the neo-value per worker (54) is that with this latter choice prices are no longer monotonically higher than the corresponding labour-values. Some prices may command more labour relative to that embodied in the respective commodity, while others transfer (surplus) value to the rest of the economy through unequal exchanges between commodities (i.e. exchange relationships non-proportional to labour-values). In the aggregate, however, the differences compensate each other like in a zero-sum game, because the neo-value distributed in circulation cannot be greater than that extracted overall in production.[11] This result is obtained by Pasinetti (1977) by setting the price of the composite commodity $\mathbf{b}(1+\sigma)$ as *numéraire* of production prices. Given the procedure through which Pasinetti constructs this composite commodity, that we have defined after Marx with "neo-value", the commodity chosen as *numéraire* has the same identical structure of that of the basket of wage commodities. This allows us to transform the economic aggregates from prices to their respective values directly starting from the normalization (54) that, for prices (55), leads to the following formalizations:

$$w(1+\sigma) = 1 \tag{57}$$

and hence

$$\sigma = \frac{1}{w} - 1 \tag{58}$$

We can obtain the same relationship between the value of labour power and the rate of surplus value expressed in value terms by equation (19) directly as a function of

[11] For some empirical evaluations of the production price deviations from the respective labour-values, see among others Tsoulfidis and Tsaliki (2019) and Mariolis and Tsoulfidis (2016) who follow Shaikh's (1998) procedure for the estimation of relative prices starting from national input-output tables.

the wage rate emerging from the price system. Consequently we can retrace the two fundamental Marxian equalities. Indeed, the global neo-value equals the monetary value added:

$$\mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x}(1+\sigma) = \mathbf{p}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x}(1+\sigma) \tag{59}$$

and the global surplus value extracted in production equals the aggregate monetary profit:

$$\sigma\mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x} = \pi_{\mathbf{\Theta}}\mathbf{p}^{\mathbf{T}}\mathbf{H}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x}(1+\sigma) \tag{60}$$

The *numéraire* chosen by Pasinetti makes it possible to make ω and w perfectly interchangeable just as it happens in the standard system. The only difference between this procedure and the one based on the Sraffa system is that in the latter we need to assume that the real wage has the same composition of the standard commodity. In the present system this assumption is superfluous. In fact, since the rate of surplus value is defined in terms of labour embodied, it remains independent of the proportions in which the commodities enter the part of net product purchased by the capitalists with their profit but depends exclusively on the volume and the structure of the basket of commodities that enter the real wage, that is the same structure of the *numéraire*-commodity opportunely constructed. This is what confers basket **b** an economic meaning that goes beyond the circumstances under which it may be considered as an "average" commodity. Furthermore, it can be highlighted that prices (41), computed in terms of labour commanded, and prices (55), evaluated in terms of labour embodied in the neo-value per worker, represent the same proportions. They only differ in the commodity chosen as *numéraire*, that is to say that they differ by a scalar multiple. This scalar multiple is exactly the term $(1+\sigma)$, that is the difference between total direct labour and the value of the labour power, i.e. the necessary labour.

8. Towards the macro-monetary perspectives of the value theory

8.1. The monetary expression of socially necessary labour time

Sraffa's standard system and Pasinetti's transformation method allowed a crucial advance in the evaluation of labour-value aggregates within the empirical sphere of the structural economic investigation. On these bases the macro-monetary approaches start, more specifically the theoretical researches collected inside the *Price of Net Product–Unallocated Purchasing Power Labour Theory of Value*

(PNP–UPP LTV).[12] If we accept the proposed interpretation of Marx's method of investigation of value and surplus value as a macro-social analysis treated in monetary terms, this leads us to reject the labour theory of value as merely a theory for determining the prices of production, but rather places us in the need to trace the categories of labour-value inside the real world's economic relationships whatever the social (i.e. institutional) rules that regulate the distribution of income and the formation of prices.

This approach proposes a solution to the problem of the articulation between the law of value and the law of exchange that is opposed to that based on the simultaneous consideration of two alternative systems for the evaluation of commodities (the system of labour-values on the one hand and that of production prices on the other), it is therefore a non-dual approach.[13] The foundation of this

[12] With this definition we refer to the independent, and then subsequently generalized, research paths of Foley (1982, 1986), Duménil (1983) and Lipietz (1982). Some reviews on the Italian wing of this interpretation (*Quelli del lavoro vivo*, "those of the living labour" that is another term for what we call here direct or abstract labour) can be found in Bellofiore (2020), Coveri (2019) and Veronese Passarella (2009) (some works are mentioned in what follows).

[13] To distinguish it from the dual, or neo-Ricardian, approach. We can explain this perspective very briefly as follows. According to the dual approach, Marx elaborates the law of value (first volume of *Capital*) on the basis of a pre-capitalist economy, that is to say that he abstracts from any social relationships proper of the capitalist mode of production and competition. In this case, money prices reflect the quantity of labour embodied in the commodities (in the terms of the present analysis, we find ourselves inside Pasinetti's natural system). Alternately, it is believed that the value theory is elaborated considering a capitalist economy (characterized by the tendency of the rates of profit uniformity) and that Marx succeeds in making the two analyses compatible only because of the strong assumption of uniform capital intensities (i.e. uniform industrial capital-labour ratios) that he introduces. From both the perspectives, the relation between the analysis of Marx's first volume, where the theory of value and surplus value is elaborated, and that of the third volume, where Marx defines the law of exchange inside a competitive capitalist economy, is acknowledge inside the contraposition of two alternative systems of evaluation of commodities: the value system and the price system, that in general, as we have seen, result to be incompatible. In other terms, it is not possible to extend the analysis of value to the situation in which it is not assumed that the prices are proportional to labour-values but instead they reflect their nature as products of capital. This is the reason why this approach tends to reject the labour theory of value in favor of a theory of relative prices of production. Some representatives of the dual approach to value and prices are Steedman (1977), Garegnani (1981) and Petri (2012) who mostly follow Samuelson's (1971) interpretation of the problem. Other critical positions regarding the PNP–UPP interpretation of the theory of surplus value are those of Screpanti (2003) and Roemer (1986). On the latter, see an anti-critique by Perri (1997).

approach is the macroeconomic interpretation of the labour theory of value and arises from the following elements. First of all, the condition of conservation of neo-value (from commodity production to circulation) must be formulated in terms of invariance of the aggregate net product and not of the gross product. As we have seen, the direct labour employed in production correctly measures the labour embodied in the net product excluding the labour embodied into the means of production. Furthermore, the value of the labour power has to be defined in monetary terms, i.e. on the basis of the purchasing power of the wage, which is supposed to be the result of the bargaining between capitalists and workers over the wage rate and more generally of the class struggle.

It is therefore of primary importance to define the *monetary expression of labour time* (MELT, i.e. the reciprocal of the value of money) of the economic system, i.e. the quantity of money necessary to command (purchase or mobilize) one unit of labour,[14] defined as the ratio between the price of the net product and the total employment. This ratio will in turn define the unit of account of the price system:

$$\frac{\mathbf{p}^{\mathrm{T}}[\mathbf{I}-\mathbf{A}]\mathbf{x}}{L} = \overline{\mathbf{p}^{\mathrm{T}}\mathbf{y}} = 1 \tag{61}$$

This normalization, which sets the price of the aggregate net product equivalent to the total employment, affirms that the law of value holds for the aggregate of the economic system. In other terms, although, as we know very well, there exist various ways of obtaining a part of the value in the form of income that do not involve the performance of direct labour in production, the value that is distributed through the exchange of goods cannot in any way be greater than the total value generated in production. Hence, “the price form of the value created by the total productive labour spent during a certain period of time is the price of the net product of the period” (Duménil and Foley 2008, p. 9).

With the exception of some very special cases, the relative prices of commodities naturally diverge from their respective labour-values, that is to say that each commodity presents itself on the market exhibiting a price that can be equal to, greater or less than the monetary equivalent of labour required to produce it. However, through a macroeconomic definition of the monetary expression of labour

[14] This definition of labour commanded differs from that introduced by Smith as labour commanded by the price of commodities. As discussed above, Smith evaluates the prices in units of wages or by normalizing with total labour (fatigue) the wage bill (its remuneration) rather than, as we do here, the surplus (its product).

time, or the absolute value of money, it emerges that every part of surplus value gained or lost in the competitive exchange between commodities compensates for the economic system as a whole, since the unit of measure adopted here allows relative prices to indicate nothing more than shares of the net product realized by each producer, exactly like it happens with Pasinetti's *numéraire* choice and transformation procedure. In this sense, the macro-normalization manages to represent the mechanism through which surplus value, once extracted inside production from labour power, is preserved into money and subsequently re-distributed among capitalists within the circulation of commodities, therefore, according to the prevailing competitive law that regulates them, that is, in the case considered by the Classical economists, the one that guarantees the long-period uniformity of industrial rates of profit.[15]

8.2. The value of the labour power and the dual nature of the wage

The attempt to express wages in terms of labour so as to deduct the rate of surplus value from it raises, at empirical level, some problems. The real wage, in fact, generally has a different composition from the basket of goods that compose the net product. This implies that, if we admit that prices diverge from the contents of labour, the quantity of labour embodied in the share of net product that is exchanged for real wages is different from the quantity of labour embodied in the latter. In a more general sense, when it is accepted that workers as individuals can spend their wages on goods of various kinds and prices or save a part of them, the value of the labour power is likely to be indeterminable or different depending on the consumption behaviours of the individual worker. Indeed, since wages are paid in money and not negotiated in terms of a specific physical basket of commodities, an appropriate definition of the value of the labour power must consider the prevailing price system when it comes to analysing a monetary economy of production like the capitalist system. To resolve the question it is sufficient to follow Marx's methodological approach. If the production of commodities is followed by their

[15] The "free competition" hypothesis that guarantees the long-period uniformity of profit rates is introduced by the Classical economists but is only one of the possible rules of exchange by which the relative prices of commodities can be determined. For a broader discussion on this point see Sinha (2016; 2022), Zambelli (2018) and Perri and Oro (2022).

circulation and social relations between classes are established before the distribution of income and the formation of prices, this is equivalent to accepting, therefore having to formally express, also that "exploitation is logically prior to consumption" (Duménil 1983, p. 444). As it is clear in Marx, the wage goods do not enter directly into production like the input-commodities do, but are consumed individually by workers, subsequently or in any case outside production, therefore they only expose their use value in the sphere of circulation. What actually enters production after wages are paid is abstract labour, i.e. human labour activity. Thus, since the neo-value is by definition equivalent to the direct labour globally performed, the wage rate applied to the latter must express a (social) relationship between two (homogeneous) flows of labour, rather than the price of one unit of labour or of a specific basket of goods.

Therefore, we can relax the Classical hypothesis that defines the wage rate in physical terms, i.e. according to a rigid theory of subsistence, to reformulate the definition of the value of labour power in monetary terms as follows:

$$\omega = w/\overline{\mathbf{p}^{\mathrm{T}}\mathbf{y}}, \qquad \text{and alternately,} \qquad w = \omega\overline{\mathbf{p}^{\mathrm{T}}\mathbf{y}} \tag{62}$$

From this perspective, the value of the labour power is defined by the quantity of labour commanded (purchased) by the monetary wage and which, given the macroeconomic definition of the monetary expression of labour time, turns to represent the fraction of the net product that goes to the workers.[16] It follows that "the labour necessary to reproduce the value of the labour power can reasonably be defined as the quantity of labour embodied in the part of the net product that exchanges for the wages consumed by the workers and that actually equals the living labour expended to produce the equivalent of the wages consumed" (Perri 1998, p. 176, translated from the Italian). On the one hand, the wage bill expresses the price form of the variable capital as "unallocated purchasing power", that provides the workers with the possibility of consuming a fraction of the surplus. On the other hand the monetary expression of surplus value, i.e. of exploitation, turns to be the aggregate profit, "which smiles at the capitalist with all the charm of a creation from nothing" (Marx 1990, Ch. 7).

[16] In this framework, the normalization introduced to define the value of the labour power (18) is substituted by normalization (62) that sets it as a function of the money wage rate and of the monetary expression of labour time, that here is chosen as *numéraire*.

The definition of wages in terms of embodied labour requires that "the dual nature of wages in the capitalist economy" emerge in the system of production prices. In fact, as we can clearly observe, the wage rate represents a monetary cost of production, at the beginning of the production process, that must necessarily be equivalent to the fraction of the national income corresponding to the wage goods consumed by the workers. Furthermore, the value of the labour power as a share of labour performed, by directly relating the standard of living of workers with their ability to produced neo-value, is coherent with Marx's theory of development based on the theory of value, that is, of the historical determination of the real wage resulting from the dynamic social relations in a capitalist economy of production.[17]

8.3. Surplus-labour and the rate of profit

At this point, the equivalence between the neo-value and the monetary value added can be retraced as follows:

$$\omega\overline{\mathbf{p}^{\mathrm{T}}\mathbf{y}}(1+\sigma)=1 \tag{63}$$

from which it naturally derives:

$$\sigma=\frac{\overline{\mathbf{p}^{\mathrm{T}}\mathbf{y}}-w\mathbf{v}^{\mathrm{T}}\mathbf{y}}{w\mathbf{v}^{\mathrm{T}}\mathbf{y}}=\frac{1}{w}-1 \tag{64}$$

[17] This is the reading of the Sraffian equations proposed by Stefano Perri (1998) after the PNP–UPP approach and according to a line of reasoning that starts from the counterfactual method, present in the first volume of *Capital* (Perri 2003) and, above all, after the consultation of the Sraffa archive (Perri 2010). It is argued that following the path of the definition of labour power simply with the labour embodied in the basket of subsistence goods would return to Sraffa's second equations, in which the wage rate is indistinguishable from the prices of other input-commodities and the surplus (in the Classical notion) is entirely appropriated by the capitalists. In this case, the rate of surplus value becomes merely a technical relation between quantities of labour embodied, while according to Marx, it must show the social relationships between classes, this is why it has to be defined as a function of the working time, namely the global abstract labour employed in production (something close to what Pasinetti does with normalization 18). In fact, that reasoning would lead into a paradoxical capital value theory, as Pasinetti (1977, Ch. 5) states. Moreover, Perri affirms that for a given monetary wage, the quantity of labour embodied in the real wage must vary when we assume that the workers are free to choose the kind of goods to consume. This latter assumption is introduced mostly for historical and theoretical reasons, namely because in the analysis of modern capitalism it is more realistic to assume a flexible theory of monetary wages instead of a rigid theory of subsistence.

that again bring us back to the relation between the wage rate and the rate of surplus value emerged from Pasinetti's method with the equation (58).
Finally, we can verify what happens to the two fundamental equalities we discussed so far. The price of the net product is:

$$\mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x}(1+\sigma) = \mathbf{p}^{\mathbf{T}}\mathbf{y}L \tag{65}$$

that turns again to be equal to the neo-value or labour embodied in the net product, while equivalence between the aggregate profit and the global surplus-labour is given by:

$$\sigma\mathbf{v}^{\mathbf{T}}\mathbf{b}\mathbf{l}^{\mathbf{T}}\mathbf{x} = \mathbf{p}^{\mathbf{T}}\mathbf{y}(1-w)L \tag{66}$$

In conclusion, the rate of profit that emerges results from the ratio between the global surplus value and the means of production evaluated in prices:

$$\pi = \frac{(1-w)L}{\mathbf{p}^{\mathbf{T}}\mathbf{H}\mathbf{y}L} = \frac{\sigma\mathbf{v}^{\mathbf{T}}\mathbf{b}}{\mathbf{p}^{\mathbf{T}}\mathbf{H}\mathbf{y}} \tag{67}$$

The expression shows the profit rate associated with a certain rate of surplus value generated in production (the numerator) and indirectly related with the technical composition of capital, i.e. the rate between capital and labour (the denominator). The profit rate will therefore adjust to any price structure for a given the distribution of the surplus that is determined by the conflicting claims inside production, i.e. determined inside the labour-value system. These claims, i.e. what Marx calls "class struggle", concern the technique of production brought into being, the length of the working time and the commodities that enter the basket of the real wage. This method allows us to generalize Pasinetti's transformation procedure for all the economic settings that we might observe at empirical level. "In reality, if we accept this interpretation, the labour theory of value is compatible with any mechanism for determining the monetary prices of commodities" (Gozzi 2002, p. 88, translated from the Italian).

9. A comparison

We have considered the connections between the system of labour-values and that of prices through different approaches. In the last three cases we could obtain the precise equivalences between the two systems, namely the solution that was based Sraffa's standard system, that elaborated along Pasinetti's solution to the Marxian transformation problem, and the macro-monetary approach, while in the case where we chose to measure the prices in terms of labour commanded the problem of the divergences of the aggregates evaluated in prices from the same evaluated in labour-values persisted. We are now in the position to do some comparisons.

The solution based on Sraffa's standard system implies that the wage is paid in terms of the standard net product, that once chosen as *numéraire*, provides a transparent and linear relationship between the distributive variables. The procedure stands on the re-construction of the basket of wage goods in such a way to be in the standard proportions. At this point the wage rate and the value of the labour power are perfectly interchangeable and the two equalities are achieved in the price system.

Pasinetti's solution to Marx's transformation problem highlights a deeper question than the mere choice of the proper *numéraire*. In fact, the real wage here is taken as given in physical terms, correctly defined within the labour-value system, and immediately converted into a share of total direct labour, i.e. the value of the labour power (18), to compute the rate of surplus value in terms of labour as well. At this point, when turning to the prices of production, the wage rate cannot but be defined as the price of the basket of wage goods. However, the *numéraire* that is chosen to measure prices cannot be the net product as observed from the transactions table, because the proportions of the actual net product differ from those of the real wage. Pasinetti constructs a composite commodity with the structure of the real wage and the volume of the net product and chooses it as *numéraire* of the system. In this way, the two equivalences are obtained in the price system because the money profit exclusively depends on the structure of the real wage (that is in fact the same of that of the chosen *numéraire*) and not on the commodities that enter the fraction of net product that remains after wages are paid. This procedure has the merit to hold in any price system, given the real wage in physical terms, as long as the *numéraire*-commodity of the net product is properly constructed.

The PNP–UPP method, reappraise the standard system procedure by reversing the analytical path proposed by Pasinetti, even though it reaches the same conclusions. The goal is to evaluate the economic aggregates in terms of labour embodied starting from the real world's transactions tables. We do not know the exact basket of wage consumed by the workers, but we do know that the labour embodied into it must be a fraction of the total direct labour, namely necessary labour. Thus, what Marx calls "monetary expression of labour time" is chosen as *numéraire*, in this way, the law of value is transposed into the price system through the measure unit, whatever the rule of price formation may be. Later, when we reasonably assume that wages are paid in money and the workers can consume it in various kinds of goods, the wage share of the net product can be considered a proper estimation of the labour embodied in the fraction of the net product that is exchanges with wages, i.e. necessary labour. The two normalizations can be applied to any price system.

What connects the three perspectives is essentially the macroeconomic relevance of the value dimension, that must naturally be separated from (even though coherent with) the money dimension of individual exchanges. In fact, the object of the analysis is the surplus, defined with the aggregate net product and correctly measured with the total employment of labour inside the economy, this is the macro condition of value conservation, i.e. the law of value. The wage rate defined in terms of the net product represents the value of the labour power as a proportion of total direct labour, hence the distribution of the surplus concerns the allocation of labour between necessary and surplus-labour, rather than the technical requirements of the wage-producing sectors. The social relationship therefore prevails over the technical relationship. Finally, the price structure and the respective profit rate are determined by the institutional mechanisms that are put forward in the second stage of the analysis, that is the commodity circulation and exchanges. In other terms, Pasinetti shows that the labour theory of value must be considered as a macroeconomic physical law of capitalist reproduction, that must remain coherent with any law of exchange and that allows the observer to abstract from price mechanisms when studying the essential features of the capitalist mode of production.

