An investigation of interregional trade network structures

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Abstract We provide empirical evidence on the network structure of trade flows between European regions and discuss the theoretical underpinning of such a structure. First, we analyze EU regional trade data using Social Network Analysis. We describe the topology of this network and compute local and global centrality measures. Finally, we consider the distribution of higher order statistics, through the analysis of local clustering and main triadic structures in the triad census of interregional trade flows. In the theoretical part, we explore the relationship between trade costs and trade links. As shown by Behrens (2004, 2005a, 2005b) in a two-region linear new economic geography (NEG) model, trade costs and the local market size determine, even with finite trade costs, unconditional autarky and unilateral trade, that is, a one-directional flow from one region to the other. Following these contributions and guided by the empirical evidence, we clarify the relationship between market competition, trade costs and the patterns of trade in a three-region NEG model. We identify a larger set of trade network configurations other the three elementary ones that occur at the dyadic level between two regions (no trade, one-way trade, reciprocated two-way trade), and relate the model with the triad census.

1 Introduction

In this paper we provide some empirical evidence on the network structure of trade flows at the regional level in Europe and we discuss the possible theoretical underpinning of such a structure. In the empirical part of the paper, we look at the EU regional trade data recently produced by the PBL Netherlands Environmental Assessment Agency (Thissen, Diodato, and Van Oort, 2013; Thissen, Van Oort, and Diodato, 2013; Thissen, Lankhuizen, and Jonkeren, 2015), and we analyze it using Social Network Analysis tools (Wasserman and Faust, 1994). We take advantage of

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both the binary structure of the European regional trade network (analyzing the presence and absence of regional trade flows) and of its weighted counterpart (making use of the distribution of the value of trade flows, measured in millions of Euros). We use the latter to construct a meaningful threshold to restrict the density of the binary structure, and, following De Benedictis and Tajoli (2011) and De Benedictis, Nenci, Santoni, Tajoli, and Vicarelli (2014), we visualize the trade network at different levels of the threshold, define and describe the topology of the network and produce some of the main local and global centrality measures for the different European regions. Finally,

> ...since the most interesting and basic questions of social structure arise with regard to triads (Hanneman and Riddle, 2005),

we account for the distribution of higher order statistics of the network, through the analysis of local clustering and the main triadic structures in the *triad census* of interregional trade flows.

Given the explicit assumption that trade costs, together with regional markets size, are as for the gravity model of international trade (De Benedictis and Tajoli, 2011; Anderson, 2011; Head and Mayer, 2014) among the main determinants of inter-regional trade flows, the network analysis of regional trade flows in Europe informs the main topological properties of the data that must be reflected in the modeling of such trade flows.

From the theoretical point of view we explore how changes in crucial parameters - especially a reduction in trade costs – may favor the creation of trade links. The theoretical framework we adopt is a three-region linear new economic geography (NEG) model. We have chosen the linear version of a NEG model to overcome a crucial weakness of the standard approach as developed in the literature beginning from Krugman (1991). Indeed, in the standard NEG model all regions trade with each other as long as trade costs are finite. This follows from the isoelastic demand function – because of the specific assumption on consumer's CES preferences – and the *ad valorem*, proportional to price, *iceberg* trade costs. In the linear version of the NEG model, as developed by Ottaviano, Tabuchi, and Thisse (2002), this is not necessarily true. As shown by Behrens (2004, 2011, 2005a) for a two-region linear NEG model, trade costs and the dimension of the local market may determine *unconditional autarky* even in the presence of finite trade costs and *asymmetric patterns of trade*, that is a one-directional flow from one region to the other. Of crucial importance is the size and density of the industrial sector, that even in the presence of symmetric bilateral trade costs, may induce differences in local prices – with a lower price in the larger market. As stated by Behrens:

> price competition and trade costs endogenously create interregional asymmetries in market access and give rise to one-way trade in differentiated products (Behrens, 2005a, p. 473).

The same result is obtained by Okubo, Picard, and Thisse (2014, OPT). These authors remark that while the NEG and trade literature stress

> the importance of trade barriers for the intensity of competition and the spatial pattern of the global economy,

it pays much less attention to the reverse relationship:

> the impact of competition on the nature and intensity of trade as well as on the location of economic activities.

OPT capture the intensity of competition in domestic markets within a linear NEG model by assuming two regions with asymmetric population sizes.

Following the above mentioned contributions, our theoretical analysis aims to clarify the relationship between the intensity of competition, trade costs and the patterns of trade. Differently from the previous literature, and guided by the empirical evidence, we will consider a three-region model.[1] This allows us to identify a larger set of trade network configurations other the three elementary ones that occur at the dyadic level between two regions (no trade, one-way trade from one region to the other, reciprocated two-way trade), and relate the model with characteristics of the triad census. We also elaborate on how the structure of a trade network can be modified as trade costs vary. In order to focus on the properties of the short-run equilibrium and on the emergence of network structures in this time framework, we exclude factor migration.

The paper is structured as follows: Section 2 presents some stylized facts on the dominant interregional trade patters in Europe. Section 3 presents the basic economic framework of the theoretical model, i.e. a three-region NEG linear model with asymmetric trade costs and all possible configurations of trade flows between two regions. In Section 3 we derive the short-run equilibrium and determine the trade costs thresholds that determine all network configurations. Section 4 reports a brief discussion and concludes.

2 Interregional trade network in Europe: some stylized facts

In this section we perform an empirical analysis of the interregional trade network in Europe in 2010 using a new dataset on constructed trade flows between the European NUTS-2 regions. The aim is to provide some stylized facts about interregional trade patterns in Europe. In particular, through a triad census analysis, we want to identify the dominant triadic types, that is the frequency of each possible triadic structure, in the directed binary European regional trade network.

Interregional trade data are noticeably missing from European regional databases. The only database on interregional trade in goods and services at the NUTS-2 territorial aggregation level, fully consistent with international trade data between the

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[1] In the NEG literature, Ago, Isono, and Tabuchi (2006) and Behrens (2011) put also forward three-region linear models à la Ottaviano, Tabuchi, and Thisse (2002). However, they limit their analysis to specific trade cost structures; moreover they do not allow for unilateral trade flows. Exceptions are Behrens (2011) and Melitz and Ottaviano (2008). The first one studies two numerical examples that, for a specific trade costs structure, may determine one-way trade flows; the latter consider, in the context of a model with heterogeneous firms, the case of identical trade costs for the three regions.

Member States and with the rest of the world, is produced by the PBL Netherlands Environmental Assessment Agency (Thissen, Diodato, and Van Oort, 2013; Thissen, Van Oort, and Diodato, 2013; Thissen, Lankhuizen, and Jonkeren, 2015). These data are estimated by essentially breaking down international trade flows and national Supply and Use Tables to the regional level (see Thissen, Lankhuizen, and Jonkeren, 2015, for an overview of the methodology used in the construction of the data). Importantly, the methodology used is a parameter-free approach and therefore deviates from earlier methods based on the gravity model that suffer from analytical inconsistencies. Unlike a gravity model estimation, the methodology stays as close as possible to observed data without imposing any geographical trade patterns. The resulting data can therefore be used as such in our trade network analysis.

Nevertheless, as pointed out by Thissen, Diodato, and Van Oort (2013), one has to keep in mind that the constructed interregional trade data are inferred from other data sources and are not measured as a flow from one region to another. Given the compatibility constraints with macro variables, some bias in the trade flows between regions inside a country or between regions of different countries, might result from the weighting procedure used in the construction of the data. In particular the number of positive trade flows is extraordinary high with respect to other international trade data at the national level, such as the Comtrade UN database. The number of zeros in the regional trade matrix is minimal, meaning that the resulting trade network is almost a fully connected one. Therefore, we opted to use the information contained in the data to distinguish the main regional trade flows from the flows being lower than a chosen threshold w. Then, we exploit only the resulting binary structure of the truncated regional trade matrix, focusing on the relative dimension of trade links rather than on the individual absolute value of trade flows between pair of regions.

The version of the *bi-regional trade database* used in our empirical work comprises $267 \times 267 = 71,289$ observations of intra- or interregional trade flows among European regions for the year 2010 (Thissen, Lankhuizen, and Jonkeren, 2015). Export and import flows (both priced free on board) are measured in values (million of euros) and divided into 6 product categories (aggregates AB, CDE, F, GHI, JK and LMNOP of the NACE rev. 1.1). For our purposes, we only use the aggregate CDE, which includes "Mining and quarrying" (Section C), "Manufacturing" (Section D) and "Electricity, gas and water supply" (Section E). The countries covered by the data are the countries of the EU-27 (Croatia is not included).

We explore this new dataset through network analysis. As in De Benedictis, Nenci, Santoni, Tajoli, and Vicarelli (2014), we visualize the trade network, define and describe the topology of the binary network and produce some of the main network's statistics (i.e. local and global centrality measures). We also calculate higher order statistics, enriching the analysis with the reports on local clustering and the triad census of inter-regional trade links.

In general terms, the fundamental unit of analysis necessary to study regional trade flows is at the dyadic level rs: if between region r - the exporting region – and region s - the importing region – trade takes place, two levels of information are recorded. The first one is about the *existence* of a trade link, and it is a binary measure that takes the value one if a trade link exists and zero otherwise. The second is about the *intensity* of the trade relation between the two regions r and s, and is a continuous measure that is conditional on the associated binary measure: if the binary measure is zero, the only possible value that the intensity of the relation can take is zero; if the binary variable is one, the intensity of the relation takes real positive values. Since trade flows are directional, it is not in general correct to impose any kind of symmetry, and the value of trade flows between r and s will not be equivalent to the value of trade flows between s and r.

Even if the fundamental unit of analysis is at the dyadic level rs, the decision of agents from region r to trade with agents of region s is not taken in isolation, but it must consider the (best) possible option of trading with region z as an alternative. This is true for both r and s: dyadic trade flows do not occur in isolation. This motivates the use of network analysis in studying the relation between r and s, extending to the n-th level the logic behind the study of the so-called *third-region* (z) effect.

More formally, a trade network $N = (V, L, P, W)$ consists of a graph $G = (V, L)$, with $V = 1, 2, ..., n$ being a set of *nodes* (the regions, labeled with the respective NUTS-2 code) and L a set of *links* between pairs of vertices (e.g., trade partnership), plus P, the additional information on the vertices, and W the additional information on the links of the graph. The additional information included in the line value function W captures the intensity of trade between r and s (in million of euros). The information on the vertices (P) assembles different properties or characteristics of the regions (regions' labels, GDP, population, and so on). As mentioned, the trade graph is a *directed graph* in nature, since $l_{rs} \in \{0, 1\}$ indicates the existence or not of some exports from region r to region s, and $l_{rs} \neq l_{sr}$.

The graph associated to the EU regional trade network, $G = (V, L)$, has an average dimension of 267 vertices ($V = 1, ..., 267$) and 70,898 trade links ($L = 1, ..., 70, 898$) out of 71,022 possible links (i.e. there are only 124 zeros, 0.27 %). Indeed, as the data is constructed, the EU regional trade network is *strongly* connected, that is almost every vertex r is reachable from every s by a direct walk. However, for many dyadic observations the amount of exports is negligible. Thus, in order to visualize the network and to compute local and global centrality measures we use the information associated with the intensity of the links to define an appropriate threshold for the selection of links, and then exploit the binary information of the resulting network.

As an example, excluding all dyadic observations lower than 25 million of euros, the remaining flows almost cover the 90% of the total intra-Europe inter-regional trade in the aggregate sector CDE (i.e. imports+exports, which amount to almost 3,000 billions of Euros) (see Table 2). Adopting a threshold of $w > 25$ the number of edges (and so the density) is substantially reduced, from 70,898 to 20,086 (from 0.998 to 0.285), and one region (PT15) appears as an isolate. The density of the truncated network indicates that the inter-regional trade network is not regular and is far from being complete, or in other terms if the heterogeneity in the strength of links is used to select their presence, this heterogeneity is reflected in the connectivity of the network.[2]

The choice of the threshold of $w > 25$, simply based on the criterion to cover the 90% of the total intra-Europe inter-regional trade, may certainly appear arbitrary. A valid assessment of the robustness of our analysis to alternative choices of the threshold therefore requires a preliminary exploration of the distribution of interregional export values to ascertain whether any discontinuity takes place in the neighborhood of 25. To perform this check, we report in figure 1 the estimated kernel density of interregional export values. The visual inspection of the graph shows a reasonably smooth distribution and does not reveal any relevant jump at the cut off of 25. This evidence supports our choice. As a "litmus test", the results of the network analysis turned out to be robust to any alternative choice of the threshold just around 25 (for example 20 or 30 millions). With these results in hand, we can safely proceed by considering the interregional trade network resulting from the application of the threshold of $w > 25$ as our *benchmark*. Moreover, we assess the sensitivity of our analysis to alternative thresholds much further from 25 (namely for $w > 500$, $w > 1000$ and $w > 2500$, accounting for about 25%, 10% and 1% of the total intra-Europe trade) (see table 2).

	Full	w>25 mln.	w>500 mln.	w>1000 mln.	w>2500 mln.
% of intra-European trade	100.0	91.0	24.4	10.0	1.3
Number of nodes (regions)	267	266	217	100	13
Number of links	70,898	20,086	834	200	12
% of zeros	0.27	71.72	98.82	99.71	99.98
Density	0.998	0.283	0.018	0.003	0.001
Degree centralization	0.002	0.717	0.988	0.997	0.999
Degree SD	0.006	0.181	0.026	0.030	0.115
Eigenvector centralization	0.001	0.572	0.894	0.894	0.583
Eigenvector SD	0.006	0.297	0.162	0.204	0.322
Clustering	0.999	0.690	0.227	0.138	0.000

Table 1 Trade network properties for different thresholds. The density of a graph is the frequency of realized edges relative to potential edges. The clustering coefficient measures the proportion of vertex triples that form triangles (transitivity).

With a threshold of $w > 500$ or higher, the percentage of inter-regional European trade covered by data used to define the trade network gets substantially reduced, and with a threshold of $w > 1000$ the majority of European regions are excluded from the analysis and appear as isolates (see also Figure 2), so that the number of zeros become exorbitant. This can be easily visualized using a *sociogram* for the different levels of threshold.

In Figure 2, each European region is represented by a node in the *topological space*. The application of a so called force-directed algorithm on the regional trade data with valued links makes regions which are strongly connected close to each others, while regions which are not connected tend to be located far apart (Freeman, 1979). However, the position of each region does not depend only on its bilateral

[2] The sub-network including links with $w > 25$ is still weakly connected, but not strongly connected, that is not every vertex r is reachable from every s by a directed walk.

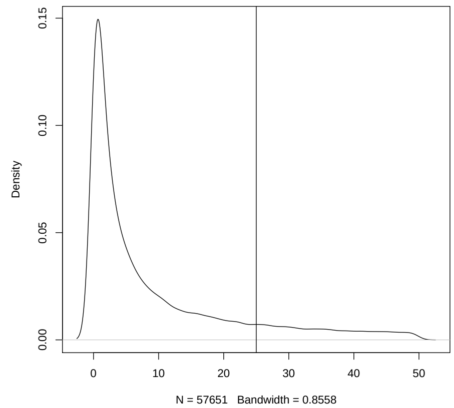

Fig. 1 Density of interregional trade flows

links but also on the indirect effect of others: the trade partners of its trade partners will contribute to determine the region's position in the network. The role of the *third-region* effect clearly emerges from the visualizations in Figure 2.

Figure 2 represents the directed network of European trade partners at the regional level in 2010. Nodes are European regions identified by NUTS-2 codes, ($P =$AT11, ..., UKN0), while links are weighted by the strength of trade flows (values in million of euros) ($W = \{\text{FR71,UKD3} = 251.69\}, \ldots, \{\text{ITC4,ITG1} = 5349.30\}$). Panels (a), (b), (c) and (d) visualize all trade links with weight $w \geq 25; 500; 1000; 2500$, respectively.

As shown in panel (d) the main European trade flows are between Italian regions (ITG1, ITC4, ITE4) and Spain (ES51), on one side, and German landers (DE11, DE12, DE21, DEA1) and France (FR10), on the other. Considering links of lesser strength (panel(c)) reinforces the impression that intra-national trade constitutes a substantial part of the structure of the European regional trade. New communities emerge (e.g. the Nordic countries, Poland, Greece) and the previous ones get reinforced by the inclusion of new links: the Italy-Spain community now includes some Portuguese, Austrian, Hungarian and Slovakian regions; and the Germany-France community is now enlarged to regions in Belgium, UK and the Netherlands. Colors indicate homogeneous cluster/community/modules of regions defined according to modularity (Newman, 2006).[3]

Regions like ITC4, DE12, DE21 are at the center of the network, while regions like FI20, PL43, EE00, at the extreme left of the visualizations in figure 2, or PT30, BG32, PT15, at the extreme right, are at the boundaries of the network structure.

[3] Networks with high modularity have dense connections between the nodes within modules but sparse connections between nodes in different modules. We use modularity to detect the community structure of the EU regional trade network. See Newman (2006) on this issue.

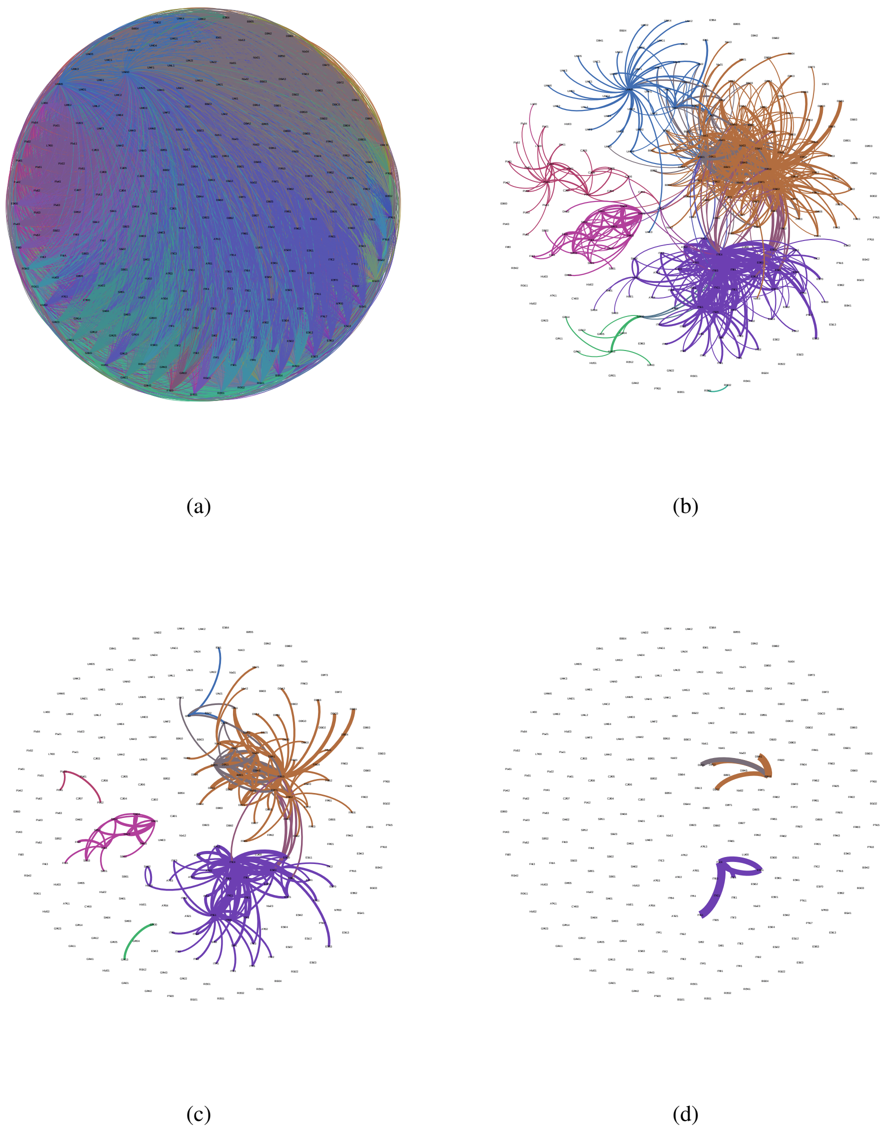

Fig. 2 Regional Trade in Europe: 2010. The figure represents the network of European trade partners at the regional level in 2010. Panel (a) visualizes all trade links with weight (exports) $w \geq 25$ (million of euros); panel (b) visualizes trade links with $w \geq 500$; panel (c) visualizes trade links with $w \geq 1000$; and panel (d) visualizes trade links with $w \geq 2500$. Regions (nodes) are identified by their NUTS-2 codes. Colors indicate homogeneous clusters defined according to modularity (with 12 clusters).

The EU regional trade network displays a core-periphery structure, with the more active regions (i.e. with higher w) at the core. Also other regions are at the center of the visualizations in figure 2: UKE3, CZ01, NL12 are visually central, but their level of w is not among the highest percentiles of the distribution of w's. Their position depends on their respective links with their major trade partners. With respect to peripheral regions they are in fact preferentially linked with central regions. As pointed out above, indeed, centrality depends on direct links but can also depend on the centrality of regional trade partners. To clarify these issues, we will report the evidence of different centrality measures.

The simplest measure of centrality of V_r is the number of its neighbors (the number of direct trade connections region r has), namely its *degree*. The standardized *degree centrality* of a vertex is its degree divided by the maximum possible degree (Wasserman and Faust, 1994; Newman, 2003; Jackson, 2008):

$$C_r^d = \frac{d_r}{n-1} = \frac{\sum_{s \neq r}^{n} l_{rs}}{n-1} \tag{1}$$

Since, in simple directed graphs like the one depicted in figure 2, a region can be both an exporter (a sender) and an importer (a receiver), we can compute both the *in-degree* of a region, $d_r = \sum_{s \neq r}^{n} d_{sr}$, as the number of incoming links (imports) to region r, and the *out-degree*, $d_r = \sum_{s \neq r}^{n} d_{rs}$, as the number of out-going links (exports) from region r towards its trade partners.

Imposing the condition $w > 25$ as a reasonable threshold that maintain the characteristics of the full network without assuming too much homogeneity between the different European regions (as shown in Table 2), standardized in-degree and out-degree distributions for the European interregional trade network in 2010 are shown in Figure 3. In both cases a strongly asymmetric and bimodal distribution emerges, suggesting that there are two distinct dominant groups of regions with low and medium standardized degrees, while a small fraction of vertices has a high in-degree (out-degree).

More specifically, the first ten central regions in terms of *in-degree* are Ìle de France (FR10), Lombardia (ITC4), Oberbayern (DE21), Stuttgart (DE11), Dusseldorf (DEA1), Arnsberg (DEA5), Koln (DEA2), Giessen (DE71), Cataluna (ES51) and Karlsruhe (DE12), with a level of $C_r^d > 0.63$ (the regions are inner linked with a little bit more than 63% of possible regional partners, with a strength of $w > 25$), with German landers at a core of European markets in terms of imports. If we look at regions as exporters, the first ten regions are, respectively, ITC4, DE21, DE11, DE71, DEA5, DE12, NL33, DEA1, DE25, DE13, with a level of $C_r^d > 0.66$ (the regions are outer linked with a little bit more than 66% of possible regional partners, with a strength of $w > 25$), with the role of Germany even more prominent. Seven out of ten regions are in the both top-tens, the more connected importing regions are also the most connected exporting regions. More broadly, in-degree and out-degree are positively correlated with a Pearson coefficient of 0.9. There are however some notable exceptions: UK12 is in the top-twenty as an importer, ranking 95th as an exporter.

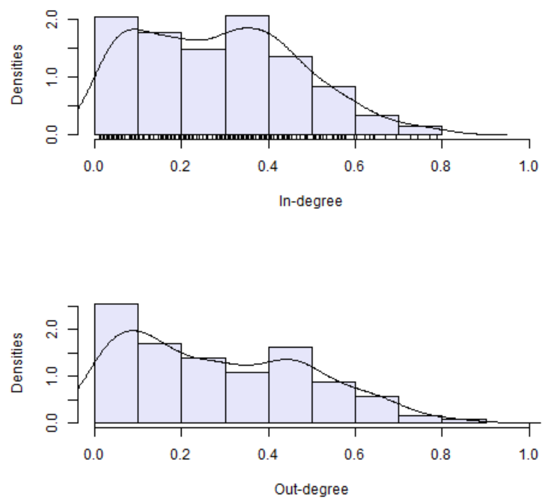

Fig. 3 In-Degree and out-degree distribution. Histogram and density plots

The scatter plot of in-degree versus out-degree (both derived using equation 1) is depicted in figure 4. The French region of Ìle de France (FR10), as previously mentioned, is the EU region with highest In-degree, a characteristic strictly associated with its level of regional GDP, highlighted by the dimension of the dot representing the position of the region in the deegre-space. The German regions of Stuttgart (DE11) and Oberbayern (DE21) and the Italian region of Lombardia (ITC4) lead the EU regions in terms of out-degree. The scatter plot clearly confirms the positive correlation between In-degree and Out-degree, but also shows the level of dispersion of the EU regional trade centralities. A notable case is the one of the Great Britain regions of Inner London (UKI1) and Outer London (UKI2) showing a level of in-degree much higher than the level of out-degree, depending on the lesser importance of regional manufacturing with respect to service production and trade.

Beyond the degree distribution itself, it is interesting to understand the manner in which regions of different degrees are linked with each other. To this end, we plot the average neighbor degree versus vertex degree (Figure 5). This plot suggests that, while there is a tendency for regions of higher degrees to link with similar regions, nodes of intermediate degree tend to link with regions of both intermediate and higher degrees. This issue can be better analyzed using a global measure of centrality, namely the eigenvector centrality.[4]

[4] The degree centrality (C_r^d) is classified as a *local measure of centrality* since it takes into consideration only the direct links of a node, its nearest neighborhood, regardless of the position of the node in the network's structure. Contrary to the local measures, *global measures of centrality* uncover the effect of others at a higher level of connection, including the direct and the *indirect*

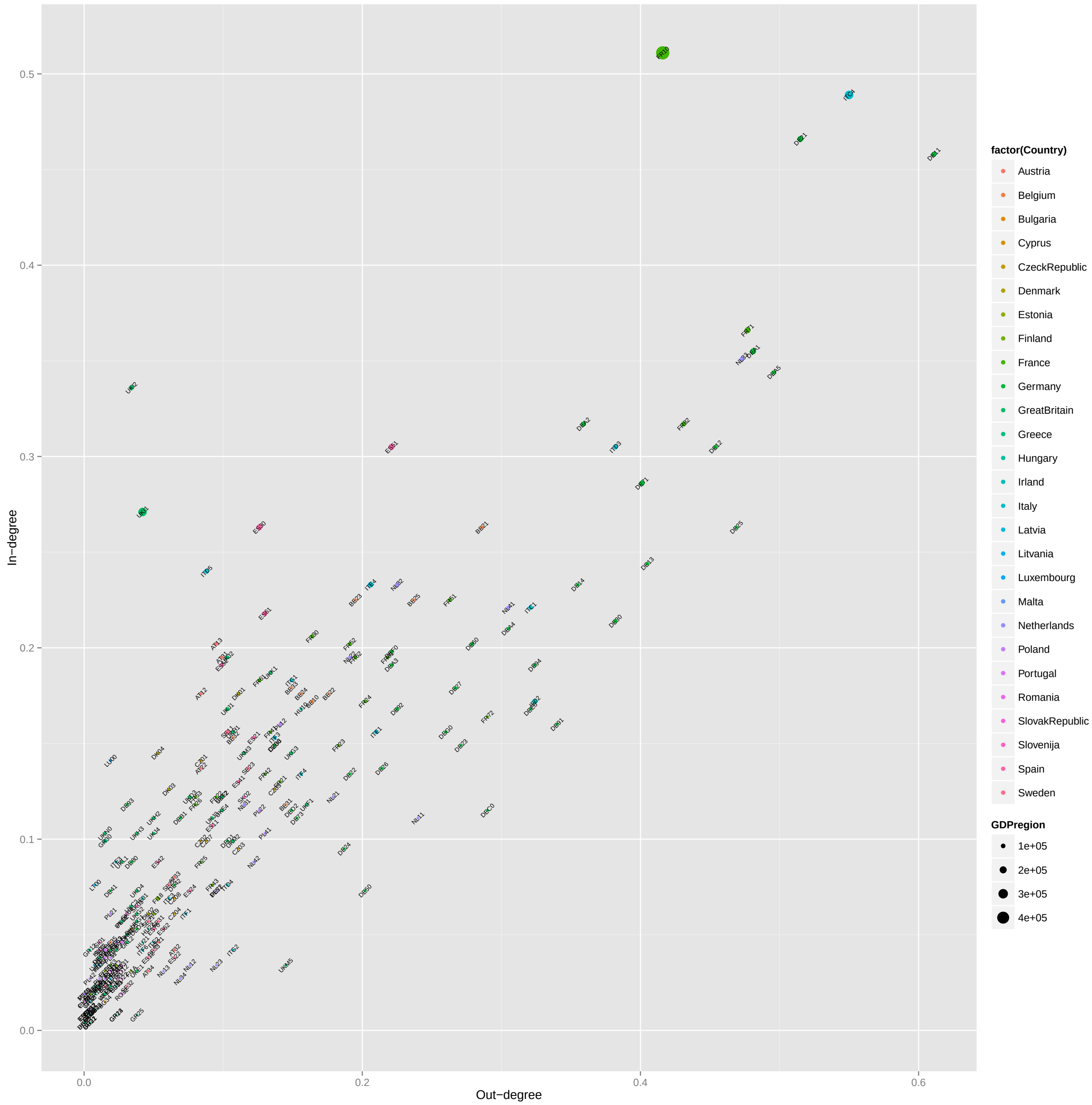

Fig. 4 Scatterplot of in-degree and out-degree centralities. The scatterplot confronts the level of in-degree with the level of out-degree for each EU region. Regions of the same country share the same color. The size of the dots is proportional to regional GDP. Dots are labeled according to NUT-2 codes.