10. Examples based on Pasinetti's numbers

The concepts discussed above can be illustrated by numerical examples based on an economy with three industrial sectors (iron, coal and wheat). In the following elaborations, we assume that the labour embodied in the real wage, i.e. the value of the labour power, remains the same whatever the rule considered for the formation and evaluation of the physical quantities and production prices. This assumption is equivalent to assert that the rate of surplus value is considered as given, so that we can try to retrace is inside alternative price systems.

Consider the following matrix of multi-sectoral coefficients, the vector of labour coefficients, and the vector of gross output:

$$\mathbf{A} = \begin{bmatrix} 0.413 & 2.571 & 0.5 \\ 0.026 & 0.285 & 0.05 \\ 0.02 & 0.285 & 0.25 \end{bmatrix}, \qquad \mathbf{l}^{\mathbf{T}} = [0.04 \quad 0.571 \quad 0.5], \qquad \mathbf{x} = \begin{bmatrix} 450 \\ 21 \\ 60 \end{bmatrix} \tag{A.1}$$

the quantities of labour embodied in one unit of each commodity (labour-values) are:

$$\mathbf{v}^{\mathbf{T}} = \mathbf{l}^{\mathbf{T}}[\mathbf{I} - \mathbf{A}]^{-1} = [0.181 \quad 1.818 \quad 0.909] \tag{A.2}$$

where the Leontief inverse is given by:

$$[\mathbf{I} - \mathbf{A}]^{-1} = \begin{bmatrix} 2.164 & 8.596 & 2.015 \\ 0.087 & 1.784 & 0.177 \\ 0.09 & 0.909 & 1.454 \end{bmatrix} \tag{A.3}$$

The vector of the net product per worker is:

$$\mathbf{y} = \frac{[\mathbf{I} - \mathbf{A}]\mathbf{x}}{L} = \begin{bmatrix} 3 \\ 0 \\ 0.5 \end{bmatrix} \tag{A.4}$$

The labour embodied in the net product, i.e. the total employment of the economic system is given by:

$$\mathbf{v}^{\mathbf{T}}\mathbf{y}L = 60 \tag{A.5}$$

When the entire net product is distributed to workers (hence with $w = 1$), each worker receives a real wage composed by 3 physical units of iron and 0.5 physical units of wheat that expressed in (direct) prices, hence in terms of labour-values, is:

$$\mathbf{v}^{\mathbf{T}}\mathbf{y} = [0.181 \quad 1.818 \quad 0.909] \begin{bmatrix} 3 \\ 0 \\ 0.5 \end{bmatrix} = 1 \tag{A.6}$$

However, we assume that the system is a capitalist economy, namely an economic system in which capitalists pay the workers the equivalent of an annual subsistence wage given by the vector:

$$\mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 0.166 \end{bmatrix} \tag{A.7}$$

The value of the labour power, i.e. the labour embodied in the real wage is given by:

$$\mathbf{v}^{\mathbf{T}}\mathbf{b} = [0.181 \quad 1.818 \quad 0.909] \begin{bmatrix} 2 \\ 0 \\ 0.166 \end{bmatrix} = 0.515 \tag{A.8}$$

This means that each worker works for himself over half the total working time ($\omega = \mathbf{v}^{\mathbf{T}}\mathbf{b}$), to produce 2 physical units of iron and 0.166 physical units of wheat, while for the rest of the time he/she works for the capitalist that appropriates the remaining surplus, composed by 1 physical unit of iron and 0.333 physical units of wheat. The rate of surplus value that emerges, and that measures the degree of exploitation of the working class, is the following:

$$\sigma = \frac{1}{\omega} - 1 = 94.11\% \tag{A.9}$$

The surplus value per worker is therefore:

$$\sigma \mathbf{v}^{\mathbf{T}}\mathbf{b} = 94.11\%[0.181 \quad 1.818 \quad 0.909]\begin{bmatrix} 2 \\ 0 \\ 0.166 \end{bmatrix} = 0.484 \tag{A.10}$$

The same results can be obtained in another way, that can be useful to apply. We can consider the following labour-value system of homogeneous equations:

$$\mathbf{v}^{\mathbf{T}}[\mathbf{I} - \sigma\mathbf{b}\mathbf{l}^{\mathbf{T}}[\mathbf{I} - (\mathbf{A} + \mathbf{b}\mathbf{l}^{\mathbf{T}})]^{-1}] = \mathbf{0}^{\mathbf{T}} \tag{A.11}$$

and find the root σ that fulfils the condition $\det[\mathbf{I} - \sigma\mathbf{b}\mathbf{l}^{\mathbf{T}}[\mathbf{I} - (\mathbf{A} + \mathbf{b}\mathbf{l}^{\mathbf{T}})]^{-1}] = 0$.

The labour-values, or direct prices, are obtained by the dominant left-hand eigenvector associated with matrix $[\mathbf{I} - \sigma\mathbf{b}\mathbf{l}^{\mathbf{T}}[\mathbf{I} - (\mathbf{A} + \mathbf{b}\mathbf{l}^{\mathbf{T}})]^{-1}]$.

10.1. Smith's "labour commanded" prices

Following Sraffa, and unlike Marx, we choose to compute the rate of profit on the means of production only. Thus, the rate of profit is determined by the economically meaningful solutions for the following system of homogeneous price equations:

$$\mathbf{p}^{\mathbf{T}}[\mathbf{I} - \pi\mathbf{\Theta}] = 0 \tag{A.12}$$

where $\mathbf{\Theta} = \mathbf{A}[\mathbf{I} - (\mathbf{A} + \mathbf{b}\mathbf{l}^{\mathbf{T}})]^{-1}$. The matrix of social reproduction coefficients is given by:

$$\mathbf{b}\mathbf{l}^{\mathbf{T}} = \begin{bmatrix} 2 \\ 0 \\ 0.166 \end{bmatrix}[0.04 \quad 0.571 \quad 0.5] = \begin{bmatrix} 0.08 & 1.142 & 1 \\ 0 & 0 & 0 \\ 0.006 & 0.095 & 0.083 \end{bmatrix} \tag{A.13}$$

whose columns express the physical quantities of the commodities that are distributed to the workers of each industry proportionally to labour employed per unit of gross product.

Therefore, the augmented matrix is:

$$\mathbf{A} + \mathbf{b}\mathbf{l}^{\mathbf{T}} = \begin{bmatrix} 0.493 & 3.714 & 1.5 \\ 0.026 & 0.285 & 0.05 \\ 0.026 & 0.38 & 0.333 \end{bmatrix} \tag{A.14}$$

The maximum eigenvalue of matrix

$$\mathbf{\Theta} = \mathbf{A}[\mathbf{I} - (\mathbf{A} + \mathbf{b}\mathbf{l}^{\mathbf{T}})]^{-1} = \begin{bmatrix} 2.163 & 18.586 & 7.01 \\ 0.163 & 1.548 & 0.559 \\ 0.187 & 1.875 & 0.937 \end{bmatrix} \tag{A.15}$$

is $\lambda_m^{\mathbf{\Theta}} = 4.324$ and the rate of profit is therefore:

$$\pi_{\mathbf{\Theta}} = \frac{1}{\lambda_m^{\mathbf{\Theta}}} = 23.12\% \tag{A.16}$$

that turns to be smaller than the corresponding rate of surplus value and higher than the rate of profit that emerges when computed on total capital advanced, i.e. composed by both the means of production and the money wages, like the original Marxian price structure considered by Pasinetti (1977). We can substitute the rate of profit into the relative price equations and add a further equation defining the *numéraire* of the price system.

First, like Smith, we choose to measure prices in terms of labour commanded:

$$w = \overline{\mathbf{p}^{\mathbf{T}}\mathbf{b}} = 1 \tag{A.17}$$

and the prices measured in terms of the wage rate are:

$$\begin{aligned}\mathbf{p}^{\mathbf{T}} &= \mathbf{v}^{\mathbf{T}}[\mathbf{I} - \pi_{\mathbf{\Theta}}\mathbf{H}]^{-1} = \\ &= [0.181 \quad 1.818 \quad 0.909]\begin{bmatrix}1.503 & 3.898 & 0.961\\ 0.039 & 1.337 & 0.081\\ 0.044 & 0.405 & 1.159\end{bmatrix} = \\ &= [0.385 \quad 3.51 \quad 1.377]\end{aligned} \tag{A.18}$$

where the matrix of the vertically integrated productive capacity coefficients is:

$$\mathbf{H} = \mathbf{A}[\mathbf{I} - \mathbf{A}]^{-1} = \begin{bmatrix}1.164 & 8.596 & 2.015\\ 0.087 & 0.784 & 0.177\\ 0.09 & 0.909 & 0.454\end{bmatrix} \tag{A.19}$$

We can now compute the value added per worker in terms of labour commanded:

$$\mathbf{p}^{\mathbf{T}}\mathbf{y} = [0.385 \quad 3.51 \quad 1.377]\begin{bmatrix}3\\ 0\\ 0.5\end{bmatrix} = 1.844 \tag{A.20}$$

and the aggregate profit per worker:

$$\begin{aligned}&\pi_{\mathbf{\Theta}}\mathbf{p}^{\mathbf{T}}\mathbf{H}\mathbf{y} = \\ &= 23.12\%[0.385 \quad 3.51 \quad 1.377]\begin{bmatrix}1.164 & 8.596 & 2.015\\ 0.087 & 0.784 & 0.177\\ 0.09 & 0.909 & 0.454\end{bmatrix}\begin{bmatrix}3\\ 0\\ 0.5\end{bmatrix} = 0.844\end{aligned} \tag{A.21}$$

that of course are both higher than the corresponding magnitudes evaluated in labour-value terms. Thus, for each rate of profit higher than zero, each commodity purchases (commands) a quantity of labour that is always greater than the quantity of labour embodied into it.

10.2. Sraffa's prices and the standard commodity

Consider now the case in which the standard proportions of quantities happen to be observed in the economic system. The standard relative quantities ($\mathbf{x}_s$) are given by the dominant right-hand eigenvector associated with matrix $\mathbf{A}$. Therefore, the vector of the net standard product can be normalized in such a way that the labour embodied into it, i.e. total employment of the standard system, is equal to unity. We can construct the standard net product (i.e. the standard commodity) as follows:

$$\mathbf{y}_s = [\mathbf{I} - \mathbf{A}]\mathbf{x}_s \left(\frac{1}{\mathbf{v}^{\mathbf{T}}[\mathbf{I} - \mathbf{A}]\mathbf{x}_s}\right) =$$

$$= \begin{bmatrix} 0.586 & -2.571 & -0.5 \\ -0.026 & 0.714 & -0.05 \\ -0.02 & -0.285 & 0.75 \end{bmatrix} \begin{bmatrix} 0.991 \\ 0.081 \\ 0.101 \end{bmatrix} \left(\frac{1}{0.136}\right) = \begin{bmatrix} 2.362 \\ 0.193 \\ 0.241 \end{bmatrix} \tag{A.22}$$

and we set the price of this composite commodity equal to unity.

In addition, we assume that the wage payment occurs in terms of the standard commodity ($\mathbf{b} = \mathbf{b}_s$), that is equivalent of considering the wage rate as the fraction of the standard net product that goes to labour and, at the same time, also as the fraction of the total working time that is devoted to produce the basket of wage goods, i.e. necessary labour. Due to the assumption above, the standard real wage can be defined as follows:

$$\mathbf{b}_s = \mathbf{y}_s\omega = \begin{bmatrix} 2.362 \\ 0.193 \\ 0.241 \end{bmatrix} 0.515 = \begin{bmatrix} 1.217 \\ 0.099 \\ 0.124 \end{bmatrix} \tag{A.23}$$

The augmented Calling $\mathbf{p}_{\mathbf{\Theta}s}^{\mathbf{T}}$ the non-normalized price structure, i.e. the dominant left-hand eigenvector associated with matrix $\mathbf{\Theta}_s = \mathbf{A}[\mathbf{I} - (\mathbf{A} + \mathbf{b}_s\mathbf{l}^{\mathbf{T}})]^{-1}$, the prices of production in terms of the standard net product are:

$$\mathbf{p}^{\mathbf{T}} = \left(\frac{1}{\mathbf{p}_{\mathbf{\Theta}s}^{\mathbf{T}}\mathbf{y}_s}\right)\mathbf{p}_{\mathbf{\Theta}s}^{\mathbf{T}} =$$

$$= \left(\frac{1}{0.506}\right)[0.101 \quad 0.926 \quad 0.362] =$$

$$= [0.2 \quad 1.828 \quad 0.714] \tag{A.24}$$

We can compute the wage rate in terms of the standard net product by doing:

$$w_s = \mathbf{p}^{\mathbf{T}}\mathbf{b}_s = [0.2 \quad 1.828 \quad 0.714] \begin{bmatrix} 1.217 \\ 0.099 \\ 0.124 \end{bmatrix} = 0.515 \tag{A.25}$$

In this way, we obtain the linear relationship between the wage rate (equal to the value of the labour power) and the rate of profit that emerges from the modified standard system:

$$\pi_s = R(1 - w_s) = 48.25\%(1 - 0.515) = 23.39\% \qquad (A.26)$$

where $R = \left(\frac{1}{\lambda_m^{\mathbf{A}}}\right) - 1$ is the maximum rate of profit, i.e. the standard ratio, that can be computed from the maximum eigenvalue associated with matrix $\mathbf{A}$.[18] The two fundamental Marxian equalities are verified inside the standard system as follows. The monetary value added per worker in terms of the net standard commodity is:

$$\mathbf{p}^{\mathbf{T}}\mathbf{y}_s = [0.2 \quad 1.828 \quad 0.714]\begin{bmatrix}2.362\\0.193\\0.241\end{bmatrix} = 1 \qquad (A.27)$$

that is equal to total abstract labour employed in the economic system thanks to the normalizations adopted for the prices with the standard net product and for this latter with total employment set equal to unity. On the other hand, the aggregate profit per worker is given by:

$$\pi_s\mathbf{p}^{\mathbf{T}}\mathbf{H}\mathbf{y}_s =$$

$$= 23.39\%[0.2 \quad 1.828 \quad 0.714]\begin{bmatrix}1.164 & 8.596 & 2.015\\0.087 & 0.784 & 0.177\\0.09 & 0.909 & 0.454\end{bmatrix}\begin{bmatrix}2.362\\0.193\\0.241\end{bmatrix} = 0.484 \qquad (A.28)$$

that is equivalent to the surplus value per worker.

10.3. Marx's prices and the neo-value

Third, we choose to measure Marx's prices in terms of a composite commodity that has the same composition of the real wage but has the same volume of the net product per worker, we called this commodity "neo-value" per worker to distinguish it from the net product per worker in the proportions observed from the input-output table defined with $\mathbf{y}$. The neo-value per worker is therefore:

$$\mathbf{b}(1+\sigma) = \begin{bmatrix}2\\0\\0.166\end{bmatrix}(1 + 94.11\%) = \begin{bmatrix}3.882\\0\\0.323\end{bmatrix} \qquad (A.29)$$

and we set its price equal to unity. The price solutions that we obtain are given by:

[18] Of course, the same rate of profit can be obtained directly from the dominant eigenvalue associated with matrix $\mathbf{\Theta}_s$, hence $\pi_s = 1/\lambda_m^{\mathbf{\Theta}s}$.

$$\mathbf{p}^{\mathrm{T}} = \left(\frac{1}{\mathbf{p}_{\Theta}^{\mathrm{T}}\mathbf{b}(1+\sigma)}\right)\mathbf{p}_{\Theta}^{\mathrm{T}} =$$
$$= \left(\frac{1}{0.512}\right)[0.101 \quad 0.926 \quad 0.363] =$$
$$= [0.198 \quad 1.808 \quad 0.709] \tag{A.30}$$

In this way, we turn one of the two inequalities into an equality, namely we set the price of the neo-value per worker equal to total employment. The latter sentence can be verified simply as follows:

$$\mathbf{p}^{\mathrm{T}}\mathbf{b}(1+\sigma) = [0.198 \quad 1.808 \quad 0.709]\begin{bmatrix} 3.882 \\ 0 \\ 0.323 \end{bmatrix} = 1 \tag{A.31}$$

for which the wage rate becomes:

$$w = \mathbf{p}^{\mathrm{T}}\mathbf{b} = [0.198 \quad 1.808 \quad 0.709]\begin{bmatrix} 2 \\ 0 \\ 0.166 \end{bmatrix} = 0.515 \tag{A.32}$$

that is equal of the labour embodied in the real wage, i.e. necessary labour. The aggregate profits in money terms are therefore:

$$\pi_{\Theta}\mathbf{p}^{\mathrm{T}}\mathbf{H}\mathbf{b}(1+\sigma) =$$
$$= 23.12\%[0.198 \quad 1.808 \quad 0.709]\begin{bmatrix} 1.164 & 8.596 & 2.015 \\ 0.087 & 0.784 & 0.177 \\ 0.09 & 0.909 & 0.454 \end{bmatrix}\begin{bmatrix} 3.882 \\ 0 \\ 0.323 \end{bmatrix} = 0.484 \tag{A.33}$$

This is the way Pasinetti, albeit through different price formation rules, arrives to a general transformation method of labour-values into production prices. This procedure keeps the two fundamental equivalences of neo-value with money value added and of surplus value with money profit.[19]

[19] In an alternative way, the price computation can be done like we have elaborated above. First, we substitute π_{Θ} inside the price equation (A.12) and then we compute the wage rate in terms of the chosen *numéraire*, i.e. the neo-value:

$$w = [\mathbf{v}^{\mathrm{T}}[\mathbf{I} - \pi_{\Theta}\mathbf{H}]^{-1}\mathbf{b}(1+\sigma)]^{-1} = 0.515 \tag{A.32b}$$

that is equal to the labour embodied inside the wage goods and that we can substitute in the price equations as follows:

$$\mathbf{p}^{\mathrm{T}} = w\mathbf{v}^{\mathrm{T}}[\mathbf{I} - \pi_{\Theta}\mathbf{H}]^{-1} =$$
$$= 0.515[0.181 \quad 1.818 \quad 0.909]\begin{bmatrix} 1.503 & 3.898 & 0.961 \\ 0.039 & 1.337 & 0.081 \\ 0.044 & 0.405 & 1.159 \end{bmatrix} =$$
$$= [0.198 \quad 1.808 \quad 0.709] \tag{A.30b}$$

10.4. Macro-monetary method

Finally, we choose to measure prices in terms of the composite commodity of the net product per worker as directly observed from the input-output table:

$$\frac{\mathbf{p}^{\mathrm{T}}[\mathbf{I}-\mathbf{A}]\mathbf{x}}{L} = \overline{\mathbf{p}^{\mathrm{T}}\mathbf{y}} = 1 \tag{A.34}$$

Additionally, we consider the money wage rate observed in the price system to represent the share of net product per worker that the workers can purchase with their wage, hence:

$$w = \omega\overline{\mathbf{p}^{\mathrm{T}}\mathbf{y}} = 0.515 \tag{A.35}$$

that is equivalent of constructing the vector of the real wage in such a way that its commodity proportions are the same of those of the net product.[20] Calling $\mathbf{p}_{\mathbf{\Theta}m}^{\mathrm{T}}$ the non-normalized price structure, i.e. the dominant left-hand eigenvector associated with matrix $\mathbf{\Theta}_m = \mathbf{A}[\mathbf{I}-(\mathbf{A}+\mathbf{b}_m\mathbf{l}^{\mathrm{T}})]^{-1}$, the price solutions that are obtained in terms of the chosen *numéraire* are the following:[21]

$$\begin{aligned}\mathbf{p}^{\mathrm{T}} &= \left(\frac{1}{\mathbf{p}_{\mathbf{\Theta}m}^{\mathrm{T}}\mathbf{y}}\right)\mathbf{p}_{\mathbf{\Theta}m}^{\mathrm{T}} = \\ &= \left(\frac{1}{0.486}\right)[0.102 \quad 0.928 \quad 0.358] = \\ &= [0.21 \quad 1.908 \quad 0.736]\end{aligned} \tag{A.36}$$

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[20] The equivalent procedure that we have adopted with the standard system, that stands again on a non-monetary, but physical, definition of the wage rate, is the following. We can slightly modify the definition of the real wage in such a way that it becomes a basket of commodities that has the same structure of the net product ($\mathbf{b} = \mathbf{b}_m$):

$$\mathbf{b}_m = \mathbf{y}\omega = \begin{bmatrix}3\\0\\0.5\end{bmatrix}0.515 = \begin{bmatrix}1.545\\0\\0.257\end{bmatrix} \tag{A.35b}$$

Once we normalize the relative prices with the net product per worker through equation (A.36), we can obtain the following money wage rate:

$$w = \mathbf{p}^{\mathrm{T}}\mathbf{b}_m = [0.21 \quad 1.908 \quad 0.736]\begin{bmatrix}1.545\\0\\0.257\end{bmatrix} = 0.515 \tag{A.35c}$$

that is equivalent to the labour embodied in the real wage as previously defined. This further construction does not change the final result obtained with a monetary definition of the value of the labour power (A.35). It is worth to mention the fact that this is precisely the procedure through which Perri (2003) discusses the compatibility between Marx's theory of surplus and Sraffa's price equations starting from Marx's counterfactual method.