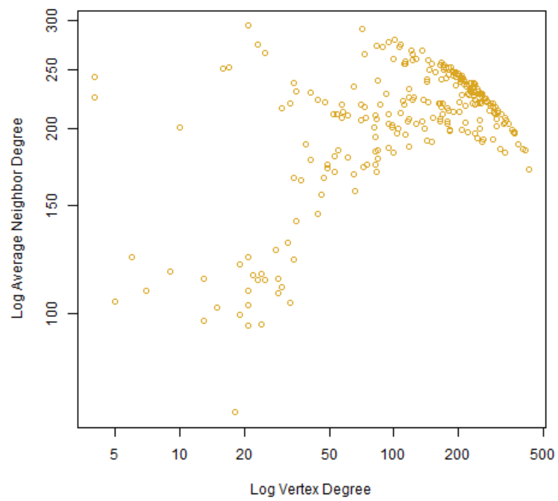

Fig. 5 Average neighbor degree versus vertex degree (log-log scale)

Figure 6 shows the Eigenvector centrality distribution for the European inter-regional trade network. Again a right skewed bimodal distribution emerges. The ten most central regions (in terms of Eigenvector centrality) are, in order, Ìle de France (FR10), Oberbayern (DE21), Lombardia (ITC4), Stuttgart (DE11), Dusseldorf (DEA1), Arnsberg (DEA5), Koln (DEA2), Karlsruhe (DE12), Darmstadt (DE71) and Rhone-Alpes (FR71).

Last but not least, we explore the new dataset through network analysis to evaluate to what extent two regions that both trade with a third region are likely to trade with each other as well. This notion corresponds to the social network concept of *transitivity* and can be captured numerically through an enumeration of the proportion of vertex triples that form triangles (i.e., all three vertex pairs are connected by edges), typically summarized in a so-called *clustering* coefficient. Table 2 shows that, with $w > 25$, about 70% of the connected triples close to form triangles. With higher thresholds of exports ($w > 500$, $w > 1000$ and $w > 2500$), this fraction steeply decreases.

More deeply, the role of the third region can be studied (and properties can be tested) using the *Triad Census* (the count of the various type of triads in the network) as a tool (Wasserman and Faust, 1994). Classical triad census analysis applies to single, directed and binary network, and we will follow the tradition in this respect.

effect of potentially all nodes in the network. In particular, the *eigenvector centrality* captures the idea that the more central the neighbors of a vertex are, the more central that vertex itself is. In other words, eigenvector centrality gives greater weight to a node the more it is connected to other highly connected nodes. Thus, it is often interpreted as measuring a node's network importance.

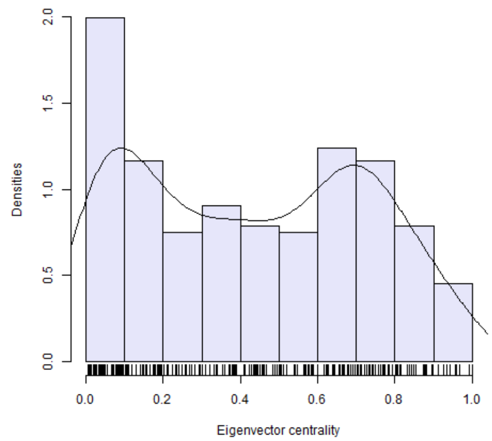

Fig. 6 Eigenvector centrality distribution

Taking the three nodes V_s, V_r and V_k, where $s \neq r \neq k$, we can call them a *triple*, if we also consider the presence or absence of links between the different nodes we have a *triad*. T_{srk} is the triad involving V_s, V_r and V_k. If the network is composed of n nodes, there are $\binom{n}{3} = \frac{n(n-1)(n-2)}{6}$ triads. In the EU regional trade network there are, therefore, 3,136,805 triads.

As far as possible realizations of triads, since there are three nodes in a triad, and each node can be connected to two other nodes, this give rise to 6 possible links. Since each link can be present or absent, there are $2^6 = 64$ possible realizations of the triads. Excluding isomorphic cases (e.g. if V_s, V_r and V_k are not linked, T_{srk}, T_{rks} and T_{ksr} are isomorphic), we remain with 16 isomorphism classes for 64 different triad states. These classes, represented in figure 7, range from the null subgraph to the subgraph in which all three dyads formed by the vertices in the triad have mutual directed links. The figure is from Wasserman and Faust (1994, p. 566) as reproduced in De Nooy, Mrvar, and Batagelj (2011). The different classes are labeled with as many as four characters, according to the M-A-N labeling scheme of Holland and Leinhardt (1970), where the first character gives the number of Mutual dyads in the triad, the second the Asymmetric ones, the third the number of Null dyads, and lastly, the forth one, if present, is used to distinguish further among the types (e.g. the two 030 triads - panel 9 and 10 in figure 7 - can be distinguished by the transitivity of the dyad 9 and the cyclic links of dyad 10). The four letters in the forth character are “U” (for up), “D” (for down), “T” (for transitive) and “C” (for cyclic).

For every network it is possible to calculate the frequencies of the 16 classes. In Table 2 we report the triad census for the European regional trade network at

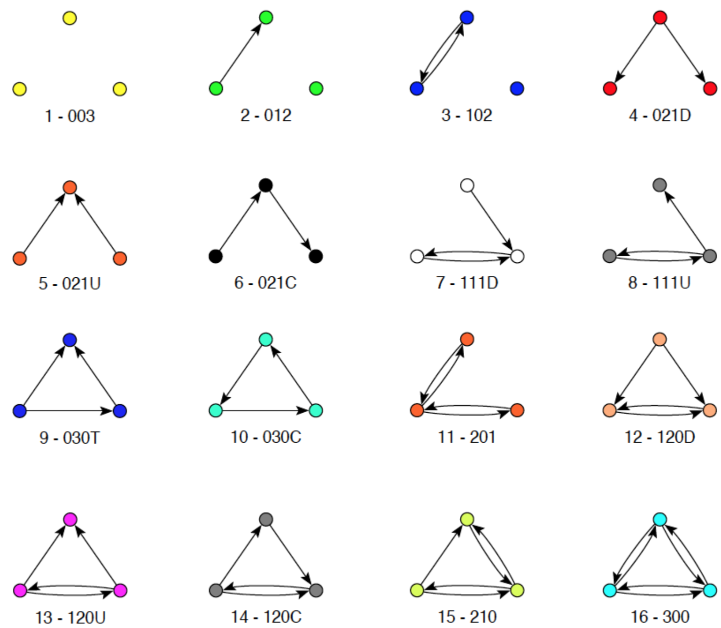

Fig. 7 Triads in a digraph. The figure is from Wasserman and Faust (1994, p. 566) as reproduced in De Nooy, Mrvar, and Batagelj (2011). The triples of directional relations are called *triads*. Among the numbers at the bottom of each panel, the first one is progressive from 1 to 16 and indicates all the possible cases of triads, while the second is the M-A-N labeling scheme of Holland and Leinhardt (1970): the first character gives the number of Mutual dyads in the triad, the second the Asymmetric ones, the third the number of Null dyads, and lastly, the forth one, if present, is used to distinguish further among the types (e.g. 003 triad as 0 Mutuals, 0 Asymmetrics and 3 Nulls).

different levels of threshold w. Every triadic census, reported in columns 4 to 8, is calculated excluding isolated regions and zero valued edges, e.g., in the last column of Table 2 the triad census is calculated for the 13 regions and 12 edges of the trade network with $w > 2500$. Here we discuss only our preferred structure (e.g., $w > 25$ mln.).

The fifth column in Table 2 shows that the large majority of triads in the EU regional trade is represented by empty-graph structures (corresponding to the 003 MAN code, row 1 in Table 2); followed by single and mutual edges (012 and 102 MAN codes, rows 2 and 3 inTable 2); and then by stars (in-stars 021U, out-stars 021D, and especially mutual stars, 201 MAN code, rows 5, 4 and 11 in Table 2). One noteworthy characteristic of the EU regional trade network is the prevalence of mutual edge + (double) Out structures (111U and 120U MAN codes, rows 8 and 13 in Table 2) over mutual edge + (double) In structures (111D and 120D MAN codes, rows 7 and 12 in Table 2). It seems that when two regions establish a mutual trade relationship this fosters them to export to, more that to import from, a third region. This tendency persists at different level of threshold w (as can be seen in columns

	MAN code	Figure	Class	Full	w>25 mln.	w>500 mln.	w>1000 mln.	w>2500 mln.
1	003		Empty graph	14	1,040,546	1,549,950	147,072	186
2	012		Single edge	0	487,976	88,052	10,440	73
3	102		Mutual edge	182	737,954	30,906	2,907	6
4	021D		Out-star	1	32,297	2,665	327	3
5	021U		In-star	0	20,621	806	175	10
6	021C		Line	0	29,254	1,407	179	3
7	111D		Mutual edge + In	0	87,483	1,314	164	1
8	111U		Mutual edge + Out	0	152,936	2,562	276	4
9	030T		Transitive	1	13,299	151	11	0
10	030C		Cycle	0	664	7	0	0
11	201		Mutual-star	7,808	157,167	965	95	0
12	120D		Mutual edge + double In	121	17,185	63	2	0
13	120U		Mutual edge + double Out	248	36,178	158	19	0
14	120C		Mutual edge + Cycle	21	16,694	131	4	0
15	210		Almost complete graph	15,645	125,402	338	23	0
16	300		Complete graph	3,112,764	145,904	105	6	0

Table 2 Triad census. Every triadic census (columns 4-8) is calculated excluding isolated regions and zero valued edges.

6-8 in Table 2). This aspect of the EU regional trade network will be discussed and theoretically motivated in the subsequent sections.

Overall, the evidence emerging from our analysis suggests interesting insights on the interregional trade network in Europe, which also inform our theoretical model described in the following sections. First, it emerges that the interregional trade network (and, thus, the European economic integration) is far from being complete since most regions do not trade (or trade with a very low intensity) with all other regions, but they rather select their partners. This first stylized fact clearly emerges once we neglect bilateral trade flows lower than 25 millions of euros. Second, interregional trade flows, partners and links in Europe are strongly heterogeneous, with a relatively small number of regions playing a central role in the network structure, both in terms of number of links and amount of intra-Europe trade flows accounted. In particular, our findings clearly show that distance matters also in trade between regions and that the emerging clusters are characterized by geographic proximity. Specifically, the national homogeneity of clusters gives evidence that national borders are relevant for regional analysis of EU trade flows. Finally, from the triadic census analysis it emerges that the tendency to reciprocate trade links (i.e., closing triangles) is limited and mutual edge patterns tend to prevail.

3 General Framework

In this Section, we build a three-region linear new economic geography (NEG) model along the lines of the evidence emerged from the empirical analysis of the EU regional trade network.

3.1 Basic Assumptions

The economy is composed of three regions (labeled suitably r, s and k). There are two sectors, Agriculture (A-sector) and Manufacturing (M-sector). Workers and entrepreneurs are the two types of agents operating in the economy, each of them is endowed with a factor of production, unskilled labor (L-factor) and human capital (E-factor).[5] L can be used in both sectors; whereas E is specific to M. Production in the A-sector involves a homogeneous good, whereas in the M-sector the output consists of N differentiated varieties. The three regions are symmetric – they have the same endowment of L and are characterized by the same production technology and consumption preferences – except for their distance. This translates into regional differences in trade costs.

3.2 Production

The A-sector is characterized by perfect competition and constant returns to scale. The production of 1 unit of the homogeneous good requires only unskilled workers as an input. Without loss of generality, we assume that 1 unit of labor gives 1 unit of output. The A-good is also chosen as numéraire. In the M-sector, instead, (Dixit-Stiglitz) monopolistic competition and increasing returns prevail. In this sector, identical firms produce differentiated varieties with the same technology involving a fixed component, 1 entrepreneur, and a variable component, η units of unskilled labor for each unit of the differentiated variety. Total cost TC for a firm i producing q_i output units corresponds to:

$$TC(q_i) = \pi_i + w\eta q_i$$

where w is the wage rate, π_i represents the remuneration of the entrepreneur and, when the zero profit condition holds, the operating profit. Given consumers' preference for variety (see below) and increasing returns, each firm will always produce a variety different from those produced by the other firms (no economies of scope

[5] A crucial difference between human and, for example, knowledge capital is that the former is embodied into the owner, whereas the second is separated. In a NEG model, this difference enters into play only when factor migration is allowed. When human capital is considered, changes in real incomes alters the migration choice also via the so-called "price index effect" so that changes in local prices may affect the long-run distribution of the industrial sector. Instead, when knowledge capital is concerned, factor movements are only driven by regional nominal profit differentials. A new economic geography (NEG) model in which the mobile factor is human capital (or, alternatively, skilled labor or entrepreneurship) is known as Footloose Entrepreneur (FE) model (developed originally in Forslid and Ottaviano, 2003); a NEG model in which the mobile factor is separated from the owner (such as physical or knowledge capital) is labeled Footloose Capital (FC) model (developed firstly in Martin and Rogers, 1995). As mentioned above, this distinction becomes relevant moving from the short to the long-run when factor migration is allowed. Even if the basic structure of the model is equivalent for FC and FE models, we consider the factor specific to the M-sector an entrepreneur.

are allowed). Moreover, since one entrepreneur is required for each manufacturing firm, the total number of firms/varieties, N, always equates the total number of entrepreneurs, $E = N$. Denoting by λ_r the share of entrepreneurs located in region r the number of regional varieties produced in that region is:

$$n_r = \lambda_r N = \lambda_r E.$$

3.3 Utility function

Following Ottaviano, Tabuchi, and Thisse (2002), to represent individual preferences we adopt a quasi-linear utility function. As we shall see, a crucial difference of this modeling strategy with respect to the standard CES approach is a CIF price (price at destination) which falls as the number of local competing firms rises. This implies a stronger dispersion force (given by the competition effect). Another difference is that it allows to highlight alternative patterns of trade (autarky, one-way, two-way trade) as shown by Behrens (2004, 2005b, 2011) and by Okubo, Picard, and Thisse (2014) for the case of two-region economies.

The utility function is composed of a quadratic part defining the preferences across the M goods and a linear component for the consumption of the A-good:

$$U = \alpha \sum_{i=1}^{N} c_i - \left(\frac{\beta - \delta}{2}\right) \sum_{i=1}^{N} c_i^2 - \frac{\delta}{2} \left(\sum_{i=1}^{N} c_i\right)^2 + C_A \tag{2}$$

where c_i is the consumption choice concerning the variety i; α represents the intensity of preferences for the manufactured varieties, with $\alpha > 0$; δ represents the degree of substitutability across those varieties, $\delta > 0$; and where the taste for variety is measured by the (positive) difference $\beta - \delta > 0$.

The representative consumer's budget constraint is:

$$\sum_{i=1}^{N} p_i c_i + C_A = y + \overline{C}_A \tag{3}$$

where $\overline{C}_A$ is the individual endowment of the agricultural good which is assumed sufficiently large to allow for positive consumption of this good in equilibrium; p_i is the price of variety i inclusive of transport costs and y is the income of the individual agent (unskilled worker or entrepreneur).

3.4 Trade costs

The three regions constitute the nodes of a network economy in which the links are the flows of commodities (esp. those produced in the M-sector). The existence, direction and magnitude of these flows depend on the size of trade costs. There are three types of flows/links between two regions – labeled let's say r and s – a one directional link from r to s, where r is the exporting region and s the importing region; a one directional link from s to r, where s is the exporting region and r the importing region; and a bidirectional link between r and s, where both regions export to and import from the other. Including also the possibility of no links, the maximum number of possible network structures involving three regions is 64; excluding the "isomorphic" cases this number reduces to 16 (see the discussion on triad census above).

Differently from Ago, Isono, and Tabuchi (2006), according to whom the three regions are equally spaced along a line, and from Behrens (2011), according to whom two regions in a trade bloc are at the same distance from a third outside region, we assume that the distance between regions is not necessarily the same. Moreover, we assume identical bilateral trade costs – so that the cost of trading industrial commodities from r to s and from s to r is identical, i.e $T_{rs} = T_{sr}$ – and no cost of trading goods within a region – that is, $T_{rr} = 0$. Moreover, we do not assume an *a priori* specific trade costs configuration,[6] that is, $T_{rs} \neq T_{rk}$ and/or $T_{rk} \neq T_{sk}$ and/or $T_{rs} \neq T_{sk}$.

Letting r, s, and k the three regions under consideration, the trade cost matrix can be written as:

$$\begin{pmatrix} 0 & T_{rs} & T_{rk} \\ T_{rs} & 0 & T_{sk} \\ T_{rk} & T_{sk} & 0 \end{pmatrix}$$

4 Short-run equilibrium

We limit our analysis to the equilibrium that emerges in the short run, contingent to given regional shares of entrepreneurs $(\lambda_r, \lambda_s, \lambda_k)$, leaving for future work the analysis of the entrepreneurial migration processes that characterize the long run.

4.1 Equilibrium determination

Solving for C_A the budget constraint (3), substituting into the utility function (2) and then differentiating with respect to c_i, we obtain the following first-order conditions $(i = 1, ..., N)$:

$$\frac{\partial U}{\partial c_i} = \alpha - (\beta - \delta)\, c_i - \delta \sum_{i=1}^{N} c_i - p_i = 0$$

[6] A specific configuration could emerge after empirical analysis. We could have, for example, a "hub and spoke" structure by letting: $T_{rs} \leq T_{rk} \leq T_{sk}$, with $T_{rs} \neq T_{rk}$ and/or $T_{rk} \neq T_{sk}$. This would stress the locational advantage of region r (the "hub") with respect to s and k (the "spokes").

from which

$$p_i = \alpha - (\beta - \delta)\, c_i - \delta \sum_{i=1}^{N} c_i$$

The linear demand function is

$$\begin{aligned} c_i(p_1, ..., p_N) &= \frac{\alpha}{(N-1)\delta + \beta} - \frac{1}{\beta - \delta} p_i + \frac{\delta}{(\beta - \delta)[(N-1)\delta + \beta]} \sum_{i=1}^{N} p_i \\ &= a - (b + cN) p_i + cP \end{aligned}$$

where $P = \sum_{i=1}^{N} p_i$, $0 \leq p_i \leq \widetilde{p} \equiv \frac{a+cP}{b+cN}$ and

$$a \equiv \frac{\alpha}{(N-1)\delta + \beta}, \quad b \equiv \frac{1}{(N-1)\delta + \beta}, \quad c \equiv \frac{\delta}{(\beta - \delta)[(N-1)\delta + \beta]}.$$

The indirect utility is given by:

$$V = S + y + \overline{C}_A$$

where S corresponds to the consumer's surplus:

$$\begin{aligned} S &= U(c(p_i), i \in [0, N]) - \sum_{i=1}^{N} p_i c_i(p_i) - C_A \\ &= \frac{a^2 N}{2b} + \frac{b + cN}{2} \sum_{i=1}^{N} p_i^2 - aP - \frac{c}{2} P^2 \end{aligned}$$

The consumer's demand originating from region s – but it could be region r or region k, after a suitable change in the subscripts – for a good produced in region r – but it could be region s or k – is

$$c_{rs} = a - (b + cN) p_{rs} + cP_s$$

where c_{rs} is the demand of a consumer living in region s for a good produced in region r; p_{rs} is the price of a good produced in region r and consumed in region s; and P_s is the price index in region s, with

$$P_s = n_r p_{rs} + n_s p_{ss} + n_k p_{ks.}$$

Notice that, following from the assumption of symmetric behavior of firms, prices differ across regions – segmenting markets – only because of trade costs.

Short-run equilibrium requires that in each segmented market demand equals supply:

$$c_{rs} = q_{rs}$$

where q_{rs} is the output produced in region r – but it could be region s or k – that is brought to a market in region s – but it could be region r or k.

In order to derive the short-run solutions, we only consider region r (but the same reasoning applies to region s or k after a suitable change in the subscripts). The operating profit of a representative firm in region r is:

$$\begin{aligned}\pi_r = \; & (p_{rr}-\eta)q_{rr}(L_r+\lambda_r E)\\ + \; & (p_{rs}-\eta-T_{rs})q_{rs}(L_s+\lambda_s E)\\ + \; & (p_{rk}-\eta-T_{rk})q_{rk}(L_k+\lambda_k E).\end{aligned}$$

From the profit maximization procedure and market segmentation, considering further than $N=E$, the first-order conditions follow:

$$\begin{aligned}\frac{\partial \pi_r}{\partial p_{rr}} &= [a+\eta(b+cE)+cP_r-2p_{rr}(b+cE)](L_r+\lambda_r E)=0\\ \frac{\partial \pi_r}{\partial p_{rs}} &= [a+(\eta+T_{rs})(b+cE)+cP_s-2p_{rs}(b+cE)](L_s+\lambda_s E)=0\\ \frac{\partial \pi_r}{\partial p_{rk}} &= [a+(\eta+T_{rk})(b+cE)+cP_k-2p_{rk}(b+cE)](L_k+\lambda_k E)=0\end{aligned}$$

Taking into account trade costs and letting $\widetilde{p}_r=\frac{a+cP_r}{b+cN}>\eta$, profit-maximizing prices correspond to

$$p_{rr}=\frac{a+cP_r+\eta(b+cE)}{2(b+cE)}=\frac{\widetilde{p}_r}{2}+\frac{\eta}{2} \tag{4}$$

$$p_{rs}=\begin{cases}\frac{a+cP_s+(\eta+T_{rs})(b+cE)}{2(b+cE)}=\frac{\widetilde{p}_s}{2}+\frac{\eta}{2}+\frac{T_{rs}}{2} & if\ T_{rs}\le\widetilde{p}_s-\eta\\ \widetilde{p}_s & if\ T_{rs}>\widetilde{p}_s-\eta\end{cases} \tag{5}$$

$$p_{rk}=\begin{cases}\frac{a+cP_k+(\eta+T_{rk})(b+cE)}{2(b+cE)}=\frac{\widetilde{p}_k}{2}+\frac{\eta}{2}+\frac{T_{rk}}{2} & if\ T_{rk}\le\widetilde{p}_k-\eta\\ \widetilde{p}_k & if\ T_{rk}>\widetilde{p}_k-\eta\end{cases} \tag{6}$$

where p_{rr} is the price that a firm located in r sets in its own market, p_{rs} the price that such a firm sets in market s, p_{rk} the price set in market k, $\widetilde{p}_s=\frac{a+cP_s}{b+cE}$ the reservation price of a consumer living in region s and $\widetilde{p}_k=\frac{a+cP_k}{b+cE}$ that of a consumer living in region k.

Using the demand and price functions, we can write:

$$q_{rr}=(b+cE)(p_{rr}-\eta) \tag{7}$$

$$q_{rs}=\begin{cases}(b+cE)(p_{rs}-\eta-T_{rs}) & if\ T_{rs}\le\widetilde{p}_s-\eta\\ 0 & if\ T_{rs}>\widetilde{p}_s-\eta\end{cases} \tag{8}$$

$$q_{rk}=\begin{cases}(b+cE)(p_{rk}-\eta-T_{rk}) & if\ T_{rk}\le\widetilde{p}_k-\eta\\ 0 & if\ T_{rk}>\widetilde{p}_k-\eta\end{cases} \tag{9}$$

According to expressions (5)-(6) and (8)-(9), if a firm located in r quotes in market s (or in market k), a price larger than the reservation price for consumers resident in s (or in k), the export from region r to region s (or k) is zero.[7] The boundary condition for trade as reported in these expression is crucial for the following analysis.

The indirect utility for r is given by

$$V_r = S_r + y + \overline{C}_A$$

where S_r corresponds to the consumer's surplus:

$$S_r = \frac{a^2 E}{2b} + \frac{b + cE}{2}\left(\lambda_r p_{rr}^2 + \lambda_s p_{sr}^2 + \lambda_k p_{kr}^2\right) E - aP_r - \frac{c}{2} P_r^2$$

We group all the sixteen possible network structures created by the trade flows between the regions into four cases: *i*) no trade occurs between all the regions; *ii*) one-way or two-way trade occurs between region r and s and region k is in autarky; *iii*) one-way or two-way trade occurs between regions r and s and r and k, but regions s and k do not trade with each other (what is called in the triad census terminology a "star" structure); *iv*) one-way or two-way trade occurs between any two-regions in the economy. For all network structures we derive the relevant conditions as determined by the relationship between trade costs and the distribution of the industrial activity. Some specific cases will be developed in some detail to understand the effects on the three regions of the creation of a new link. As we shall see, there is a one-to-one correspondence between the trade network structures and the triad census taxonomy so that we can match each trade network configuration to a triad as reported in Figure 7.

4.2 Case 1) All autarkic regions

First we consider the case in which all the regions are in autarky. This corresponds to triad 1 in Figure 7. Due to the isomorphic properties, we focus only on region r.

Region r is in autarky when conditions $T_{rs} > \widetilde{p}_s - \eta$ and $T_{rk} > \widetilde{p}_k - \eta$ apply. Using the equations (4), (5) and (6) and the expression for the reservation price, these "no-trade" conditions can be written as:

$$T_{rs} > \frac{2(a - \eta b)}{2b + c\lambda_s E} = \widetilde{T}_s \quad and \quad T_{rk} > \frac{2(a - \eta b)}{2b + c\lambda_k E} = \widetilde{T}_k$$

where $\frac{\partial \widetilde{T}_s}{\partial \lambda_s} = -\frac{2(a-\eta b)}{(2b + c\lambda_s E)^2} cE < 0$ and $\frac{\partial \widetilde{T}_k}{\partial \lambda_k} = -\frac{2(a-\eta b)}{(2b + c\lambda_k E)^2} cE < 0$.

[7] See Behrens (2004, 2005b, 2011). On the empirical relevance of the zero in the trade flow matrix, see Melitz (2003).

These results have two important implications: *i*) a lower degree of local competition increases the likelihood of interregional trade – making more permeable the local market in s (or in k) for firms located in r; *ii*) reducing distance – i.e. trade costs – between regions r and s (or between r and k) has a similar impact. That is, closer regions have a more accessible market.

If trade costs are too high, then no trade occurs among the three regions and firms only sell in the local market. From (4), (5) and (6), considering the linear demand non-negativity constraint, we obtain the price index for region r:

$$P_r = \frac{a(2-\lambda_r) + \eta\lambda_r(b+cE)}{2b + c\lambda_r E}E$$

and the equilibrium prices and quantities:[8]

$$\begin{aligned}
p_{rr} &= \frac{a+\eta(b+c\lambda_r E)}{2b+c\lambda_r E} \quad q_{rr} = (b+cE)(p_{rr}-\eta) \\
p_{rs} &= 2p_{ss} - \eta \quad q_{rs} = 0 \\
p_{rk} &= 2p_{kk} - \eta \quad q_{rk} = 0
\end{aligned}$$

As shown by these expressions, the equilibrium prices and quantities depend negatively on the number of local firms λ_r. This is a manifestation of the so-called local competition effect: the larger is the number of firms competing in the local market, the lower the price firms are able to set (and the smaller the output they are able to sell).

Taking into account our symmetry assumption, according to which the regions are endowed with the same number of L, the equilibrium short-run profit for a firm located in region r, which sells only to the local market, corresponds to:[9]

$$\pi_r = (p_{rr}-\eta)^2(b+cE)\left(\frac{L}{3} + \lambda_r E\right) = (b+cE)\left(\frac{a-\eta b}{2b+c\lambda_r E}\right)^2\left(\frac{L}{3} + \lambda_r E\right)$$

Finally, we obtain the indirect utility of an entrepreneur resident in r:

$$V_r = S_r + \pi_r + \overline{C}_A = \frac{(a-\eta b)^2(b+cE)[3\lambda_r E(3b + c\lambda_r E) + 2bL]}{6b(2b+c\lambda_r E)^2} + \overline{C_A}$$

[8] Note that analogous expressions can be obtained for s or for k by simple switching r and s or r and k.

[9] Notice that the assumption of identical workers population has no significant impact on the short-run analysis and it can be easily removed; whereas in the long run, it determines the regional entrepreneurial shares and, via these shares, the trade flows.

4.3 Case 2) Trade only occurs between two regions

We now consider the case where (one-way or two-way) trade occurs but only between two regions. This implies that, because of high trade costs, the third region has no trade links with the other two. We focus on the possible links between regions r and s; the region in autarky is, therefore, k. Due to the isomorphic properties, the same analysis applies for the links between regions r and k (with s in autarky) or regions s and k (with r in autarky).

There are two subcases: 2.A) one-way trade from region r to region s (isomorphic to the link going in the opposite direction from s to r) corresponding to triad 2 in Figure 7; and 2.B) two-way trade between r and s, corresponding to triad 3.

Considering the subcase 2.A), from equations (4) and (5)-(6), we deduce that one-way trade from region r to region s occurs as long as:

$$\frac{2(a-\eta b)}{2b+c\lambda_r E} = \widetilde{T}_r < T_{rs} \leq \widetilde{T}_s = \frac{2(a-\eta b)}{2b+c\lambda_s E} \tag{10}$$

That is, the region with the larger share of entrepreneurs has a higher chance to be the exporting region, taking advantage of the lower competition in the destination market.

When only one-way trade from region r to region s is allowed, then firms in region r are able to sell both in their local market and in the outside market s. Firms in region s, as before, only produce for the local market but now they have to compete not only with each other but also with firms located in r.

We also derive the conditions of "no trade" between regions r and k and between regions s and k:

$$\max\left(\frac{2(a-\eta b)}{2b+c\lambda_r E}, \frac{2(a-\eta b)}{2b+c\lambda_k E}\right) = \max\left(\widetilde{T}_r, \widetilde{T}_k\right) < T_{rk}, \tag{11}$$

$$\max\left(\frac{2(a-\eta b)+cE\lambda_r T_{rs}}{2b+c\left(\lambda_r+\lambda_s\right)E}, \frac{2(a-\eta b)}{2b+c\lambda_k E}\right) = \max\left(\widetilde{T}_{ks}, \widetilde{T}_k\right) < T_{sk}. \tag{12}$$

Looking at the expression (11), we note that the boundary condition for a one-directional link from region k to region s is more restrictive than before, $\widetilde{T}_{ks} < \widetilde{T}_s$ since $T_{rs} < \widetilde{T}_s$, taking into account the competition coming from the third region r. Moreover, from these conditions, it follows that the distance from r to s is shorter than the distance from s to k ($T_{rs} < T_{sk}$). This can be proven considering that (i) $T_{rs} \leq \widetilde{T}_{ks}$ from the condition $T_{rs} \leq \widetilde{T}_s$ in (10) and (*ii*) $\widetilde{T}_{ks} < T_{sk}$ from (12). Putting together (*i*) and (*ii*), we have $T_{rs} < T_{sk}$.