[21] The re-formulation of the price structure is required by the change in the definition of the real wage $\mathbf{b}_m$ in such a way to be proportional to the actual net product, see footnote (20).

In this way, the equivalence between the net product and total direct labour can be verified as follows:

$$\omega \mathbf{p}^{\mathrm{T}}\mathbf{y}(1+\sigma) =$$

$$= 0.515[0.21 \quad 1.908 \quad 0.736]\begin{bmatrix} 3 \\ 0 \\ 0.5 \end{bmatrix}(1+94.11\%) = 1 \qquad (\mathrm{A}.37)$$

Due to the normalization adopted for prices and the definition of the wage rate, the same relationship between the wage and the rate of surplus value can be achieved:

$$\sigma = \frac{1}{w} - 1 = 94.11\% \qquad (\mathrm{A}.38)$$

and the total profit per worker comes again to be equivalent to the surplus value per worker:

$$\overline{\mathbf{p}^{\mathrm{T}}\mathbf{y}} - w\mathbf{v}\mathbf{y} = 0.484 \qquad (\mathrm{A}.39)$$

However, since we added a further constraint to the price system, that is the equalization between the wage rate and the share of abstract labour embodied in the real wage, the profit rate associated with such income distribution will differ from that emerged from the (maximum) rate of profit associated with matrix $\mathbf{\Theta}$. The profit rate is determined by the ratio between the total surplus value and the price of the means of production per worker, i.e. ratio between capital and labour or capital intensity:

$$\pi_m = \frac{1-w}{\mathbf{p}^{\mathrm{T}}\mathbf{H}\mathbf{y}} = \frac{0.484}{1.983} = 24.43\% \qquad (\mathrm{A}.40)$$

The results are summarized in Table 1. The two fundamental Marxian equivalences are verified in all the price systems considered with the exception of the case in that they are determined in terms of labour commanded, i.e. in terms of the wage rate. In Sraffa's price system, the price of the standard net product is set as the *numéraire* and wages are assumed to be paid in terms of standard commodity. In Pasinetti's procedure for Marx's prices computation, the composite commodity neo-value equivalent to the abstract labour employed is constructed and is considered as the *numéraire* of the prices. With the macro-monetary approach, it is the price of net product per worker that is chosen as the *numéraire* and the value of the labour power is assumed to be defined in monetary rather than physical terms.

Table 1. Production prices, wage rates, profit rates and economic aggregates with alternative measure units.

System	*Numéraire*	Iron	Coal	Wheat	Wage rate (V.L.P)	Profit (Surplus value)	Capital (Indirect labour)	Profit rate (Surplus value rate)
Direct prices (Labour-values)	$\mathbf{v}^{\mathbf{T}}\mathbf{y} = 1$	0.181	1.818	0.909	0.515	0.484	1.909	94.11%
Smith prices	$w = \mathbf{p}^{\mathbf{T}}\mathbf{b} = 1$	0.385	3.51	1.377	1	0.844	3.65	23.12%
Sraffa prices	$\mathbf{p}^{\mathbf{T}}\mathbf{y}_s = 1$	0.2	1.828	0.714	0.515	0.484	2.072	23.39%
Marx prices	$\mathbf{p}^{\mathbf{T}}\mathbf{b}(1+\sigma) = 1$	0.198	1.808	0.709	0.515	0.484	2.096	23.12%
PNP–UPP prices	$\mathbf{p}^{\mathbf{T}}\mathbf{y} = 1$	0.21	1.908	0.736	0.515	0.484	1.983	24.43%

Notes: Regarding the magnitudes that refer to direct prices, i.e. the labour-value system, they are expressed in terms of labour embodied. This is the reason of the different nomenclature expressed between brackets in the last four columns; therefore, the wage rate represents the value of the labour power.

11. Concluding remarks

The critical viewpoints on Marx's labour theory of value reject its utility into the determination of prices. Indeed, we have shown that we can move from one system to the other without loss of the correspondence between the economic aggregates, namely the aggregate net product, the monetary profit and the share of the surplus that goes to the workers. However, we do not know yet whether labour-values determine the prices, or prices determine the values, because both of them systems can be derived autonomously simply starting from the technical coefficients of production, i.e. the vertically integrated productive capacity coefficients matrix and the vertically integrated labour coefficients vector expressed.

If we accept, instead, the macro-monetary meaning of the value theory, we position ourselves in the position to reconcile the law of value with that of exchange rather than prefer one to the other as two alternative devices for evaluating the physical quantities of commodities. From such perspective, we only need the construct the proper transformation procedure according to the theoretical framework that the preliminary assumptions are related with. Pasinetti's method along Sraffa's framework contributes to reappraise the labour theory of value as elaborated from the Classical economists from the macroeconomic perspective of the analysis. What follows are the main conclusion that can be drawn from previous investigation.

(i) The labour theory of value must be considered as a macroeconomic theory describing the specific capitalist mode of production of commodities by means of commodities. Its relevance stands in the abstraction of the market rules that regulate the formation of monetary prices, being the latter a part of the analysis of commodity circulation. From the macro perspective, the law of value must be defined on the basis of the aggregate net product, excluding the labour embodied in means of production.

(ii) The transformation method not only concerns the proper choice of the *numéraire*, but the latter must be defined in accordance with the chosen assumption introduced to define the value of the labour power, i.e. the real wage. When we start from the subsistence theory of wages, then the net product chosen as *numéraire* is in fact what we called neo-value, namely a composite commodity with the same volume of the net product and the same proportions of the real wage previously given in physical terms. In the case where we start from the aggregate definition of the value of money, i.e. the money expression of labour time, then we are assuming that the basket of wage goods has the same proportion of the net product as it is, on the basis of the societal advancement of the money wage bargained in the labour market. Finally, if the standard net product is chosen as *numéraire*, than the real wage is modified in such a way to expose the standard proportions.

(iii) The mechanisms of price formation and income distribution belong to the later stage of economic investigation, i.e. when we enter the sphere of commodity circulation between social classes and among individual producers. These rules can be multiple, and independent of the value system; however, we have shown, through the lenses of Pasinetti, how monetary aggregates can be treated at a level of abstraction that allows the investigation of the essential features of the capitalist mode of production: the length of the working time, the technique brought into being and the composition of the surplus produced by the social abstract labour.

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CHAPTER TWO

Production prices, income distribution and the value structure inside the Italian economy: An input-output investigation

Abstract

We exploit the input-output table of the Italian economy for the year 2018 to investigates the price effects induced by income distribution variations and their relationships with direct prices based on Classical labour-value. We evaluate the distance of production prices and direct prices from observed market prices and estimate the corresponding wage-profit frontier and observed income distribution. We then perform some simulations of the key indicators to study the possible trajectories through which we can observe the price effects induced by variations in income distribution, notably the production-direct price percent differences, the sectoral capital-labour ratios and the sectoral output-capital ratios.

Keywords: Production prices, Income distribution, Value theory, Standard system, Critical output-capital relationships.
JEL Classification: B51, C67, D57.

1. Introduction

The investigation of the relationships between value and income distribution has been central for Classical political economy and the modern approaches. The Classical economists argue that the "natural prices" act as a centre of gravity towards which the market prices fluctuate. The labour theory of value, according to them, ultimately explain the prices by arguing that they are proportional to the labour embodied (i.e. labour time required by the production of commodities). However, Classical economists realize that prices cannot be proportional to labour embodied when they exploit different techniques that present non-uniform relationships between capital and labour. Consequently the transformation problem, in Marx, and the problem of the invariable measure of value, in Ricardo, remained open. After the publication of Sraffa (1960), and notably after the contribution of Pasinetti (1977, Ch. 5) that translated it into input-output language, there has been a reappraisal of the value theory from the Classical political economy perspective.

Shaikh (1984, 1998, 2016, and Shaikh et al. 2020) shows that prices can be computed at empirical level starting from national input-output tables and observes that the deviations of the market prices from prices of production were about 20-25 percent in the US economy and 7-9 percent in the Italian economy. Ochoa (1989) uses input-output tables of the US economy from 1947 to 1972 and observes that market pries deviate from labour-values by 12-14 percent and from production prices by 14-17 percent. Shaikh and Ochoa conclude that these findings suggest that labour-values can be reasonably considered as approximation of observed market prices. Since then, a long literature covered this field of research studying the input-output tables for Greece (Tsoulfidis and Tsaliki 2019; Tsoulfidis and Mariolis 2007), China (Mariolis and Tsoulfidis 2009; Cheng and Li 2020), Yugoslavia (Petrović 1987; 1991), Brazil (da Silva 1991), South Korea (Tsoulfidis and Rieu 2006) and for multiple countries considered at once (Işıkara and Mokre 2021). All these contributions provided empirical validation of the labour theory of value to be tested and used to evaluate the value structure and income distribution of an economic system. The following contributions aims to expand and update the field of research on price-value differences and is mainly focused on the investigation of the value structure of the Italian economy for the year 2018.

The essay is organized as follows. In section 2 we develop the basic devices for the investigation, namely a linear schema of production, the respective labour-value and direct price system, the production price system and the standard system. In section 3 we analyse the main empirical findings on value structure and income distribution inside the Italian economy. In section 4 we perform some simulations of income distribution variations and their impact on the key indicators: the price-value percent difference, the sectoral capital-labour ratios and the sectoral output-capital ratios. Concluding remarks follow.

2. Method and theory

2.1. A linear schema of production

Production of commodities occurs according to the dominant technological paradigm assumed to be viable, that is defined by the fixed proportions of inputs (capital and labour) that are required to produce one unit of output. The physical quantity produced can be accounted as follows:

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{y} \tag{1}$$

which solves for the output vector:

$$\mathbf{x} = [\mathbf{I} - \mathbf{A}]^{-1}\mathbf{y} \tag{2}$$

We call $\mathbf{x} = [x_j]$ the physical quantity of gross product or output, $\mathbf{A} = [a_{ij}]$ (where a_{ij} is the quantity of good i necessary to produce one unit of product of sector j) the technical coefficients of production that explain the inter-sectoral flows of intermediate consumptions, and $\mathbf{y} = [\mathbf{I} - \mathbf{A}]\mathbf{x}$ the quantity of commodities that compose the final demand, or net product (i.e. what remains after we subtract from the gross output all the commodities needed to replace the means of production) and consequently is made available for final uses, namely consumption or investment. The linear operator $[\mathbf{I} - \mathbf{A}]^{-1}$ is the Leontief inverse matrix whose columns capture the inter-sectoral flows of commodities that are put in motion in the system to satisfy the final demand.

2.2. Labour-value system and direct prices

In Classical political economy, labour-values represent the labour embodied in each commodity, that is the quantity of (homogeneous) labour directly or indirectly required to produce the surplus or net product. Calling $\boldsymbol{\lambda}^{\mathbf{T}}$ the vector of labour-values, we can write:

$$\boldsymbol{\lambda}^{\mathbf{T}}\mathbf{x} = \boldsymbol{\lambda}^{\mathbf{T}}\mathbf{A}\mathbf{x} + \mathbf{l}^{\mathbf{T}}\mathbf{x} \tag{3}$$

which solves for the vector of values:

$$\boldsymbol{\lambda}^{\mathbf{T}} = \mathbf{l}^{\mathbf{T}}[\mathbf{I} - \mathbf{A}]^{-1} \tag{4}$$

Labour-values correspond to what Pasinetti (1973) calls vertically integrated labour coefficients, that explain the two components of labour that enter in production indirectly ($\boldsymbol{\lambda}^{\mathbf{T}}\mathbf{A}\mathbf{x}$, i.e. the labour required to reintegrate the means of production), and directly, namely the actual employment of the economy ($\mathbf{l}^{\mathbf{T}}\mathbf{x}$, i.e. the labour required to satisfy the final demand).

When we compute the labour embodied in the surplus, we can immediately observe that the labour required to produce the net product is exactly equal to the total direct labour, namely the actual employment in present period of production (L), or what Marx calls *living labour*:

$$\boldsymbol{\lambda}^{\mathbf{T}}\mathbf{y} = \mathbf{l}^{\mathbf{T}}[\mathbf{I} - \mathbf{A}]^{-1}[\mathbf{I} - \mathbf{A}]\mathbf{x} = \mathbf{l}^{\mathbf{T}}\mathbf{x} = L$$

Since we want to compare labour quantities with monetary prices, we must adopt a conversion rate that transforms labour-values into money values. For such purpose, we define what Marx calls the *monetary expression of labour time* (MELT) as the relationship between the net product evaluated at market prices (calibrated with a horizontal summation vector $\mathbf{e}^{\mathbf{T}}$, i.e. all the elements are equal to unity) and the labour embodied in the same magnitude.[22] By multiplying this rate by the vector of labour-values $\boldsymbol{\lambda}^{\mathbf{T}}$ we obtain a vector of direct prices $\mathbf{v}^{\mathbf{T}}$ based on labour-values:

$$\mathbf{v}^{\mathbf{T}} = \left(\frac{\mathbf{e}^{\mathbf{T}}\mathbf{y}}{\boldsymbol{\lambda}^{\mathbf{T}}\mathbf{y}}\right)\boldsymbol{\lambda}^{\mathbf{T}} \tag{5}$$

that explain a set of exchange ratios “that are directly proportional to labour-values where the constant proportionality relates to the money unit to a unit of labour time (i.e. worker-hours/dollars)” (Ochoa 1989).

[22] The choice of the numéraire varies among researchers and theoretical approaches. For example, Ochoa (1989) and Shaikh (1998) choose the price of the gross output, Petrović (1991) chooses the price of the basket of consumption goods, while Foley (1982) and Duménil (1983) suggest to choose the price of the net product with which da Silva (1991) provides an empirical input-output analysis of price-value deviations.

2.3. Production prices and income distribution

Moving to the price system, we assume that prices resolve into wages, profits and material costs, the latter being represented by circulating capital flows which turn over at each period of production. Hence, we exclude both the stock of capital goods and any capital depreciation. Furthermore, we assume that the institutional settings of the market lead both the profit and the wage rate to be uniform across sectors, hence the economic system runs in a perfect competition regime. We shall write:

$$\mathbf{p}^{\mathbf{T}} = \mathbf{p}^{\mathbf{T}}\mathbf{A}(1+\pi) + w\mathbf{l}^{\mathbf{T}} \tag{6}$$

The above expression states that wages are paid *post factum* by applying the wage rate w to the quantity of labour employed in the period that is represented by the vector of labour coefficients $\mathbf{l}^{\mathbf{T}}$. It also states that the rate of profit π is computed on the money value of circulating capital, after the manner of Sraffa (1960) and that it is uniform across sectors. After a few algebraical steps, system (6) can be rewritten as follows:

$$\mathbf{p}^{\mathbf{T}}[\mathbf{I}-\mathbf{A}] = w\mathbf{l}^{\mathbf{T}} + \pi\mathbf{p}^{\mathbf{T}}\mathbf{A} \tag{7}$$

$$\mathbf{p}^{\mathbf{T}} = w\mathbf{v}^{\mathbf{T}} + \pi\mathbf{p}^{\mathbf{T}}\mathbf{H} \tag{8}$$

where $\mathbf{H} = \mathbf{A}[\mathbf{I}-\mathbf{A}]^{-1}$ is the matrix of vertically integrated productive capacity, whose jth column represents the amount of each input-commodity that is directly or indirectly required to produce one unit of final good. In this way we have a clear illustration of how prices essentially resolve themselves into wages and profits like stated by Smith.[23] System (8) allows us to configurate the particular set of exchange ratios in two opposite situations. In the case in which the net product goes entirely to wages, hence with $\pi = 0$, we easily find that production prices are proportional to labour-values expressed in money terms, hence to direct prices $\mathbf{p}^{\mathbf{T}} = W\mathbf{v}^{\mathbf{T}}$. In this case, wages are at their maximum level ($\max w = W$) and they absorb the entire purchasing power of the commodities, i.e. prices are proportional to the respective labour-values. In the opposite case, with $w = 0$, the economy experiences the maximum rate of profit, and system (8) becomes $\mathbf{p}^{\mathbf{T}}[\mathbf{I} - \Pi\mathbf{H}] = 0$ where the maximum rate of profit ($\max \pi = \Pi$) must be positive in order to assure the reproducibility of the economy. The economic system is viable *ex hypothesis*, therefore Π must be positive We know that the maximum profit rate emerges from

[23] Cf. Pasinetti (1973), Kurz and Salvadori (1997).

the Perron-Frobenius theorem resulting from the reciprocal of the dominant eigenvalue associated with matrix $\mathbf{H}$, that is real, positive and associated with a non-negative left-hand eigenvector that assures non-negative prices ($\Pi = 1/\lambda_m^{\mathbf{H}}$). On the basis of the P-F theorem, we also know that matrix $\mathbf{H}$ is non-negative, because the viability of the economic system implies that the Leontief inverse $[\mathbf{I}-\mathbf{A}]^{-1}$ is non-negative as well, hence the general solution of system (8), for any positive $\pi < \Pi$, will be the following:

$$\mathbf{p}^{\mathbf{T}} = w\mathbf{v}^{\mathbf{T}}[\mathbf{I}-\pi\mathbf{H}]^{-1} \tag{9}$$

that brings non-negative prices and inverse and monotonic relationship between the profit rate π and the wage rate w, as proved by Sraffa (1960). The wage rate that emerges from system (9) for any positive π, can be expressed as follows, once we choose the price of the net product as *numéraire* ($\mathbf{p}^{\mathbf{T}}\mathbf{y} = 1$):

$$w = \frac{\mathbf{e}^{\mathbf{T}}\mathbf{y}}{\mathbf{v}^{\mathbf{T}}[\mathbf{I}-\pi\mathbf{H}]^{-1}\mathbf{y}} \tag{10}$$

that, due to the choice of the numéraire with the price of the net product, results to be equal to the share of the surplus that goes to labour, hence $w < 1$. Furthermore, the following normalization holds for any price changes induced by variations in income distribution:

$$\mathbf{p}^{\mathbf{T}}\mathbf{y} = \mathbf{v}^{\mathbf{T}}\mathbf{y} = \mathbf{e}^{\mathbf{T}}\mathbf{y} \tag{11}$$

that state the market prices, the direct prices and the prices of production are evaluated in the same measure unit.