The price indexes when only one-way trade from r to s occurs are:

$$P_r = \frac{a(2-\lambda_r)+\eta\lambda_r(b+cE)}{2b+c\lambda_r E}E$$

$$P_s = \frac{a[2-(\lambda_r+\lambda_s)]+[\eta(\lambda_r+\lambda_s)+T_{rs}\lambda_r](b+cE)}{2b+c(\lambda_r+\lambda_s)E}E$$

$$P_k = \frac{a(2-\lambda_k)+\eta\lambda_k(b+cE)}{2b+c\lambda_k E}E$$

and the equilibrium prices and quantities:

$$\begin{aligned}
p_{rr} &= \frac{a+\eta(b+c\lambda_r E)}{2b+c\lambda_r E} \quad q_{rr} = (b+cE)(p_{rr}-\eta)\\
p_{ss} &= \frac{a+\eta[b+c(\lambda_r+\lambda_s)E]+\frac{T_{rs}}{2}c\lambda_r E}{2b+c(\lambda_r+\lambda_s)E} \quad q_{ss} = (b+cE)(p_{ss}-\eta)\\
p_{kk} &= \frac{a+\eta(b+c\lambda_k E)}{2b+c\lambda_k E} \quad q_{kk} = (b+cE)(p_{kk}-\eta)\\
p_{rs} &= p_{ss}+\frac{T_{rs}}{2} \quad q_{rs} = (b+cE)(p_{rs}-\eta-T_{rs})\\
p_{rk} &= 2p_{kk}-\eta \quad q_{rk} = 0\\
p_{sr} &= 2p_{rr}-\eta \quad q_{sr} = 0\\
p_{sk} &= 2p_{kk}-\eta \quad q_{sk} = 0\\
p_{kr} &= 2p_{rr}-\eta \quad q_{kr} = 0\\
p_{ks} &= 2p_{ss}-\eta \quad q_{ks} = 0
\end{aligned}$$

As these equations show, the creation of a one-directional trade link from region r to region s affects negatively the price applied by local firms in region s. Moreover, the competition effect is reinforced since now they have to compete not only with the other local firms but also with the firms located in region r.

Due to lack of space, from now on we do not derive explicit results for profits and indirect utilities – given that, especially the second ones become more and more complicated as the number of links increases. Brief comments on the short-run effects of a link creation on all the regions will be provided for some of the trade network configurations. Indeed, as we shall see, the creation of a link may have an effect not only on the two regions involved but also on the third. Analogously, the presence of a third region may alter the effect of the new link on the two regions directly involved. This is a version of the so-called “third-region” effect.

The equilibrium profits for the case of one-way trade from r to s are:

$$\begin{aligned}
\pi_r &= (b+cE)\left[(p_{rr}-\eta)^2\left(\frac{L}{3}+\lambda_r E\right)+(p_{rs}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_s E\right)\right]\\
\pi_s &= (b+cE)\left[(p_{ss}-\eta)^2\left(\frac{L}{3}+\lambda_s E\right)\right]\\
\pi_k &= (b+cE)(p_{kk}-\eta)^2\left(\frac{L}{3}+\lambda_k E\right).
\end{aligned}$$

The effect of the creation of a one-directional link (i.e. one-way trade or exports) from region r to region s has a positive effect on the welfare of region r (the exporting region). Indeed, the opening of the new market (the one in region s) – and the generation of the profit accruing from it – causes an increase in the overall profit of the r-firms (i.e. the firms located in region r). The impact on the welfare of region s (the importing region), instead, is ambiguous due to the counterbalancing of two opposite effects: the one on the consumer's surplus – that we call "the surplus effect" – which is positive and the one on profits – that we call "the profit effect" – which is negative. The first is induced by the larger availability of manufactured goods traded in the local market; whereas the second by the stronger competition that the s-firms have to suffer in the local market coming from the r-firms. The "surplus effect", which is smaller at the beginning of the integration process, may overcome the "profit effect", with further trade liberalization, depending on parameter values and on the regional distribution of the industrial sector.

Moving on to the subcase 2.B), when trade costs are reduced enough, two-way trade between region r and s is allowed, implying that with respect to the previous situation, now also firms in s are able to compete in both markets. Two-way trade from region r to region s occurs as long as

$$T_{rs} < \min\left(\frac{2(a-\eta b)}{2b+c\lambda_r E}, \frac{2(a-\eta b)}{2b+c\lambda_s E}\right) = \min\left(\widetilde{T}_r, \widetilde{T}_s\right). \tag{13}$$

For future reference, we also derive the conditions for "no trade" between region r and k and between region s and k:

$$\max\left(\frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c(\lambda_r+\lambda_s)E}, \frac{2(a-\eta b)}{2b+c\lambda_k E}\right) = \max\left(\widetilde{T}_{kr}, \widetilde{T}_k\right) < T_{rk}, \tag{14}$$

$$\max\left(\frac{2(a-\eta b)+cE\lambda_r T_{rs}}{2b+c(\lambda_r+\lambda_s)E}, \frac{2(a-\eta b)}{2b+c\lambda_k E}\right) = \max\left(\widetilde{T}_{ks}, \widetilde{T}_k\right) < T_{sk}. \tag{15}$$

First of all we notice that, due to additional competition coming from the s-firms operating in the market in region r, also the condition for a one directional link from region k to region r is more restrictive. Second, from the above conditions, it follows that the trade distance between r and s is the shortest, that is, $T_{rs} < \min(T_{rk}, T_{sk})$. This can be proven considering that, as shown before, $T_{rs} < T_{sk}$. Moreover, consider that (i) $T_{rs} < \widetilde{T}_{kr}$ from the condition $T_{rs} \leq \widetilde{T}_r$ in (13); and (ii) $\widetilde{T}_{kr} < T_{rk}$ from (14). Putting together (i) and (ii), we have that $T_{rs} < T_{rk}$ as well.

The price indexes when two-way trade from r to s occurs are:

$$P_r = \frac{a[2-(\lambda_r+\lambda_s)] + [\eta(\lambda_r+\lambda_s) + T_{sr}\lambda_s](b+cE)}{2b+c(\lambda_r+\lambda_s)E}E$$

$$P_s = \frac{a[2-(\lambda_r+\lambda_s)] + [\eta(\lambda_r+\lambda_s) + T_{sr}\lambda_r](b+cE)}{2b+c(\lambda_r+\lambda_s)E}E$$

$$P_k = \frac{a(2-\lambda_k) + \eta\lambda_k(b+cE)}{2b+c\lambda_k E}E$$

and the equilibrium prices and quantities:

$$\begin{aligned}
p_{rr} &= \frac{a+\eta[b+c(\lambda_r+\lambda_s)E] + \frac{T_{rs}}{2}c\lambda_s E}{2b+c(\lambda_r+\lambda_s)E} \quad q_{rr} = (b+cE)(p_{rr}-\eta) \\
p_{ss} &= \frac{a+\eta[b+c(\lambda_r+\lambda_s)E] + \frac{T_{rs}}{2}c\lambda_r E}{2b+c(\lambda_r+\lambda_s)E} \quad q_{ss} = (b+cE)(p_{ss}-\eta) \\
p_{kk} &= \frac{a+\eta(b+c\lambda_k E)}{2b+c\lambda_k E} \quad q_{kk} = (b+cE)(p_{kk}-\eta) \\
p_{rs} &= p_{ss} + \frac{T_{rs}}{2} \quad q_{rs} = (b+cE)(p_{rs}-\eta-T_{rs}) \\
p_{rk} &= 2p_{kk} - \eta \quad q_{rk} = 0 \\
p_{sr} &= p_{rr} + \frac{T_{rs}}{2} \quad q_{sr} = (b+cE)(p_{sr}-\eta-T_{rs}) \\
p_{sk} &= 2p_{kk} - \eta \quad q_{sk} = 0 \\
p_{kr} &= 2p_{rr} - \eta \quad q_{kr} = 0 \\
p_{ks} &= 2p_{ss} - \eta \quad q_{ks} = 0
\end{aligned}$$

As these equations show, the effect of the link creation from s to r affects p_{rr}: compared with the previous case, a firm located in r applies a lower price in the local market and faces competition not only from local firms but also from those located in region s.

The equilibrium profits for the case of two-way trade between r and s are:

$$\begin{aligned}
\pi_r &= (b+cE)\left[(p_{rr}-\eta)^2\left(\frac{L}{3}+\lambda_r E\right) + (p_{rs}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_s E\right)\right] \\
\pi_s &= (b+cE)\left[(p_{ss}-\eta)^2\left(\frac{L}{3}+\lambda_s E\right) + (p_{sr}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_r E\right)\right] \\
\pi_k &= (b+cE)(p_{kk}-\eta)^2\left(\frac{L}{3}+\lambda_k E\right)
\end{aligned}$$

The effect of the creation of a “link back” from region s to region r, generating two-way trade between the two regions, has an ambiguous impact on region r due the counterbalancing of the two effects mentioned above: The first effect, on profit, is negative – due to the additional competition in the local market coming from the s-firms –; whereas the second, on the consumer’s surplus, is positive due to the larger availability of manufactured commodities for the r-consumers (the consumers living in region r). The impact on region s, instead, is positive due to the additional profits accruing to the s-firms from the market located in r.

4.4 Case 3) One of the regions trade with the other two but the other two do not trade with each other.

This third case includes six possible network structures collected in two groups. To the first group, composed of three cases, belong those structures characterized by the existence of one-directional links only, that is, by one-way trade flows only; to the second group belong those structures characterized by one or two bidirectional links. In what follows, we assume that the region r always trade with the other two but these, regions s and k, do not trade with each other. The conditions determining the first group of network structures are listed below:

3.A.1 One-way trade from region r to region s and from region r to region k – corresponding to triad 4 in Figure 7 — occurs as long as:

$$\frac{2(a-\eta b)}{2b+c\lambda_r E} = \widetilde{T}_r < T_{rs} \leq \widetilde{T}_s = \frac{2(a-\eta b)}{2b+c\lambda_s E}$$
$$\frac{2(a-\eta b)}{2b+c\lambda_r E} = \widetilde{T}_r < T_{rk} \leq \widetilde{T}_k = \frac{2(a-\eta b)}{2b+c\lambda_k E}$$

3.A.2 One-way trade from region r to region s and from region k to region r – corresponding to triad 6 in Figure 7 – occurs as long as:

$$\frac{2(a-\eta b)}{2b+c\lambda_r E} = \widetilde{T}_r < T_{rs} \leq \widetilde{T}_s = \frac{2(a-\eta b)}{2b+c\lambda_s E}$$
$$\frac{2(a-\eta b)}{2b+c\lambda_k E} = \widetilde{T}_k < T_{rk} \leq \widetilde{T}_r = \frac{2(a-\eta b)}{2b+c\lambda_r E}$$

3.A.3. One-way trade from region s to region r and from region k to region r – corresponding to Triad 5 in Figure 7 – occurs as long as:

$$\frac{2(a-\eta b)}{2b+c\lambda_s E} = \widetilde{T}_s < T_{rs} \leq \widetilde{T}_{sr} = \frac{2(a-\eta b)+cE\lambda_k T_{rk}}{2b+c(\lambda_r+\lambda_k)E}$$
$$\frac{2(a-\eta b)}{2b+c\lambda_k E} = \widetilde{T}_k < T_{rk} \leq \widetilde{T}_{kr} = \frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c(\lambda_r+\lambda_s)E}$$

As before, the inequalities holding for cases 3.A.1-3.A.2 confirm that the region with the smaller (larger) share of entrepreneurs has a higher chance to be the importing (exporting) region. Looking at case 3.A.3, we note that the boundary condition for a one-directional link from region s (k) to region r takes into account the competition coming from the the third region k (s).

An important remark is useful at this stage: there are many ways in which new links can be added to an existing network structure, taking into account all the isomorphic configurations. Therefore the way we are proceeding (the order we are using) in our analysis – adding one link after the other and looking at the consequences (for example on prices, profits and short-run welfare) of a new link – is not the only possible.

In what follows we study in detail the case 3.A.1 (leaving the analysis of cases 3.2 and 3.A.3 to another contribution).

The price indexes when one-way trade from r to s and from to r to k occurs are:

$$P_r = \frac{a(2-\lambda_r)+\eta\lambda_r(b+cE)}{2b+c\lambda_r E}E$$

$$P_s = \frac{a[2-(\lambda_r+\lambda_s)]+[\eta(\lambda_r+\lambda_s)+T_{rs}\lambda_r](b+cE)}{2b+c(\lambda_r+\lambda_s)E}E$$

$$P_k = \frac{a[2-(\lambda_r+\lambda_k)]+[\eta(\lambda_r+\lambda_k)+T_{rk}\lambda_r](b+cE)}{2b+c(\lambda_r+\lambda_k)E}E$$

and the equilibrium prices and quantities:

$$\begin{aligned}
p_{rr} &= \frac{a+\eta(b+c\lambda_r E)}{2b+c\lambda_r E} \quad q_{rr} = (b+cE)(p_{rr}-\eta)\\
p_{ss} &= \frac{a+\eta[b+c(\lambda_r+\lambda_s)E]+\frac{T_{rs}}{2}c\lambda_r E}{2b+c(\lambda_r+\lambda_s)E} \quad q_{ss} = (b+cE)(p_{ss}-\eta)\\
p_{kk} &= \frac{a+\eta[b+c(\lambda_r+\lambda_k)E]+\frac{T_{rk}}{2}c\lambda_r E}{2b+c(\lambda_r+\lambda_k)E} \quad q_{kk} = (b+cE)(p_{kk}-\eta)\\
p_{rs} &= p_{ss}+\frac{T_{rs}}{2} \quad q_{rs} = (b+cE)(p_{rs}-\eta-T_{rs})\\
p_{rk} &= p_{kk}+\frac{T_{rk}}{2} \quad q_{rk} = (b+cE)(p_{rk}-\eta-T_{rk})\\
p_{sr} &= 2p_{rr}-\eta \quad q_{sr} = 0\\
p_{sk} &= 2p_{kk}-\eta \quad q_{sk} = 0\\
p_{kr} &= 2p_{rr}-\eta \quad q_{kr} = 0\\
p_{ks} &= 2p_{ss}-\eta \quad q_{ks} = 0
\end{aligned}$$

As we have seen before, looking at the price applied by firms located in region k, the creation of a one directional link from r to s reduces p_{kk} because now local firms located in region k have to face additional competition from the firms located in r.

The equilibrium profits for the case of one-way trade from r to s and from to r to k are:

$$\begin{aligned}\pi_r = \quad & (p_{rr}-\eta)^2\left(\frac{L}{3}+\lambda_r E\right)(b+cE)\\ + \quad & (p_{rs}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_s E\right)(b+cE)\\ + \quad & (p_{rk}-\eta-T_{rk})^2\left(\frac{L}{3}+\lambda_k E\right)(b+cE)\end{aligned}$$

$$\pi_s = (b+cE)(p_{ss}-\eta)^2\left(\frac{L}{3}+\lambda_s E\right)$$

$$\pi_k = (b+cE)(p_{kk}-\eta)^2\left(\frac{L}{3}+\lambda_k E\right)$$

Concerning the welfare analysis, adding a one-directional link from r to k has a positive effect on r with an increase in profits for firms and in the surplus for consumers located in that region. As before, the effect of trade liberalization on k is ambiguous with an initial reduction in welfare (with respect to the case of autarky). With further trade liberalization welfare increases and it may rise above the autarky level depending on parameter values and on the distribution of entrepreneurs.