2.4. The Sraffian standard system

When the profit rate is positive, the relationships between the latter, the wage rate and the prices of production are complicated by the fact that a change in income distribution leads to a dual and "ambiguous" effect: (a) the variation of the relative prices arising from the necessity to restore the balance within the corresponding sector; and (b) the variation in the price of the *numéraire* commodity arising from the necessity to restore its own sectoral balance.[24] Moreover, the linear relationship between wages and profits cannot be shown because when income distribution varies, it leads both the wage rate and the relative price of capital required to

[24] Cf. Bellino (2004).

produce the *numéraire* commodity to change (in previous case, the price of the means of production required to produce the net product change with the distribution).

Sraffa shows that there exists a composite commodity (or sector) whose price does not need to vary when the wage rate changes due to a variation of income distribution. The standard system is defined as an economic system in which "the various commodities are represented among its aggregate means of production *in the same proportions* as they are among its products" (Sraffa 1960, p. 19). The standard system provides the proportions of inputs that ,multiplied by the standard ratio between the sectoral surplus and the means of production, gives the output proportions. Thus, in order to obtain the quantity vector is necessary to fix the scale that, applied to such proportions, determines the connection between the standard and the actual system. Following Sraffa, we choose to normalize the net standard commodity to total employment:

$$[\mathrm{I} - (1 + \Pi)\mathbf{A}]\mathbf{x}_s = \mathbf{0} \tag{12}$$

$$\mathbf{l}^{\mathbf{T}}\mathbf{x}_s = L \tag{13}$$

where Π is maximum rate of profit, i.e. the uniform "standard ratio" between the sectoral physical surplus and the respective means of production, and $\mathbf{x}_s$ results to be the right-hand eigenvector associated with matrix $\mathbf{A}$.

$$\mathbf{y}_s = [\mathbf{I} - \mathbf{A}]\mathbf{x}_s \left(\frac{\mathbf{v}^{\mathbf{T}}\mathbf{y}}{\mathbf{v}^{\mathbf{T}}[\mathbf{I} - \mathbf{A}]\mathbf{x}_s} \right) \tag{14}$$

Consequently, by using equation (8), we can compute the price of the standard net product as follows:

$$\mathbf{p}^{\mathbf{T}}\mathbf{y}_s = w_s\mathbf{v}^{\mathbf{T}}\mathbf{y}_s + \pi\mathbf{p}^{\mathbf{T}}\mathbf{H}\mathbf{y}_s \tag{15}$$

that we know, by definition, to be equivalent in magnitude when evaluated in terms of labour embodied, i.e. direct prices. Hence, we can write:

$$\mathbf{v}^{\mathbf{T}}\mathbf{y}_s = w_s\mathbf{v}^{\mathbf{T}}\mathbf{y}_s + \frac{\pi}{\Pi}\mathbf{v}^{\mathbf{T}}\mathbf{y}_s \tag{16}$$

Dividing both sides of the above equation by $\mathbf{v}^{\mathbf{T}}\mathbf{y}_s$, we obtain the following expressions:

$$w_s = 1 - \frac{\pi}{\Pi}, \qquad \text{or alternately,} \qquad \pi = \Pi(1 - w_s) \tag{17}$$

that represent the famous opposite and linear "standard" relationship defined by Sraffa (1960, Ch. 4). The standard commodity provides an invariable measure that assures the isolation of the feedback effects (the *numéraire* price effect and the capital effect) within the analyses of value and income distribution. In other words, the changes in the distribution of income can be measured independently from the relative prices of production. A particular wage share (wage rate) observed from the actual economic system and associated with a certain profit rate, that is converted in terms of standard commodity, corresponds to a different profit rate emerging from the standard system. Only in the special case in which the wage rate is absent in the actual system ($w = 0$), the rate of profit that emerges equals the standard ratio, i.e. the maximum rate of profit prevails..

The above procedures allow us to obtain the following equalizations:

$$\mathbf{p}^{\mathbf{T}}\mathbf{y}_s = \mathbf{v}^{\mathbf{T}}\mathbf{y}_s = \mathbf{v}^{\mathbf{T}}\mathbf{y} = \mathbf{e}^{\mathbf{T}}\mathbf{y} \quad (18)$$

Normalization (18) shows the connection between the actual and the standard system, that is the global labour employed in the economy within the period of production and, in the meantime, shows instead that the price of the net product is equal to the global employment of the economy due to the normalization of the labour-values (direct prices) to the market value of the net product. Although relative prices will normally differ from labour-values for any positive profit rate, some of them might be higher or lower than the respective labour-value, but at aggregate level every deviation compensate, and the money value of the net product remains equal to the monetary expression of the labour time expended in production.[25]

3. Empirical findings from the Italian economy for the year 2018

The empirical investigation that follows exploits a symmetrical industry-by-industry. input-output table of 63-order from the domestic Italian economy for the year 2018. The dataset is provided by the ISTAT (2021). A preliminary analysis of a descriptive nature can be carried out by visualizing in parallel the direct prices (prices proportional to labour-values, defined in section 2.2.) and the production prices (defined in section 2.3), keeping that the market prices are set equal to unity. The results are summarized in Table 1.

The mean absolute difference between market and production prices is 2.89%. The mean absolute difference between production and direct prices is 15.87%. The mean absolute difference between market and direct prices is 11.88%. This brings very

[25] Cf. Foley (1982) and Duménil (1983).

close results to those obtained by the existing literature discussed above. We can state that statistically, the production price system can represent most of the structure of market prices.

Consider the direct prices. When an industry exhibits a direct price equal to unity it means that the market price of the industry is directly proportional to the respective embodied labor. When a sector has a direct price higher than unity, it means that the market price under-represents the quantity of labour embodied in the commodities produced by the sector, i.e. the latter transfers labour-value to the rest of the economy. When a sector exhibits a direct price lower than unity, it means that the sector considered captures more value than that generated in its own production process, i.e. the labour embodied in its product is less than the labour embodied in the goods for which this product is exchanged. For the year 2018, in the Italian economy there are twenty-five sectors (out of sixty-three) that have a direct price between 0.9 and 1.1 (i.e. that have an absolute difference between market prices and direct prices lower than 10%). There are ten sectors that have a direct price significantly higher that 1.2 (vegetable and animal production, fishing, retail, accommodation services, selection of personnel, investigation, social care, computer repair, personal services, activities of family as employers). These sectors expose a market-direct price difference lower than −20%. There are eleven sectors that have a direct price lower than 0.8 (mining activity, coke and petroleum derivatives, pharmaceutical products, supply of gas and electricity, telecommunications, financial services, insurance and pension funds, real estate, scientific research and development, rental business, public administration and defence). These sectors expose a market-direct price deviation higher than 20%.

Consider now the prices of production. They're evaluated according to the uniformity condition of the rates of profit. The difference between direct and production prices can be interpreted like that for market, minding that here the hypothetical situation of inter-sectoral capital mobility is introduced. When the production price of a sector is higher than unity, it means that free capital mobility and perfect competition would be beneficial for the sector, while instead it transfers value to the rest of the economy due to the possible entry barriers make market prices deviate from the prices of production. When a sector exposes a production price lower than unity, it means that the real market structure allows the sector to gain value relatively to the perfect competition situation. We observed that,

statistically, the difference between production and market prices is very low (under 3%). This means that in the Italian economy there was a tendency of the sectoral rates of profit to equalise across sectors and the market power of individual commodities is low. There are five sectors (41, 44, 47, 55, 56) that expose significantly high percent difference between market and production prices. These are respectively the financial services, real estate, research and development, instruction, and health services. These sectors transfer value to the other sectors if free capital mobility prevails in the economy while instead they benefit from the present market conditions that determine the actual distribution of the surplus among sectors. The alternative price to price differences are displayed in Figure 1.
The difference between production and direct prices, depends on the profit component of production prices and its intensity, i.e. its sign, is determined by the differential capital-labour ratios across sectors. We can verify this fact by comparing Figure 1(a), showing the percent differences between production and direct prices, with Figure 2, where the vertically integrated capital-labour ratios are displayed ($\mathbf{p}^{\mathbf{T}}\mathbf{H}/\mathbf{v}^{\mathbf{T}}$). The impact of the variations of income distribution, i.e. an increase in the profit rate, on the production prices mainly depend by the ratio between the price of the means of production and the labour embodied in the surplus according to technical characteristics of the sectors (i.e. if they are above or below the capital intensity of the *numéraire*-commodity-producing sector, that is the net product). On the other hand, the difference between market and production prices, or market and direct prices, are exclusively determined by the prevailing institutional settings that take the economic system away from the hypothetical situation of profit rates uniformity. All these mechanisms are considered outside the framework. The capital-intensive sectors are mainly those of the manufacturing and transport, that show in fact positive production-direct price percent differences in the period considered, as they are above the average capital intensity (Figure 2).
In Figure 3 we display the wage-profit frontiers estimated. We compare the standard relationship (17), which is always linear, with the same curve evaluated in terms of the actual net product that presents a significant concavity within the intermediate levels of the relative profit rate (π/Π).

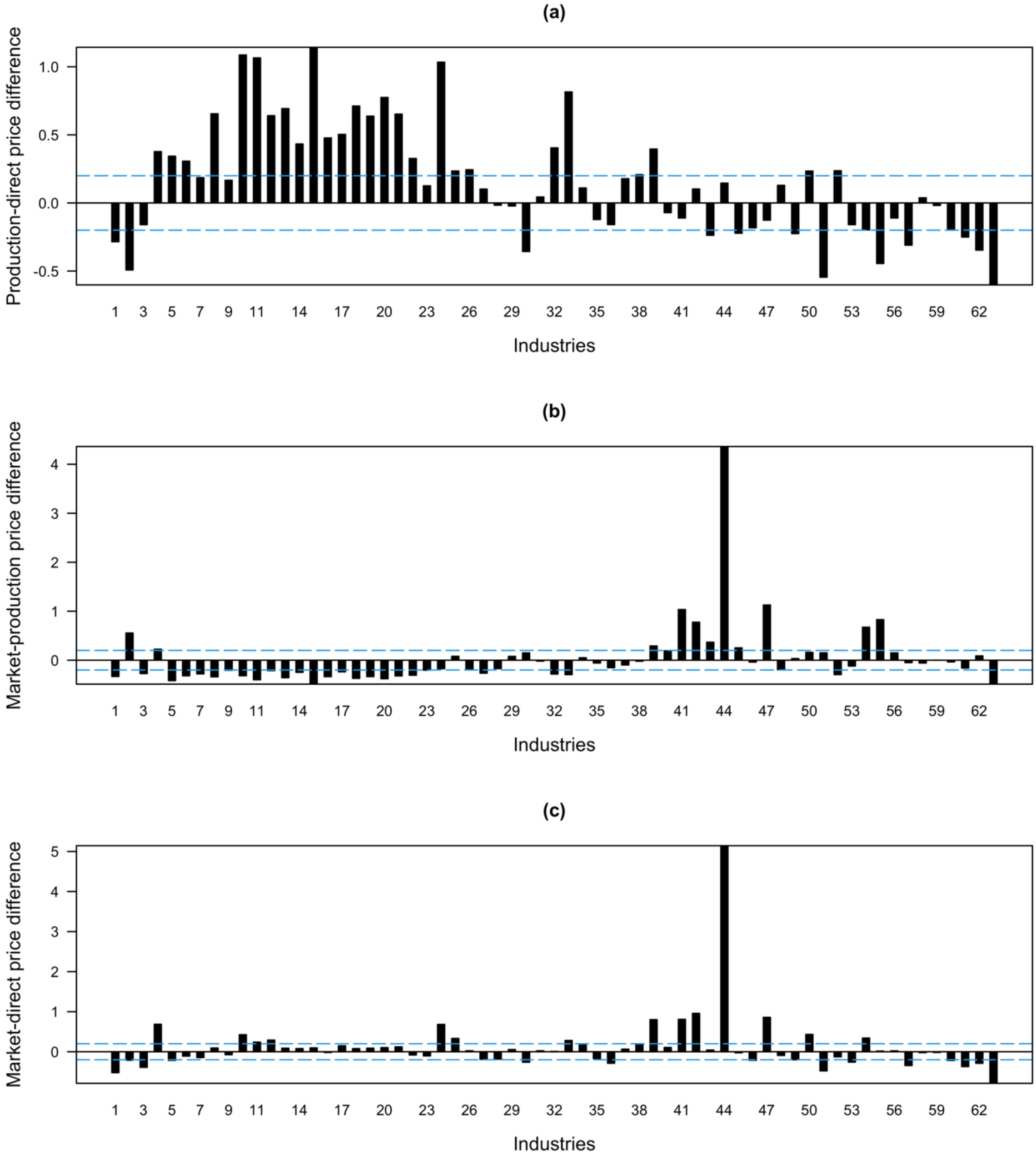
Table 1. Sectoral direct prices and production prices.

N.	Industry name	Direct prices (Labour-values)	Production prices
1	Vegetable and animal production and hunting	2.093	1.497
2	Forestry	1.266	0.643
3	Fishing and aquaculture	1.635	1.374
4	Mining activity	0.593	0.817
5	Food, beverages and tobacco	1.276	1.715
6	Textile industries, clothing	1.122	1.466
7	Wood and cork industry	1.170	1.387
8	Paper making	0.914	1.513
9	Printing and reproduction on recorded media	1.079	1.258
10	Manufacture of coke and petroleum derivatives	0.701	1.464
11	Manufacture of chemicals	0.806	1.665
12	Manufacture of pharmaceutical products	0.774	1.271
13	Manufacture of rubber and plastic items	0.918	1.554
14	Processing of non-metallic minerals	0.926	1.327
15	Metallurgical activities	0.911	1.953
16	Manufacture of metal products	1.018	1.504
17	Manufacture of computer/electronic products	0.871	1.309
18	Manufacture of electrical equipment	0.927	1.587
19	Manufacture of machinery and equipment	0.918	1.503
20	Manufacture of motor vehicles, trailers	0.906	1.608
21	Manufacture of other means of transport	0.890	1.471
22	Furniture manufacturing	1.087	1.442
23	Repair and installation of machines	1.114	1.255
24	Supply of electricity, gas, steam and a.c.	0.594	1.209
25	Collection, treatment and supply of water	0.750	0.926
26	Management of sewage networks and rubbish	0.975	1.213
27	Buildings	1.234	1.359
28	Vehicle trade and repair	1.225	1.205
29	Wholesale	0.949	0.928
30	Retail	1.353	0.870
31	Land and pipeline transport	0.976	1.019
32	Sea and water transport	0.992	1.394
33	Airplane transport	0.780	1.416
34	Warehousing and transport support	0.857	0.952
35	Postal services and courier business	1.207	1.060
36	Accommodation services; catering services	1.403	1.180
37	Publishing activities	0.939	1.107
38	Film production business	0.845	1.021
39	Telecommunications	0.555	0.774

40	Computer programming and consultancy	0.903	0.839
41	Provision of financial services	0.552	0.491
42	Insurance and pension funds	0.510	0.563
43	Financial services and insurance business	0.958	0.731
44	Real estate activities	0.162	0.186
45	Legal activities and accounting	1.026	0.798
46	Architecture and engineering activities	1.270	1.039
47	Scientific research and development	0.538	0.470
48	Advertising and market research	1.099	1.242
49	Other professional activities	1.249	0.968
50	Rental and leasing business	0.698	0.861
51	Search and selection of personnel	1.911	0.869
52	Activities of travel agency services	1.145	1.415
53	Investigation and surveillance services	1.349	1.134
54	Public administration and defence	0.745	0.597
55	Instruction	0.983	0.546
56	Health services activities	0.978	0.870
57	Social care	1.524	1.051
58	Creative, artistic and entertainment activities	1.022	1.060
59	Sports and entertainment activities	1.017	0.998
60	Activities of membership organizations	1.278	1.035
61	Computer repair	1.588	1.190
62	Other personal service businesses	1.406	0.919
63	Activities of families as employers	4.729	1.883

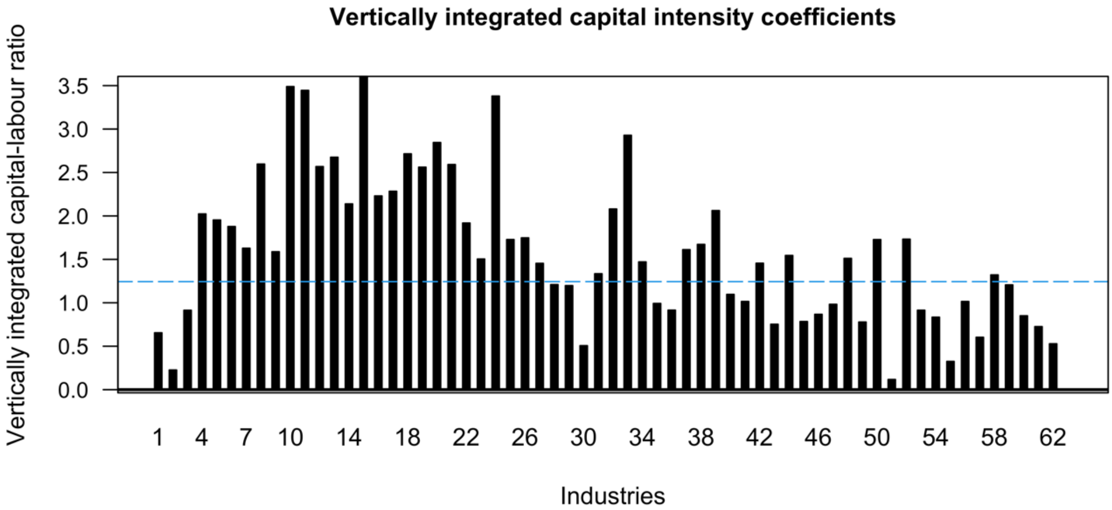
Notes: *Numéraire*: $\mathbf{p}^{\mathrm{T}}\mathbf{y} = 1$. Data: ISTAT Input-Output Tables (2021), own calculations.

Figure 1. Production-direct price percent difference (a), market-production price percent difference (b), market-direct price percent difference (c).



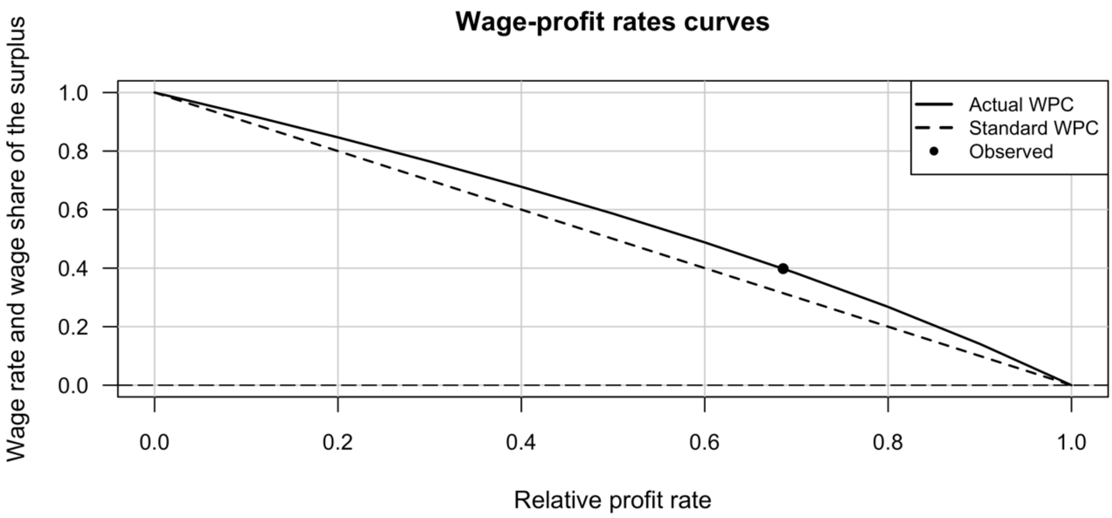
Notes: *Numéraire*: $\mathbf{p}^{\mathrm{T}}\mathbf{y} = 1$. Dashed lines (blue): $\pm 20\%$ difference threshold. Data: ISTAT Input-Output Tables (2021), own calculations.

Figure 2. Empirical estimation of sectoral capital-labour ratios.



Notes: *Numéraire*: $\mathbf{p}^{\mathbf{T}}\mathbf{y} = 1$. Dashed line (blue): Average capital intensity: $\mathbf{p}^{\mathbf{T}}\mathbf{H}\mathbf{y}/\mathbf{v}^{\mathbf{T}}\mathbf{y} = 1.243$. Data: ISTAT Input-Output Tables (2021), own calculations.

Figure 3. Empirical estimation of the wage-profit frontiers.



Notes: *Numéraires*: $\mathbf{p}^{\mathbf{T}}\mathbf{y} = 1$ and $\mathbf{p}^{\mathbf{T}}\mathbf{y}_s = 1$. Actual wage rate: $w = 0.398$. Actual rate of profit: $\pi = 48.38\%$. Actual relative rate of profit: $\pi/\Pi = 68.55\%$. Data: ISTAT Input-Output Tables (2021), own calculations.

4. Price effects induced by income distribution variations

The investigation of how the variations in income distribution induce the changes in production prices was central in Classical political economy. According to Ricardo (1951, Ch. 4) and Marx (1993, Ch. 11) argue that the changes in production prices can be, in general, small and predictable. Notably, the hypothesis states that, for a positive and increasing rate of profit, the price changes are affected by the sectoral capital intensity, i.e. capital-labour ratio, with respect to the economy's average. The argument is developed as follows. When the profit rate is null ($\pi = 0$) prices of production are identical to direct prices. It is assumed a fall in the wage rate that leads to an increase of the profit rate, however, the impact of the increasing profit rate is not uniform across sectors. In fact, profits increase but less in the capital-intensive industries (i.e. sectors with a capital intensity higher than the average) than in the labour-intensive industries (i.e. sectors with a capital intensity lower than the average), leading to a short-period non-uniformity of the profit rates. When the mechanisms of adjustment of capital mobility towards the "most profitable" sectors ensure that the profit rates become uniform in the long period, then the prices of the capital-intensive sectors will increase, while the prices of the labour-intensive sectors will decrease. Therefore, Ricardo and Marx share the idea that prices change monotonically to the upward and downward directions depending on the sectoral capital intensity with respect to the average.

Sraffa (1960) changes the perspective from which looking at the problem, by arguing that a variation in income distribution activate complex feedback effects that can change the trajectories imposed by the sectoral capital-labour ratios. The actual economy is characterised by inter-sectoral relations. It follows that together with the distribution of income, the revaluation of capital can interfere with the trajectory undertaken by a certain price of production to a change in the distribution. Some industry can be defined capital-intensive when the profit rate is absent while becoming progressively labour-intensive for positive and increasing profit rates. Therefore, Sraffa shows that the price effects cannot be predictable like Ricardo and Marx believe.

Pasinetti (1977, Ch. 5) follows Sraffa's considerations and shows in analytical terms the directions that relative prices can take when affected by the variations in income distribution (i.e. the rate of profit). He concludes that ultimately, the price trajectories can be observed as the summation of two distinct effects: the "capital intensity effect", that will be positive (negative) for all the sectors that exploit capital-intensive (labour-intensive) production processes, in relation to the sector producing the *numéraire*-commodity; and the "price effect", that can be either positive or negative depending on the overall inter-sectoral relationships of the economy. Therefore the latter effect can strengthen or weaken the capital intensity effect.

We can take advantage of the procedure that employs the standard commodity, illustrated by Mariolis and Tsoulfidis (2009), to show Pasinetti's decomposition of the price movements induced by income distribution variations.[26] Consider the prices of production evaluated in terms of the standard net product. The price system becomes:

$$\mathbf{p}^{\mathbf{T}} = \left(1 - \frac{\pi}{\Pi}\right)\mathbf{v}^{\mathbf{T}} + \pi\mathbf{p}^{\mathbf{T}}\mathbf{H} \tag{19}$$

We can arrange this expression in such a way to obtain a measure of the percentage difference of the prices of production evaluated in terms of standard net product from the respective direct price (labour-value):

$$\left(\frac{\mathbf{p}^{\mathbf{T}}}{\mathbf{v}^{\mathbf{T}}}\right) - \mathbf{e} = \pi\left[\frac{\mathbf{p}^{\mathbf{T}}\mathbf{H}}{\mathbf{v}^{\mathbf{T}}} - \Pi^{-1}\right] \tag{20}$$

where we can observe, between the square brackets, the ratio between capital and labour, i.e. the vertically integrated capital intensity, and the capital intensity of the standard system that is independent of prices. By deriving the expression above with respect the rate of profit, we obtain:

$$\begin{aligned} \frac{\mathrm{d}\left[\left(\frac{p_j}{v_j}\right) - 1\right]}{\mathrm{d}\pi} &= \pi\left(\frac{\mathrm{d}k_j}{\mathrm{d}\pi}\right) + \left(k_j - \Pi^{-1}\right) = \\ &= k_j\left[\left(\frac{\mathrm{d}k_j}{\mathrm{d}\pi}\right)\frac{\pi}{k_j} + \left(1 - \frac{\Pi^{-1}}{k_j}\right)\right] \end{aligned} \tag{21}$$

where $k_j = \mathbf{pH}_j/\mathbf{v}_j$ is the term that synthesize the sectoral capital-labour ratios. Between the square brackets we have a representation of the complexity of price movements to income distribution as a summation between two distinct effects. The second term expresses the capital intensity effect, or "Ricardo-Marx effect", while

[26] Cf. Tsoulfidis and Tsaliki (2019, Ch. 4), Mariolis and Tsoulfidis (2016).

the first term expresses the elasticity of the sectoral capital intensities to changes in the rate of profit that we call price effect, or "Sraffa effect".

From equation (21) we can observe the two opposite situations of the percentage distances of production prices from value. When the profit rate is absent ($\pi = 0$) the deviation will result to be the following:

$$\frac{\mathrm{d}\left[\left(\frac{p_j}{v_j}\right) - 1\right]}{\mathrm{d}\pi} = k_j(0) - \Pi^{-1} \tag{22}$$

In the opposite case, when the maximum profit rate prevails ($\pi = \Pi$) then the deviation of the prices from values will be at its maximum:

$$\frac{\mathrm{d}\left[\left(\frac{p_j}{v_j}\right) - 1\right]}{\mathrm{d}\pi} = \Pi k_j(\Pi) \tag{23}$$

When the capital intensity of one sector is above the standard capital intensity (Π^{-1}) then the Ricardo-Marx effect will be positive, and the inverse holds for the labour-intensive sectors. However, the price effect can be positive or negative. When it is positive it strengthens the capital intensity effect, while when it is negative the final outcome depends on the relative strengths of the two effects.

The results of this computation are shown in Figure 5. At first sight, we observe how the Ricardo-Marx effect is dominant in all industries, being most of the trajectories near-linear starting from the proportionality to direct prices and progressively moving towards the maximum profit rate. However, the Sraffa feedback effect is not completely negligible. In fact, some sectors (5, 6, 38 and 52) end up crossing the equality line of production-direct prices for high levels of the relative profit rate and therefore they represent the exception to the dominance of capital intensity effect. Notably, sectors 5 and 6 cross the line at $\pi/\Pi = 90\%$; sector 38 crosses it at $\pi/\Pi = 50\%$; sector 52 nearby $\pi/\Pi = 60\%$. These trajectories are highlighted in Figure 8(a).

Further considerations on the dominant effects of prices changes to income distribution can be provided by looking at the sectoral capital intensities, i.e. vertically integrated capital-labour ratios, at each level of the relative profit rate. These results are displayed in Figure 6. Notably, we can compare the movements of prices with the respective trajectories of the capital-labour ratios in relation to its "distance" from the average capital intensity, that is reflected by the inverse of the maximum profit rate, in fact $\Pi^{-1} = \mathbf{p}^{\mathrm{T}}\mathbf{y}_s/\mathbf{p}^{\mathrm{T}}\mathbf{H}\mathbf{y}_s$. In general, we observe that the sectors that, when prices and labour-values are proportional, are above the standard capital intensity, they tend to remain above and in some cases they continue to diverge upward, see for examples Figure 6(d). On the other hand, for initial levels of capital-labour ratios close to the average, the sectors are more likely to change their technical characterization from capital- to labour-intensive process and vice versa. Here we can observe few cases of the former movements, namely sectors (5, 25, 38, 42, 50, 52) while none of the sector experiences the latter. The trajectories of these exceptional industries are highlighted in Figure 8(b). The more the sectoral capital intensities are distant from the average, the less they are affected by the price changes induced by income distribution variations.

Another way to study the price movements is suggested by Sraffa's formalization. An increase in the wage rate immediately induces the inverse variation in the corresponding profit rate, so the prices adjust to these changes. However, the output prices affect the price of the means of production that will have to change in order to ensure the uniformity of profit rates. Therefore, as Shaikh (2012) advances, the sectoral relationships between output and capital are crucial to observe the complexity of price movements. We start once again by considering the prices of production evaluated in terms of the standard net product and compute the critical output-capital ratios:

$$\frac{\mathbf{p}^{\mathbf{T}}}{\mathbf{p}^{\mathbf{T}}\mathbf{H}} = \left(1 - \frac{\pi}{\Pi}\right)\frac{\mathbf{v}^{\mathbf{T}}}{\mathbf{p}^{\mathbf{T}}\mathbf{H}} + \pi \tag{24}$$

From this expression, we can visualize these ratios in the two opposite situations of income distribution. The output-capital ratios explain their particular values according to each sector, hence, when $\pi = 0$, we have:

$$\frac{\mathbf{p}^{\mathbf{T}}(0)}{\mathbf{p}^{\mathbf{T}}(0)\mathbf{H}} = \frac{\mathbf{v}^{\mathbf{T}}}{\mathbf{v}^{\mathbf{T}}\mathbf{H}} \tag{25}$$

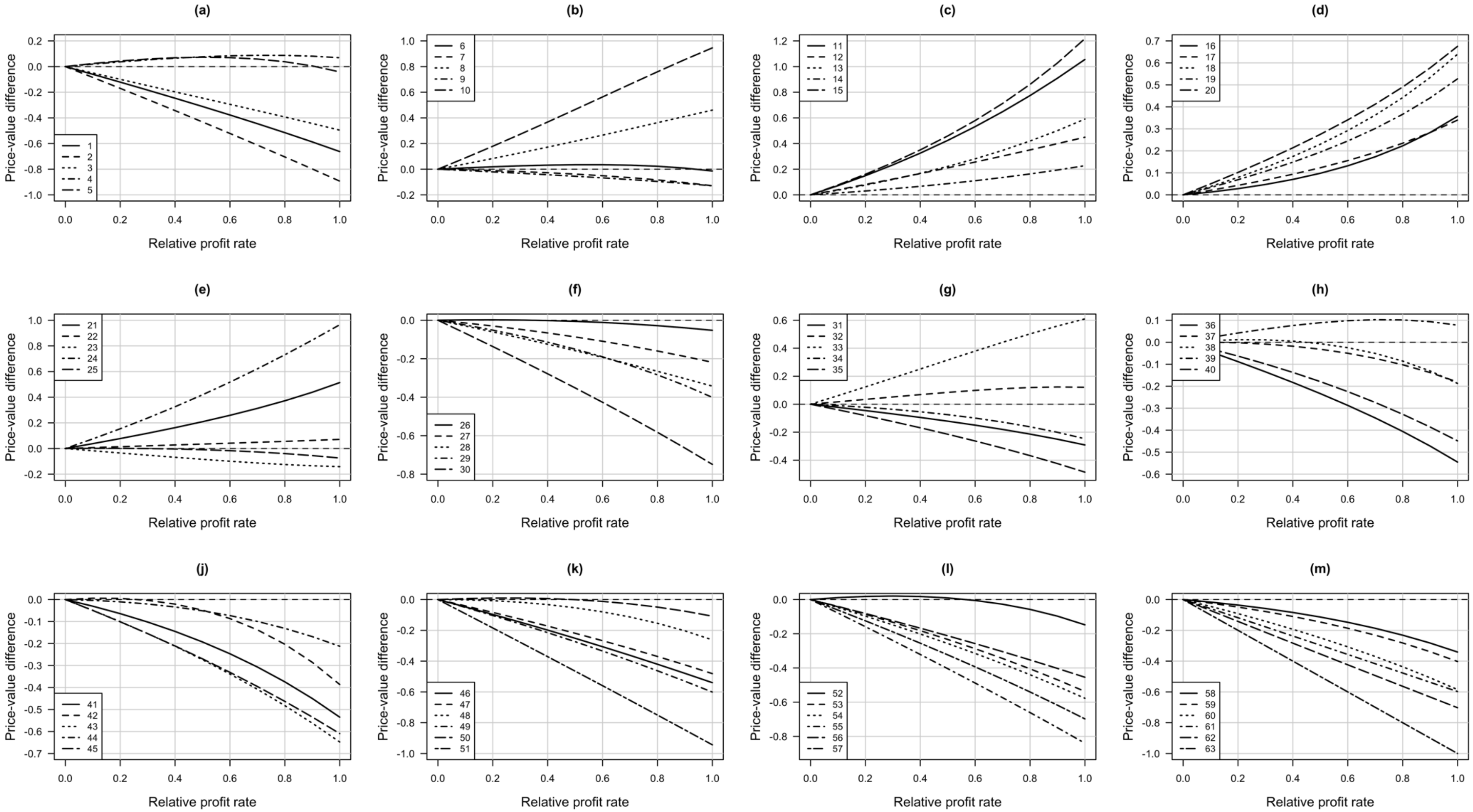
that is when production prices are proportional to labour-values, the output-capital relationships equal the ratios between the sectoral labour embodied in the surplus and the labour embodied in the respective means of production per output unit. In the opposite case, hence with $\pi = \Pi$, the vertically integrated output-capital ratio become the following:

$$\frac{\mathbf{p}^{\mathbf{T}}(\Pi)}{\mathbf{p}^{\mathbf{T}}(\Pi)\mathbf{H}} = \frac{\mathbf{v}^{\mathbf{T}}\mathbf{y}_s}{\mathbf{v}^{\mathbf{T}}\mathbf{H}\mathbf{y}_s} = \Pi \tag{26}$$

that, as we can observe, become uniform across sectors and equal to the relationship between the value embodied in the net product and the labour embodied in the means of production inside the standard system. Of course, this relationship holds in the standard system even if we consider the same magnitudes in terms of physical quantities. The question now is the movements through which these sectoral relationships behave starting from their peculiar values, determined by the technique exploited in the process, and progressively converging to the uniform standard ratio, i.e. when the maximum profit rate prevails. In Sraffa's argument, these movements are not predictable because of the non-uniformity of the capital-labour ratio, as discussed above, they can even deviate from the original directions presenting maximums and minimums or crossing the value of Π at intermediate structures of income distribution.

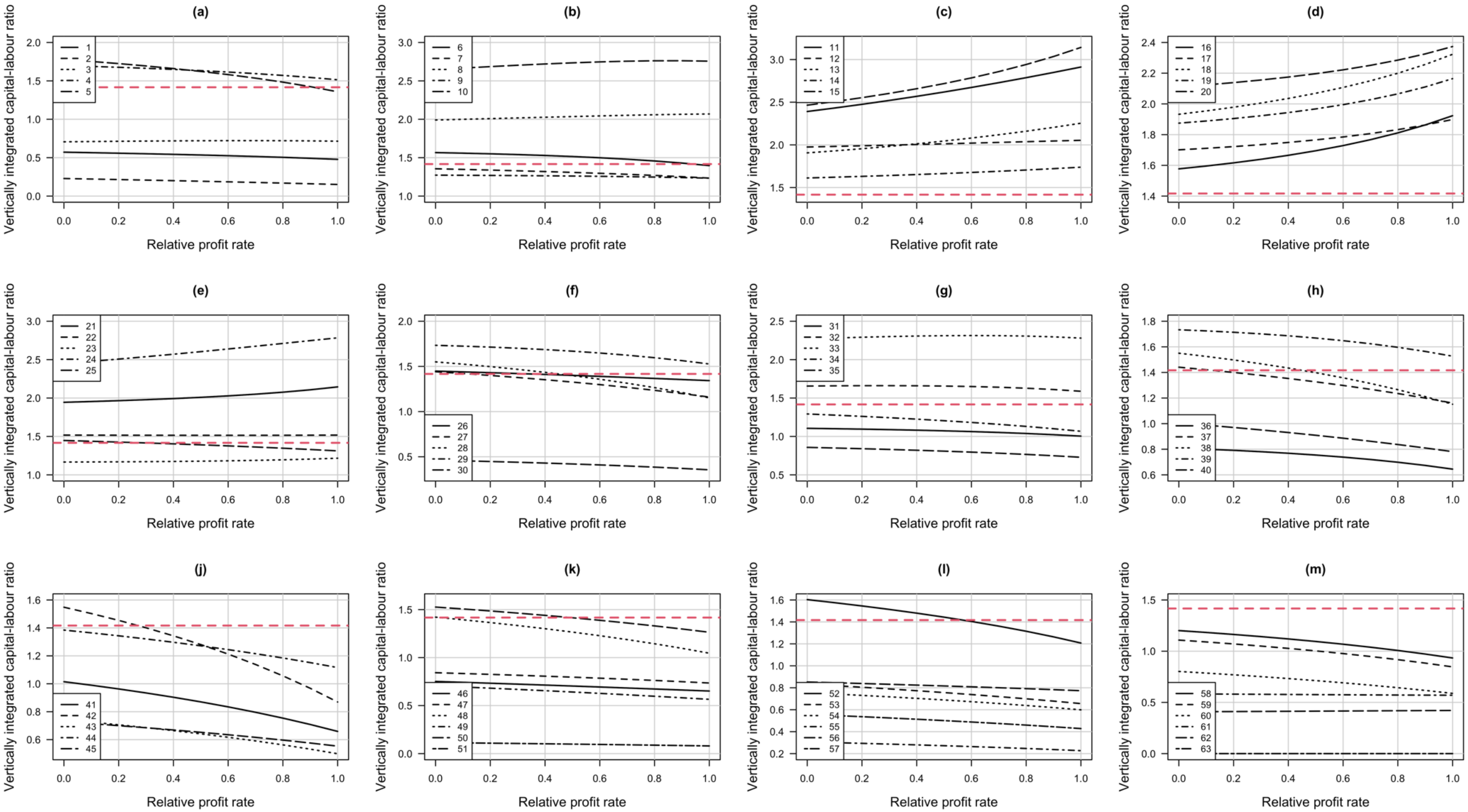
The results of the computations are displayed in Figure 7. As we can observe, almost all of the sectors show near-linear trajectories in the respective output-capital ratios towards the uniform standard ratio. The Sraffa price effects that actually leads to disturbances to the capital intensity effect can be visualized more clearly in Figure 8(c) in which we can find some of the sectors appears among the exceptional industries that change the technical categorization from capital- to labour-intensive for growing levels of the rate of profit. Notably, some sectors (25, 26, 37, 42, 50 and 52) cross the Π line for positive rates of profit and present maximums at levels above a relative profit rate of 60% and then shifts down to the standard ratio.