Turning to the case in which one or two bidirectional links exist, three are the possible configurations. The corresponding "trade conditions" are reported below:

3.B.1. Two-way trade between r and s and one-way trade from r to k – corresponding to triad 8 in Figure 7 – occurs as long as:

$$T_{rs} \le \min\left(\frac{2(a-\eta b)}{2b+c\lambda_r E}, \frac{2(a-\eta b)}{2b+c\lambda_s E}\right) = \min\left(\widetilde{T}_r, \widetilde{T}_s\right) \tag{16}$$

$$\frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c(\lambda_r+\lambda_s)E} = \widetilde{T}_{kr} < T_{rk} \le \widetilde{T}_k = \frac{2(a-\eta b)}{2b+c\lambda_k E} \tag{17}$$

Moreover, for future reference, we add the condition of "no trade" between s and k:

$$T_{sk} > \max\left(\frac{2(a-\eta b)+cE\lambda_r T_{rk}}{2b+c(\lambda_r+\lambda_k)E}, \frac{2(a-\eta b)+cE\lambda_r T_{rs}}{2b+c(\lambda_r+\lambda_s)E}\right) = \max\left(\widetilde{T}_{sk}, \widetilde{T}_{ks}\right) \tag{18}$$

3.B.2. Two-way trade between r and s and one-way trade from k to r – corresponding to triad 7 in Figure 7 – occurs as long as:

$$T_{rs} \le \min\left(\frac{2(a-\eta b)}{2b+c\lambda_r E}, \frac{2(a-\eta b)+cE\lambda_k T_{rk}}{2b+c(\lambda_r+\lambda_k)E}\right) = \min\left(\widetilde{T}_r, \widetilde{T}_{sr}\right) \tag{19}$$

$$\frac{2(a-\eta b)}{2b+c\lambda_k E} = \widetilde{T}_k < T_{rk} \le \widetilde{T}_{kr} = \frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c(\lambda_r+\lambda_s)E} \tag{20}$$

Moreover, for future reference, we add the condition of "no trade" between s and k:

$$T_{sk} > \max\left(\frac{2(a-\eta b)}{2b+c\lambda_k E}, \frac{2(a-\eta b)+cE\lambda_r T_{rs}}{2b+c(\lambda_r+\lambda_s)E}\right) = \max\left(\widetilde{T}_k, \widetilde{T}_{ks}\right) \tag{21}$$

3.B.3. Two-way trade between r and s and between r and k – corresponding to triad 11 in Figure 7 – occurs as long as:

$$T_{rs} \leq \min\left(\frac{2(a-\eta b)}{2b+c\lambda_r E}, \frac{2(a-\eta b)+cE\lambda_k T_{rk}}{2b+c(\lambda_r+\lambda_k)E}\right) = \min\left(\widetilde{T}_r, \widetilde{T}_{sr}\right)$$
$$T_{rk} \leq \min\left(\frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c(\lambda_r+\lambda_s)E}, \frac{2(a-\eta b)}{2b+c\lambda_k E}\right) = \min\left(\widetilde{T}_{rk}, \widetilde{T}_k\right)$$

We study in some detail cases 3.B.1 and 3.B.3. Considering case 3.B.1, the price indexes when two-way trade between r to s and one-way trade from r to k occur are:

$$P_r = \frac{a[2-(\lambda_r+\lambda_s)]+[\eta(\lambda_r+\lambda_s)+T_{rs}\lambda_s](b+cE)}{2b+c(\lambda_r+\lambda_s)E}E$$
$$P_s = \frac{a[2-(\lambda_r+\lambda_s)]+[\eta(\lambda_r+\lambda_s)+T_{rs}\lambda_r](b+cE)}{2b+c(\lambda_r+\lambda_s)E}E$$

$$P_k = \frac{a[2-(\lambda_r+\lambda_k)]+[\eta(\lambda_r+\lambda_k)+T_{rk}\lambda_r](b+cE)}{2b+c(\lambda_r+\lambda_k)E}E$$

and the equilibrium prices and quantities:

$$p_{rr} = \frac{a+\eta[b+c(\lambda_r+\lambda_s)E]+\frac{T_{rs}}{2}c\lambda_s E}{2b+c(\lambda_r+\lambda_s)E} \quad q_{rr} = (b+cE)(p_{rr}-\eta)$$
$$p_{ss} = \frac{a+\eta[b+c(\lambda_r+\lambda_s)E]+\frac{T_{rs}}{2}c\lambda_r E}{2b+c(\lambda_r+\lambda_s)E} \quad q_{ss} = (b+cE)(p_{ss}-\eta)$$
$$p_{kk} = \frac{a+\eta[b+c(\lambda_r+\lambda_k)E]+\frac{T_{rk}}{2}c\lambda_r E}{2b+c(\lambda_r+\lambda_k)E} \quad q_{kk} = (b+cE)(p_{kk}-\eta)$$
$$p_{rs} = p_{ss}+\frac{T_{rs}}{2} \quad q_{rs} = (b+cE)(p_{rs}-\eta-T_{rs})$$
$$p_{rk} = p_{kk}+\frac{T_{rk}}{2} \quad q_{rk} = (b+cE)(p_{rk}-\eta-T_{rk})$$
$$p_{sr} = p_{rr}+\frac{T_{rs}}{2} \quad q_{sr} = (b+cE)(p_{sr}-\eta-T_{rs})$$
$$p_{sk} = 2p_{kk}-\eta \quad q_{sk} = 0$$
$$p_{kr} = 2p_{rr}-\eta \quad q_{kr} = 0$$
$$p_{ks} = 2p_{ss}-\eta \quad q_{ks} = 0$$

The equilibrium profits for the case of two-way trade from r to s and one-way trade from r to k are:

$$\begin{aligned}
\pi_r = \quad & (p_{rr}-\eta)^2\left(\frac{L}{3}+\lambda_r E\right)(b+cE) \\
+ \quad & (p_{rs}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_s E\right)(b+cE) \\
+ \quad & (p_{rk}-\eta-T_{rk})^2\left(\frac{L}{3}+\lambda_k E\right)(b+cE) \\
\pi_s = \; & (b+cE)\left[(p_{ss}-\eta)^2\left(\frac{L}{3}+\lambda_s E\right)+(p_{sr}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_r E\right)\right] \\
\pi_k = \; & (b+cE)(p_{kk}-\eta)^2\left(\frac{L}{3}+\lambda_k E\right)
\end{aligned}$$

The effect of creating a link from r (the exporting region) to k (the importing region) on these two regions – as trade costs are reduced below the threshold – are analogous to those highlighted previously. It is positive on r and ambiguous on k depending on the distribution of entrepreneurs and on parameter values; whereas the effect on the welfare of the third region – at least in the short-run – is nil.

Moving on to case 3.B.3, the price indexes when two-way trade between r and s and from r to k occur are:

$$\begin{aligned}
P_r &= \frac{a+[\eta+T_{rs}\lambda_s+T_{rk}\lambda_k](b+cE)}{2b+cE}E \\
P_s &= \frac{a[2-(\lambda_r+\lambda_s)]+[\eta(\lambda_r+\lambda_s)+T_{rs}\lambda_r](b+cE)}{2b+c(\lambda_r+\lambda_s)E}E \\
P_k &= \frac{a[2-(\lambda_r+\lambda_k)]+[\eta(\lambda_r+\lambda_k)+T_{rk}\lambda_r](b+cE)}{2b+c(\lambda_r+\lambda_k)E}E
\end{aligned}$$

and the equilibrium prices and quantities:

$$
\begin{aligned}
p_{rr} &= \frac{a+\eta(b+cE)+\left(\frac{T_{rs}}{2}\lambda_s+\frac{T_{rk}}{2}\lambda_k\right)cE}{2b+cE} \quad q_{rr}=(b+cE)(p_{rr}-\eta)\\
p_{ss} &= \frac{a+\eta[b+c(\lambda_r+\lambda_s)E]+\frac{T_{rs}}{2}c\lambda_r E}{2b+c(\lambda_r+\lambda_s)E} \quad q_{ss}=(b+cE)(p_{ss}-\eta)\\
p_{kk} &= \frac{a+\eta[b+c(\lambda_r+\lambda_k)E]+\frac{T_{rk}}{2}c\lambda_r E}{2b+c(\lambda_r+\lambda_k)E} \quad q_{kk}=(b+cE)(p_{kk}-\eta)\\
p_{rs} &= p_{ss}+\frac{T_{rs}}{2} \quad q_{rs}=(b+cE)(p_{rs}-\eta-T_{rs})...\\
p_{rk} &= p_{kk}+\frac{T_{rk}}{2} \quad q_{rk}=(b+cE)(p_{rk}-\eta-T_{rk})\\
p_{sr} &= p_{rr}+\frac{T_{rs}}{2} \quad q_{sr}=(b+cE)(p_{sr}-\eta-T_{rs})\\
p_{sk} &= 2p_{kk}-\eta \quad q_{sk}=0\\
p_{kr} &= p_{rr}+\frac{T_{rk}}{2} \quad q_{kr}=(b+cE)(p_{kr}-\eta-T_{rk})\\
p_{ks} &= 2p_{ss}-\eta \quad q_{ks}=0
\end{aligned}
$$

The equilibrium profits for the case of two-way trade between r and s and from r to k are:

$$
\begin{aligned}
\pi_r = &\quad (p_{rr}-\eta)^2\left(\frac{L}{3}+\lambda_r E\right)(b+cE)\\
+ &\quad (p_{rs}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_s E\right)(b+cE)\\
+ &\quad (p_{rk}-\eta-T_{rk})^2\left(\frac{L}{3}+\lambda_k E\right)(b+cE)\\
\pi_s &= (b+cE)\left[(p_{ss}-\eta)^2\left(\frac{L}{3}+\lambda_s E\right)+(p_{sr}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_r E\right)\right]\\
\pi_k &= (b+cE)\left[(p_{kk}-\eta)^2\left(\frac{L}{3}+\lambda_k E\right)+(p_{sk}-\eta-T_{rk})^2\left(\frac{L}{3}+\lambda_k E\right)\right]
\end{aligned}
$$

Comparing the first expression with the previous case, the effect of adding a link from k to r on the profits of region r is negative; whereas the effect on the overall welfare for region r is difficult to assess due to the counterbalancing of the negative effect on profits and the positive effect on the consumer's surplus. The effect on region s' welfare is negative: due to the negative impact on profits; whereas the effect on consumer's surplus is zero for that region. The opposite holds for region k, the overall effect on welfare is positive: this is due to the positive impact on profit; whereas also for this region the impact on the consumer's surplus is nil.

4.5 Case 4) one-way trade or two-way trade is present between any two region.

This fourth case includes seven possible network structures that can be divided into four groups: 4.A) the first group is composed of two structures characterized by the existence of only one-directional links; 4.B) the second of three structures characterized by the existence of one bidirectional link and two one-directional links; 4.C) the third of a single structure characterized by the existence of two bidirectional links and one directional link; 4.D) and the fourth of the single structure characterized by all bidirectional links. The conditions determining these network structures are reported below:

4.A. All regions are involved in one-way trade:

4.A.1 One way trade from r to s, from r to k and from s to k – corresponding to triad 9 in Figure 7 – occurs as long as:

$$\begin{aligned}
\frac{2(a-\eta b)}{2b+c\lambda_r E} &= \widetilde{T}_r < T_{rs} \leq \widetilde{T}_s = \frac{2(a-\eta b)}{2b+c\lambda_s E} \\
\frac{2(a-\eta b)}{2b+c\lambda_r E} &= \widetilde{T}_r < T_{rk} \leq \widetilde{T}_{rk} = \frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c(\lambda_s+\lambda_k)E} \\
\frac{2(a-\eta b)}{2b+c\lambda_s E} &= \widetilde{T}_s < T_{sk} \leq \widetilde{T}_{sk} = \frac{2(a-\eta b)+cE\lambda_r T_{rs}}{2b+c(\lambda_r+\lambda_k)E}
\end{aligned}$$

4.A.2 One way trade from r to s, from k to r and from s to k – corresponding to triad 10 in Figure 7 – occurs as long as:

$$\begin{aligned}
\frac{2(a-\eta b)}{2b+c\lambda_r E} &= \widetilde{T}_r < T_{rs} \leq \widetilde{T}_s = \frac{2(a-\eta b)}{2b+c\lambda_s E} \\
\frac{2(a-\eta b)}{2b+c\lambda_k E} &= \widetilde{T}_k < T_{rk} \leq \widetilde{T}_r = \frac{2(a-\eta b)}{2b+c\lambda_r E} \\
\frac{2(a-\eta b)}{2b+c\lambda_s E} &= \widetilde{T}_s < T_{sk} \leq \widetilde{T}_k = \frac{2(a-\eta b)}{2b+c\lambda_k E}
\end{aligned}$$

4.B. Two regions are involved in two-way trade with each other and in one-way trade with the third region:

4.B.1 Two-way trade between r and s, one-way trade from r to k and from s to k – corresponding to triad 13 in Figure 7 – occurs as long as:

$$T_{rs} < \min\left(\frac{2(a-\eta b)}{2b+c\lambda_r E}, \frac{2(a-\eta b)}{2b+c\lambda_s E}\right) = \min\left(\widetilde{T}_r, \widetilde{T}_s\right)$$

$$\frac{2(a-\eta b)}{2b+c\lambda_r E} = \widetilde{T}_r < T_{rk} \leq \widetilde{T}_{rk} = \frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c(\lambda_s+\lambda_k)E}$$
$$\frac{2(a-\eta b)}{2b+c\lambda_s E} = \widetilde{T}_s < T_{sk} \leq \widetilde{T}_{sk} = \frac{2(a-\eta b)+cE\lambda_r T_{rs}}{2b+c(\lambda_r+\lambda_k)E}$$

4.B.2 Region r and s are involved in two-way trade between each other; one way trade from region k to region r and from region s to region k. This structure – corresponding to triad 14 in Figure 7 – occurs as long as:

$$T_{rs} \leq \min\left(\frac{2(a-\eta b)+cE\lambda_k T_{rk}}{2b+c(\lambda_r+\lambda_k)E}, \frac{2(a-\eta b)}{2b+c\lambda_s E}\right) = \min\left(\widetilde{T_{sr}}, \widetilde{T}_s\right)$$

$$\frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c(\lambda_s+\lambda_k)E} = \widetilde{T}_{rk} < T_{rk} \leq \widetilde{T}_{kr} = \frac{2(a-\eta b)+cE\lambda_s T_{rk}}{2b+c(\lambda_r+\lambda_s)E}$$
$$\frac{2(a-\eta b)+cE\lambda_r T_{rs}}{2b+c(\lambda_r+\lambda_s)E} = \widetilde{T}_{ks} \leq T_{sk} < \widetilde{T}_k = \frac{2(a-\eta b)}{2b+c\lambda_k E}$$

4.B.3 Regions r and s are involved in two-way trade between each other; one way trade from region k to region r and from region k to region s. This structure – corresponding to triad 12 in Figure 7 – occurs as long as:

$$T_{rs} \leq \min\left(\frac{2(a-\eta b)+cE\lambda_k T_{rk}}{2b+c(\lambda_r+\lambda_k)E}, \frac{2(a-\eta b)+cE\lambda_k T_{sk}}{2b+c(\lambda_s+\lambda_k)E}\right) = \min\left(\widetilde{T}_{sr}, \widetilde{T}_{rs}\right)$$

$$\frac{2(a-\eta b)}{2b+c\lambda_k E} = \widetilde{T}_k < T_{rk} \leq \widetilde{T}_{kr} = \frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c(\lambda_r+\lambda_s)E}$$
$$\frac{2(a-\eta b)}{2b+c\lambda_k E} = \widetilde{T}_k < T_{rk} \leq \widetilde{T}_{ks} = \frac{2(a-\eta b)+cE\lambda_r T_{rs}}{2b+c(\lambda_r+\lambda_s)E}$$