Figure 5. Production-direct price percent differences induced by variations in income distribution.



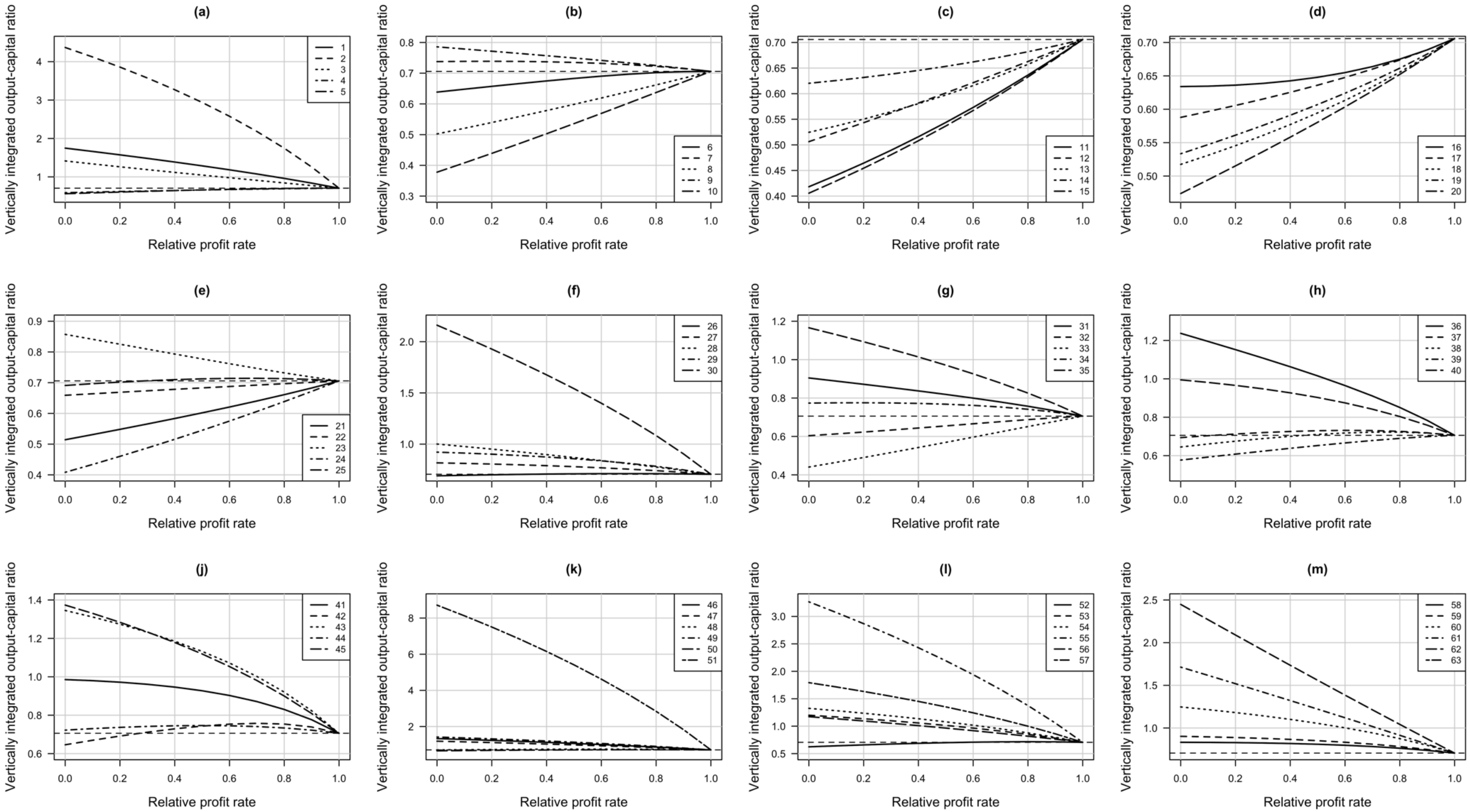
Notes: Numéraire: $\mathbf{p}^{\mathbf{T}}\mathbf{y}_s = 1$. Data: ISTAT Input-Output Tables (2021), own calculations.

Figure 6. Vertically integrated capital-labour ratio effects induced by variations in income distribution.



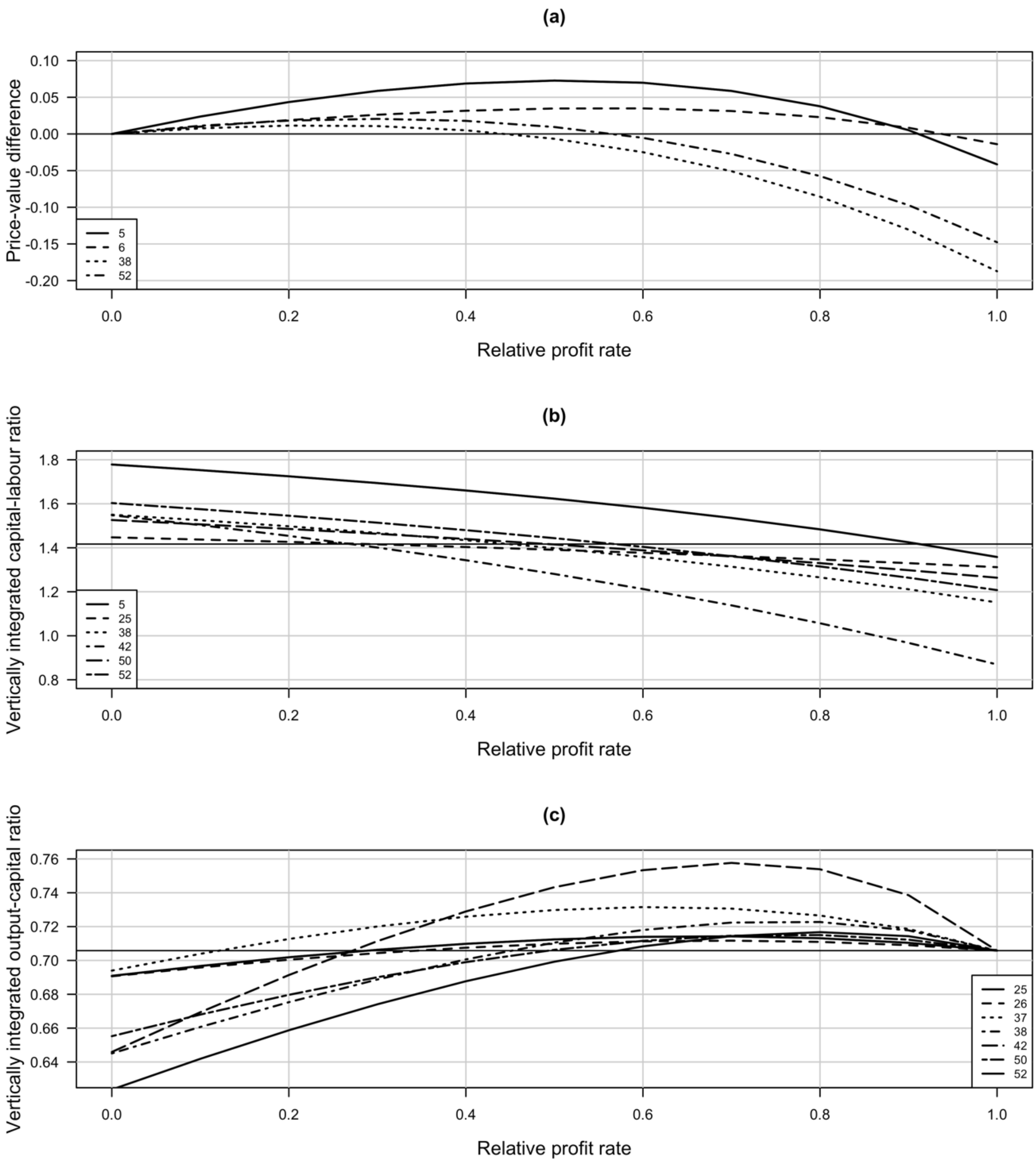
Notes: *Numéraire*: $\mathbf{p}^{\mathrm{T}}\mathbf{y}_s = 1$. Horizontal dashed line (red): standard capital intensity ($\Pi^{-1} = 1.416$). Data: ISTAT Input-Output Tables (2021), own calculations.

Figure 7. Vertically integrated output-capital relationship effects induced by income distribution variations.



Notes: *Numéraire*: $\mathbf{p}^{\mathrm{T}}\mathbf{y}_s = 1$. Straight dashed line = standard ratio ($\Pi = 70.58\%$). Data: ISTAT Input-Output Tables (2021), own calculations.

Figure 8. Production-direct price percent differences (a), vertically integrated capital-labour ratio (b) and vertically integrated output-capital ratio (c) effects induced by income distribution variations of the "exceptional sectors".



Notes: *Numéraire*: $\mathbf{p}^{\mathrm{T}}\mathbf{y}_s = 1$. Panel (b): straight line = standard capital intensity (Π^{-1} = 1.416). Panel (c): straight line = standard ratio (Π = 70.58%). Data: ISTAT Input-Output Tables (2021), own calculations.

5. Concluding remarks

We have provided an updated study of the value structure inside the Italian economy for the year 2018. Starting from the input-output tables and according to the procedures adopted by the previous literature, we could estimate the percent differences between market and direct prices, production and direct prices, market and production prices in a three-levels analysis. We found an agreement between our results and the previous ones concerning the mean absolute deviations between the three types of prices. Thus, according to the literature, production prices and direct prices can provide estimation for the observed market prices of the Italian economy (Ochoa 1989, Shaikh 1984, Tsoulfidis and Tsaliki 2019). Furthermore, we have shown that the price trajectories induced by the variation in income distribution is quite predictable in most cases, being the capital intensity effect prevailing with respect to the price effect inside each industrial sector.

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CHAPTER THREE

Self-replacing prices with alternative market structures and the methodological reappraisal of the Sraffian standard system

Abstract

It is well known that the standard commodity integrated into Sraffa's price equations as the measure unit that allows the relationship between the wage and the profit rates to be a relation between physical quantities and not between values (i.e. prices) completely loses its utility in a situation of non-uniform profit rates. The reason is that, when it is assumed that differential rates of profit persist, we can no longer consider the distribution of income and the formation of the prices independently of the system of quantities. So, what can we do with the standard commodity? It will be discussed that the role of the Sraffian standard system can be reappraised for descriptive purposes, namely it can be introduced as an analytical device to make comparisons between the real situation, in which differential profit rates can co-exist inside, and the hypothetical situation of free capital mobility and perfect competition market regime, that instead is explained by the standard relationships.

Keywords: Sraffa prices, Standard system, Non-uniform rates of profit, Market structures, Computable approaches.

JEL Classification: B51, C63, E11.

1. Introduction

The following investigation focuses on the formation of self-replacing prices in the framework of non-uniform rates of profit. The existing literature agrees that the role of the standard commodity can be neglected because, when we relax the uniformity condition of the rates of profit, its properties lose their utility of making the distributive relationship between wage and profit rates transparent, i.e. independent of the prices. This contribution advances the hypothesis of the possibility that the standard system can reacquire a methodological success for the normative purpose of economic analyses inside a framework of self-replacing prices with non-uniform rates of profit. A computational procedure is elaborated to construct the standard system as the "equivalent system" that transparently displays the necessary distribution of income that allows self-replacement by remaining independent of the level and the structure of prices, of profit rates and of any institutional mechanisms or constraints that affect their variations. Notably, we highlight the fact that the general rate of profit, based on the hypothetical situation of perfect competition, acquires a deeper theoretical meaning than the ex post measure of the average of sectoral profit rates and can be obtained within the standard system to carry out comparative assessments on the observed market structure.

The second section reconstruct an essential review of the economic literature regarding the integration of non-uniform rates of profit inside the self-replacing price systems. The third section explains the basic framework that we consider in the analysis, concerning Sraffa's formulation of relative prices and the construction of the standard system starting from the actual economic system. The fourth section advances a new computation procedure to show the normative relevance that the standard system can maintain inside a framework of differential profit rates. The fifth section presents some numerical examples in which alternative market structures are considered, namely the case where perfect competition prevails, where the energy sector operates under the monopoly regime, where differential monopoly degrees persist and, finally, where the institutional settings of the society lead to the nationalization of the energy sector. Concluding remarks follow.

2. Relative prices and non-uniform rates of profit: a literature review

The uniformity condition of industrial profit rates inside price systems starts with the traditional theoretical framework of the Classical economists who elaborate the concept of natural price (normal price or price of production). According to the Classical political economy, market prices are those exchange relationships that fluctuate in the short period due to changes in demand and supply of productive factors and labour. In fact, Ricardo (1951), in his fourth chapter entitled “On Natural and Market prices”, defines the market prices as disequilibrium prices. The forces exercised by demand and supply do not contribute to the determination of natural prices but make the economic system gravitate towards them. The law of gravitation towards uniform rates of profit is the condition for the formation of the prices of production (natural prices), that show the long-period industrial positions that ensures the self-replacing state of the economic system. Such vision is justified by the fact that a perfect competition market regime is assumed to prevail in the long period. Capital mobility drives the producers to invest into the activities that maximise their respective rates of profit. This mobility generally stops towards the positions in which the sectoral profit rates are equalised. To explain these mechanisms, it can be useful to come back to Ricardo’s text:[27]

> Let us suppose that all commodities are at their natural price, and consequently that the profits of capital in all employments are exactly at the same rate, or differ only so much as, in the estimation of the parties, is equivalent to any real or fancied advantage which they possess or forego. Suppose now that a change of fashion should increase the demand for silks, and lessen that for woollens; their natural price, the quantity of labour necessary to their production, would continue unaltered, but the market price of silks would rise, and that of woollens would fall; and consequently the profits of the silk manufacturer would be above, whilst those of the woollen manufacturer would be below, the general and adjusted rate of profits. Not only the profits, but the wages of the workmen, would be affected in these employments. This increased demand for silks would however soon be supplied, by the transference of

[27] For a broader discussion on this interpretation of Ricardo’s theory, see Perri (1988).

> capital and labour from the woollen to the silk manufacture; when the market prices of silks and woollens would again approach their natural prices, and then the usual profits would be obtained by the respective manufacturers of those commodities. (Ricardo 1951, pp. 90-91).

Sraffa (1960) brings the uniformity condition of profit rates among the assumptions for building his system of relative prices. Sraffa does not explicitly elaborate a theory of income distribution, therefore, the fact that he brings the condition from the Classical theoretical framework (as it is believed by some interpretations of Sraffa's work, for example that of Garegnani 1976) is still a controversial matter.[28] In fact, the question of the possibility of integrating differential rates of profit into Sraffa's price equations was already under investigation after Sraffa's publication.

Nardozzi (1974), Grillo (1976) and Giannini (1976) analyse the condition of self-replacement of the economic system in the cases where the profit rates are positive but non-uniform.

Pasinetti (1981, 1988) studies the "natural features of a growing economic system". He states that, in a framework where the growth rate of the economy has to be maintained at the "natural level", i.e. equal to the rate of growth of the working population, then the rate of profit must ensure, for each sector, the financing of new investment and therefore must be equal to the natural growth rate. Consequently, the set of natural rates of profit will be non-uniform because it depends on the structure of the growth rates of sectoral final demand (which depend on the population growth rate and on the sectoral consumption per capita growth rates).[29]

Sylos Labini (1967) considers a further motivation for integrating differential rates of profit inside the price system. The procedure is justified by the hypothesis of the presence of "entry barriers" across sectors because the industries exercise a certain

[28] Cf. Sinha (2016), Sinha and Dupertuis (2009), Dupertuis and Sinha (2009), and Zambelli (2018). It can be worth mentioning the fact that these contributions gave rise to a debate among three different views regarding the Sraffa system and the meaning of uniform rates of profit, that was hosted by the journal *History of Economic Ideas* (see Pignalosa and Trabucchi 2022a, 2022b; Perri and Oro 2022a, 2022b and Sinha 2022). For the purpose of the present analysis it is not necessary to go any further with the discussion, while for a critical review of Sinha's interpretation, see Levrero (2019).

[29] Formally, the natural profit rates designed by Pasinetti are expressed with $\pi_j = g + r_j$, where the first term is the working population growth rate and the second the (non-uniform) rates of growth of consumption per capita.

degree of monopoly inside the market. Sylos Labini states that, in fact, the perfect competition (or free capital mobility) condition, considered by the Classical economists, and the monopoly market regime are two extreme special cases of a more general form of market in which differential monopoly degrees can co-exist.
Stirati (2010), from a perspective close to that of Sylos Labini, studies at empirical level the national accounts of the Italian economy to provide some stylized facts of the relationship between the recent changes in functional income distribution (i.e. the fall of the wage share) and the increase of the profit rates inside those sectors that have been involved into the privatization regime of the 1990s-2000s, notably the transport and communications and the financial intermediation sectors.
Bellino and Serrano (2017) study the conditions that ensure the market prices to converge, in the long period, towards normal or production prices, that are defined, after the Classical manner, as those exchange relationships that imply the uniformity condition of the sectoral rates of profit.
Sinha (2016) opposes to the interpretation of the Sraffa system that follows Garegnani's perspective, by asserting that the market price mechanisms do not ensure the long-period gravitation of the economic system towards production (or normal) prices. Consequently, Sinha advances the necessity of a restatement of the uniformity condition of the profit rates inside the price equations from the Classical political economy theoretical perspective.
Zambelli (2018) studies alternative market structure inside the Sraffa price system in order to visualize the algebraic space in which prices can position themselves according to a certain (functional and sectoral) income distribution or a certain market structure (i.e. non-uniform rates of profit). Zambelli mentions the properties of the standard system in the appendix of his work but neglects their utility inside the investigation on alternative market forms.
The study of the self-replacing prices with non-uniform profit rates can function as a bridge to the other Post-Keynesian and Kaleckian approaches. In fact, within such framework, one can well integrate the principle of effective demand and the short-period analyses inside the investigation of industrial position and income distribution. This argument is developed by Roncaglia (1995). However, we remain inside a stationary framework of investigation, namely abstracting from any variation in the volume or the proportions of the quantities produced.

What is missing in the literature is the methodological reappraisal of the Sraffian standard system inside a framework of non-uniform sectoral profit rates. It is well known that the standard commodity integrated into Sraffa's price equations as the measure unit that allows the relationship between the wage and the profit rates to be a relation between physical quantities and not between values (i.e. prices), completely loses its utility in a situation of non-uniform profit rates. The reason is that, when it is assumed that differential rates of profit persist, we can no longer consider the distribution of income and the formation of the prices independently of the system of quantities. So, what can we do with the standard commodity? It will be discussed that the role of the Sraffian standard system can be reappraised for descriptive purposes, namely it can be introduced as an analytical device to make comparisons between the real market situation, in which differential profit rates can co-exist inside, and the hypothetical situation of free capital mobility and perfect competition market regime, that instead is explained by the standard relationships.

3. The basic framework

3.1. Self-replacing prices and income distribution

Consider an economic system with a number of industries that produce goods necessary for both production and consumption using capital goods that are replaced in each period. The technique exploited by the economy can be represented with a square matrix of multi-sectoral commodity exchanges $\mathbf{A} = [a_{ji}]$, that we assume to be viable, and with a vertical vector of sectoral employment shares $\mathbf{L} = [l_i]$. The described economic system can be formulated in matrix notation as follows:[30]

$$\mathbf{Bp} = (\mathbf{I} + \mathbf{\Pi})\mathbf{Ap} + \mathbf{L}w \tag{1}$$

where $\mathbf{p}$ is the vector of production prices, w is the uniform wage rate, $\mathbf{B} = \mathrm{diag}(\mathbf{b})$ is the square matrix whose diagonal elements represent the physical quantities of gross final product of each sector, $\mathbf{\Pi} = \mathrm{diag}(\boldsymbol{\pi})$ is the square matrix whose diagonal

[30] All vectors are vertical, while the horizontal vectors are transposed and therefore marked with superscript $\mathbf{T}$. Vectors are indicated in bold lowercase letters, while bold uppercase letters indicate matrices (except for $\mathbf{L}$ that is a vertical vector of sectoral shares). The scalars are signed with italic letters.

elements represent the sectoral rates of profit (and all the other elements are zero), while $\mathbf{I}$ is the identity matrix.

The viability condition of the economic system ensures the existence of a physical surplus that is available for consumption after that the means of production have been subtracted from the final gross product. We can define the net product or surplus in physical terms as follows:

$$\mathbf{y} = [\mathbf{B} - \mathbf{A}]^{\mathbf{T}}\mathbf{e} \tag{2}$$

where $\mathbf{e}$ is the unit or summation vector (each element is equal to one).

Following Sraffa, we choose this composite commodity as *numéraire* (i.e. measure unit) of the relative price system:

$$\mathbf{y}^{\mathbf{T}}\mathbf{p} = 1 \tag{3}$$

The solutions for the prices and the wage rate are obtained, given the vector of sectoral profit rates ($\boldsymbol{\pi}$), as follows:

$$\begin{bmatrix} \mathbf{p} \\ w \end{bmatrix} = \begin{bmatrix} \mathbf{B} - (\mathbf{I} + \mathbf{\Pi})\mathbf{A} & -\mathbf{L} \\ \mathbf{y}^{\mathbf{T}} & 0 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \tag{4}$$

that, once we expand the inverted matrix, become respectively:

$$w = [\mathbf{y}^{\mathbf{T}}[\mathbf{B} - (\mathbf{I} + \mathbf{\Pi})\mathbf{A}]^{-1}\mathbf{L}]^{-1} \tag{5}$$

and

$$\mathbf{p} = [\mathbf{B} - (\mathbf{I} + \mathbf{\Pi})\mathbf{A}]^{-1}\mathbf{L}w \tag{6}$$

In this way, the prices (6) and the wage rate (5) result to represent the shares of surplus through which the commodities that remain after the replacement of the means of production are distributed between social classes (workers and capitalists) and among individual producers (or sectors) at the end of each period production. Therefore, the self-replacement condition (1) can be formulated as follows:

$$\mathbf{y}^{\mathbf{T}}\mathbf{p} = \begin{bmatrix} [\mathbf{B} - \mathbf{A}]\mathbf{p} - \mathbf{L}w \\ \mathbf{e}^{\mathbf{T}}\mathbf{L}w \end{bmatrix} = \begin{bmatrix} \mathbf{\Pi}\mathbf{A}\mathbf{p} \\ \mathbf{e}^{\mathbf{T}}\mathbf{L}w \end{bmatrix} \tag{7}$$

The expression above states that given the structure of the sectoral rates of profit, and opportunely chosen the *numéraire* of price equations to be the net product of the economy, we can obtain a set of prices and wage rate that allows the economic system to self-replace while distributing the surplus to productive sectors (proportionally to the means of production invested $\mathbf{Ap}$) and to the workers (proportionally to the labour employed $\mathbf{e}^{\mathbf{T}}\mathbf{L}$).[31]

[31] Cf. Pasinetti (1977, Ch. 5) and Bellino (2021, Ch. 8).

3.2. *The Sraffian standard system*

The relationships between prices and income distribution are very complex. In fact, the variation of all the prices induced by the changes in income distribution leads to the problem of the measure unit. As we have seen in system (4), the determination of prices requires the fixing of the price of one of the commodities produced as *numéraire* of the system. The fact that all the prices vary with income distribution has the consequence that the variation of the price of one commodity cannot be certainly connected to the effects induced by the wage or the profit rate because it is possible that the price variation is induced by the feedback effects on the price of the commodity that has been chosen as *numéraire*.

Sraffa shows that we can build and introduce a composite commodity (or sector), the standard commodity, whose price does not need to vary when the wage rate changes due to a variation of income distribution. The standard system is defined as an economic system where "the various commodities are represented among its aggregate means of production in the same proportions as they are among its products" (Sraffa 1960, p. 19). This system exploits the same technique of the actual economic system, employs the same quantity of labour, produces the same commodities but in the particular proportions that ensure the uniformity of the physical ratios between sectoral physical surplus the same commodities used as means of production. These conditions are sufficient, according to Sraffa, to define the actual and the standard system as two equivalent systems. We can represent the standard system associated with the actual economic system as follows:

$$\mathbf{B}_s - (1 + R)\mathbf{A}_s = \mathbf{0} \tag{8}$$

$$\mathbf{e}^{\mathbf{T}}\mathbf{L}_s = \mathbf{e}^{\mathbf{T}}\mathbf{L} \tag{9}$$

where R is maximum rate of profit, i.e. the uniform "standard ratio" between the vector of sectoral physical surplus and the respective quantities used as means of production. This is the peculiarity of the standard system.

We can take advantage of these properties by choosing the standard net product as the *numéraire* of the self-replacing prices:

$$\mathbf{y}_s^{\mathbf{T}}\mathbf{p} = 1 \tag{10}$$

Then, like Sraffa, we set the employment of the standard system equal to unity:[32]

[32] This is a redundant equation because we have defined labour in terms of sectoral employment shares, hence the summation of these shares is already equal to unity.

$$\mathbf{e}^{\mathbf{T}}\mathbf{L}_s = 1 \tag{11}$$

Then, multiplying the price equations (6) by $\mathbf{B}_s$ and doing the proper computations, we obtain the following accounting identity:

$$1 = \pi\mathbf{A}_s\mathbf{p} + w_s \tag{12}$$

where w_s is the wage rate in terms of standard commodity, that results to be the wage share of the standard net product, and $\pi < R$ is the uniform rate of profit that emerges from the "modified" standard system, i.e. when the (surplus) wage rate appears. The result is that the relationship between the rate of profit and the wage rate measured in terms of the standard net product (i.e. the wage share of the surplus inside the standard system) is completely independent of prices:

$$w_s = 1 - \frac{\pi}{R}, \qquad \text{and alternately,} \qquad \pi = R(1 - w_s) \tag{13}$$

The use of the standard commodity as measure unit for relative prices makes the relationship between the wage and the profit rate transparent and allows to study the distribution of income in terms of physical quantities. A particular wage share (wage rate) observed from the actual economic system and associated with a certain profit rate, that is then converted in terms of standard commodity, corresponds to a different profit rate emerging from the standard system. Only in the special case in which the wage rate is absent in the actual system ($w = 0$), the rate of profit that emerges equals the standard ratio, i.e. the maximum rate of profit prevails inside the actual system (therefore we have $\Pi = \frac{\mathbf{y}^{\mathbf{T}}\mathbf{p}(\Pi)}{\mathbf{Ap}(\Pi)} = R$).[33]

4. The meaning of the standard system inside price equations with heterogeneous rates of profit

4.1. The problem with the standard commodity as measuring device

The uniformity of the rates of profit is a necessary hypothesis for the significance of the standard commodity as the measure unit of price equations. In fact, when the economic system presents non-uniform profit rates, such as the equations (4), it is no longer possible to make use of the standard commodity. The reason is that, when the rates of profit are non-uniform, the determination of prices can no longer be distinguished from the system of quantities produced. As shown, the utility of the

[33] For a deeper discussion on the properties of the standard commodity, see Bellino (2004).

standard commodity as measure unit for prices lies in the fact that it is possible to evaluate the net product, and consequently any income distribution, through a relation between physical quantities. The standard relationship expressed by equations (13), therefore, allows to compute the uniform rate of profit given the wage rate in terms of standard commodity and vice versa independently of prices. Consequently, in the case of non-uniform profit rates, the only way to obtain a linear relationship between the wage and the profit rates is to compute the average rate of profit of the actual system, that is possible by using the actual quantities produced. In terms of the present analysis, by evaluating the self-replacing prices with differential profit rates in terms of the standard net product, the accounting identity (12) becomes:

$$1 = \mathbf{\Pi A}_s\mathbf{p} + w_s \tag{12b}$$

The only way to use this expression in order to determine the prices (6) in terms of standard net product is that of defining the average rate of profit (π_a) inside the standard system as follows:

$$\mathbf{\Pi A}_s\mathbf{p} = \pi_a \mathbf{A}_s\mathbf{p} \tag{12c}$$

so that we can obtain the following standard relationship:

$$\pi_a = \pi = R(1 - w_s) \tag{13b}$$

However, as we have seen, to determine π_a we need to use to the actual quantities of commodities, in fact the average rate of profit emerging from the actual system is, by definition, $\pi_a = \frac{\mathbf{\Pi Ap}}{\mathbf{Ap}}$, and the latter is equal to π, emerging from the standard system, only when the actual system produces the commodities in the standard proportions. Therefore, the equality between the actual average rate and the rate of profit emerging from the standard system is satisfied only when the actual system produces the quantities in the same proportions of the standard system. It is not possible to keep the determination of prices distinct from that of quantities and the standard commodity can no longer be used as measure unit.[34] However, we can reappraise the standard system inside a framework in which the rates of profit

[34] This is the reason why the uniformity of profit rates inside the Sraffa system is often interpreted as a necessary hypothesis of the whole framework. The demonstration can be found, in a more detailed treatment, in Giannini (1976), Nardozzi (1977) and more recently Sinha (2016) (see also Sinha and Dupertuis 2009 and Dupertuis and Sinha 2009). The conditions of positivity of the prices in the cases of differential profit rates were already studied by Grillo (1976).

uniformity condition is not satisfied in the actual economic system. Notably, the standard system can be used as a logical-analytical device to advance in the investigation of the ruling structure of market power across sectors.

4.2. A methodological reappraisal of the standard system

Suppose that the institutional settings of the economic system allow a limitation to capital mobility and market competition such that each industry, according to the respective monopoly degree, sets the rate of profit and appropriate a certain share of the surplus that can be lower or higher than the one that would be allowed by free capital mobility, i.e. when the uniformity condition of the profit rates applies. The technical requirements that allow the system to be in a self-replacing state concern, in this framework, the fact that the rates of profit must at least one of them higher than zero, because of the viability condition. In this case, it is clear that the average rate of profit that emerges from the observed market structure is different from the rate of profit that emerges in the standard system when we consider the wage rate to be paid in terms of standard commodity. To distinguish the two concepts, we call "average rate of profit" essentially the aggregate profit rate of the actual system, that is simply a statistical *ex post* evaluation of the average value of the sectoral profit rates (and hence, it remains, as a measure, dependent on actual profit rates and prices). Instead, we call "general rate of profit" the rate that emerges in the actual system when the condition of the uniformity of profit rates across sectors prevails, that is precisely equal to the rate of profit emerging inside the standard system given the (actual) wage rate after that we opportunely convert it in terms of standard commodity. The reason is that the standard ratio, and hence any profit rate inside the standard system, is independent of the prices and of the profit rates that are observed in the actual system, and consequently, it remains independent of any other of the (outside) mechanisms that determine their value. In other terms, once we obtain the real purchasing power of the wage rate associated with a certain market structure, we can convert it in terms of standard commodity. The profit rate that emerges in the standard system from such functional distribution of the surplus will essentially reflect the "necessary" sectoral distribution, i.e. the general profit rate, that allows the self-replacement of the economy independently of the mechanisms that determine the actual profit rates and prices.

We can formalize these concepts as follows. Given a certain market structure, that can be synthesized into a particular vector of sectoral rates of profit ($\boldsymbol{\pi}$) coming from the outside mechanisms, we can immediately obtain, as shown in previous section (equations 3 and 4), the set of prices and wage rate that allows the self-replacement of the economy. However, we also can convert the wage rate associated with such market setting in terms of standard commodity as follows:

$$w_s = [\mathbf{y}_s^{\mathbf{T}}[\mathbf{B}_s - (\mathbf{I} + \mathbf{\Pi})\mathbf{A}_s]^{-1}\mathbf{L}_s]^{-1} \tag{14}$$

Then, by substituting the "standard wage rate" inside the standard relation (13) we obtain the general rate of profit, i.e. the profit rate that emerges in the (modified) standard system once the surplus wage rate appears:

$$\pi = R(1 - w_s) \tag{15}$$

This rate of profit shows the hypothetical and uniform level of the sectoral profit rates in the case where capital mobility (inside the actual system) is not limited by differential market power or entry barriers across industries. The general profit rate is evidently different from the average rate of profit emerging from the actual system (i.e. when we evaluate the prices in terms of the actual net product).

Once we know the wage in terms of standard commodity and the general profit rate associated with it, we are able to compute the prices in terms of standard commodity:

$$\mathbf{p} = [\mathbf{B}_s - (\mathbf{I} + \mathbf{\Pi}_s)\mathbf{A}_s]^{-1}\mathbf{L}_s w_s \tag{16}$$

that allow us to obtain a complete, and transparent, comparison between the actual economic system and the respective standard system in which the uniformity condition of the sectoral rates of profit prevails, given the wage rate or share (notice that in the above expression, $\mathbf{\Pi}_s = \mathrm{diag}(\boldsymbol{\pi}_s)$ as determined in equation 15).

More specifically, we can compute the deviation factors that allow to visualize the fractions of profit rate gained (when above the zero) or lost (when below the zero) by each industry relatively to the hypothetical situation of free inter-sectoral capital mobility (reflected by profit rate emerging from the standard system):

$$\boldsymbol{\delta} = \boldsymbol{\pi} - \boldsymbol{\pi}_s \tag{17}$$

This vector of sectoral deviation factors therefore provides an estimation of the differential degrees of monopoly expressed in direct relationship with the general rate of profit, and not with the average, whose structure takes the actual economic system away from perfect competition market regime any time the values of $\boldsymbol{\delta}$ are different from zero.

We can compare this procedure with that elaborated in Perri and Oro (2022a). It was discussed that, unlike Sinha (2016, appendix to chapter 7) does, the sectoral deviation factors must be related to the general profit rate and not to the average profit rate, being the former a truly objective estimation of the uniform rate of profit and the latter merely an *ex post* statistical evaluation of the profit rates that depends itself on prices and sectoral non-uniform profit rates.[35] Consequently, the standard ratio was used to formalize the differential degrees of monopoly exercised by each industry inside the market ($\boldsymbol{\delta}$), considered exogenous. In fact, the divergence between each sectoral profit rate and the general rate of profit provides a measure of the "distance" of the actual market structure from the free (or perfect) competition market regime. It was highlighted that we cannot consider all the deviation factors completely as given from outside because we have to respect some constraint. That is to say, the market power is a relative concept inside the surplus theory as long as each sector influences the prices and the rates of profit to the extent that it has more or less power than the other sectors. Therefore, at least one sector must adjust its profit rate to the market power of the others, in other terms, it suffers the power of other industries. Once the vector of deviation factors has been properly constructed, we can determine the vector of the rates of profit and consequently the set of prices and wage rate emerging from the actual (and from the standard) economic system that is associated with such form of market.

Here, we have advanced a procedure to do the reversed path. Namely, we want to determine the objective measure of the "distance" of the actual market structure from the perfect competition regime, given a certain set of observed non-uniform sectoral rates of profit. In this framework, the constraint that must be respected involves the absolute level of the given profit rates. The choice can be quite arbitrary within the limit of which the average profit rate must be lower the maximum rate of profit, meaning that some sectors can realize a profit rate above the maximum level as long as the other sectoral profit rates are enough low to bring the average profit rate below the maximum level. By reversing the logical-analytical path that emerged from the critique of Sinha's interpretation, we can reappraise the role of the

[35] In the discussion referring to Sinha's work, these concepts have been elaborated by using Sraffa's second set of price equations, namely the system that represents a self-replacement economy in which the surplus is entirely appropriated by the capitalists in the form of profits and the wage rate is simply defined with the price of a specified basket of subsistence goods.

standard system from a descriptive perspective, namely we can still exploit its properties by integrating it as an analytical device that captures the distance of the real situation in which different forms of market and monopoly degrees can co-exist from the hypothetical situation of perfect capital mobility, defined as the necessary sectoral distribution associated with a given purchasing power of the wage rate opportunely converted into the proper measure unit.

5. Examples based on Sraffa's numbers

The concepts that we have discussed can be visualized through simple numerical examples. We start by providing the same economic system of three sectors (iron, coal, wheat) of that considered by Sraffa (1960, Ch. 4).

The multi-sectoral matrix of commodity exchanges, the sectoral shares of labour employed and the vector of gross product of the actual economic system are the following:

$$\mathbf{A} = \begin{bmatrix} 90 & 120 & 60 \\ 50 & 125 & 150 \\ 40 & 40 & 200 \end{bmatrix}, \qquad \mathbf{L} = \begin{bmatrix} \frac{3}{16} \\ \frac{5}{16} \\ \frac{8}{16} \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 180 \\ 450 \\ 480 \end{bmatrix} \tag{18}$$

The columns of matrix $\mathbf{A}$ indicate the means of production used as inputs by each industry. The first column is iron, the second is coal, the third is wheat. At the end of the period of production, the physical surplus available for social distribution and consumption is given by following vector:

$$\mathbf{y}^{\mathbf{T}} = [0 \quad 165 \quad 70] \tag{19}$$

The equivalent system, i.e. the respective standard system, that exploits the same technique and employs the same quantity of labour but produces the commodities and employs the workers in different proportions, is given by the following magnitudes:

$$\mathbf{A}_s = \begin{bmatrix} 120 & 160 & 80 \\ 40 & 100 & 120 \\ 40 & 40 & 200 \end{bmatrix}, \qquad \mathbf{L}_s = \begin{bmatrix} \frac{4}{16} \\ \frac{4}{16} \\ \frac{8}{16} \end{bmatrix}, \qquad \mathbf{b}_s = \begin{bmatrix} 240 \\ 360 \\ 480 \end{bmatrix} \tag{20}$$

The standard net product that emerges is given by:

$$\mathbf{y}_s^{\mathbf{T}} = [40 \quad 60 \quad 80] \tag{21}$$

In what follows we provide a systematic comparison between the two systems, by taking both the actual and the standard net product as measure unit for the respective price system.