4.C. One-way trade between two regions that are both involved in two-way trade with the third region: Two way trade between r and s and r and k and one way trade from s to k – corresponding to triad 15 in Figure 7 – occurs as long as:

$$T_{rs} < \min\left(\frac{2(a-\eta b)+cE\lambda_k T_{rk}}{2b+c(\lambda_r+\lambda_k)E}, \frac{2(a-\eta b)}{2b+c\lambda_s E}\right) = \min\left(\widetilde{T}_{sr}, \widetilde{T}_s\right)$$
$$T_{rk} < \min\left(\frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c(\lambda_r+\lambda_s)E}, \frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c(\lambda_s+\lambda_k)E}\right) = \min\left(\widetilde{T}_{kr}, \widetilde{T}_{rk}\right)$$

$$\frac{2(a-\eta b)+cE\lambda_r T_{rs}}{2b+c(\lambda_r+\lambda_s)E} = \widetilde{T}_{ks} \leq T_{sk} < \widetilde{T}_{sk} = \frac{2(a-\eta b)+cE\lambda_r T_{rs}}{2b+c(\lambda_r+\lambda_k)E}$$

4.D. All regions are involved in two-way trade: This structure – corresponding to triad 16 in Figure 7 – occurs as long as:

$$T_{rs} \leq \min\left(\frac{2(a-\eta b)+cE\lambda_k T_{rk}}{2b+c(\lambda_r+\lambda_k)E}, \frac{2(a-\eta b)+cE\lambda_k T_{sk}}{2b+c(\lambda_s+\lambda_k)E}\right) = \min\left(\widetilde{T}_{sr}, \widetilde{T}_{rs}\right)$$
$$T_{rk} \leq \min\left(\frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c(\lambda_r+\lambda_s)E}, \frac{2(a-\eta b)+cE\lambda_s T_{rs}}{2b+c(\lambda_s+\lambda_k)E}\right) = \min\left(\widetilde{T}_{kr}, \widetilde{T}_{rk}\right)$$
$$T_{sk} \leq \min\left(\frac{2(a-\eta b)+cE\lambda_r T_{rs}}{2b+c(\lambda_r+\lambda_s)E}, \frac{2(a-\eta b)+cE\lambda_r T_{rs}}{2b+c(\lambda_r+\lambda_k)E}\right) = \min\left(\widetilde{T}_{ks}, \widetilde{T}_{sk}\right)$$

Considering case 4.A.1, the price indexes are:

$$P_r = \frac{a(2-\lambda_r)+\eta\lambda_r(b+cE)}{2b+c\lambda_r E}E$$
$$P_s = \frac{a[2-(\lambda_r+\lambda_s)]+[\eta(\lambda_r+\lambda_s)+T_{rs}\lambda_r](b+cE)}{2b+c(\lambda_r+\lambda_s)E}E$$
$$P_k = \frac{a+(\eta+\lambda_r T_{rk}+\lambda_s T_{sk})(b+cE)}{2b+cE}E$$

and the equilibrium prices and quantities:

$$p_{rr} = \frac{a+\eta(b+c\lambda_r E)}{2b+c\lambda_r E} \quad q_{rr} = (b+cE)(p_{rr}-\eta)$$
$$p_{ss} = \frac{a+\eta[b+c(\lambda_r+\lambda_s)E]+\frac{T_{rs}}{2}c\lambda_r E}{2b+c(\lambda_r+\lambda_s)E} \quad q_{ss} = (b+cE)(p_{ss}-\eta)$$
$$p_{kk} = \frac{a+\eta(b+cE)+\left(\frac{T_{rk}}{2}\lambda_r+\frac{T_{sk}}{2}\lambda_s\right)cE}{2b+cE}$$
$$p_{rs} = p_{ss}+\frac{T_{rs}}{2} \quad q_{rs} = (b+cE)(p_{rs}-\eta-T_{rs})$$
$$p_{rk} = p_{kk}+\frac{T_{rk}}{2} \quad q_{rk} = (b+cE)(p_{rk}-\eta-T_{rk})$$
$$p_{sr} = 2p_{rr}-\eta \quad q_{sr} = 0$$
$$p_{sk} = p_{kk}+\frac{T_{sk}}{2} \quad q_{sk} = (b+cE)(p_{sk}-\eta-T_{sk})$$
$$p_{kr} = 2p_{rr}-\eta \quad q_{kr} = 0$$
$$p_{ks} = 2p_{ss}-\eta \quad q_{ks} = 0$$

The equilibrium profits for the case 4.A.1 are:

$$
\begin{aligned}
\pi_r = &\quad (p_{rr}-\eta)^2\left(\frac{L}{3}+\lambda_r E\right)(b+cE)\\
+ &\quad (p_{rs}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_s E\right)(b+cE)\\
+ &\quad (p_{rk}-\eta-T_{rk})^2\left(\frac{L}{3}+\lambda_k E\right)(b+cE)\\
\pi_s &= (b+cE)\left[(p_{ss}-\eta)^2\left(\frac{L}{3}+\lambda_s E\right)+(p_{sr}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_r E\right)\right]\\
\pi_k &= (b+cE)(p_{kk}-\eta)^2\left(\frac{L}{3}+\lambda_k E\right)
\end{aligned}
$$

According to this configuration, a creation of a link from s to k has a negative effect on profits and on welfare for r (with no effect on consumer's surplus in this region). It has a positive effect on region s profits and on this region welfare (with no effect on consumer's surplus). Finally, the effect on k is ambiguous, since the negative effect on profit is counterbalanced by the positive effect on the consumer's surplus.

Considering case 4.B.1, the price indexes are:

$$
\begin{aligned}
P_r &= \frac{a[2-(\lambda_r+\lambda_s)]+[\eta(\lambda_r+\lambda_s)+T_{rs}\lambda_s](b+cE)}{2b+c(\lambda_r+\lambda_s)E}E\\
P_s &= \frac{a[2-(\lambda_r+\lambda_s)]+[\eta(\lambda_r+\lambda_s)+T_{rs}\lambda_r](b+cE)}{2b+c(\lambda_r+\lambda_s)E}E\\
P_k &= \frac{a+(\eta+\lambda_r T_{rk}+\lambda_s T_{sk})(b+cE)}{2b+cE}E
\end{aligned}
$$

and the equilibrium prices and quantities:

$$
\begin{aligned}
p_{rr} &= \frac{a+\eta[b+c(\lambda_r+\lambda_s)E]+\frac{T_{rs}}{2}c\lambda_s E}{2b+c(\lambda_r+\lambda_s)E} \quad q_{rr}=(b+cE)(p_{rr}-\eta)\\
p_{ss} &= \frac{a+\eta[b+c(\lambda_r+\lambda_s)E]+\frac{T_{rs}}{2}c\lambda_r E}{2b+c(\lambda_r+\lambda_s)E} \quad q_{ss}=(b+cE)(p_{ss}-\eta)\\
p_{kk} &= \frac{a+\eta(b+cE)+\left(\frac{T_{rk}}{2}\lambda_r+\frac{T_{sk}}{2}\lambda_s\right)cE}{2b+cE} \quad q_{kk}=(b+cE)(p_{kk}-\eta)\\
p_{rs} &= p_{ss}+\frac{T_{rs}}{2} \quad q_{rs}=(b+cE)(p_{rs}-\eta-T_{rs})\\
p_{rk} &= p_{kk}+\frac{T_{rk}}{2} \quad q_{rk}=(b+cE)(p_{rk}-\eta-T_{rk})\\
p_{sr} &= p_{rr}+\frac{T_{rs}}{2} \quad q_{sr}=(b+cE)(p_{sr}-\eta-T_{rs})\\
p_{sk} &= p_{kk}+\frac{T_{sk}}{2} \quad q_{sk}=(b+cE)(p_{sk}-\eta-T_{sk})\\
p_{kr} &= 2p_{rr}-\eta \quad q_{kr}=0\\
p_{ks} &= 2p_{ss}-\eta \quad q_{ks}=0
\end{aligned}
$$

The equilibrium profits for the case 4.B.1 are:

$$
\begin{aligned}
\pi_r = &\quad (p_{rr}-\eta)^2\left(\frac{L}{3}+\lambda_r E\right)(b+cE)\\
+ &\quad (p_{rs}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_s E\right)(b+cE)\\
+ &\quad (p_{rk}-\eta-T_{rk})^2\left(\frac{L}{3}+\lambda_k E\right)(b+cE)\\
\pi_s = &\quad (p_{ss}-\eta)^2\left(\frac{L}{3}+\lambda_s E\right)(b+cE)\\
+ &\quad (p_{sr}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_r E\right)(b+cE)\\
+ &\quad (p_{sk}-\eta-T_{sk})^2\left(\frac{L}{3}+\lambda_k E\right)(b+cE)\\
\pi_k &= (b+cE)(p_{kk}-\eta)^2\left(\frac{L}{3}+\lambda_k E\right)
\end{aligned}
$$

In this configuration, the creation of a link from s to r has a negative effect on profits for firms located in region r and a positive effect on the surplus of consumers located in that region, with an ambiguous overall effect on welfare. The effect on welfare of region s is positive due to the increase in profits, whereas the effect on the consumer' surplus is nil. The creation of such a link has no short-run effect on region k.

Considering case 4.C, the price indexes are:

$$P_r = \frac{a+(\eta+\lambda_s T_{rs}+\lambda_k T_{rk})(b+cE)}{2b+cE}E$$
$$P_s = \frac{a[2-(\lambda_r+\lambda_s)]+[\eta(\lambda_r+\lambda_s)+T_{rs}\lambda_r](b+cE)}{2b+c(\lambda_r+\lambda_s)E}E$$
$$P_k = \frac{a+(\eta+\lambda_r T_{rk}+\lambda_s T_{sk})(b+cE)}{2b+cE}E$$

and the equilibrium prices and quantities:

$$p_{rr} = \frac{a+\eta(b+cE)+\left(\frac{T_{rs}}{2}\lambda_s+\frac{T_{rk}}{2}\lambda_k\right)cE}{2b+cE} \quad q_{rr} = (b+cE)(p_{rr}-\eta)$$
$$p_{ss} = \frac{a+\eta[b+c(\lambda_r+\lambda_s)E]+\frac{T_{rs}}{2}c\lambda_r E}{2b+c(\lambda_r+\lambda_s)E} \quad q_{ss} = (b+cE)(p_{ss}-\eta)$$
$$p_{kk} = \frac{a+\eta(b+cE)+\left(\frac{T_{rk}}{2}\lambda_r+\frac{T_{sk}}{2}\lambda_s\right)cE}{2b+cE} \quad q_{kk} = (b+cE)(p_{kk}-\eta)$$
$$p_{rs} = p_{ss}+\frac{T_{rs}}{2} \quad q_{rs} = (b+cE)(p_{rs}-\eta-T_{rs})$$
$$p_{rk} = p_{kk}+\frac{T_{rk}}{2} \quad q_{rk} = (b+cE)(p_{rk}-\eta-T_{rk})$$
$$p_{sr} = p_{rr}+\frac{T_{rs}}{2} \quad q_{sr} = (b+cE)(p_{sr}-\eta-T_{rs})$$
$$p_{sk} = p_{kk}+\frac{T_{sk}}{2} \quad q_{sk} = (b+cE)(p_{sk}-\eta-T_{sk})$$
$$p_{kr} = p_{rr}+\frac{T_{rk}}{2} \quad q_{kr} = (b+cE)(p_{kr}-\eta-T_{rk})$$
$$p_{ks} = 2p_{ss}-\eta \quad q_{ks} = 0$$

The equilibrium profits for the case 4.C are:

$$\begin{aligned}
\pi_r = \quad & (p_{rr}-\eta)^2\left(\frac{L}{3}+\lambda_r E\right)(b+cE) \\
+ \quad & (p_{rs}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_s E\right)(b+cE) \\
+ \quad & (p_{rk}-\eta-T_{rk})^2\left(\frac{L}{3}+\lambda_k E\right)(b+cE) \\
\pi_s = \quad & (p_{sr}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_r E\right)(b+cE) \\
+ \quad & (p_{ss}-\eta)^2\left(\frac{L}{3}+\lambda_s E\right)(b+cE) \\
+ \quad & (p_{sk}-\eta-T_{sk})^2\left(\frac{L}{3}+\lambda_k E\right)(b+cE)
\end{aligned}$$

$$\pi_k = \left[(p_{kr}-\eta-T_{rk})^2\left(\frac{L}{3}+\lambda_r E\right)+(p_{kk}-\eta)^2\left(\frac{L}{3}+\lambda_k E\right)\right](b+cE)$$

According to this configuration, compared to the case 4.B.1, the creation of a link from k to r determines for region r a reduction in profits; however, since the consumer's surplus in this region is increased, the overall effect on region r welfare is ambiguous. The effect on region s's welfare is negative due to the impact of stronger competition in r on the profits of s-firms accruing from the market in r. Finally, the effect on k is positive due to the profits accruing to k-firms from the market in r.

Considering case 4.D, the price indexes are:

$$P_r = \frac{a+(\eta+\lambda_s T_{rs}+\lambda_k T_{rk})(b+cE)}{2b+cE}E$$
$$P_s = \frac{a+(\eta+\lambda_r T_{rs}+\lambda_k T_{sk})(b+cE)}{2b+cE}E$$
$$P_k = \frac{a+(\eta+\lambda_r T_{rk}+\lambda_s T_{sk})(b+cE)}{2b+cE}E$$

and the equilibrium prices and quantities:

$$p_{rr} = \frac{a+\eta(b+cE)+\left(\frac{T_{rs}}{2}\lambda_s+\frac{T_{rk}}{2}\lambda_k\right)cE}{2b+cE} \quad q_{rr}=(b+cE)(p_{rr}-\eta)$$
$$p_{ss} = \frac{a+\eta(b+cE)+\left(\frac{T_{rs}}{2}\lambda_r+\frac{T_{sk}}{2}\lambda_k\right)cE}{2b+cE} \quad q_{ss}=(b+cE)(p_{ss}-\eta)$$
$$p_{kk} = \frac{a+\eta(b+cE)+\left(\frac{T_{rk}}{2}\lambda_r+\frac{T_{sk}}{2}\lambda_s\right)cE}{2b+cE} \quad q_{kk}=(b+cE)(p_{kk}-\eta)$$
$$p_{rs} = p_{ss}+\frac{T_{rs}}{2} \quad q_{rs}=(b+cE)(p_{rs}-\eta-T_{rs})$$
$$p_{rk} = p_{kk}+\frac{T_{rk}}{2} \quad q_{rk}=(b+cE)(p_{rk}-\eta-T_{rk})$$
$$p_{sr} = p_{rr}+\frac{T_{rs}}{2} \quad q_{sr}=(b+cE)(p_{sr}-\eta-T_{rs})$$
$$p_{sk} = p_{kk}+\frac{T_{sk}}{2} \quad q_{sk}=(b+cE)(p_{sk}-\eta-T_{sk})$$
$$p_{kr} = p_{rr}+\frac{T_{rk}}{2} \quad q_{kr}=(b+cE)(p_{kr}-\eta-T_{rk})$$
$$p_{ks} = p_{ss}+\frac{T_{sk}}{2} \quad q_{ks}=(b+cE)(p_{ks}-\eta-T_{sk})$$

The equilibrium profits for the case 4.D are:

$$\begin{aligned}
\pi_r = &\ (p_{rr}-\eta)^2\left(\frac{L}{3}+\lambda_r E\right)(b+cE)\\
+ &\ (p_{rs}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_s E\right)(b+cE)\\
+ &\ (p_{rk}-\eta-T_{rk})^2\left(\frac{L}{3}+\lambda_k E\right)(b+cE)\\
\pi_s = &\ (p_{sr}-\eta-T_{rs})^2\left(\frac{L}{3}+\lambda_r E\right)(b+cE)\\
+ &\ (p_{ss}-\eta)^2\left(\frac{L}{3}+\lambda_s E\right)(b+cE)\\
+ &\ (p_{sk}-\eta-T_{sk})^2\left(\frac{L}{3}+\lambda_k E\right)(b+cE)\\
\pi_k = &\ (p_{kr}-\eta-T_{rk})^2\left(\frac{L}{3}+\lambda_r E\right)(b+cE)\\
+ &\ (p_{ks}-\eta-T_{sk})^2\left(\frac{L}{3}+\lambda_s E\right)(b+cE)\\
+ &\ (p_{kk}-\eta)^2\left(\frac{L}{3}+\lambda_k E\right)(b+cE)
\end{aligned}$$

According to this configuration, the creation of a link from k to s has the following short-run effect on the three regions welfare: The effect on region r's welfare is negative since the impact of stronger competition in s reduces the profits that are accruing to r-firms from the market in s. The effect on s is ambiguous: indeed the negative effect on profits is counterbalanced by the increase in the surplus of consumers living in that region. Finally, the effect on k is positive given to the profits accruing to the k-firms originating from the market in s.

4.6 From theory back to triad census - a special case as an example.

In the previous subsection, we derived conditions on the trade costs for the occurrence of different triad patterns. In this subsection, we illustrate how these conditions can be used to corroborate the empirically found regularities in the trade network as summarized in Table 2.

One striking result of the triad census was that triad 3 (Mutual edge) and triad 8 (Mutual edge + Out) are much more often found than triad 7 (Mutual edge + In), which means that when two regions establish a mutual trade relationship this fosters them to export on, more than to import from, a third region. Theory suggests that more trade links come into existence with lower trade costs. This leads to the following interpretation: If in a triplet of regions all trade costs are very high, no trade occurs. If in a triplet of regions, bilateral trade costs between regions r and s are lower, than mutual trade occurs between those two regions. If in a triplet of regions, trade costs between another pair of regions – say between r and k – are lower as well, than unilateral trade from region r to region k is much more often found (resulting in triad 8) than trade in the opposite direction (which would lead to triad 7). There is an economic rationale for this difference that relies on third country effects, i.e. on the network structure: Exporting from region r to region k is comparatively easy, because in region k no third country competition is yet present (there is no trade from s to k). Trade in the opposite direction - i.e. exporting from region k to region r - is more difficult, since in region r also competing firms from the third country, i.e. from region s, are already present.

In order to focus on these network effects, we assume as simplification a uniform distribution of firms, i.e. $\lambda_r = \lambda_s = \lambda_k = \frac{1}{3}$. Then, the conditions for the occurrence of triad 8 (16), (17) and (18) reduce to

$$T_{rs} \leq \min\left(\frac{2(a-\eta b)}{2b+c\frac{1}{3}E}, \frac{2(a-\eta b)}{2b+c\frac{1}{3}E}\right)$$

$$\frac{2(a-\eta b)+cE\frac{1}{3}T_{rs}}{2b+c\frac{2}{3}E} < T_{rk} \leq \frac{2(a-\eta b)}{2b+c\frac{1}{3}E}$$

$$T_{sk} > \max\left(\frac{2(a-\eta b)+cE\frac{1}{3}T_{rk}}{2b+c\frac{2}{3}E}, \frac{2(a-\eta b)+cE\frac{1}{3}T_{rs}}{2b+c\frac{2}{3}E}\right)$$

The second equation can only hold if

$$\frac{2(a-\eta b)+cE\frac{1}{3}T_{rs}}{2b+c\frac{2}{3}E} < \frac{2(a-\eta b)}{2b+c\frac{1}{3}E}$$

which can be easily transformed into the first condition, that is also the condition (13) holding for triad 3, i.e. for bilateral trade. Therefore, if bilateral trade between region r and region s exists and if T_{rk} sufficiently falls, than exports from region r to region k may start - triad 8 may come into existence.

Instead, Triad 7 is characterized by the conditions (19), (20) and (21), that by setting $\lambda_r = \lambda_s = \lambda_k = \frac{1}{3}$ simplify to :

$$T_{rs} \leq \min\left(\frac{2(a-\eta b)}{2b+c\frac{1}{3}E}, \frac{2(a-\eta b)+cE\frac{1}{3}T_{rk}}{2b+c\frac{2}{3}E}\right)$$

$$\frac{2(a-\eta b)}{2b+c\frac{1}{3}E} = \widetilde{T}_k < T_{rk} \leq \widetilde{T}_{kr} = \frac{2(a-\eta b)+cE\frac{1}{3}T_{rs}}{2b+c\frac{2}{3}E}$$

$$T_{sk} > \max\left(\frac{2(a-\eta b)}{2b+c\frac{1}{3}E}, \frac{2(a-\eta b)+cE\frac{1}{3}T_{rs}}{2b+c\frac{2}{3}E}\right)$$

The second equation can only hold if

$$\frac{2(a-\eta b)}{2b+c\frac{1}{3}E} < \frac{2(a-\eta b)+cE\frac{1}{3}T_{rs}}{2b+c\frac{2}{3}E}$$

which can be transformed into

$$\frac{2(a-\eta b)}{\left(2b+c\frac{1}{3}E\right)} < T_{rs}$$

which contradicts the first equation. Therefore, triad 7 is not a possible outcome of a reduction in T_{rk}.

Summing up, for an equal distribution of firms, we can show that a reduction in T_{rk} may lead to triad 8, while triad 7 is not possible. The result might change for unequal distributions, but this analysis is left for further studies.

5 Final Remarks

Some stylized facts emerging from the network analysis of interregional trade flows have confirmed that the European economic integration is still largely incomplete: most regions do not trade with any other region, but they rather select their partners, and a relatively small number of regions play a central role in the network structure. Moreover, the triad census analysis has revealed that the large majority of triads in the EU regional trade is represented by empty-graph structures (autarky), followed by single and mutual edges. This suggests that regions select their partners engaging mostly in bilateral trade. In addition, when two regions, say r and s, establish a mutual trade relationship, a third region, say k, is more likely to participate as an importer rather than as an exporter.

In order to shed more light to the processes that shape the specific network structure, we used a three-region footloose entrepreneur model. This model stresses the fact that an integrated market, for example the one composed of regions r and s, is more difficult to access than a non-integrated market (for example that represented by region k) due to stronger competition. Therefore, for region r or s is easier to export towards k (with an outward link) than the other way round (for k to export towards r or s, with an inward link). Using the model, we derived explicit conditions on the bilateral trade cost for the occurrence of each of the 16 possible triads. For a special case, we exemplified how these conditions can be used to draw interferences on the network structure. Based on this evidence, it has been possible to envisage a specific sequence of links generation. This sequence starts from the case of full autarky, with no links; it proceeds to the creation of a one-directional link, for example from r to s; next, a bidirectional link between r and s comes into existence, then a further link towards a third region k and, finally, a second bidirectional link, for example between r and k is created. Implicit in this sequence, there is a corresponding reduction of trade costs, which characterizes the process of European integration. However, this can be assessed only by looking at time series data and we leave this to future work.

Appendix

AT11	Burgenland	FI19	Lansi-Suomi	PL43	Lubuskie
AT12	Niederosterreich	FI1A	Pohjois-Suomi	PL51	Dolnoslaskie
AT13	Wien	FI20	Aland	PL52	Opolskie
AT21	Karnten	FR10	Ile de France	PL61	Kujawsko-Pomorskie
AT22	Steiermark	FR21	Champagne-Ardenne	PL62	Warminsko-Mazurskie
AT31	Oberosterreich	FR22	Picardie	PL63	Pomorskie
AT32	Salzburg	FR23	Haute-Normandie	PT11	Norte
AT33	Tirol	FR24	Centre	PT15	Algarve
AT34	Vorarlberg	FR25	Basse-Normandie	PT16	Centro (PT)
BE10	Region de Bruxelles	FR26	Bourgogne	PT17	Lisboa
BE21	Prov. Antwerpen	FR30	Nord - Pas-de-Calais	PT18	Alentejo
BE22	Prov. Limburg (B)	FR41	Lorraine	PT20	Regio Autnoma dos Aores
BE23	Prov. Oost-Vlaanderen	FR42	Alsace	PT30	Regio Autnoma da Madeira

BE24	Prov. Vlaams Brabant
BE25	Prov. West-Vlaanderen
BE31	Prov. Brabant Wallon
BE32	Prov. Hainaut
BE33	Prov. Liege
BE34	Prov. Luxembourg (B)
BE35	Prov. Namur
CZ01	Praha
CZ02	Stredni Cechy
CZ03	Jihozapad
CZ04	Severozapad
CZ05	Severovychod
CZ06	Jihovychod
CZ07	Stredni Morava
CZ08	Moravskoslezko
DE11	Stuttgart
DE12	Karlsruhe
DE13	Freiburg
DE14	Tubingen
DE21	Oberbayern
DE22	Niederbayern
DE23	Oberpfalz
DE24	Oberfranken
DE25	Mittelfranken
DE26	Unterfranken
DE27	Schwaben
DE30	Berlin
DE41	Brandenburg - Nordost
DE42	Brandenburg - Sdwest
DE50	Bremen
DE60	Hamburg
DE71	Darmstadt
DE72	Giessen
DE73	Kassel
DE80	Mecklenburg-Vorpommern
DE91	Braunschweig
DE92	Hannover
DE93	Luneburg
DE94	Weser-Ems
DEA1	Dusseldorf
DEA2	Koln
DEA3	Munster
DEA4	Detmold
DEA5	Arnsberg
DEB1	Koblenz
DEB2	Trier
DEB3	Rheinhessen-Pfalz
DEC0	Saarland
DED1	Chemnitz
DED2	Dresden
DED3	Leipzig
DEE0	Sachsen-Anhalt
DEF0	Schleswig-Holstein
DEG0	Thringen
DK01	Hovedstadsreg
DK02	Ost for Storeblt
DK03	Syddanmark
DK04	Midtjylland
DK05	Nordjylland
EE00	Eesti
ES11	Galicia
ES12	Principado de Asturias
ES13	Cantabria
ES21	Pais Vasco
ES22	Foral de Navarra
ES23	La Rioja
ES24	Aragon
FR43	Franche-Comte
FR51	Pays de la Loire
FR52	Bretagne
FR53	Poitou-Charentes
FR61	Aquitaine
FR62	Midi-Pyrenees
FR63	Limousin
FR71	Rhone-Alpes
FR72	Auvergne
FR81	Languedoc-Roussillon
FR82	Provence-Alpes-Cote d Azur
FR83	Corse
GR11	Anatoliki Makedonia Thraki
GR12	Kentriki Makedonia
GR13	Dytiki Makedonia
GR14	Thessalia
GR21	Ipeiros
GR22	Ionia Nisia
GR23	Dytiki Ellada
GR24	Sterea Ellada
GR25	Peloponnisos
GR30	Attiki
GR41	Voreio Aigaio
GR42	Notio Aigaio
GR43	Kriti
HU10	Kozep-Magyarorszag
HU21	Kozep-Dunantul
HU22	Nyugat-Dunantul
HU23	Del-Dunantul
HU31	eszak-Magyarorszag
HU32	eszak-Alfold
HU33	Del-Alfold
IE01	Border Midlands
IE02	Southern and Eastern
ITC1	Piemonte
ITC2	Valle dAosta Vallee dAoste
ITC3	Liguria
ITC4	Lombardia
ITD1	Bolzano-Bozen
ITD2	Provincia Autonoma Trento
ITD3	Veneto
ITD4	Friuli-Venezia Giulia
ITD5	Emilia-Romagna
ITE1	Toscana
ITE2	Umbria
ITE3	Marche
ITE4	Lazio
ITF1	Abruzzo
ITF2	Molise
ITF3	Campania
ITF4	Puglia
ITF5	Basilicata
ITF6	Calabria
ITG1	Sicilia
ITG2	Sardegna
LT00	Lietuva
LU00	Luxembourg (Grand-D)
LV00	Latvija
MT00	Malta
NL11	Groningen
NL12	Friesland
NL13	Drenthe
NL21	Overijssel
NL22	Gelderland
NL23	Flevoland
NL31	Utrecht
NL32	Noord-Holland
SE11	Stockholm
SE12	ostra Mellansverige
SE21	Sydsverige
SE22	Norra Mellansverige
SE23	Mellersta Norrland
SE31	ovre Norrland
SE32	Smland med oarna
SE33	Vstsverige
SK01	Bratislavsk kraj
SK02	Zapadne Slovensko
SK03	Stredne Slovensko
SK04	Vchodne Slovensko
UKC1	Tees Valley and Durham
UKC2	Northumberland Tyne and Wear
UKD1	Cumbria
UKD2	Cheshire
UKD3	Greater Manchester
UKD4	Lancashire
UKD5	Merseyside
UKE1	East Riding, North Lincolnshire
UKE2	North Yorkshire
UKE3	South Yorkshire
UKE4	West Yorkshire
UKF1	Derbyshire and Nottinghamshire
UKF2	Leicestershire Rutland
UKF3	Lincolnshire
UKG1	Herefordshire Worcestershire
UKG2	Shropshire and Staffordshire
UKG3	West Midlands
UKH1	East Anglia
UKH2	Bedfordshire Hertfordshire
UKH3	Essex
UKI1	Inner London
UKI2	Outer London
UKJ1	Berkshire Bucks Oxfordshire
UKJ2	Surrey East and West Sussex
UKJ3	Hampshire and Isle of Wight
UKJ4	Kent
UKK1	Gloucestershire Wiltshire
UKK2	Dorset and Somerset
UKK3	Cornwall and Isles of Scilly
UKK4	Devon
UKL1	West Wales and The Valleys
UKL2	East Wales
UKM2	North Eastern Scotland
UKM3	Eastern Scotland
UKM5	South Western Scotland
UKM6	Highlands and Islands
UKN0	Northern Ireland
BG31	Severozapaden
BG32	Severen tsentralen
BG33	Severoiztochen
BG34	Yugoiztochen
BG41	Yugozapaden
BG42	Yuzhen tsentralen
CY00	Kypros/K?br?s
SI01	Vzhodna Slovenija
SI02	Zahodna Slovenija
RO11	Nord-Vest
RO12	Centru
RO21	Nord-Est
RO22	Sud-Est
RO31	Sud Muntenia
RO32	Bucure?ti Ilfov
RO41	Sud-Vest Oltenia
RO42	Vest

ES30	Comunidad de Madrid	NL33	Zuid-Holland
ES41	Castilla y Leon	NL34	Zeeland
ES42	Castilla-la Mancha	NL41	Noord-Brabant
ES43	Extremadura	NL42	Limburg (NL)
ES51	Cataluna	PL11	Ldzkie
ES52	Comunidad Valenciana	PL12	Mazowieckie
ES53	Illes Balears	PL21	Malopolskie
ES61	Andalucia	PL22	Slaskie
ES62	Region de Murcia	PL31	Lubelskie
ES63	Ceuta (ES)	PL32	Podkarpackie
ES64	Melilla (ES)	PL33	Swietokrzyskie
ES70	Canarias (ES)	PL34	Podlaskie
FI13	Ita-Suomi	PL41	Wielkopolskie
FI18	Etela-Suomi	PL42	Zachodniopomorskie

Table 3: Regional NUTS-2 classification

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