5.1. The case where perfect competition prevails

We can consider, initially, the traditional case in which the economy exposes uniform rates of profit across sectors, hence, when the perfect competition market regime prevails. Once the price of the net product has been chosen as *numéraire* for both the actual and the standard system and by performing the procedure of operations that has been discussed above, the economy shows the results that are reported in Table 1 that can be directly compared with the results of the respective standard system, shown in Table 2.

Table 1. Actual system (case 1).

Industry	Prices	Rates of profit (%)	Surplus shares (%)	Gross product	Means of production
Iron	11.14	**6.22**	10.97	2.005	1.764
Coal	4.455	**6.22**	10.45	2.004	1.681
Wheat	3.784	**6.22**	8.58	1.816	1.38
Aggregate		6.22*	30	5.826	4.826

Notes: *Numéraire*: $\mathbf{y}^\mathbf{T}\mathbf{p} = 1$. Wage rate and wage share of the surplus: $w = 0.7$. Prices are multiplied by 10^3. (*) Average rate of profit. The economic aggregates are expressed at their value.

Table 2. Standard system (case 1).

Industry	Prices	Rates of profit (%)	Surplus shares (%)	Gross product	Means of production
Iron	10.968	6.22	14.4	2.632	2.316
Coal	4.386	6.22	8.23	1.579	1.324
Wheat	3.725	6.22	8.45	1.788	1.359
Aggregate			31.1	6	5

Notes: *Numéraire*: $\mathbf{y}_s^\mathbf{T}\mathbf{p} = 1$. Wage rate and wage share of the surplus: $w_s = 0.689$. Prices are multiplied by 10^3. The economic aggregates are expressed at their value.

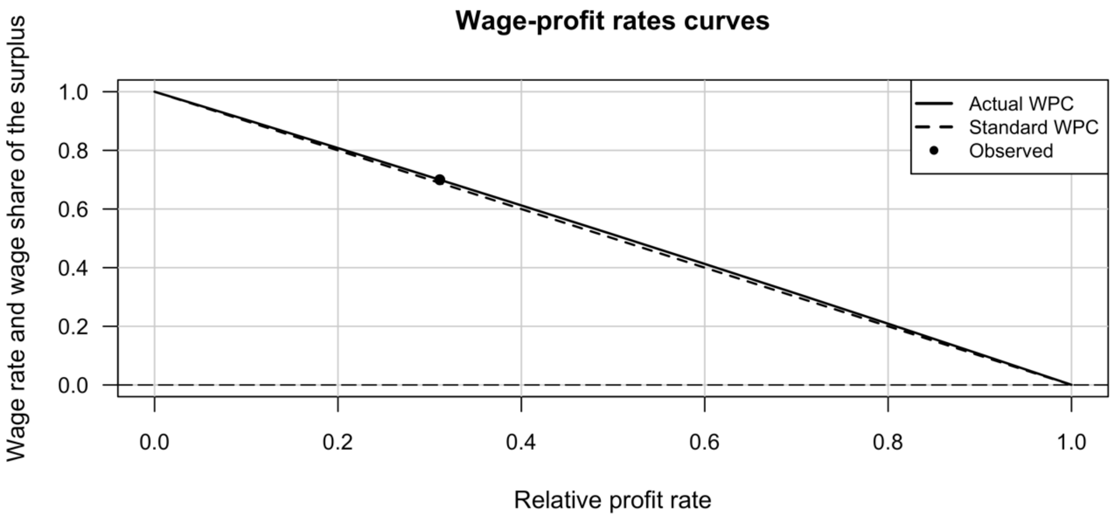
From this example, we can observe that a given profit rate is associated with different wage rates (measured respectively in terms of the actual and the standard net product) depending on which system we are looking at, because of the non-uniform sectoral relationships between the surplus and the means of production observed inside the actual system. The deviation factors are in this case, obviously, all equal to zero:

$$\boldsymbol{\delta}_1^{\mathbf{T}} = [0 \quad 0 \quad 0] \tag{22}$$

therefore, we can visualize all the possible income distributions between social classes and among industries for any given uniform profit rate lower than the maximum profit rate ($\pi < \Pi$).

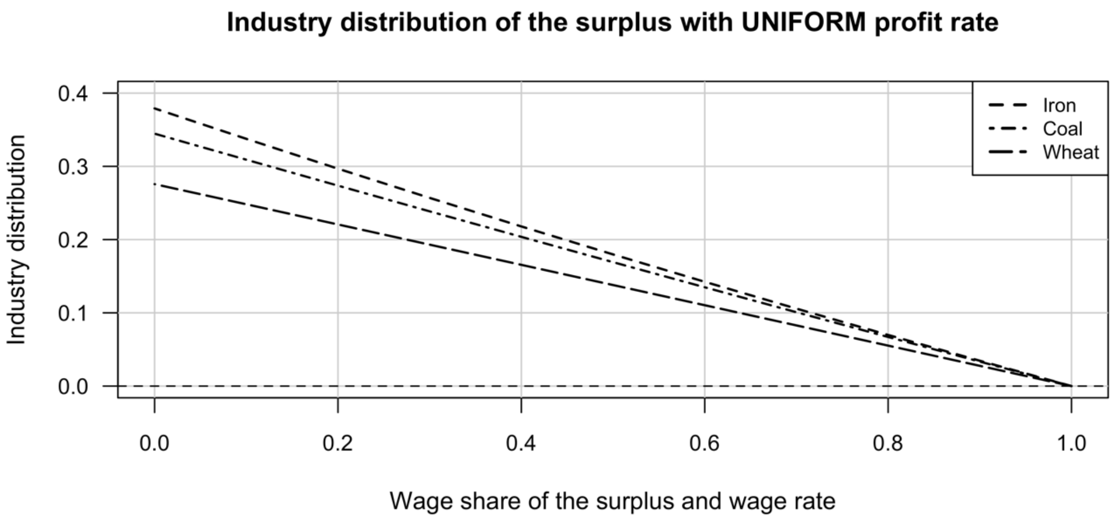
In Figure 1 we observe that the wage-profit curves, emerging respectively from the actual and the standard system, appear to be very close to each other even in the intermediate levels of the profit rate, while the standard curve is always perfectly linear. However, the distribution of the surplus between wages and profits changes when we transpose the given profit rate from the actual to the standard system and vice versa. Together with functional distribution, also the industry distribution of the aggregate profit changes in the two systems, notably, the proportions in which commodities compose the surplus in the actual system lead to a re-balancing of the distribution, mainly from the iron to the coal sector, in the transition from the standard to the actual system. This can be clearly visualized in the next figures.

Figure 1. Income distribution with uniform profit rate.



Notes: *Numéraire*: $\mathbf{y}^{\mathbf{T}}\mathbf{p} = 1$ and $\mathbf{y}_s^{\mathbf{T}}\mathbf{p} = 1$. Relative profit rate: $\pi/\Pi = 31.1\%$.

Figure 2. Income distribution in the actual system with uniform profit rate.



Notes: *Numéraire*: $\mathbf{y}^{\mathbf{T}}\mathbf{p} = 1$.

Figure 3. Income distribution in the standard system with uniform profit rate.

Notes: *Numéraire*: $\mathbf{y}_s^{\mathbf{T}}\mathbf{p} = 1$.

Figure 2 shows the sectoral distribution of the surplus at any given functional distribution in the case of profit rates uniformity. It is convenient to look at the same curves as they appear in the standard system, that are displayed in Figure 3. As we have seen in the table, at each level of the wage rate the iron-producing sector loses shares of surplus in favour to the coal-producing sector, while the wheat-producing sector does not deviate much from the standard distribution. In the case of uniform profit rates, the difference between the two system depends on the sectoral relationships between the physical surplus and the means of production. The non-uniformity of these relationships in the actual system makes the variations of relative prices induced by changes in income distribution very complex. However, the standard proportions make possible to obtain a linear relationship between the distributive variables that is independent of prices and hence, it remains independent also of the (institutional) mechanisms that allow the profit rates to be non-uniform.

5.2. The case where the energy sector operates (alone) under the monopoly regime

Consider now the case where the coal-producing sector manages to exercise a market power that it increases the share of surplus appropriated by the sector, while the other sectors continue to realize equal rates of profit. The results exposed by the market structure is displayed in Table 3, that can be compared to the results referring to the respective standard system, that is displayed in Table 4. The results of the actual system when the general profit rate prevails are shown in Table 5.

Table 3. Actual system (case 2).

Industry	Prices	Rates of profit (%)	Surplus shares (%)	Gross product	Means of production
Iron	11.101	**6.22**	10.95	1.998	1.76
Coal	4.469	**7.22**	12.1	2.011	1.676
Wheat	3.75	**6.22**	8.54	1.8	1.373
Aggregate		6.55*	31.59	5.809	4.809

Notes: *Numéraire*: $\mathbf{y}^{\mathbf{T}}\mathbf{p} = 1$. Wage rate and wage share of the surplus: $w = 0.684$. Prices are multiplied by 10^3. (*) Average rate of profit. The economic aggregates are expressed at their value.

Table 4. Standard system (case 2).

Industry	Prices	Rates of profit (%)	Surplus shares (%)	Gross product	Means of production
Iron	10.986	6.48	15.02	2.636	2.317
Coal	4.385	6.48	8.58	1.578	1.324
Wheat	3.717	6.48	8.8	1.784	1.358
Aggregate			32.42	6	5

Notes: *Numéraire*: $\mathbf{y}_s^{\mathrm{T}}\mathbf{p} = 1$. Wage rate and wage share of the surplus: $w_s = 0.675$. Prices are multiplied by 10^3. The economic aggregates are expressed at their value.

Table 5. Actual system with uniform profit rates (case 2).

Industry	Prices	Rates of profit (%)	Surplus shares (%)	Gross product	Means of production
Iron	11.166	6.48	11.45	2.009	1.766
Coal	4.457	6.48	10.9	2.005	1.682
Wheat	3.778	6.48	8.95	1.813	1.38
Aggregate			31.31	5.829	4.829

Notes: *Numéraire*: $\mathbf{y}^{\mathrm{T}}\mathbf{p} = 1$. Wage rate and wage share of the surplus: $w = 0.686$. Prices are multiplied by 10^3. The economic aggregates are expressed at their value.

The investigation of the market structure can be done as follows. The transition from the actual system to the standard system is required essentially to discover the profit rate that would prevail in the actual system if the uniformity condition of profit rates were fulfilled, i.e. the general profit rate. As can be observed, the general rate of profit is different, and in this case lower, than the average profit rate that is shown in Table 3, even though it plays no theoretical role but only a statistical one. The comparison must be done with the results emerging from the actual system when the general profit rate substitutes the differential structures of profit rates and generate the price solutions evaluated in the same terms of the actual system, displayed in Table 5. From the actual market structure to the perfect competition situation the coal-producing sector, the monopolistic industry, transfers shares of the surplus to the rest of the economy. This is not the whole story, because also the functional distribution of the surplus has changed. In fact, the aggregate profit share decreases while transitioning from monopolistic into perfect competition. The consequence is that the perfect competition regime leads to a higher income distributed to labour relatively to the actual market situation in which the coal-producing sector runs into a monopoly regime, as the wage rate associated with the general profit rate is higher than the that emerging from the actual market structure, both measured in the same terms that are shares the (actual) net product.

The deviation factors that emerge and that provide a measure of the market structure of the economy are given by:

$$\boldsymbol{\delta}_2^{\mathrm{T}} = [-0.264 \quad 0.735 \quad -0.264] \tag{23}$$

where we can observe the symmetrical losses of the iron-producing and the wheat-producing sectors relatively to the perfect competition regime and the relative gain of the coal-producing sector for exercising a degree of monopoly within the market.[36]

5.3. *The case where differential monopoly degrees and industrial entry barriers persist*

Consider now an almost random set of sectoral rates of profit, chosen of course by respecting the logical constraints exposed in previous section. Each industry practices "entry barriers" in the sense that they act in the market according to the respective monopoly degrees that are related to those of the other sectors. The results for the actual economic system are displayed in Table 6, while those for the respective standard system are displayed in Table 7. The situation to be compared with the actual situation is shown in Table 8.

Table 6. Actual system (case 3).

Industry	Prices	Rates of profit (%)	Surplus shares (%)	Gross product	Means of production
Iron	10.767	$\mathbf{4.22}$	7.3	1.938	1.73
Coal	4.483	$\mathbf{8.22}$	13.61	2.017	1.656
Wheat	3.717	$\mathbf{5.22}$	7.06	1.784	1.353
Aggregate		5.88*	27.98	5.74	4.74

Notes: *Numéraire*: $\mathbf{y}^{\mathrm{T}}\mathbf{p} = 1$. Wage rate and wage share of the surplus: $w = 0.72$. Prices are multiplied by 10^3. (*) Average rate of profit. The economic aggregates are expressed at their value.

[36] All the deviation factors displayed are multiplied by 100.

Table 7. Standard system (case 3).

Industry	Prices	Rates of profit (%)	Surplus shares (%)	Gross product	Means of production
Iron	10.923	5.55	12.84	2.621	2.312
Coal	4.387	5.55	7.36	1.579	1.325
Wheat	3.747	5.55	7.56	1.798	1.361
Aggregate			27.77	6	5

Notes: *Numéraire*: $\mathbf{y}_s^{\mathrm{T}}\mathbf{p} = 1$. Wage rate and wage share of the surplus: $w_s = 0.722$. Prices are multiplied by 10^3. The economic aggregates are expressed at their value.

Table 8. Actual system uniform rates of profit (case 3).

Industry	Prices	Rates of profit (%)	Surplus shares (%)	Gross product	Means of production
Iron	11.075	5.55	9.76	1.993	1.758
Coal	4.448	5.55	9.33	2.001	1.679
Wheat	3.799	5.55	7.67	1.823	1.38
Aggregate			26.76	5.819	4.819

Notes: *Numéraire*: $\mathbf{y}^{\mathrm{T}}\mathbf{p} = 1$. Wage rate and wage share of the surplus: $w = 0.732$. Prices are multiplied by 10^3. (*) Average rate of profit. The economic aggregates are expressed at their value.

The average rate of profit is, once again, higher than the general profit rate, and both of them proportionally determine the profit share of the surplus, meaning that perfect competition goes in favour of the workers, while the differential monopoly degrees that we have considered lead to a higher overall profit share.

The deviation factors now indicate the profit increase of the coal-producing sector compared to the loss of the iron-producing and the wheat-producing sectors:

$$\boldsymbol{\delta}_3^{\mathrm{T}} = [-1.334 \quad 2.665 \quad -0.334] \tag{25}$$

Here the competition among producers does not affect the functional income distribution significantly, while the general rate of profit remains lower than the average profit rate.

5.4. The case where the institutional settings of the society lead to the nationalization of the energy sector

Consider now the case where the ruling institutions nationalize and consequently manage the coal-producing sector. We can assume that such settings are translated into the fact that the exchanges of the energy sector do not require a positive profit rate, in other terms, the price of coal now simply accounts for the price of the means of production used by the energy sector and the wages paid to the workers employed in the latter. The results relative to the actual system are displayed in Table 9, while those relative to the standard system are displayed in Table 10. The situation to be compared with the actual situation is shown in Table 11.

Table 9. Actual system (case 4).

Industry	Prices	Rates of profit (%)	Surplus shares (%)	Gross product	Means of production
Iron	11.386	**6.22**	11.12	2.049	1.788
Coal	4.365	**0**	0	1.964	1.714
Wheat	3.996	**6.22**	8.89	1.918	1.429
Aggregate		4.14*	20	5.932	4.932

Notes: *Numéraire*: $\mathbf{y}^{\mathbf{T}}\mathbf{p} = 1$. Wage rate and wage share of the surplus: $w = 0.8$. Prices are multiplied by 10^3. (*) Average rate of profit. The economic aggregates are expressed at their value.

Table 10. Standard system (case 4).

Industry	Prices	Rates of profit (%)	Surplus shares (%)	Gross product	Means of production
Iron	10.858	4.57	10.55	2.606	2.307
Coal	4.389	4.57	6.06	1.58	1.326
Wheat	3.778	4.57	6.24	1.813	1.365
Aggregate			22.87	6	5

Notes: *Numéraire*: $\mathbf{y}_s^{\mathbf{T}}\mathbf{p} = 1$. Wage rate and wage share of the surplus: $w_s = 0.771$. Prices are multiplied by 10^3. The economic aggregates are expressed at their value.

Table 11. Actual system with uniform rate of profit (case 4).

Industry	Prices	Rates of profit (%)	Surplus shares (%)	Gross product	Means of production
Iron	10.981	4.57	8	1.976	1.75
Coal	4.439	4.57	7.67	1.997	1.677
Wheat	3.821	4.57	6.31	1.834	1.381
Aggregate			21.99	5.808	4.808

Notes: *Numéraire*: $\mathbf{y}^{\mathrm{T}}\mathbf{p} = 1$. Wage rate and wage share of the surplus: $w = 0.78$. Prices are multiplied by 10^3. (*) Average rate of profit. The economic aggregates are expressed at their value.

The results here are logical: the average rate of profit is in this case lower than the general rate of profit and associated with a lower overall profit share relatively to that emerging from the perfect competition market regime. The nationalization, i.e. the absence of the profit rate inside the coal-producing sector, brings a higher level of the wage rate relatively to the situation in which perfect competition prevails, meaning that now the actual economic system is not only less competitive at the market level, but it is also "less capitalist" concerning the institutional settings that determine the market structure and the functional distribution of income between wages and profits. Nonetheless, the other sectors clearly benefit, even though individually, from the nationalization of the energy sector.

The deviation factors show the advantage of the sectors that remain private-owned (and realize equal rates of profit):

$$\boldsymbol{\delta}_4^{\mathrm{T}} = [1.645 \quad -4.57 \quad 1.645] \tag{24}$$

The factors of the iron-producing and the wheat-producing industries are symmetrical because we choose to replicate the situation displayed in case 1 by simply turning to zero the profit rate of the coal sector.

6. Concluding remarks

This contribution aimed to provide an additional analytical device for the investigation of alternative market structures that involved a reappraisal of the Sraffian standard system. Given the properties of the standard commodity, we could estimate the general rate of profit, i.e. the profit rate associated with a perfect competition and free capital mobility situation, as a different concept from the average rate of profit as the statistical evaluation of the differential profit rates observed. Consequently, we could compare the real situations showing alternative market structures with the hypothetical situation in which the uniformity condition of the profit rate is fulfilled. The output of the investigation provided the deviation factors showing the gains and losses of each sector relatively to the situation emerged from the standard system, namely when the general profit rate prevails. Accordingly we could analyse different scenarios and the implications on both industry-level and functional income distribution. The procedure advanced had been applied for the descriptive analysis of alternative market structures but it can be applied to further research and frameworks. For example, it can be useful for normative purposes, notably in order to obtain a desired set of industrial profit rates structure that can channel resources to some sectors rather than to the others.

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